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MRTD: New Time Domain Schemes Based on Multiresolution Analysis

by

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Brief Summary of Performed Study

1.0 Abstract

The application of multiresolution analysis directly to Maxwell's equations results in new time domain schemes with unparalleled properties. This time domain approach, MRTD (MultiResolution Time Domain Method), allows for the development of schemes which are based on scaling functions only or on a combination of scaling functions and wavelets for the development of a variable griding. The dispersion of the MRTD schemes compared to the conventional FDTD Yee's scheme shows an excellent capability to approach the exact solution with negligible error for sampling rates which approach the Nyquist limit. Furthermore, due to the weak-interaction properties of the wavelets, MRTD schemes allow for time/space-adaptive grids. These recent developments in time domain techniques at the University of Michigan have strongly indicated the potential of MRTDs in creating a major impact to the area of computational electromagnetics [10,11]. MRTD is not a new methodology. It is a correct and accurate generalization of the conventional discretization approaches. It provides the correct mathematical frame for solving problems in time domain and allows for the understanding of important issues in time-domain computational electromagnetics.

2.0 Introduction to MRTD

The finite-difference time-domain method (FDTD) has proven to be a powerful numerical technique in electromagnetic field computations [1,2]. However, despite its simplicity and modeling versatility, the technique suffers from serious limitations due to the substantial computer resources required to model electromagnetic problems with medium or large computational volumes. In addition, the FDTD method cannot provide the accuracy required for computer simulations of time-dependent electromagnetic interactions in electrically long regions or in regions which contain non-linear materials. Such simulations are very important for integrated device modeling, especially in relation to the design of non-linear photonic devices. The above limitations have always made it a matter of great interest to improve the efficiency of Yee's FDTD scheme and have led researchers to the development of hybrid combinations of FDTD with other propagation methods [3,4] and higher order FDTD schemes based on Yee's grid [5]. The method of moments provides a mathematically correct approach for the discretization of integral and partial differential equations [6]. Its application to the discretization of Maxwell's partial differential equations has provided the field theoretical foundation for TLM [7,8]. In addition, it has been shown recently [9] that Yee's FDTD scheme can be derived from the same approach when using pulse functions for the expansion of the unknown fields. Since the method of moments allows for the use of any complete and orthonormal set, the choice of an appropriate expansion set may lead to new powerful time domain schemes. The application of the method of moments using scaling and wavelet functions, known as multiresolution analysis (MRA), has been applied to Maxwell's partial differential equations and has lead to novel and powerful time domain schemes [10] and [11].

In a MRTD scheme the electromagnetic fields are represented by a two-fold expansion in scaling and wavelet functions with respect to space. The expansion in terms of scaling functions allows for a correct modeling of smoothly-varying electromagnetic fields. In regions characterized by strong field variations or field singularities, additional field sampling points are introduced by incorporating wavelets in the field expansions. These additional points are introduced only at specific locations, thus, allowing for a variable grid capability. The use of different families of functions leads to various time domain schemes. The exclusive use of scaling functions provides a variety of conventional schemes including FDTD and TLM. The MRTDs which have been recently developed at Michigan have used pulse functions as expansion and testing functions in the time domain in order to obtain a two-step finite difference scheme with respect to time.

MRTD schemes based on cubic spline Battle-Lemarie scaling and wavelet functions have been developed and applied to a variety of problems. An extensive discussion of these derivations is presented in [10]. This orthonormal wavelet expansion has already been applied successfully to the computation of electromagnetic-field problems in the frequency domain and results have been presented for both 2-D and 3-D problems [17,18]. The Battle-Lemarie scaling and wavelet functions do not have compact support, thus the MRTD schemes have to be truncated with respect to space (see Figures 1,2). However, this disadvantage is offset by the low-pass and band-pass characteristics in spectral domain, allowing for an a priori estimate of the number of resolution levels necessary for a correct field modeling. Furthermore, for this type of scaling and wavelet functions, the evaluation of the moment method integrals during the discretization of Maxwell's PDEs is simplified due to the existence of closed form expressions in spectral domain and simple representations in terms of cubic spline functions in space domain. The use of non-localized basis functions cannot accommodate localized boundary conditions and cannot allow for a localized modeling of material properties. To overcome this difficulty, the image principle is used to model perfect electric and magnetic boundary conditions. As for the description of material parameters, the constitutive relations are discretized accordingly and the relationships between the electric/magnetic flux and the electric/magnetic field are given by two matrix equations.

FIGURE 1. Battle-Lemarie Scaling Function and its Fourier Transforms.

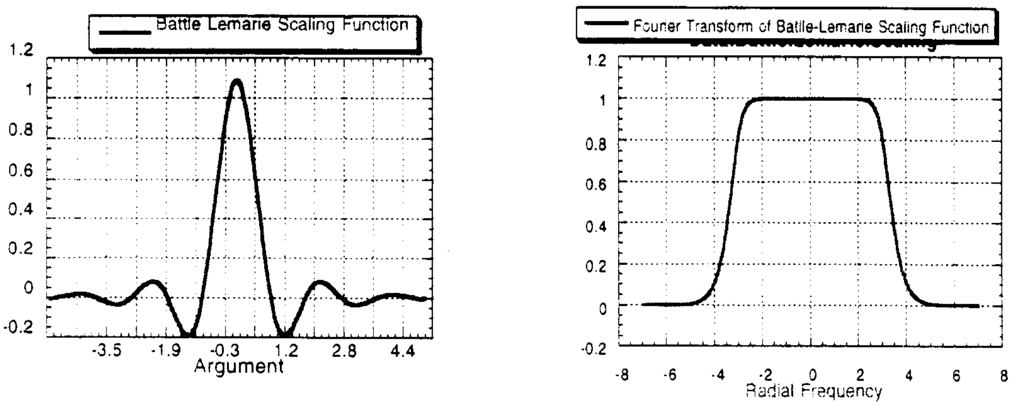
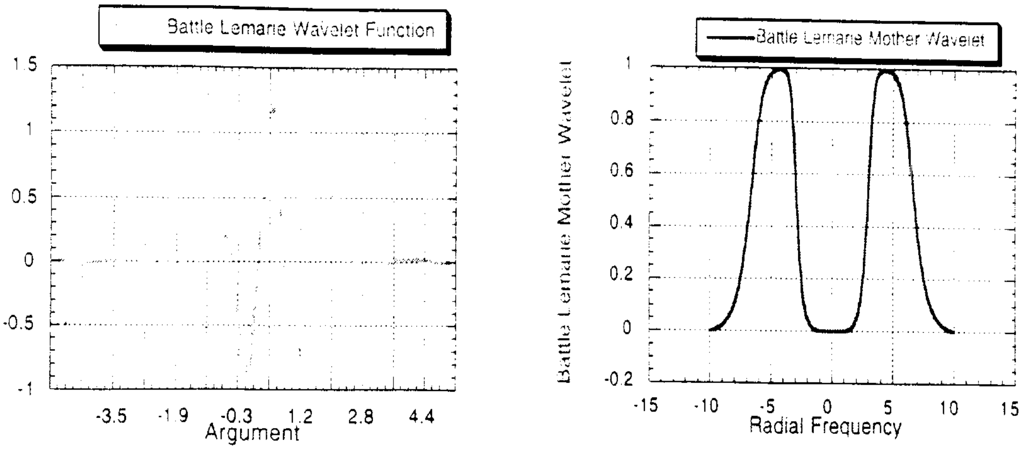


FIGURE 2. Battle-Lemarie Mother Wavelet and its Fourier Transforms.



