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# Moving Target Azimuth Velocity Estimation for the MASA Mode Based on Sequential SAR Images 

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#### Abstract

A novel azimuth velocity estimation method is proposed based on the multiple azimuth squint angles (MASA) imaging mode, acquiring sequential SAR images with different squint angles and time lags. The MASA mode acquisition geometry is given first, and the effect of target motion on azimuth offset and slant range offset is discussed in detail. Then, the azimuth velocity estimation accuracy is analyzed, considering the errors caused by registration, defocusing and range velocity. Moreover, the interaction between target azimuth velocity and range velocity is studied for a better understanding of the azimuth velocity estimation error caused by range velocity. With the proposed error compensation step, the new method can achieve a very high accuracy in azimuth velocity estimation, as verified by experimental results based on both simulated data and the TerraSAR-X data.


Index Terms-Synthetic aperture radar (SAR), velocity estimation, multiple azimuth squint angles, sequential images

## I. Introduction

THe rapid development of space-borne synthetic aperture radar (SAR) techniques, such as antenna azimuth pattern steering, allows for innovative applications to meet the rising demand in monitoring, recognition, and classification. By increasing the azimuth pattern steering angle span, several advanced imaging modes, including TOPS, inverse TOPS, sliding spotlight and staring spotlight [1]-[4], have been proposed for a higher geometry resolution or a wider range coverage, and adopted in some advanced SAR satellites, such as ALOS-2 [5], Sentinel-1 [6] and TerraSAR-X [7]. In addition, improved azimuth steering span is also beneficial to moving target detection and velocity estimation. Our research is focused on the innovative imaging mode and the corresponding processing method for moving target azimuth velocity estimation.

Moving target velocity estimation has been studied for a long time in the SAR community. The effects of target motion on

[^0]position change, resolution distortion and shadow feature were analyzed in [8]-[11], and several classic methods were proposed. For example, by using the information of position change, a velocity estimation method was proposed based on evaluation of the temporal correlation in SAR images [12]. However, the accuracy of this method is poor, and it only works for point-like moving target. Based on resolution distortion caused by target motion, refocusing methods were proposed, such as the shear averaging (SA) method [13] and the method based on the velocity correlation function (VCF) [14], [15]. The SA method is sensitive to azimuth velocity, but incapable of range velocity estimation. Compared with the SA method, the VCF method is more efficient, and can estimate velocity in both azimuth and range directions. Nonetheless, since resolution distortion is normally not obvious enough, it is difficult for both methods to achieve a high accuracy.

With the development of multiple-channel and distributed SAR systems, methods with displacement phase center antenna (DPCA) [16], along track interferometry (ATI) [17] and space time adaptive processing (STAP) [18] were proposed. STAP is the best method in theory, but it needs prior knowledge of clutter and requires a significant amount of computing resources, and therefore becomes impractical in many cases. Compared with STAP, DPCA and ATI are easier to implement with a moderate computation load. However, DPCA is limited by a strict relationship between pulse repetition frequency (PRF) and phase center interval. As for ATI, velocity estimation accuracy can be affected easily by both noise and correlation of the multi-channel data. As a result, the DPCA method is usually used in combination with ATI (short for DPCA-ATI) for improved velocity estimation accuracy [19], [20]. However, DPCA-ATI is only suitable for range velocity estimation, and does not work for the azimuth one.

Regarding azimuth velocity estimation, a novel imaging mode was presented for moving target indication, namely the bidirectional mode (short for BiDi ) [21]-[23], which is based on an azimuth beam pattern with two major lobes pointing to different directions simultaneously. As a result, the same area is scanned twice as the sensor passes with a short along track separation in-between the two acquisitions. In the observed area, the position of stationary targets will not change between these two images, but a position offset will be clearly visible for
moving target and easy to be detected. However, since signals from different directions are received at the same time, they should be separated in the Doppler spectral domain, which needs a significantly increased PRF. With an increased PRF, the range coverage will become narrow, leading to reduction in observing efficiency. Moreover, the effect of target range velocity on azimuth velocity estimation is ignored, which will lead to a significant estimation error, especially with a high range velocity.

In this paper, a novel azimuth velocity estimation method is proposed, in combination with the multiple azimuth squint angles (MASA) imaging mode. In the MASA mode, the same area will be observed with different azimuth squint angles by employing the azimuth steering technique [24], [25], acquiring sequential SAR images with different time lags. Different from the BiDi mode, the MASA mode receives signals from different azimuth squint angles at different time, and does not need to increase PRF to separate signals in the Doppler frequency domain, leading to a much wider range coverage than the BiDi mode. Moreover, the MASA mode can obtain more than two images, which helps improving the velocity estimation accuracy by an averaging operation. Furthermore, expressions for azimuth estimation errors due to defocusing and range velocity are derived, and compensation methods are provided. The performance of the proposed method is verified by both simulated data and the real TerraSAR-X image data.

This paper is organized as follows. In Sec. II, the MASA mode is introduced, and the target motion effects are analyzed based on acquisition geometry. In Sec. III, the proposed velocity estimation method is given in detail, and azimuth velocity estimation errors are also analyzed, especially with respect to range velocity. Experimental results are provided in Sec. IV, and conclusions are drawn in Sec. V.

## II. Multiple Azimuth Squint Angles Imaging

In this section, the MASA mode is introduced based on acquisition geometry. Combined with the MASA mode, target motion effects caused by azimuth velocity and range velocity are analyzed. Moreover, the interaction between azimuth velocity and range velocity is discussed in detail to show its effects on sequential SAR images, providing the theoretical foundation for the velocity estimation method presented in Section III.

## A. Acquisition Geometry

Fig. 1 shows the MASA acquisition geometry, taking three observations as an example. In the first observation, from $T_{1}$ to $\mathrm{T}_{2}$, the target area is observed with an azimuth squint angle $\varphi_{1}$ , and the moving target position is in $P_{1}$. With a time lag $t_{\text {lag } 1,2}$ and $t_{\text {lag } 2,3}$, the second and third observations are performed, with azimuth squint angles $\varphi_{2}$ and $\varphi_{3}$, respectively. Correspondingly, the target moves to the positions $P_{2}$ and $P_{3}$ . Three sequential SAR images are acquired by the satellite, with the moving target appearing at different positions. Note that the number of observations is dependent on azimuth
steering span. The larger the azimuth steering span is, the more observations we can make for the same area, leading to more accurate estimation results through the averaging operation.


Fig. 1. MASA imaging mode acquisition geometry with three observations.

## B. Doppler Characteristic Analysis due to Target Motion

The target motion effect on SAR image has been analyzed before, including the effects on resolution [8], range smear [9], sharpness [13], spectrum shift [20], and so on. However, they are mainly discussed based on boresight acquisition. In the MASA mode, sequential SAR images are acquired with different azimuth squint angles, and the effects will be different. Moreover, in the MASA mode, velocity is estimated by pixel offsets among sequential SAR images, which are affected by the imaging process. Therefore, the effects caused by the image formation process should be taken into account as well. In this part, Doppler characteristics, azimuth time offset and slant range offset are derived in detail, supported by simulation analysis.

In order to avoid significant defocusing, a moderate resolution is usually used for moving target indication in the MASA mode. With moderate resolution, considering a target moving with a constant azimuth velocity $\mathrm{v}_{\mathrm{a}}$ and range velocity $\mathrm{v}_{\mathrm{r}}$ shown in Fig. 2, the instantaneous slant range variation with time can be written as

$$
\begin{equation*}
\mathrm{r}(\mathrm{t})=\sqrt{\left(\mathrm{r}_{0} \sin \varphi+\mathrm{v}_{\mathrm{a}} \mathrm{t}-\mathrm{v}_{\mathrm{s}} \mathrm{t}\right)^{2}+\left(\mathrm{r}_{0} \cos \varphi \sin \theta+\mathrm{v}_{\mathrm{r}} \mathrm{t}\right)^{2}+\left(\mathrm{r}_{0} \cos \varphi \cos \theta\right)^{2}} \tag{1}
\end{equation*}
$$

where $r_{0}$ is the reference slant range at $t=0, v_{s}$ is radar effective velocity, $\theta$ is the elevation angle, $\varphi$ is the azimuth squint angle, $t$ is the azimuth time. Moreover, the azimuth squint angle is defined to be positive for forward looking, while negative for backward looking. With respect to the boresight, the azimuth squint angle is zero.


Fig. 2. Geometry model of the moving target.
Based on (1), the Doppler centroid and Doppler frequency modulated rate are derived as follows:

$$
\begin{align*}
& \mathrm{f}_{\mathrm{d}}=\left.\frac{2}{\lambda} \frac{\mathrm{dr}(\mathrm{t})}{\mathrm{dt}}\right|_{\mathrm{t}=0}=\mathrm{f}_{\mathrm{d} 0}+\Delta \mathrm{f}_{\mathrm{d}}  \tag{2}\\
& \mathrm{f}_{\mathrm{r}}=\left.\frac{2}{\lambda} \frac{\mathrm{~d}^{2} \mathrm{r}(\mathrm{t})}{\mathrm{dt}^{2}}\right|_{\mathrm{t}=0}=\mathrm{f}_{\mathrm{r} 0}+\Delta \mathrm{f}_{\mathrm{r}} \tag{3}
\end{align*}
$$

where $\lambda$ is wavelength, $f_{d 0}$ and $f_{r 0}$ represent the Doppler centroid frequency and Doppler frequency modulated rate of stationary target, and $\Delta f_{d}$ and $\Delta f_{r}$ are the corresponding variations caused by target motion. We also have

$$
\begin{gather*}
\mathrm{f}_{\mathrm{d} 0}=-\frac{2}{\lambda} \mathrm{v}_{\mathrm{s}} \sin \varphi  \tag{4}\\
\mathrm{f}_{\mathrm{r} 0}=\frac{2 \mathrm{v}_{\mathrm{s}}^{2}}{\lambda \mathrm{r}_{0}} \cos ^{2} \varphi  \tag{5}\\
\Delta \mathrm{f}_{\mathrm{d}}=\Delta \mathrm{f}_{\mathrm{d}, \mathrm{v}_{\mathrm{a}}}+\Delta \mathrm{f}_{\mathrm{d}, \mathrm{v}_{\mathrm{r}}}  \tag{6}\\
\Delta \mathrm{f}_{\mathrm{r}}=\Delta \mathrm{f}_{\mathrm{r}, \mathrm{v}_{\mathrm{a}}}+\Delta \mathrm{f}_{\mathrm{r}, \mathrm{v}_{\mathrm{r}}}+\Delta \mathrm{f}_{\mathrm{r}, \mathrm{v}_{\mathrm{a}}, \mathrm{v}_{\mathrm{r}}} \tag{7}
\end{gather*}
$$

In order to analyze the azimuth and range velocities separately, we define

$$
\begin{gather*}
\Delta \mathrm{f}_{\mathrm{d}, \mathrm{v}_{\mathrm{a}}}=\frac{2 \sin \varphi}{\lambda} \mathrm{v}_{\mathrm{a}}  \tag{8a}\\
\Delta \mathrm{f}_{\mathrm{d}, \mathrm{v}_{\mathrm{r}}}=\frac{2 \sin \theta \cos \varphi}{\lambda} \mathrm{v}_{\mathrm{r}}  \tag{8b}\\
\Delta \mathrm{f}_{\mathrm{r}, \mathrm{v}_{\mathrm{a}}}=\frac{2}{\lambda \mathrm{r}_{0}}\left(-2 \mathrm{v}_{\mathrm{a}} \mathrm{v}_{\mathrm{s}} \cos ^{2} \varphi+\mathrm{v}_{\mathrm{a}}^{2} \cos ^{2} \varphi\right) \approx-\frac{4}{\lambda \mathrm{r}_{0}} \mathrm{v}_{\mathrm{a}} \mathrm{v}_{\mathrm{s}} \cos ^{2} \varphi  \tag{9a}\\
\Delta \mathrm{f}_{\mathrm{r}, \mathrm{v}_{\mathrm{r}}}=\frac{2}{\lambda \mathrm{r}_{0}}\left(2 \mathrm{v}_{\mathrm{r}} \mathrm{v}_{\mathrm{s}} \sin \theta \sin \varphi \cos \varphi+\mathrm{v}_{\mathrm{r}}^{2} \cos ^{2} \theta+\mathrm{v}_{\mathrm{r}}^{2} \sin ^{2} \theta \sin ^{2} \varphi\right) \\
\approx \frac{4}{\lambda \mathrm{r}_{0}} \mathrm{v}_{\mathrm{r}} \mathrm{v}_{\mathrm{s}} \sin \theta \sin \varphi \cos \varphi  \tag{9b}\\
\Delta \mathrm{f}_{\mathrm{r}, \mathrm{v}_{\mathrm{a}}, v_{\mathrm{r}}}=-\frac{4}{\lambda \mathrm{r}_{0}}\left(\mathrm{v}_{\mathrm{r}} \mathrm{v}_{\mathrm{a}} \sin \theta \sin \varphi \cos \varphi\right) \approx 0 \tag{9c}
\end{gather*}
$$

Based on (8a) and (8b), the Doppler spectrum shift caused by azimuth velocity and range velocity are independent, and it will
result in an offset in azimuth. With respect to the Doppler frequency modulated rate variation, $\Delta \mathrm{f}_{\mathrm{r}, \mathrm{v}_{\mathrm{a}}, v_{r}}$ is basically zero and its effect can be ignored. In the case of small squint angle, compared with $\Delta f_{r, v_{r}}, \Delta f_{r, v_{\mathrm{a}}}$ is the dominant factor in SAR image defocusing.

Fig. 3 shows the peak to peak value of residual quadratic phase error due to target motion, using the parameters listed in Table. I.