Complete dispersion analyses of the MRTD schemes including applications to 3-D problems and comparisons to Yee's FDTD scheme are given in [10] and show the superiority of MRTDs to all other existing discretization techniques. Specifically, the results show the capability of the MRTD method to provide excellent accuracy with up two points per wavelength which is the Nyquist sampling limit. The use of Battle Lemarie scaling and wavelet functions has provided very efficient solutions to open and shielded circuit problems (see **Appendix B**). Figure 4 shows field calculations for the even mode excited in coupled strips operating in an open environment as shown in Figure 3. The open boundaries have been accounted for by incorporating PML regions within the MRTD technique. The use of MRTD allowed for the placement of the matching layer right at the planar lines while it provided very high accuracy and high computational efficiency. Figure 5 shows the even-mode electric field distributions for a similar coupled strip geometry operating in a shielded environment as shown in Figure 6 [22] (see **Appendix B**). As with Battle-Lemarie functions, the application of the Haar basis functions (see Figure 7) has led into the development of a multigrid FDTD technique [23, 24] and has demonstrated the capability for a spatially adaptive grid in 2-dimensional waveguide and transmission line problems. Figure 8 shows the field distribution in a two-dimensional shielded stripline. On the same figure the spatially adaptive grid utilized in this problem is shown. For more details on this development see **Appendices A and F**.

FIGURE 3. Open Coupled Strip Line

FIGURE 4. Even-Mode Field Distribution in the Structure of Figure 3

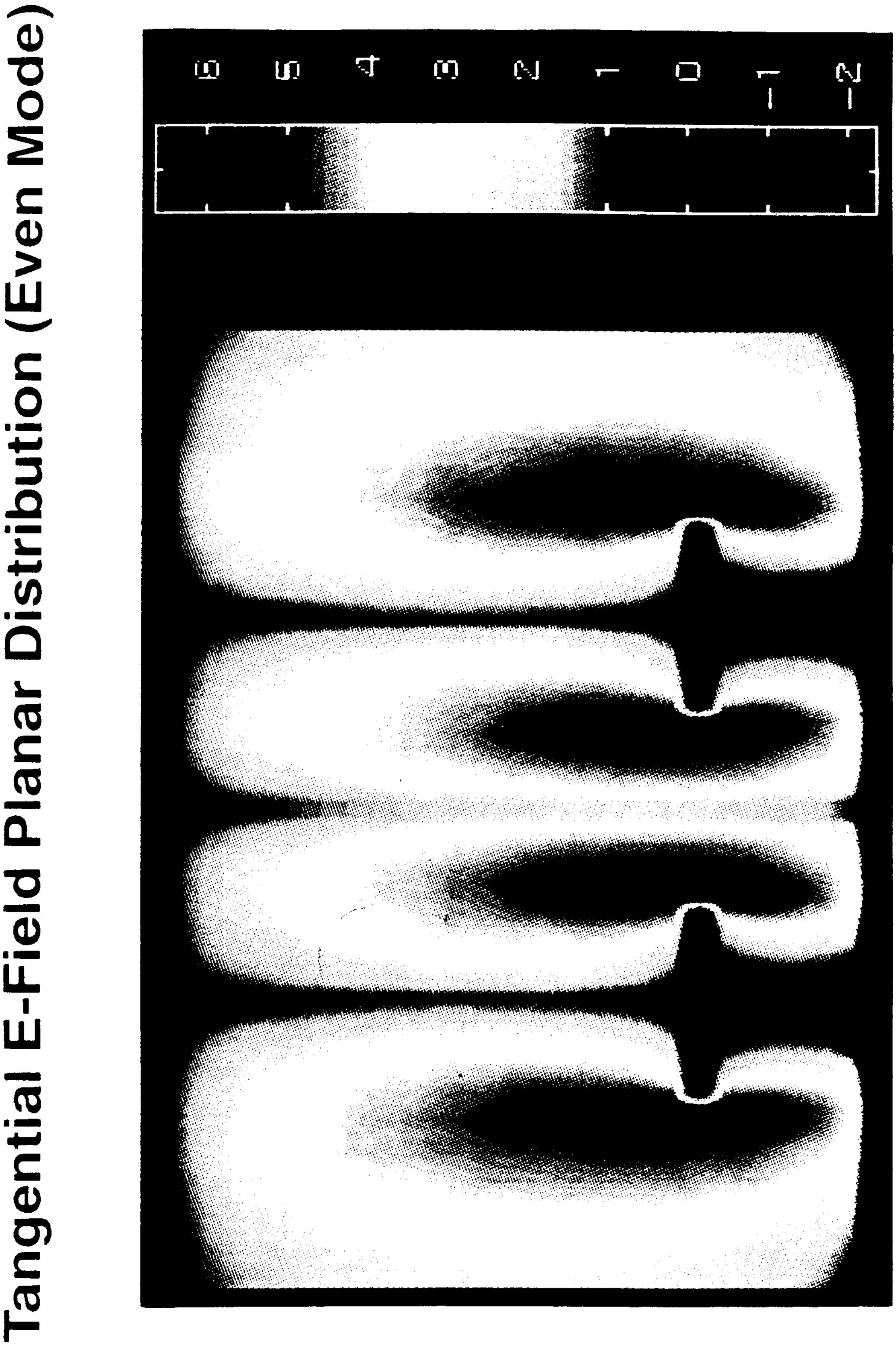
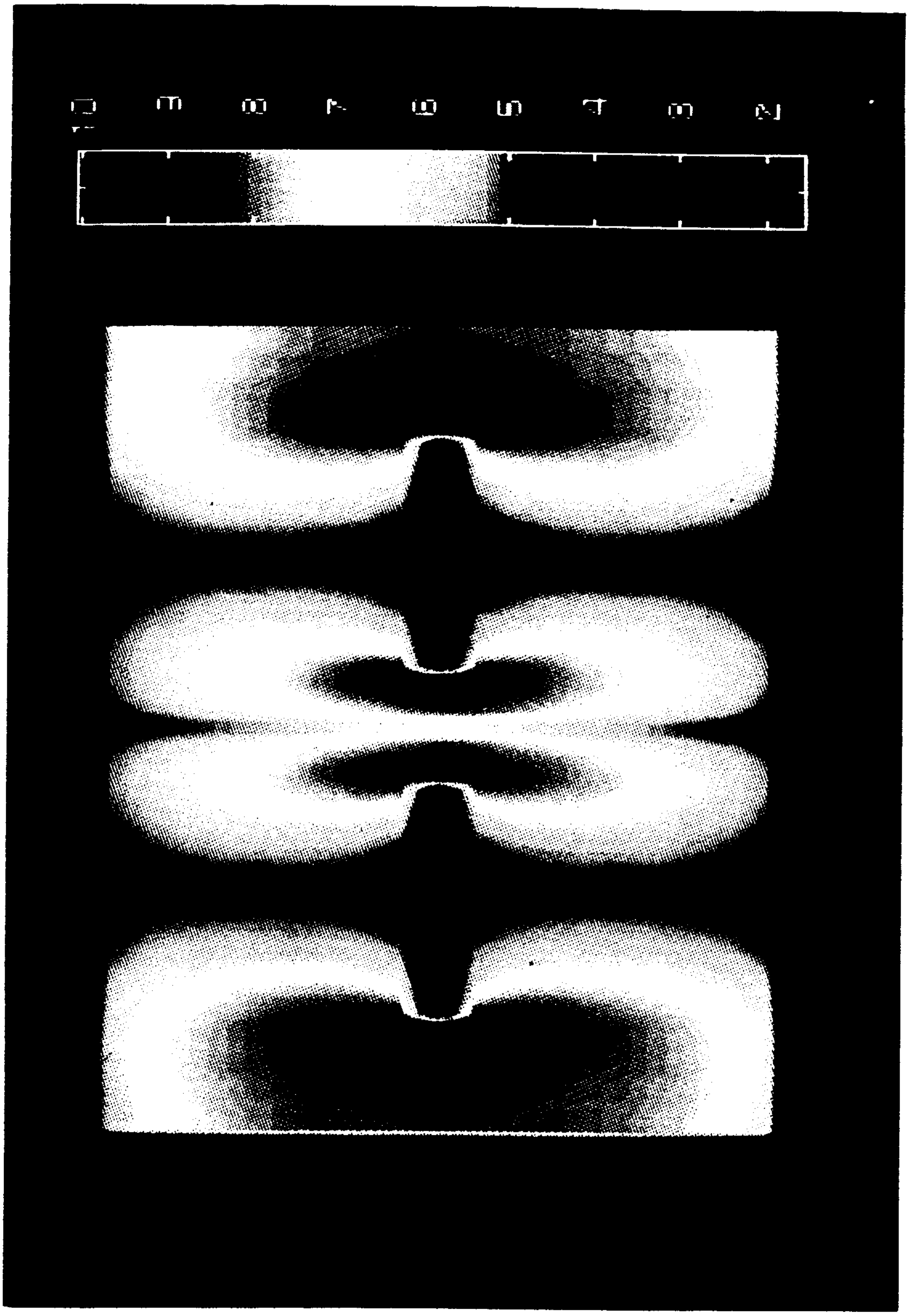
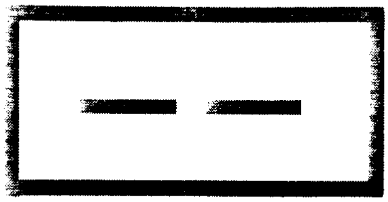