TABLE I
Simulation Parameters

| Parameters | Value |
| :---: | :---: |
| Wavelength | 0.03 m |
| Elevation | $35.0^{\circ}$ |
| Orbit height | 525 km |
| PRF | 5000 Hz |
| Aperture time | 0.43 s |
| Effective velocity | $7500 \mathrm{~m} / \mathrm{s}$ |



Fig. 3. The peak to peak value of residual quadratic phase error by target motion with $30 \mathrm{~m} / \mathrm{s}$ azimuth velocity and $30 \mathrm{~m} / \mathrm{s}$ range velocity. Azimuth velocity introduces a large quadratic phase error, resulting in significant azimuth resolution deterioration (red solid line). The range velocity results to a small quadratic phase error, whose effects on resolution can be ignored (blue dashed line). Quadratic phase error caused by interactions between azimuth and range velocities is nearly zero (black dashdot line).

As shown in Fig. 3, the Doppler frequency modulated rate variation will lead to image defocusing. When the variation is large enough, pattern distortion of the impulse response function (IRF) will occur, causing a position measurement error. This phenomenon will be discussed in Sec. III.

## C. Azimuth Time Offset

Due to the target motions, the Doppler spectrum shift will introduce a time offset in azimuth

$$
\begin{equation*}
\Delta \mathrm{t}_{\mathrm{A}}=-\frac{\Delta \mathrm{f}_{\mathrm{d}}}{\mathrm{f}_{\mathrm{r}}}=-\frac{\Delta \mathrm{f}_{\mathrm{d}, \mathrm{v}_{\mathrm{a}}}+\Delta \mathrm{f}_{\mathrm{d}, \mathrm{v}_{\mathrm{r}}}}{\mathrm{f}_{\mathrm{r}}}=\Delta \mathrm{t}_{\mathrm{A}, \mathrm{v}_{\mathrm{a}}}+\Delta \mathrm{t}_{\mathrm{A}, \mathrm{v}_{\mathrm{r}}} \tag{10}
\end{equation*}
$$

where $\Delta t_{A, v_{\mathrm{a}}}$ and $\Delta \mathrm{t}_{\mathrm{A}, \mathrm{v}_{\mathrm{r}}}$ are azimuthal time offset caused by $\mathrm{v}_{\mathrm{a}}$ and $\mathrm{v}_{\mathrm{r}}$, respectively:

$$
\begin{array}{r}
\Delta \mathrm{t}_{\mathrm{A}, \mathrm{v}_{\mathrm{a}}}=-\frac{2 \mathrm{v}_{\mathrm{a}}}{\lambda \cdot \mathrm{f}_{\mathrm{r}}} \sin \varphi \approx-\frac{\mathrm{v}_{\mathrm{a}}}{\mathrm{v}_{\mathrm{s}}^{2} \cos ^{2} \varphi} \mathrm{r}_{0} \sin \varphi \\
\Delta \mathrm{t}_{\mathrm{A}, \mathrm{v}_{\mathrm{r}}}=-\frac{2 \mathrm{v}_{\mathrm{r}} \sin \theta}{\lambda \cdot \mathrm{f}_{\mathrm{r}}} \cos \varphi \approx-\frac{\mathrm{v}_{\mathrm{r}}}{\mathrm{v}_{\mathrm{s}}^{2} \cos \varphi} \mathrm{r}_{0} \sin \theta \tag{11b}
\end{array}
$$

With the parameters in Table. I, azimuthal time offset caused by $\mathrm{v}_{\mathrm{a}}=10 \mathrm{~m} / \mathrm{s}$ and $\mathrm{v}_{\mathrm{r}}=10 \mathrm{~m} / \mathrm{s}$ is shown in Fig. 4.


Fig. 4. Azimuth time offset. (a) azimuth velocity leads to a backward offset in forward looking acquisition, and a forward offset in backward looking acquisition; (b) range velocity leads to a backward offset in both forward and backward looking acquisition.

It can be seen that, azimuth offset caused by range velocity is symmetric, which means forward and backward looking will lead to the same offset with the same absolute value of azimuth squint angle. In contrast to $\Delta \mathrm{t}_{\mathrm{A}, \mathrm{v}_{\mathrm{r}}}, \Delta \mathrm{t}_{\mathrm{A}, \mathrm{v}_{\mathrm{a}}}$ is anti-symmetric, and causes a smaller azimuth offset with the same squint angle. However, $\Delta \mathrm{t}_{\mathrm{A}, \mathrm{v}_{\mathrm{r}}}$ varies little with the azimuth squint angle.

## D. Slant Range Offset

Target motion also affects range history, causing a slant range offset, which raises two major issues. Firstly, in SAR image, position of the moving target is away from its real positon along the slant range direction. Secondly, the moving target will be located at a wrong range bin. Consequently, mismatched Doppler parameters are used for azimuth compression during the image formation process, which will cause an extra time offset in azimuth.

Because $f_{d 0} \square \Delta f_{d}$, slant range offset can be approximated as follows:

$$
\begin{equation*}
\Delta \mathrm{s}_{\mathrm{r}} \approx \frac{\lambda}{2}\left(\mathrm{f}_{\mathrm{d}}+\frac{1}{2} \mathrm{f}_{\mathrm{r}} \Delta \mathrm{t}_{\mathrm{A}}\right) \Delta \mathrm{t}_{\mathrm{A}} \approx \frac{\lambda}{2} \mathrm{f}_{\mathrm{d} 0} \Delta \mathrm{t}_{\mathrm{A}} \tag{12}
\end{equation*}
$$

Then, the slant range offsets caused by $v_{a}$ and $v_{r}$ are given by

$$
\begin{gather*}
\Delta \mathrm{s}_{\mathrm{r}, \mathrm{v}_{\mathrm{a}}}=-\frac{\mathrm{f}_{\mathrm{d} 0}}{\mathrm{f}_{\mathrm{r}}} \mathrm{v}_{\mathrm{a}} \sin \varphi \approx \frac{\mathrm{v}_{\mathrm{a}}}{\mathrm{v}_{\mathrm{s}}} \mathrm{r}_{0} \frac{\sin ^{2} \varphi}{\cos ^{2} \varphi}  \tag{13a}\\
\Delta \mathrm{~s}_{\mathrm{r}, \mathrm{v}_{\mathrm{r}}}=-\frac{\mathrm{f}_{\mathrm{d} 0}}{\mathrm{f}_{\mathrm{r}}} \mathrm{v}_{\mathrm{r}} \sin \theta \cos \varphi \approx \frac{\mathrm{v}_{\mathrm{r}}}{\mathrm{v}_{\mathrm{s}}} \mathrm{r}_{0} \frac{\sin \varphi}{\cos \varphi} \sin \theta \tag{13b}
\end{gather*}
$$

Using the parameters in Table. I, slant range offset caused by $\mathrm{v}_{\mathrm{a}}=10 \mathrm{~m} / \mathrm{s}$ and $\mathrm{v}_{\mathrm{r}}=10 \mathrm{~m} / \mathrm{s}$ is shown in Fig. 5.

(a) Slant range offset caused by azimuth velocity

(b) Slant range offset caused by range velocity

Fig. 5. Slant range offset. (a) slant range offset caused by azimuth velocity shows a quadratic increase with azimuth squint angle. (b) slant range offset caused by range velocity varies linearly with azimuth squint angle.

For the azimuth velocity effect on slant range offset, it shows a quadratic increase with azimuth squint angle, leading to the same slant range offset in forward looking and backward looking acquisition, with the absolute value of the same squint angle. However, range velocity varies linearly, and it introduces a significantly larger slant range offset compared with azimuth velocity.