FIGURE 5. Even-Mode Electric Field Distribution for the Structure of Figure 6



Tangential E-Field Planar Distribution (Even Mode)

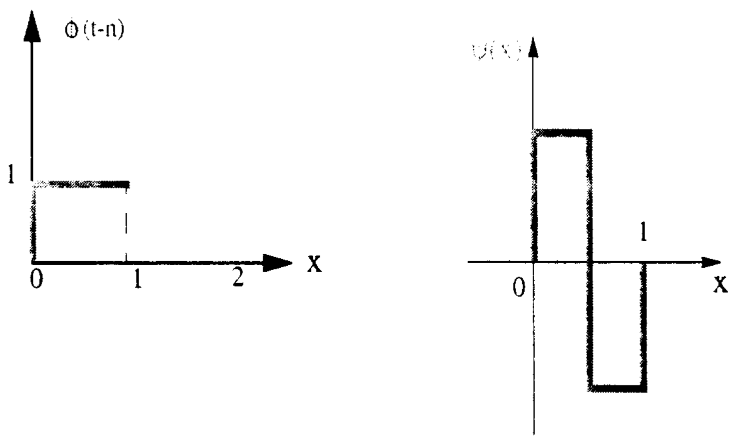
FIGURE 6. Shielded Coupled Strip Line



3.0 Time/Space Adaptive MRTD Schemes

The major advantage of the use of Multiresolution analysis to time domain is the capability to develop time and space adaptive schemes for an open as well as shielded space, and its application to linear and non-linear problems. The effectiveness of the technique is measured in terms of its accuracy and computational efficiency in comparison to conventional time domain techniques (FDTD, TLM). The development of a spatially and temporally adaptive MRTD method is based on the property of wavelet expansion functions to interact weakly and allow for a spatial sparsity that may vary with time as needed through a thresholding process [12-19]. The availability of such an adaptive method is extremely important for the accurate modeling of sharp field variations of the type encountered in beam focusing in nonlinear optics, wave propagation through narrow slits and apertures etc. A brief presentation of the fundamental steps required for such a adaptive grid is given in the following section.

FIGURE 7. Haar Scaling Functions and Wavelets



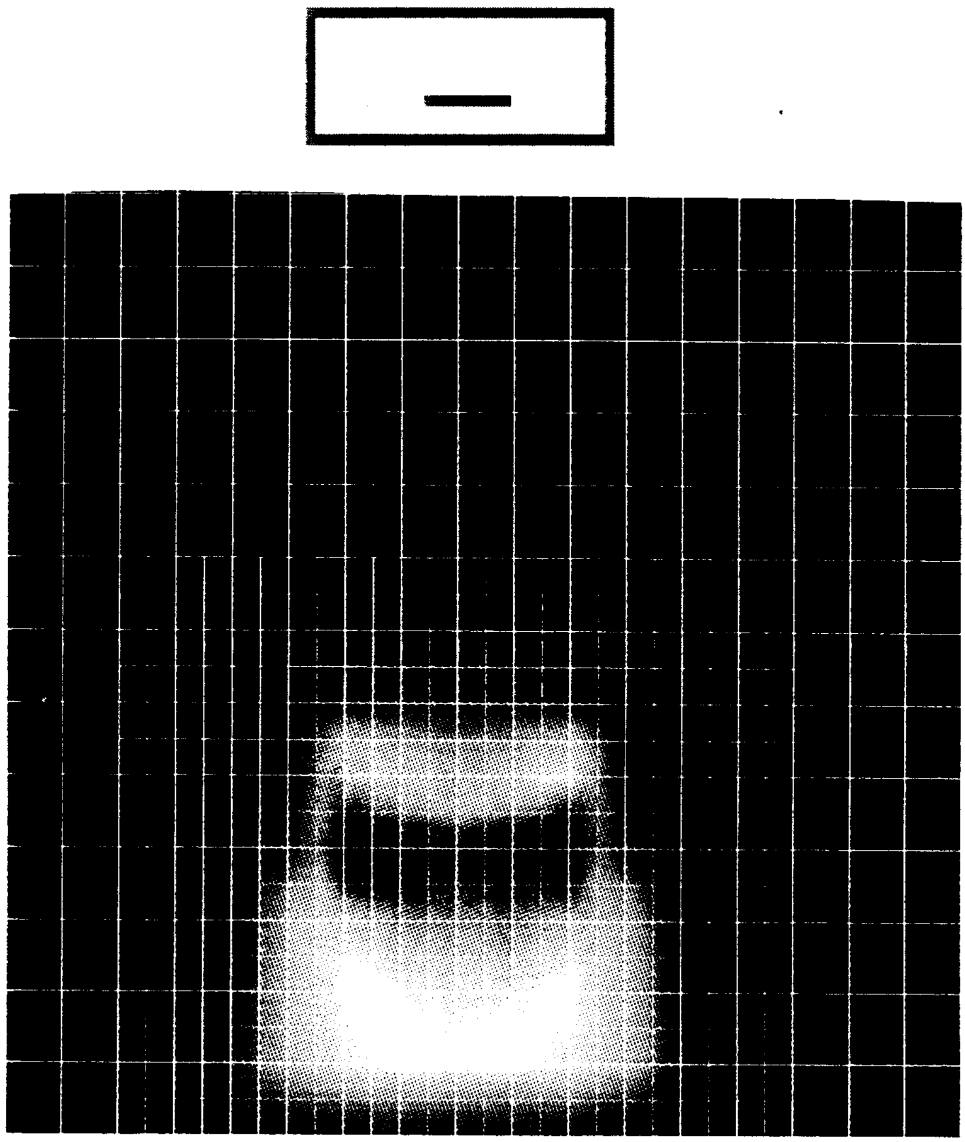
The use of multiresolution analysis for adaptive grid computations for PDEs has been suggested by Perrier and Basdevant [20] and Liantdrat and Tchamitchian [21]. To describe the basic ideas of such an adaptive scheme for Maxwell's hyperbolic system, let us cast Maxwell's equations in one spatial dimension in the form:

$$\frac{\partial}{\partial t}\hat{u} = A\hat{u} \tag{1}$$

where $\hat{u} = (E(z,t), H(z,t))^t$, and A is the operator:

$$A = \begin{bmatrix} 0 & -\varepsilon(z)^{-1}\frac{\partial}{\partial z} \\ -\mu(z)^{-1}\frac{\partial}{\partial z} & 0 \end{bmatrix} \tag{2}$$

FIGURE 8. **Field Distribution in a Printed Stripline Using MRTD Based on Haar Functions**



The numerical solution of (1) subject to initial conditions and appropriate boundary conditions at the two boundary points is sought. Following appropriate derivations, the above equation can be written in the following form:

$$M\ |U\rangle = 0 \tag{3}$$

where

$$M = \begin{bmatrix} \varepsilon T^{\dagger}{}_h d_t & T^{\dagger}{}_h D_z \\ Z^{\dagger}{}_h D_z & \mu Z^{\dagger}{}_h d_t \end{bmatrix} \tag{4}$$

In equation 4, Z_h, T_h are half shift operators for the spatial and temporal coordinates z, t respectively and $Z^{\dagger}{}_h, T^{\dagger}{}_h$ are their Hermitian conjugates. Furthermore, d_t, D_z are difference operators given by the following equations:

$$d_t = \frac{1}{\Delta t}(T^{\dagger}{}_h - T_h) \tag{5}$$

and

$$D_z = \frac{1}{\Delta z}\left(Z^{\dagger}{}_h \sum_{i=-9}^{8} \alpha_{\varphi}(i) Z^{-i} + \sum_{i=-9}^{8} \alpha_{\psi}(i) Z^{-i}\right) \tag{6}$$

where

$$Z^{\dagger} = Z^{-1}\ . \tag{7}$$

In equation 6, $\alpha_{\varphi}(i), \alpha_{\psi}(i)$ are the coefficients associated with the scalar functions or wavelet expansion functions respectively. Since M is represented by block of band matrices, it can be shown that the domain where the field coefficients of $u^{(n+1)}$ are non-negligible is at most equal to the corresponding domain of $u^{(n)}$ plus the width of the bands in matrix M that represent the operator A in the wavelet basis. If N_{φ}, N_{ψ} are the total number of nonzero scaling and wavelet field coefficients (grid points), the number of operations required to compute M is of the order of O($N_{\varphi} + N_{\psi}$). As it has been shown in [10,11], the total number of scaling grid points can be as low as two per wavelength.

However, the resolution required in the wavelet grid points is determined by the nature of the boundaries in the problem of interest.

We are now ready to describe the method suggested in [20,21] to adapt in space and time the wavelet grid and thus follow the sharp features of the waves as they develop and/or move on the grid. At each time step we keep both the wavelet field values that are larger than a given threshold as well as the adjacent values. An adjacent wavelet field value is defined on the basis of the wavelet resolution level(s) incorporated in the solution. The development of an appropriate definition can be considered. Let $G_{(n)}$ be the wavelet field values (grid points) which are kept and represent the approximate solution at the nth time step. From these let us call fundamental the wavelet coefficients that are greater than the threshold and adjacent the ones as defined above. From equation 3 we compute the wavelet field coefficients of $u^{(n+1)}$ corresponding to the fundamental and adjacent coefficients that constitute grid $G_{(n)}$. We then adjust $G_{(n)}$ by changing into "fundamental" those field coefficients that are greater than the threshold and changing into adjacent their adjacent ones. This process creates the new grid $G_{(n+1)}$. We project u onto the space corresponding to $G_{(n+1)}$ and we are ready for the next step update. Clearly the basic assumption behind this algorithm is that during a time Δt, the domain of the fundamental field coefficients does no spread beyond its border of adjacent coefficients.

This method has already been applied to a variety of circuit problems [25] and is described in detail in **Appendices C, D** and **E**. Figure 10 shows the adaptive mesh following the pulse exciting the parallel plate waveguide structure shown in Figure 9, with a dense dielectric slab placed perpendicular to the parallel plates. This figure clearly shows the capability of the mesh to discretize based on field intensity and not on geometry. (see Appendix)

FIGURE 9. Parallel Plate Waveguide Structure

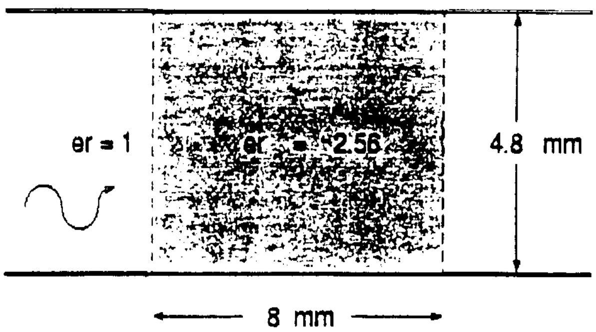
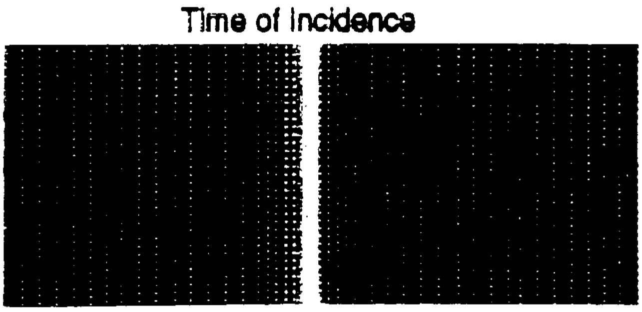
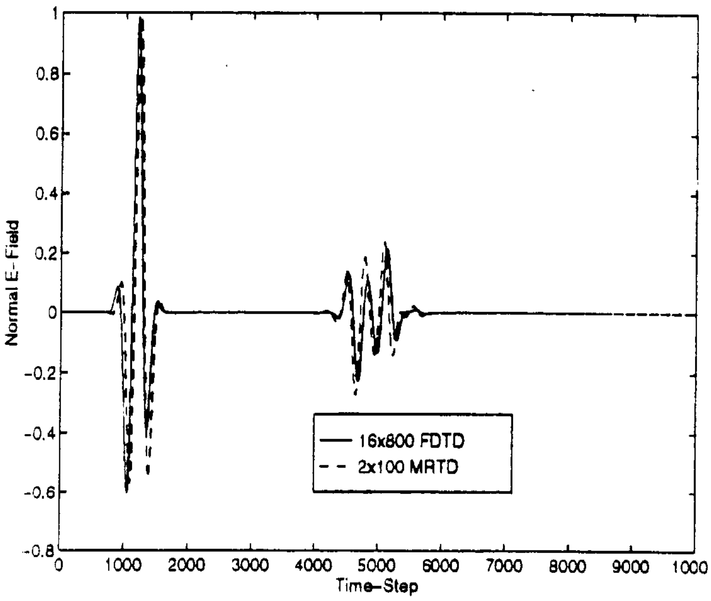


FIGURE 10. Time Adaptive Meshing





Normal E-field (Time-Domain).

APPENDIX A

Appendix A:
An FDTD Multigrid Based on MRTD

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AN FDTD MULTIGRID BASED ON MRTD:

Introduction

The use of wavelets in the method of moments for the solution of integral equations in frequency domain has been known since the 1993 Antennas and Propagation Symposium in Ann Arbor, MI [1]. Recent publications have demonstrated that the application of the method of moments directly to Maxwell's equations allows as well for the use of multiresolution analysis in the time domain [2], [3], [4]. In fact, multiresolution time domain (MRTD) schemes based on Battle-Lemarie scaling functions have shown to exhibit highly linear dispersion characteristics. In comparison to conventional FDTD, savings in computation time and memory of one and two orders of magnitude have been reported in [2]. Depending on the choice of basis functions, several different schemes result, each one carrying the signature of the basis functions used in MRA. It is also important to note that the design of an MRTD scheme can be accomplished using one's own application-specific basis functions.

The objective of this work is to develop an FDTD multigrid using the Haar wavelet basis. It will be shown that MRTD technique using Haar scaling functions results in the FDTD technique. Motivation for this work stems from the theory of MRA which says that a function which is expanded in terms of scaling functions of a lower resolution level, m1, can be improved to a higher resolution level, m2, by using wavelets of the intermediate levels. In other words, expanding a function using scaling function of resolution level m1 and wavelets up to resolution level m2 gives the same accuracy as expanding the function using just the scaling functions of resolution m2. However, the use of wavelet expansions has major implications in memory savings due to the fact that the wavelet expansion coefficients are significant only in areas of rapid field variations. This allows for the capability to discard wavelet expansion coefficients where they are not significant thereby leading to significant economy in memory. Different resolutions of wavelets can be combined so as to locally improve the accuracy of the approximation of the unknown function. This, combined with the fact that wavelet coefficients are significant only at abrupt field variations and discontinuities allows MRTD to lend itself very naturally to a Multigrid capability.