Moreover, slant range offset caused by target velocity leads to range bin variation. For example, assuming range bin $p_{r}=c / 2 / f_{s}$ is equal to $2 m$. According to the parameters listed in Table. I, the resultant range bin variation is analyzed and shown in Fig. 6.

(a) Slant range offset caused by azimuth velocity

(b) Slant range offset caused by range velocity

Fig. 6. Slant range offset on the variation of range bins. (a) Slant range offset caused by azimuth velocity leads to one range bin variation with squint angle $2.8^{\circ}, 2.0^{\circ}$ and $1.6^{\circ}$ corresponding azimuth velocity $10 \mathrm{~m} / \mathrm{s}, 20 \mathrm{~m} / \mathrm{s}$ and $30 \mathrm{~m} / \mathrm{s}$; (b) With respect to range velocity, one range bin variation will occur with squint angle $0.24^{\circ}, 0.12^{\circ}$ and $0.08^{\circ}$, corresponding to range velocity $10 \mathrm{~m} / \mathrm{s}, 20 \mathrm{~m} / \mathrm{s}$ and $30 \mathrm{~m} / \mathrm{s}$, respectively.

Regarding azimuth velocity, with the azimuth squint angle $\varphi=3^{\circ}$, it causes only one, two and three range bin variations, corresponding to $\mathrm{v}_{\mathrm{a}}=10 \mathrm{~m} / \mathrm{s}, \mathrm{v}_{\mathrm{a}}=20 \mathrm{~m} / \mathrm{s}$ and $\mathrm{v}_{\mathrm{a}}=30 \mathrm{~m} / \mathrm{s}$. However, with respect to range velocity, it will lead to the change of dozens of range bins. As a consequence, mismatched Doppler parameters are used in image formation process, resulting in a noticeable azimuth time offset and the corresponding azimuth velocity estimation error.

## III. Velocity Estimation

In this section, the proposed azimuth velocity estimation method is provided in detail based on the MASA mode. Its performance is analyzed by considering different factors including range velocity, registration and defocusing.

## A. The Proposed Method and Range Velocity Effects

Azimuth time offset is determined by the Doppler centroid frequency shift [26]. The azimuth pixel offset $\Delta n_{a}$ can be obtained by

$$
\begin{equation*}
\Delta \mathrm{n}_{\mathrm{a}} \cdot \Delta \mathrm{t}_{\mathrm{s}}=-\frac{\Delta \mathrm{f}_{\mathrm{d}, \text { total }}}{\mathrm{f}_{\mathrm{r}}}=-\frac{\Delta \mathrm{f}_{\mathrm{d}, \mathrm{v}_{\mathrm{a}}}+\Delta \mathrm{f}_{\mathrm{d}, \mathrm{v}_{\mathrm{r}}}+\Delta \mathrm{f}_{\mathrm{d}, \text { mis }, \mathrm{v}_{\mathrm{r}}}+\Delta \mathrm{f}_{\mathrm{d}, \mathrm{mis}, \mathrm{v}_{\mathrm{a}}}}{\mathrm{f}_{\mathrm{r}}} \tag{14}
\end{equation*}
$$

where $\Delta \mathrm{t}_{\mathrm{s}}$ is the pixel sampling time interval and $\Delta \mathrm{f}_{\mathrm{d} \text {,total }}$ is the total Doppler centroid frequency shift. $\Delta \mathrm{f}_{\mathrm{d}, \text { total }}$ is composed of four parts, including $\Delta f_{d, v_{a}}$ (caused by azimuth velocity), $\Delta f_{d, v_{r}}$ (caused by range velocity), $\Delta f_{d, \text { mis }, v_{\mathrm{a}}}$ (caused by parameter mismatch due to azimuth velocity) and $\Delta f_{d, \text { mis }, v_{r}}$ (caused by parameter mismatch due to range velocity).

According to (14), since azimuth pixel offsets are determined by both azimuth and range velocities, it is impossible to estimate target velocity using azimuth pixel offsets with a single SAR image. Fortunately, we can employ sequential SAR images for target motion estimation by using the difference of azimuth pixel offset. However, challenge arises in the separation of effects of azimuth velocity and range velocity.

First, we analyze the effects caused by range velocity and Doppler parameter mismatch on azimuth velocity estimation,
deriving the differentiations of $\Delta f_{d, v_{r}}, \Delta f_{d, \text { mis }, v_{r}}$ and $\Delta f_{d, \text { mis }, v_{\mathrm{a}}}$ as follows (see Appendix).

$$
\begin{align*}
\begin{aligned}
\mathrm{d}\left[\Delta \mathrm{f}_{\mathrm{d}, \mathrm{v}_{\mathrm{r}}}\right] & = \\
& \frac{\partial \Delta \mathrm{f}_{\mathrm{d}, \mathrm{v}_{\mathrm{r}}}}{\partial \varphi} \mathrm{~d} \varphi+\frac{\partial \Delta \mathrm{f}_{\mathrm{d}, \mathrm{v}_{\mathrm{r}}}}{\partial \theta} \mathrm{~d} \theta \\
& =-\frac{2 \mathrm{v}_{\mathrm{r}}}{\lambda} \sin \theta \sin \varphi \Delta \varphi+\frac{2 \mathrm{v}_{\mathrm{r}}}{\lambda} \cos \theta \cos \varphi \Delta \theta \\
\mathrm{~d}\left[\Delta \mathrm{f}_{\mathrm{d}, \text { mis }, \mathrm{v}_{\mathrm{r}}}\right] & =\frac{\partial \Delta \mathrm{f}_{\mathrm{d}, \text { mis }, \mathrm{v}_{\mathrm{r}}}}{\partial \mathrm{r}} \frac{\partial \mathrm{r}}{\partial \varphi}+\frac{\partial \Delta \mathrm{f}_{\mathrm{d}, \text { mis }, \mathrm{v}_{\mathrm{r}}}}{\partial \mathrm{r}} \frac{\partial \mathrm{r}}{\partial \theta} \\
& \approx \frac{2 \mathrm{v}_{\mathrm{r}}}{\lambda} \sin \theta \sin \varphi \Delta \varphi+\frac{2 \mathrm{v}_{\mathrm{r}}}{\lambda} \cos \theta \frac{\sin ^{2} \varphi}{\cos \varphi} \Delta \theta \\
\mathrm{~d}\left[\Delta \mathrm{f}_{\mathrm{d}, \text { mis }, \mathrm{v}_{\mathrm{a}}}\right] & =\frac{\partial \Delta \mathrm{f}_{\mathrm{d}, \text { mis }, \mathrm{v}_{\mathrm{a}}}}{\partial \mathrm{r}} \frac{\partial \mathrm{r}}{\partial \varphi}=\frac{4 \mathrm{v}_{\mathrm{a}}}{\lambda} \frac{\sin ^{2} \varphi}{\cos ^{3} \varphi} \Delta \varphi \approx 0
\end{aligned} \tag{15}
\end{align*}
$$

where $\Delta \varphi$ and $\Delta \theta$ are the difference of azimuth squint angles and elevation angles of two arbitrary acquisitions.