As mentioned above, the FDTD equations can be derived by applying the method of moments to Maxwell's equations using pulses as basis functions. Since pulse functions are the scaling functions in the Haar system, the FDTD technique can be considered as a specific MRTD scheme. Based on these ideas, the multigrid for the first resolution level is derived in the following sections. This is then followed by the performance of dispersion analysis to demonstrate the improvement by adding the multigrid to the regular FDTD. The 2D MRTD scheme based on Haar basis functions is then developed and applied to solve for the Electromagnetic fields in a waveguide and a shielded stripline. The results obtained are compared with those computed using conventional FDTD technique. It will be shown that the wavelet coefficients are significant only at locations with abrupt field variations. This facilitates in obtaining accurate solutions by combining the wavelet and scaling coefficients only in regions where the wavelet coefficients are significant (discontinuities).

Derivation of the FDTD Multigrid Using the Haar System

As it is known in the literature, the Haar system is generated by a scaling function $\Phi(x)$ and a mother wavelet $\Psi(x)$ given below:

$$\Phi(x) = \begin{matrix} 1 & for\, 0 \leq x \leq 1 \\ 0 & elsewhere \end{matrix}$$

and

$$\Psi(x) = \begin{cases} 1 & for\, 0 \leq x < \frac{1}{2} \\ -1 & for\, \frac{1}{2} \leq x < 1 \\ 0 & otherwise \end{cases}$$

Fig.1 shows these two generating functions. In the regular FDTD scheme, the electric and magnetic fields are expanded in terms of pulse functions as shown in the equation below:

$$E(x,t) = \Sigma_{k,m}\ {}_kE_m\ h_k(t)\ h_m(x)$$

where ${}_kE_m$ is an unknown constant and $h_k(t)$ and $h_m(x)$ are the pulse functions centered at t_k and x_m.

Applying this expansion to the one dimensional wave equation and using the method of moments (Galerkin's technique) we obtain the following discretized equation:

$$\frac{\Delta x}{c\Delta t}({}_{k+1}E_m -_{k-1} E_m) =_k E_{m+1} -_k E_{m-1}$$

which can also be denoted as:

$$D_tE =_k E_{m+1} -_k E_{m-1}$$

where D_t is the differential operator in the time domain. This is the regular FDTD discretization scheme with the E and corresponding H components located at the same nodes. To apply multiresolution analysis, the scaling functions and wavelets of the Haar system are both used as shown below :

$$E(x,t) = \Sigma_{k,m}h_k(t)[{}_kE_m^{\phi}\Phi_m(x) +_k E_m^{\psi}\Psi_m(x)]$$

Using Galerkin's method, two discretized equations are obtained for the one dimensional wave equation. Next, a dispersion analysis is performed in order to compare the characteristics of new scheme with that of the traditional FDTD technique in terms of the dispersion errors. A similar approach can be adopted for the 2D and 3D cases to perform the discretization.

Dispersion Analysis

The dispersion relation of the FDTD and the MRTD scheme is calculated from the solution of the eigenvalue problem after transforming the equations in the frequency domain [5]

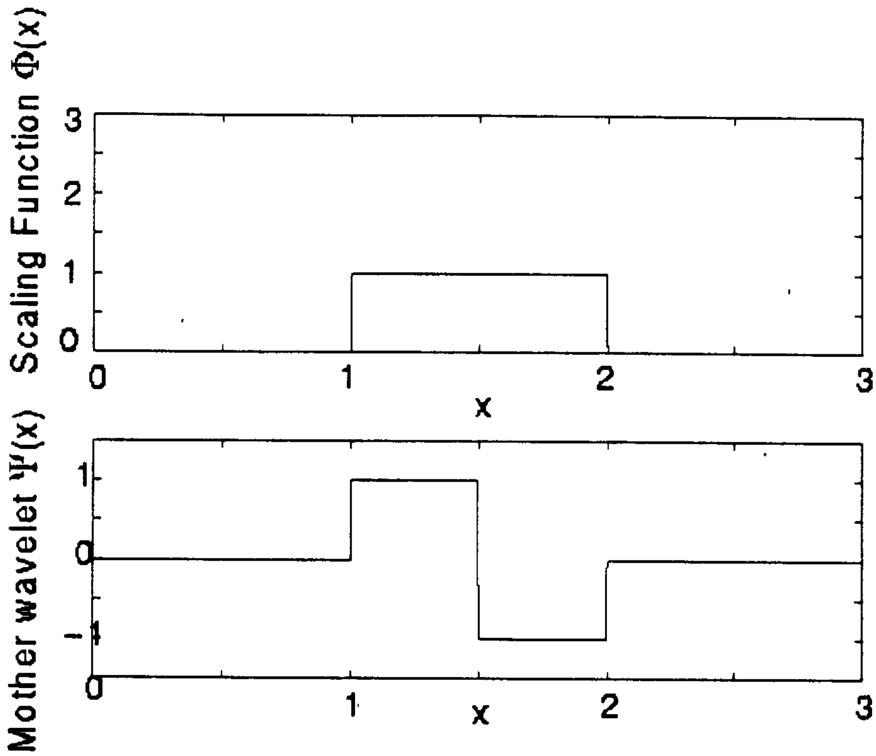

Figure 1: Scaling and Wavelet Functions.

- [6]. For the 1D wave equation discussed above, the FDTD dispersion relation is shown below:

$$\frac{\Delta x}{c\Delta t}sin(\frac{\Omega}{2}) = sin(\frac{\chi}{2})$$

where

$$\Omega = \omega\Delta t$$

and

$$\chi = \Delta x k_x$$

This equation is also derived when using Yee's cell. In the equations above, Δx and Δt are the space and time steps respectively and k_x is the magnitude of the wave vector. The dispersion relation of the first order resolution MRTD mesh (FDTD multigrid) for the 1D wave is given by the following equation:

$$\frac{\Delta x}{c\Delta t}sin(\frac{\Omega}{2}) = 2sin(\frac{\chi}{4})$$

Fig.2 shows plots of the normalized wave vector component χ as a function of the normalized frequency Ω for the ideal case, the FDTD technique and MRTD scheme. From the figure it can be seen that the dispersion curve of the MRTD scheme is much more linear and closer to the ideal than the FDTD scheme.

Similar analysis has been performed for a 2D problem where the wavelets were applied in one direction and only scaling functions in the other direction. As expected, the dispersion relation is the same as that of the FDTD scheme in the direction in which no wavelets are applied while the dispersion curve substantially improves in the direction in which the wavelets are used. Figs. 3 and 4 show plots of χ as a function of Ω for different directions and validate the claims above.

The 2D-MRTD scheme Consider the following 2-D scalar equation obtained from Maxwell's H-curl equation:

$$\epsilon\frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} + \beta H_y \quad (1)$$

This equation can be rewritten in a differential operator form as shown below:

$$L_1(f_1(x,y,t)) + L_2(f_2(x,y,t)) = g \quad (2)$$

where L_1 and L_2 are the operators and f_1(x,y,t) and f_2(x,y,t) represent the electric/magnetic fields. We now expand the fields using a Haar based MRA with scaling functions ϕ and wavelet functions ψ The field expansion can be represented as follows:

$$\begin{aligned} f(x,y,t) &= [A][\phi(x)\phi(y)] + [B][\phi(x)\psi(y)] \\ &+[C][\psi(x)\phi(y)] + [D][\psi(x)\psi(y)] \end{aligned} \quad (3)$$

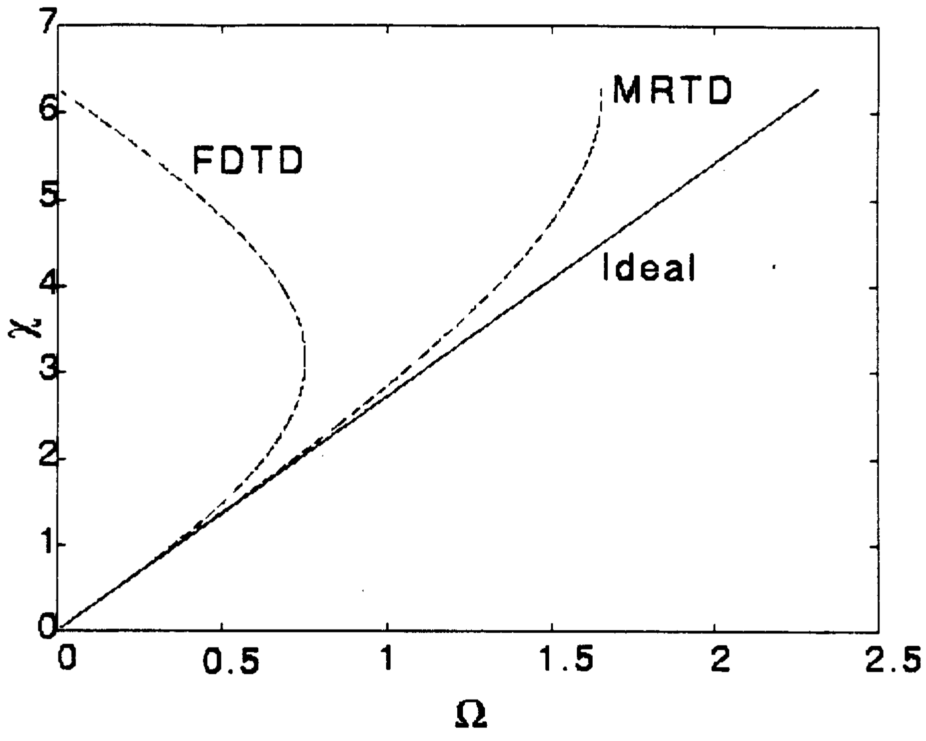

Figure 2: Dispersion Plots for 1D.

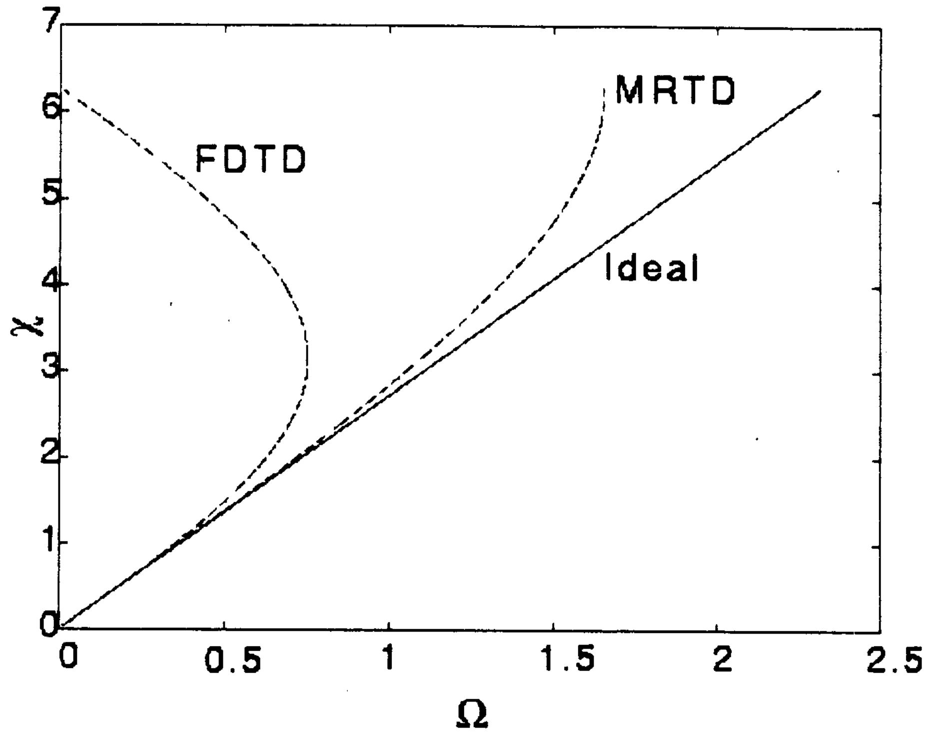

Figure 3: Dispersion Plots for 2D in (1,0,0) direction.

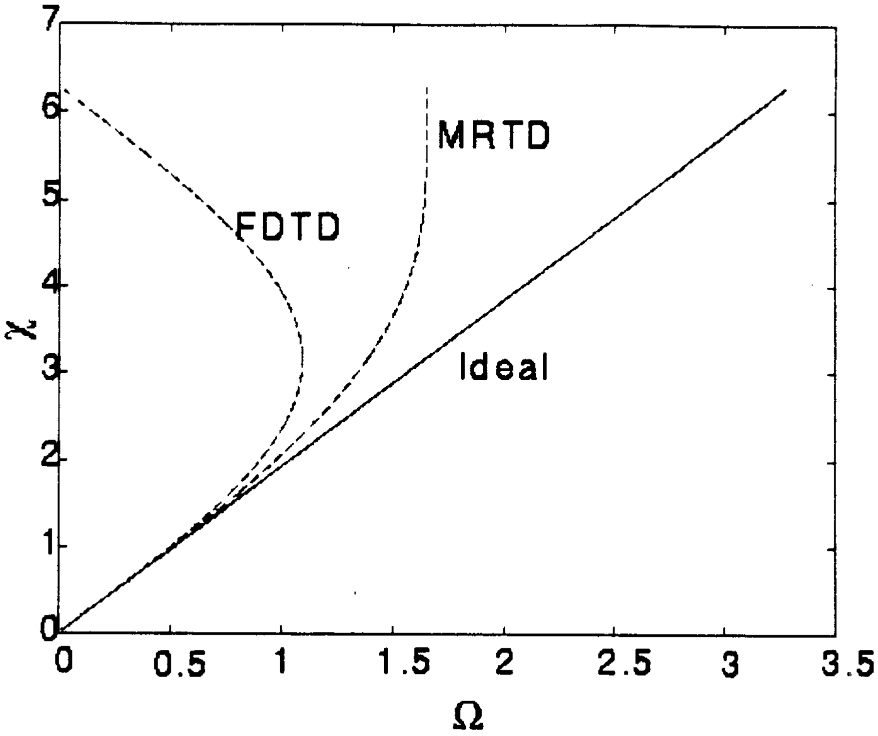

Figure 4: Dispersion Plots for 2D in (1,1,0) direction.

where $[\phi(x)\phi(y)]$, $[\phi(x)\psi(y)]$, $[\psi(x)\phi(y)]$ and $[\psi(x)\psi(y)]$ represent matrices whose elements are the corresponding basis functions in the computation domain of interest and [A], [B], [C], [D] represent the matrices of the unknown coefficients which give information about the fields and their derivatives.

Application of Galerkin's technique leads to 4 schemes which can be represented as follows:

$$< [\phi\phi].L_1(f_1) + L_2(f_2) >=< [\phi\phi].g >: \phi\phi Scheme \tag{4}$$

$$< [\phi\psi].L_1(f_1) + L_2(f_2) >=< [\phi\psi].g >: \phi\psi Scheme \tag{5}$$

$$< [\psi\phi].L_1(f_1) + L_2(f_2) >=< [\psi\phi].g >: \psi\phi Scheme \tag{6}$$

$$< [\psi\psi].L_1(f_1) + L_2(f_2) >=< [\psi\psi].g >: \psi\psi Scheme \tag{7}$$

From this system, we obtain a set of simultaneous discretized equations. For the first resolution level of Haar wavelets, the above four schemes decouple and coupling can be achieved only through the excitation term and the boundaries.

The shielded structures analyzed here are terminated at Perfect Electric Conductors (PEC) and the boundary conditions are obtained by applying the natural boundary condition for the electric field on a PEC as shown below:

$$\begin{aligned} &E_t^{\phi\phi}\phi(x)\phi(y) + E_t^{\phi\psi}\phi(x)\psi(y) + E_t^{\psi\phi}\psi(x)\phi(y) + \\ &+E_t^{\psi\psi}\psi(x)\psi(y) = 0....At PEC. \end{aligned} \tag{8}$$

where $E_t^{\phi\phi}$, $E_t^{\phi\psi}$, $E_t^{\psi\phi}$ and $E_t^{\psi\psi}$ are the scaling and wavelet coefficients of the tangential electric field at the boundary nodes.