Regarding the differentiation of $\Delta f_{d, \text { mis }, v_{a}}$, it is very small. As shown in Fig. 6, it cannot even cause a range bin variation with small squint angle acquisition. Moreover, the effect can be cancelled by choosing two images with the squint angle $\varphi$ and $-\varphi$. As a result, Doppler parameter mismatch caused by azimuth can be ignored.

As for range velocity, it will cause azimuth velocity estimation errors. Adding (15) and (16) together, we can obtain the residual Doppler centroid frequency shift, given by

$$
\begin{equation*}
\Theta_{\mathrm{v}_{\mathrm{r}} \rightarrow \mathrm{v}_{\mathrm{a}}}=\frac{2 \mathrm{v}_{\mathrm{r}}}{\lambda} \cos \theta \cos \varphi \Delta \theta+\frac{2 \mathrm{v}_{\mathrm{r}}}{\lambda} \cos \theta \frac{\sin ^{2} \varphi}{\cos \varphi} \Delta \theta=\frac{2 \mathrm{v}_{\mathrm{r}}}{\lambda} \frac{\cos \theta}{\cos \varphi} \Delta \theta \tag{18}
\end{equation*}
$$

For details of the proposed velocity estimation method, we consider two arbitrary SAR images $S_{i}$ and $S_{j}$, with squint angles $\varphi_{\mathrm{i}}$ and $\varphi_{\mathrm{j}}$, respectively. As shown in Fig. 7, we can obtain the following equation.


Fig. 7. Azimuth time offset between two acquisitions with azimuth squint angles $\varphi_{\mathrm{i}}$ and $\varphi_{\mathrm{j}}$. Azimuth time offset is composed of three parts, including time offset in every image and time lag between two acquisitions.

$$
\begin{equation*}
\mathrm{N}_{\mathrm{a}}(\mathrm{i}, \mathrm{j}) \cdot \Delta \mathrm{t}_{\mathrm{s}}=\Delta \mathrm{n}_{\mathrm{a}}(\mathrm{i}) \Delta \mathrm{t}_{\mathrm{s}}-\Delta \mathrm{n}_{\mathrm{a}}(\mathrm{j}) \Delta \mathrm{t}_{\mathrm{s}}+\mathrm{t}_{\text {lag }}(\mathrm{i}, \mathrm{j}) \tag{19}
\end{equation*}
$$

where $N_{a}(i, j)$ is the difference of pixel offset between $S_{i}$ and $\mathrm{S}_{\mathrm{j}}$ in azimuth, and $\mathrm{t}_{\text {lag }}(\mathrm{i}, \mathrm{j})$ is the time lag between two acquisitions.

$$
\begin{equation*}
\Delta \mathrm{t}_{\mathrm{lag}}(\mathrm{i}, \mathrm{j}) \approx \frac{\mathrm{r}_{\mathrm{i}} \sin \varphi_{\mathrm{i}}}{\mathrm{v}_{\mathrm{s}}^{2}} \mathrm{v}_{\mathrm{a}}-\frac{\mathrm{r}_{\mathrm{j}} \sin \varphi_{\mathrm{j}}}{\mathrm{v}_{\mathrm{s}}^{2}} \mathrm{v}_{\mathrm{a}} \tag{20}
\end{equation*}
$$

Substituting (14) and (20) into (19), we have

$$
\begin{align*}
\mathrm{N}_{\mathrm{a}}(\mathrm{i}, \mathrm{j}) \Delta \mathrm{t}_{\mathrm{s}} & =\Delta \mathrm{n}_{\mathrm{a}}(\mathrm{j}) \Delta \mathrm{t}_{\mathrm{s}}-\Delta \mathrm{n}_{\mathrm{a}}(\mathrm{i}) \Delta \mathrm{t}_{\mathrm{s}}+\Delta \mathrm{t}_{\text {lag }}(\mathrm{i}, \mathrm{j}) \\
& =\mathrm{v}_{\mathrm{a}}\left[\frac{\mathrm{r}_{\mathrm{i}}}{\mathrm{v}_{\mathrm{s}}^{2}} \frac{\sin \varphi_{\mathrm{i}}\left(1+\cos ^{2} \varphi_{\mathrm{i}}\right)}{\cos ^{2} \varphi_{\mathrm{i}}}-\frac{r_{\mathrm{j}}}{\mathrm{v}_{\mathrm{s}}^{2}} \frac{\sin \varphi_{\mathrm{j}}\left(1+\cos ^{2} \varphi_{\mathrm{j}}\right)}{\cos ^{2} \varphi_{\mathrm{j}}}\right]+\frac{\Theta_{\mathrm{v}_{\mathrm{t}} \rightarrow \mathrm{v}_{\mathrm{a}}}}{\mathrm{f}_{\mathrm{r}}} \tag{21}
\end{align*}
$$

Then, the azimuth velocity estimation result and the corresponding estimation error are given by

$$
\begin{align*}
& \hat{\mathrm{v}}_{\mathrm{a}}(\mathrm{i}, \mathrm{j})=\frac{\mathrm{v}_{\mathrm{s}}^{2} \cdot \mathrm{~N}_{\mathrm{a}}(\mathrm{i}, \mathrm{j}) \cdot \Delta \mathrm{t}_{\mathrm{s}}}{\left[\frac{\mathrm{r}_{\mathrm{i}} \sin \varphi_{\mathrm{i}}\left(1+\cos ^{2} \varphi_{\mathrm{i}}\right)}{\cos ^{2} \varphi_{\mathrm{i}}}-\frac{\mathrm{r}_{\mathrm{j}} \sin \varphi_{\mathrm{j}}\left(1+\cos ^{2} \varphi_{\mathrm{j}}\right)}{\cos ^{2} \varphi_{\mathrm{j}}}\right]}  \tag{22}\\
& \hat{\mathrm{v}}_{\mathrm{a}, \text { error }}(\mathrm{i}, \mathrm{j})=-\frac{\mathrm{v}_{\mathrm{r}} \mathrm{r}_{\mathrm{i}} \cos \theta_{\mathrm{i}} \Delta \theta}{\cos ^{3} \varphi_{\mathrm{i}}\left[\frac{\mathrm{r}_{\mathrm{i}} \sin \varphi_{\mathrm{i}}\left(1+\cos ^{2} \varphi_{\mathrm{i}}\right)}{\cos ^{2} \varphi_{\mathrm{i}}}-\frac{\mathrm{r}_{\mathrm{j}} \sin \varphi_{\mathrm{j}}\left(1+\cos ^{2} \varphi_{\mathrm{j}}\right)}{\cos ^{2} \varphi_{\mathrm{j}}}\right]} \tag{23}
\end{align*}
$$