The above equations are discretized by the use of Galerkin's method which results in a set of matrix equations of order N = M+1 where M is the order of the considered wavelet resolutions. These equations are solved simultaneously with the discretized Maxwell's equations to numerically apply the correct boundary conditions.

Applications of 2D FDTD Multigrid and Results The 2-D MRTD scheme derived above has been applied to analyze the Electromagnetic fields in a waveguide and a shielded stripline.

(a) Waveguide: An empty waveguide with cross-section of 12.7 x 25.4 mm is chosen. A coarse 5 x 8 mesh is used to discretize this mesh and 2D MRTD technique was applied to analyze the fields in this geometry. Fig. 5 shows the amplitudes of the wavelet and scaling coefficients of the electric field obtained by using MRTD technique. From this figure it can be seen that only the $\phi\phi$ and $\phi\psi$ coefficients make a significant contribution to the field and that the contribution of $\psi\phi$ and $\psi\psi$ is negligible. From the computed coefficients, the total field is reconstructed using an appropriate combination of the scaling and significant wavelet coefficients. For the waveguide chosen here, elimination of the wavelet coefficients that have no significant contribution leads to 480 unknowns. The reconstructed field obtained by this mesh has the same accuracy as that of a 10 x 16 FDTD mesh with 960 unknowns which is in agreement with the theory of MRA. Fig. 6 shows the results of this comparison and demonstrates that the use of multigrid scheme provides a 50% economy in memory.

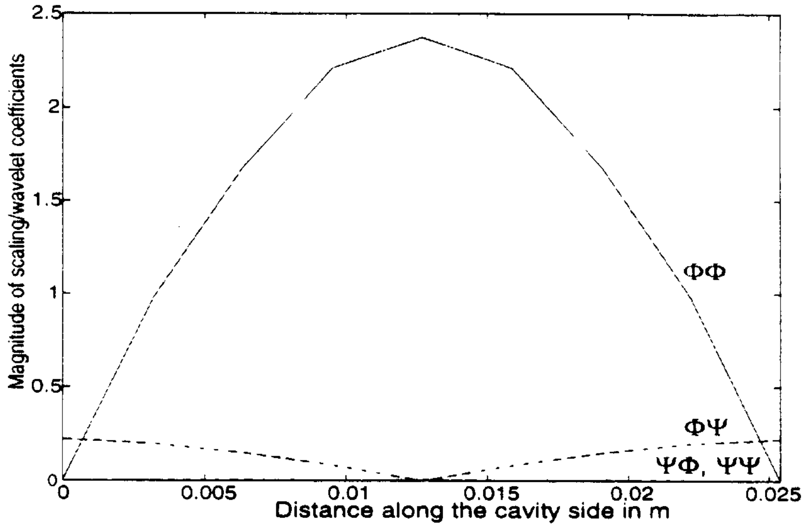

Figure 5: Amplitudes of Scaling and Wavelet Coefficients in a Waveguide.

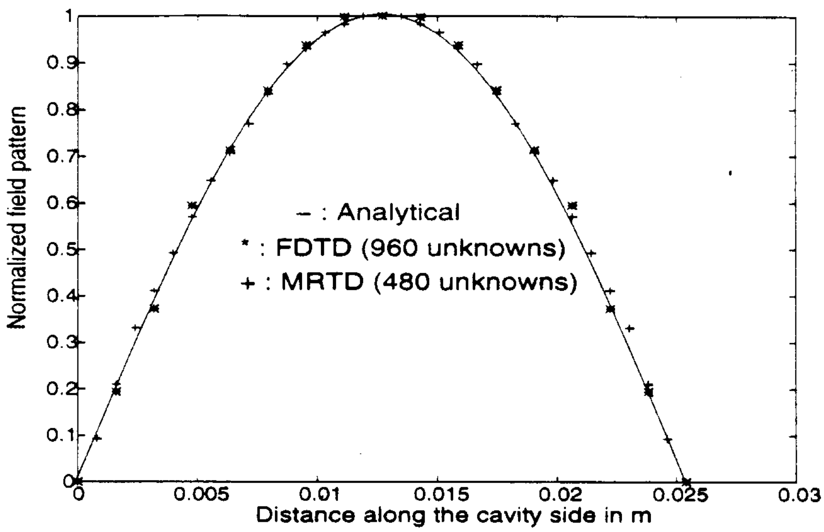

Figure 6: Comparison of MRTD , FDTD and Analytical Fields in a Waveguide.

(b) Shielded Stripline : Next, a stripline of width 1.27mm is considered. It is enclosed in a cavity of area 12.7 x 12.7 mm so that the side walls are sufficiently far away to not affect the propagation. The strip is placed 12.7mm from the ground. A 40 x 40 mesh is used to analyze the fields in this geometry with the 2D MRTD technique. Fig. 7 shows the derived scaling and wavelet coefficients of the fields just below the strip. From the figure, it can be seen that among the wavelet coefficients, only ψ_0 makes a significant contribution close to the vicinity of the strip where the field variation is rather abrupt. Fig. 8 shows the comparision of the total reconstructed field in the 40 x 40 MRTD mesh with that of a 40 x 40 and 80 x 80 FDTD mesh. From the figure it is clear that the field computed by 40 x 40 MRTD mesh using only the significant wavelet coefficients follows the results of the finer 80x80 mesh very closely, demonstrating once again the significant economy in memory as illustrated in Table 1. Fig. 9 shows the Normal Electric field plot of the strip and the variable mesh resulting from MRTD.

Table 1: Comparison of the memory requirements in FDTD and MRTD techniques

Technique	Unknown Coeff.
40x40 FDTD	9600
40x40 MRTD	11328
80x80 FDTD	38400

Conclusion : For the first time in literature, a mathematically correct approach for a FDTD multigrid has been given. Since FDTD is based on the expansion of the unknown fields in pulse functions, the principles of multiresoltion analysis allows for a consistent additional field expansion in terms of Haar wavelets. Due to the additional wavelets, the resulting MRTD scheme exhibits dispersion characteristics with much less dispersion error than the traditional FDTD scheme. The Haar wavelet based 2D MRTD scheme that has been developed here has been applied to analyse the fields in a waveguide and a shielded stripline. The wavelet coefficients obtained are significant only in regions of rapid field variations. Thus the FDTD multigrid capability using MRTD technique has demonstrated significant economy in memory.

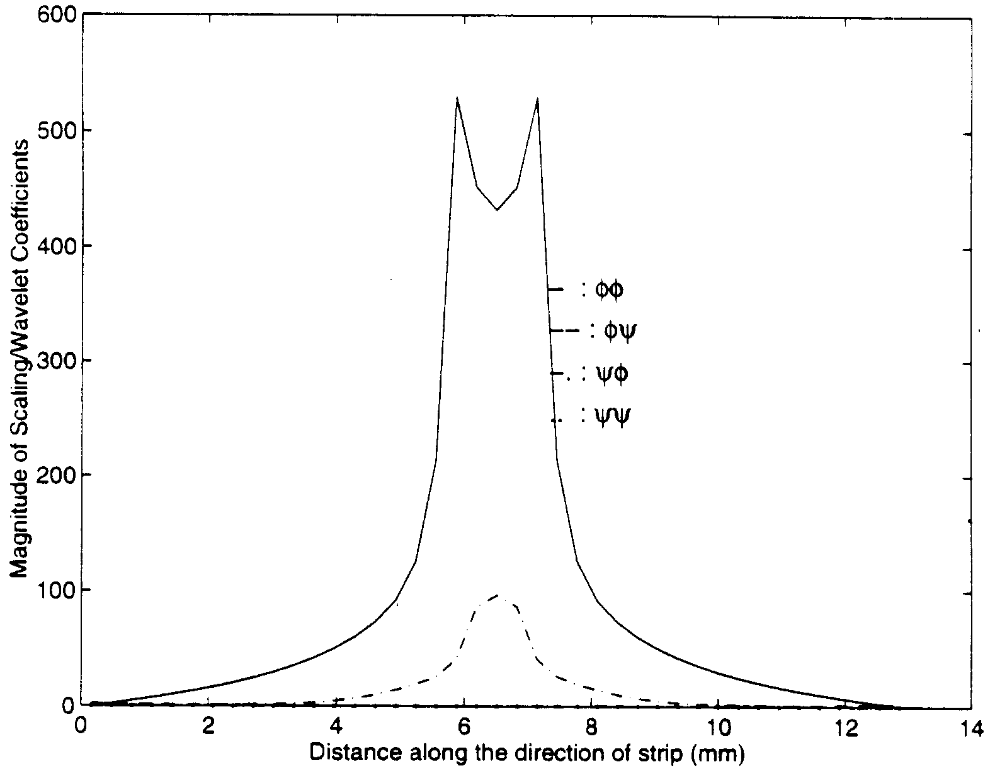

Figure 7: Amplitudes of Scaling and Wavelet Coefficients of a Shielded Stripline.