If we choose two acquisitions with the azimuth squint angles $-\varphi_{\mathrm{i}}$ and $\varphi_{\mathrm{i}}, \hat{\mathrm{v}}_{\mathrm{a}}(\mathrm{i}, \mathrm{j})$ and $\hat{\mathrm{v}}_{\mathrm{a}, \text { error }}(\mathrm{i}, \mathrm{j})$ can be rewritten as:

$$
\begin{align*}
& \hat{\mathrm{v}}_{\mathrm{a}}(-\mathrm{i}, \mathrm{i})=\frac{\mathrm{v}_{\mathrm{s}}^{2} \cdot \mathrm{~N}_{\mathrm{a}}(\mathrm{i}, \mathrm{j}) \cdot \Delta \mathrm{t}_{\mathrm{s}} \cdot \cos ^{2} \varphi}{2 \mathrm{r} \sin \varphi\left(1+\cos ^{2} \varphi\right)}  \tag{24}\\
& \hat{\mathrm{v}}_{\mathrm{a}, \text { error }}(-\mathrm{i}, \mathrm{i}) \approx-\frac{\cos \theta \cdot \Delta \theta}{\sin 2 \varphi\left(1+\cos ^{2} \varphi\right)} \cdot \mathrm{v}_{\mathrm{r}} \tag{25}
\end{align*}
$$

The estimation error due to $\mathrm{v}_{\mathrm{r}}$ is shown in Fig.8. So, in order to obtain an accurate azimuth velocity estimation result, the error caused by range velocity should be compensated, especially for the case with a small azimuth velocity and a large range velocity.


Fig. 8. Azimuth velocity estimation error change with range velocity from $0 \mathrm{~m} / \mathrm{s}$ to $30 \mathrm{~m} / \mathrm{s}$ using two images, with azimuth squint angles $\varphi=-3^{\circ}$ and $\varphi=3^{\circ}$.

## B. Registration Effect on Azimuth Velocity Estimation

Image registration error has direct influence on azimuth velocity estimation accuracy. In order to illustrate the effect due to one pixel registration error, using the parameters listed in Table. I, we choose ten different combinations of two images.

One image is acquired with the azimuth squint angle $-5^{\circ}$, and the other one with the azimuth squint angle varying from $-4^{\circ}$ to $5^{\circ}$.

As shown in Fig. 9, a larger difference in squint angle or a longer time lag is needed for reducing the effect caused by image registration error. Assuming the registration error is random, it can be reduced by calculating the expectation of $\hat{v}_{\mathrm{a}}(\mathrm{i}, \mathrm{j})$.

$$
\begin{equation*}
\hat{v}_{a}=E\left[\hat{v}_{a}(i, j)\right] \approx \frac{\sum_{i=1}^{n} \sum_{j=i+1}^{n} \hat{v}_{a}(i, j)}{C_{n}^{2}} \tag{26}
\end{equation*}
$$

where $n$ is the number of sequential SAR images, $E[\cdot]$ is the expectation operation and C denotes the combination operation.


Fig. 9. Azimuth velocity estimation error due to one pixel registration error. Ten different combinations of two images are selected, with the difference of squint angle from $1^{\circ}$ to $10^{\circ}$, which introduces an azimuth velocity error from $0.05 \mathrm{~m} / \mathrm{s} \sim 0.5 \mathrm{~m} / \mathrm{s}$.

## C. Defocusing Effect on Azimuth Velocity Estimation

As mentioned in Sec. II, target motion in azimuth is the major factor for defocusing, due to large residual phase error caused by it. When the phase error is large enough, pattern distortion of IRF will occur. As shown in Fig. 10, when residual phase error reaches $1.5 \pi$, the main lobe is cut into two parts. Consequently, the image will become blurry, leading to possible position measurement error.


Fig. 10. Azimuth velocity estimation error due to defocusing. Residual phase error not only causes loss in gain, but also pattern distortion. And, the position of maximum value will change, leading a position measurement error.

In order to mitigate the effect of defocusing, a refocusing
operation is adopted for fast moving target detection and velocity estimation. Taking $1.5 \pi$ as the judgment threshold value,

$$
\begin{equation*}
\left|\pi \Delta \mathrm{f}_{\mathrm{r}, \mathrm{v}_{\mathrm{a}}}\left(\frac{\mathrm{~T}}{2}\right)^{2}\right|=\left|\pi \frac{1}{\lambda \mathrm{r}_{0}} \mathrm{v}_{\mathrm{a}} \mathrm{v}_{\mathrm{s}} \cos ^{2} \varphi \frac{\mathrm{v}_{\mathrm{s}}}{\rho_{\mathrm{a}} \mathrm{f}_{\mathrm{r} 0}} \mathrm{~T}\right|<1.5 \pi \tag{27}
\end{equation*}
$$

Then,

$$
\mathrm{v}_{\mathrm{a}, \text { threshold }}<3 \frac{\mathrm{p}_{\mathrm{a}}}{\mathrm{~T}}
$$

where $p_{a}$ is azimuth resolution, and $T$ is synthetic aperture time. It means that target movement cannot exceed 3 time of $\rho_{\mathrm{a}}$ during the synthetic aperture time; otherwise, there will be a significant pattern distortion, resulting in an error of nearly three pixels. However, in the case of low signal-to-clutter ratio (SCR), due to the gain loss of moving target, the background clutter may have noticeable effect on pixel offset measurement. So, a smaller judgment threshold value should be used according to the SCR.

If the estimated azimuth velocity result is larger than $\mathrm{v}_{\mathrm{a}, \text { threshold }}$, a refocusing operation for moving target is required by transforming the image into the Doppler domain, using the phase compensation function

$$
\begin{equation*}
\Omega\left(\mathrm{f}_{\mathrm{a}}\right)=\exp \left\{\pi \mathrm{f}_{\mathrm{a}}^{2} \frac{\Delta \mathrm{f}_{\mathrm{r}, \mathrm{v}_{\mathrm{a}}}}{\mathrm{f}_{\mathrm{r} 0}\left(\mathrm{f}_{\mathrm{r} 0}+\Delta \mathrm{f}_{\mathrm{r}, \mathrm{v}_{\mathrm{a}}}\right)}\right\}=\exp \left\{-2 \pi \frac{\hat{\mathrm{v}}_{\mathrm{a}}}{\mathrm{v}_{\mathrm{s}}} \frac{\mathrm{f}_{\mathrm{a}}^{2}}{\mathrm{f}_{\mathrm{r} 0}+\Delta \mathrm{f}_{\mathrm{r}, \mathrm{v}_{\mathrm{a}}}}\right\} \tag{28}
\end{equation*}
$$

where $f_{a}$ is azimuth frequency. After phase compensation, the azimuth velocity estimation method is implemented again for a more accurate value.

The flowchart of the proposed method is shown in Fig. 11, where the classic chirp scaling approach is adopted [27], for focusing targets at their corresponding closest slant ranges.


Fig. 11. Flowchart of the proposed azimuth velocity estimation method.

## IV. EXPERIMENTAL RESULTS

## A. Results with Simulated Data

Experiments are performed first by simulation data, with
parameters listed in Table. I. Five sequential images are used, with the azimuth angles $-5^{\circ},-3^{\circ}, 0^{\circ}, 3^{\circ}$ and $5^{\circ}$. So, ten different combinations ( $\mathrm{C}_{5}^{2}=10$ ) can be used for velocity estimation, and the mean value is used in the following analysis.

Fig. 12 shows the azimuth velocity estimation results with respect to real azimuth velocity from $1 \mathrm{~m} / \mathrm{s}$ to $30 \mathrm{~m} / \mathrm{s}$ and range velocity is $0 \mathrm{~m} / \mathrm{s}$. With $\mathrm{p}_{\mathrm{a}}=3 \mathrm{~m}$ and $\mathrm{T}=0.43 \mathrm{~s}, \mathrm{v}_{\mathrm{a} \text {,threshold }}$ is $21 \mathrm{~m} / \mathrm{s}$ according to (27). Consequently, when azimuth velocity is up to $22 \mathrm{~m} / \mathrm{s}$, it leads to $0.4 \mathrm{~m} / \mathrm{s}$ estimation error due to pattern distortion. After defocusing compensation, the azimuth velocity estimation accuracy is significantly improved, with an error within $0.08 \mathrm{~m} / \mathrm{s}$ at azimuth velocity $30 \mathrm{~m} / \mathrm{s}$.


Fig. 12. Azimuth velocity estimation error due to defocusing.
In addition to defocusing, range velocity also has direct influence on azimuth velocity estimation accuracy, especially in case of relatively larger range velocity compared with azimuth velocity. Fig. 13 shows the azimuth velocity estimation error, with range velocity varying from $1 \mathrm{~m} / \mathrm{s}$ to $30 \mathrm{~m} / \mathrm{s}$. The larger the range velocity is, the greater the estimation error is. When range velocity is up to $30 \mathrm{~m} / \mathrm{s}$, it results in an estimation error of nearly $0.1 \mathrm{~m} / \mathrm{s}$.


Fig. 13. Azimuth velocity estimation error due to range velocity.
As can be seen from the above simulation results, a more accurate estimation can be obtained by the proposed method, especially by compensating the error due to defocusing and the range velocity effect.

## B. Results with TerraSAR-X Staring Spotlight Data

In order to further verify the proposed method, the TerraSAR-X staring spotlight data is utilized, which has an azimuth resolution of 0.25 m , with a large azimuth steering span of $-2.2^{\circ} \sim 2.2^{\circ}$ [4], [7]. The staring spotlight image is transformed into the Doppler domain, and the valid azimuth frequency spectrum is divided into $n$ sub-spectrums. Since the
azimuth frequency coincides with the azimuth squint angle, each sub-spectrum corresponds to the image acquired with different azimuth squint angles. So, sequential SAR images can be obtained by applying Inverse Fast Fourier Transform (IFFT) on each sub-spectrum, as shown in Fig. 14.


Fig. 14. Sequential images generation using the staring spotlight image.
As shown in Fig. 15, the Shenyangbei railway station staring spotlight image is used, and the parameters are listed in Table. II .

TABLE II
TERRASAR-X SHENYANGBEI STARING IMAGE PARAMETERS

| Image Parameters | 0.031 m |
| :---: | :---: |
| Wavelength | $24.07^{\circ}$ |
| Incidence angle | 514 km |
| Orbit height | 4998.45 Hz |
| PRF | 329.6 MHz |
| Signal sampling rate | 558.470 km |
| Scene center slant range | 0.588 m |
| Slant Range Resolution | 0.23 m |
| Azimuth Resolution | 42400 Hz |
| Image azimuth sampling rate |  |
| Processing Parameters | $22.18^{\circ}$ |
| Elevation | $7337 \mathrm{~m} / \mathrm{s}$ |
| Effective velocity | 32236 Hz |
| Azimuth valid frequency | 5 |
| Number of sequential images | $-1.56^{\circ}{ }^{\circ}-0.78^{\circ}, 0^{\circ}$, |
| Squint angel of sequential images | $0.78^{\circ}, 1.56^{\circ}$ |
| Time lags between sequential 1 and | 4.14 s |
| sequential 5 | 6448 Hz |
| Sub-image azimuth sampling rate | 0.455 m |
| Slant range spacing | 1.138 m |
| Azimuth spacing |  |



Fig. 15. The TerraSAR-X Shenyangbei railway station staring spotlight image. A high-speed train is moving from right to left, causing a pixel offset in azimuth. The shadow indicates the real position of high-speed train.

In Fig. 15, a high-speed train is moving, and the shadow represents its real position. The shadow movement is clearly visible and can be measured to calculate the velocity as the real velocity of the train. Fig. 16 shows the pixel offset of the shadow in following images. The velocity of shadow is calculated according to the pixel offset, slant range spacing, azimuth spacing and time lag, and the result for azimuth velocity is $2.124 \mathrm{~m} / \mathrm{s}$, slant range velocity is $-7.028 \mathrm{~m} / \mathrm{s}$ and the range velocity is $-17.227 \mathrm{~m} / \mathrm{s}$.

Fig. 17 shows the high-speed train pixel offset in the sequential SAR images, with 15 pixel offset in azimuth. Taking this azimuth offset into (22), the azimuth velocity estimation result is $2.057 \mathrm{~m} / \mathrm{s}$. Moreover, considering the range velocity effect that causes a $0.041 \mathrm{~m} / \mathrm{s}$ error, the modified estimation is $2.098 \mathrm{~m} / \mathrm{s}$, which is almost the same as the value calculated according to shadow movement, again verifying the effectiveness of the proposed method.


Fig. 16. Shadow pixel offset in sequential images, taking image 1 as the benchmark with 0 pixel offset, and calculating the pixels in slant range and azimuth respectively.


Fig. 17. High-speed offset in sequential images, with 15 pixels in azimuth.

## V. CONCLUSION

In this paper, an effective azimuth velocity estimation
method has been proposed based on the MASA imaging mode. Combined with the acquisition geometry of the MASA mode, target motion effects were analyzed in detail, including the azimuth offset and slant range offset. Based on the slant range offset, the effect of Doppler parameters mismatch during the image formation process was also discussed. The estimation error was studied considering factors including registration, defocusing and range velocity, with compensation methods presented to improve the accuracy. The effectiveness of the proposed method was demonstrated by experimental results using both simulated data and the TerraSAR-X data. However, due to target motion, moving target will be focused at a wrong position in a SAR image. For example, a moving car may be overlapped with its nearby buildings in the image. If the intensity of background clutter is close to or even stronger than that of the moving target, it would be difficult to obtain the required accurate pixel offsets, resulting in degradation of azimuth velocity estimation accuracy. In order to overcome this problem, clutter suppression techniques can be employed as a topic of future research, but it is beyond the scope of this paper.

## APPENDIX

The derivation of equations (16) and (17) is given in detail in the following.

As shown in Fig. 18, the Doppler centroid frequency is given by

$$
\begin{equation*}
\mathrm{f}_{\mathrm{d} 0}=-\frac{2 \mathrm{v}_{\mathrm{s}}}{\lambda} \sin \varphi=-\frac{2 \mathrm{v}_{\mathrm{s}}}{\lambda} \frac{\mathrm{X}_{0}}{\mathrm{r}_{0}} \tag{A1}
\end{equation*}
$$

The differentiation of $f_{d 0}$ is

$$
\begin{align*}
& d\left[f_{d 0}\right]=-\frac{2 \mathrm{v}_{\mathrm{s}}}{\lambda} \mathrm{X}_{0}\left(-\frac{1}{\mathrm{r}_{0}^{2}}\right) \mathrm{dr}  \tag{A2}\\
& =-\frac{2 \mathrm{v}_{\mathrm{s}}}{\lambda} \frac{\mathrm{X}_{0}}{\mathrm{r}_{0}}\left(-\frac{1}{\mathrm{r}_{0}}\right) \mathrm{dr}=-\frac{\mathrm{f}_{\mathrm{d} 0}}{\mathrm{r}_{0}} \mathrm{dr}
\end{align*}
$$

Fig. 18. Acquisition geometry model.
Since $\Delta \mathrm{s}_{\mathrm{r}, \mathrm{v}_{\mathrm{r}}}$ is the function of $\varphi$ and $\theta$, we can calculate the differentiation of $\Delta \mathrm{f}_{\mathrm{d}, \text { mis }, v_{\mathrm{r}}}$ according to (A2),

$$
\begin{equation*}
\mathrm{d}\left[\Delta \mathrm{f}_{\mathrm{d}, \text { mis }, \mathrm{v}_{\mathrm{r}}}\right]=\frac{\partial \Delta \mathrm{f}_{\mathrm{d}, \text { mis }, \mathrm{v}_{\mathrm{r}}}}{\partial \mathrm{r}} \frac{\partial \mathrm{r}}{\partial \varphi}+\frac{\partial \Delta \mathrm{f}_{\mathrm{d}, \text { mis }, \mathrm{v}_{\mathrm{r}}}}{\partial \mathrm{r}} \frac{\partial \mathrm{r}}{\partial \theta} \tag{A3}
\end{equation*}
$$

Substituting (A2) and (13b) into (A3), we have

$$
\begin{align*}
\mathrm{d}\left[\Delta \mathrm{f}_{\mathrm{d}, \mathrm{mis}, \mathrm{v}_{\mathrm{r}}}\right. & ]
\end{aligned}=-\frac{\mathrm{f}_{\mathrm{d} 0}}{\mathrm{r}_{0}} \cdot\left(\frac{\partial \Delta \mathrm{~s}_{\mathrm{r}, \mathrm{v}_{\mathrm{r}}}}{\partial \varphi}\right)-\frac{\mathrm{f}_{\mathrm{d} 0}}{\mathrm{r}_{0}} \cdot\left(\frac{\partial \Delta \mathrm{~s}_{\mathrm{r}, \mathrm{v}_{\mathrm{r}}}}{\partial \theta}\right) \quad \begin{aligned}
& =-\frac{\mathrm{f}_{\mathrm{d} 0}}{\mathrm{r}_{0}} \cdot\left(\frac{\mathrm{v}_{\mathrm{r}}}{\mathrm{v}_{\mathrm{s}}} \mathrm{r}_{0} \frac{\sin \theta}{\cos ^{2} \varphi}\right) \Delta \varphi-\frac{\mathrm{f}_{\mathrm{d} 0}}{\mathrm{r}_{0}} \cdot\left(\frac{\mathrm{v}_{\mathrm{r}}}{\mathrm{v}_{\mathrm{s}}} \mathrm{r}_{0} \frac{\sin \varphi}{\cos \varphi} \cos \theta\right) \Delta \theta \\
& =\frac{2 \mathrm{v}_{\mathrm{s}}}{\lambda \mathrm{r}_{0}} \sin \varphi \cdot\left(\frac{\mathrm{v}_{\mathrm{r}}}{\mathrm{v}_{\mathrm{s}}} \mathrm{r}_{0} \frac{\sin \theta}{\cos ^{2} \varphi}\right) \Delta \varphi+\frac{2 \mathrm{v}_{\mathrm{s}}}{\lambda \mathrm{r}_{0}} \sin \varphi \cdot\left(\frac{\mathrm{v}_{\mathrm{r}}}{\mathrm{v}_{\mathrm{s}}} \mathrm{r}_{0} \frac{\sin \varphi}{\cos \varphi} \cos \theta\right) \Delta \theta \\
& =\frac{2 \mathrm{v}_{\mathrm{r}}}{\lambda} \sin \theta \frac{\sin \varphi}{\cos ^{2} \varphi} \Delta \varphi+\frac{2 \mathrm{v}_{\mathrm{r}}}{\lambda} \cos \theta \frac{\sin ^{2} \varphi}{\cos \varphi} \Delta \theta
\end{align*}
$$

Since $\varphi$ is small, (A4) can be approximated as follows

$$
\begin{equation*}
\mathrm{d}\left[\Delta \mathrm{f}_{\mathrm{d}, \text { mis }, \mathrm{v}_{\mathrm{r}}}\right] \approx \frac{2 \mathrm{v}_{\mathrm{r}}}{\lambda} \sin \theta \sin \varphi \Delta \varphi+\frac{2 \mathrm{v}_{\mathrm{r}}}{\lambda} \cos \theta \frac{\sin ^{2} \varphi}{\cos \varphi} \Delta \theta \tag{A5}
\end{equation*}
$$

As for $\Delta \mathrm{s}_{\mathrm{r}, \mathrm{v}_{\mathrm{a}}}$, it is only a function of $\varphi$. So the differentiation of $\Delta f_{d, \text { mis }, v_{\mathrm{a}}}$ is given by

$$
\begin{equation*}
\mathrm{d}\left[\Delta \mathrm{f}_{\mathrm{d}, \mathrm{mis}, \mathrm{v}_{\mathrm{a}}}\right]=\frac{\partial \Delta \mathrm{f}_{\mathrm{d}, \mathrm{mis}, \mathrm{v}_{\mathrm{a}}}}{\partial \mathrm{r}} \frac{\partial \mathrm{r}}{\partial \varphi} \tag{A6}
\end{equation*}
$$

In the same way, substituting (A2) and (13a) into (A6), we have

$$
\begin{align*}
\mathrm{d}\left[\Delta \mathrm{f}_{\mathrm{d}, \mathrm{mis}, \mathrm{v}_{\mathrm{a}}}\right] & =-\frac{\mathrm{f}_{\mathrm{d} 0}}{\mathrm{r}_{0}} \cdot\left(\frac{\partial \Delta \mathrm{~s}_{\mathrm{r}, \mathrm{v}_{\mathrm{a}}}}{\partial \varphi}\right)=-\frac{\mathrm{f}_{\mathrm{d} 0}}{\mathrm{r}_{0}} \cdot\left(\frac{\mathrm{v}_{\mathrm{a}}}{\mathrm{v}_{\mathrm{s}}} \mathrm{r}_{0} \frac{2 \sin \varphi}{\cos ^{3} \varphi}\right) \Delta \varphi \\
& =\frac{2 \mathrm{v}_{\mathrm{s}}}{\lambda \mathrm{r}_{0}} \sin \varphi \cdot\left(\frac{\mathrm{v}_{\mathrm{a}}}{\mathrm{v}_{\mathrm{s}}} \mathrm{r}_{0} \frac{2 \sin \varphi}{\cos ^{3} \varphi}\right) \Delta \varphi  \tag{A7}\\
& =\frac{4 \mathrm{v}_{\mathrm{a}}}{\lambda} \frac{\sin ^{2} \varphi}{\cos ^{3} \varphi} \Delta \varphi \approx 0
\end{align*}
$$

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