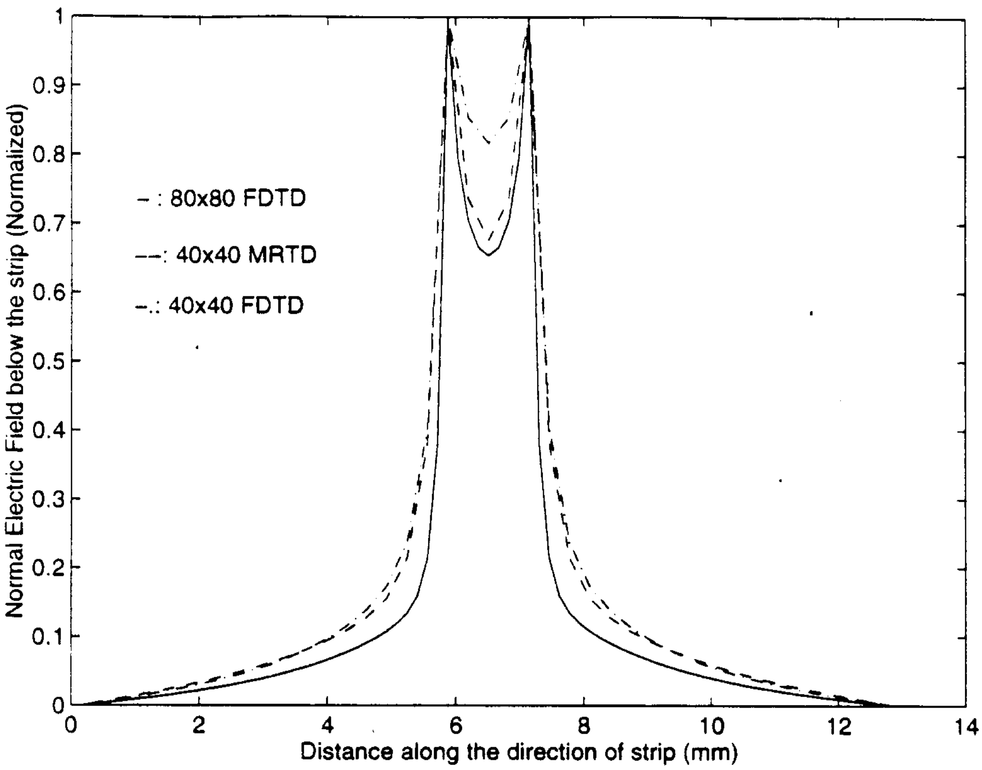

Figure 8: Comparison of Normal Electric Field under a stripline using MRTD and FDTD techniques.

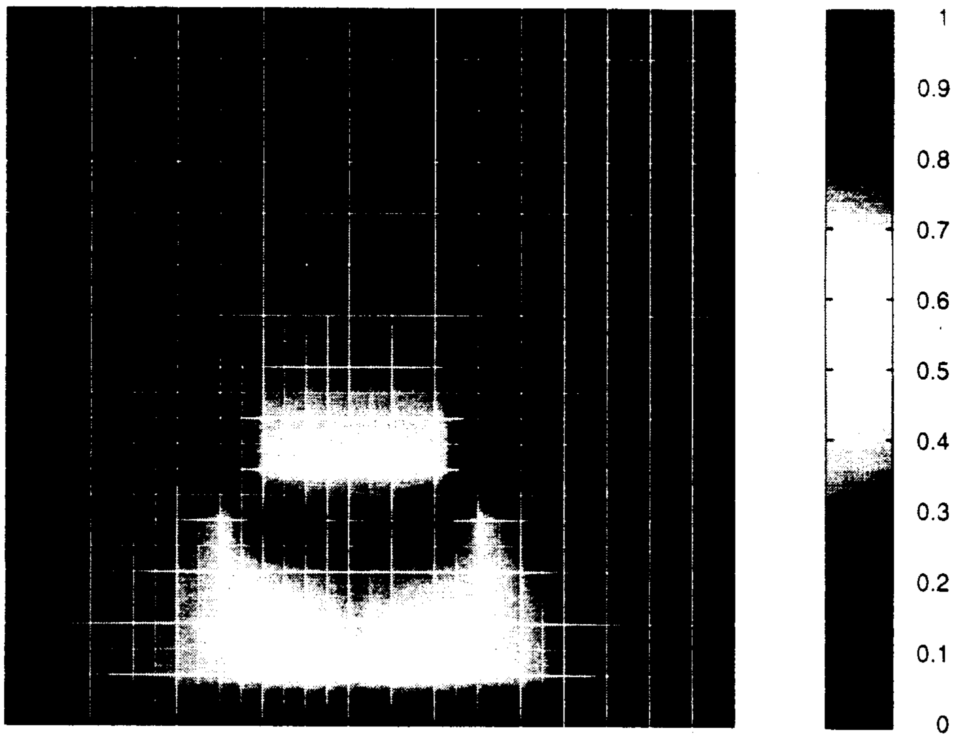

Figure 9: FDTD Multigrid and Field Plot of the Stripline.

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APPENDIX B

SPACE- AND TIME- ADAPTIVE GRIDDING USING MRTD TECHNIQUE

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Abstract- The MRTD scheme is applied to the analysis of waveguide problems. Specifically, the field pattern and the S-parameters of a dielectric-loaded parallel-plate waveguide are calculated. The use of wavelets enables the implementation of a space- and time-adaptive gridding technique. The results are compared to those obtained by use of the conventional FDTD scheme to indicate considerable savings in memory and computational time.

I Introduction

Recently a new technique has been successfully applied [1-4] to a variety of microwave problems and has demonstrated unparalleled properties. This technique is derived by the use of multiresolution analysis for the discretization of the time-domain Maxwell's equations. The multiresolution time domain technique (MRTD) based on Battle Lemarie functions has been applied to linear as well as nonlinear propagation problems. The PML absorbing boundary condition has been generalized in order to analyze open planar structures. MRTD has demonstrated savings in time and memory of two orders of magnitude. In addition, the most important advantage of this new technique is its capability to provide space and time adaptive gridding without the problems that the conventional FDTD is encountering. This is due to the use of two separate sets of basis functions, the scaling and wavelets and the capability to threshold the field coefficients due to the excellent conditioning of the formulated mathematical problem.

In this paper, a space/time adaptive gridding algorithm based on the MRTD scheme is proposed and applied to the waveguide problems. As an example, the propagation of a Gabor pulse in a partially-filled parallel-plate waveguide is simulated and the S-parameters are evaluated. Wavelets are placed only at locations where the EM fields have significant values, creating a space- and time- adaptive dense mesh in regions of strong field variations, while maintaining a much coarser mesh elsewhere.

II The 2D-MRTD scheme

For simplicity the 2D-MRTD scheme for the TM_z modes will be used herein. To derive the 2D-MRTD scheme, the field components are represented by a series of cubic spline Battle-Lemarie [5] scaling and wavelet functions to the longitudinal direction in space and pulse functions in time. After inserting the field expansions in Maxwell's equations, we sample them using pulse functions in time and scaling/wavelet functions in space domain.

As an example, sampling $\partial D_x/\partial t, = -\partial H_y/\partial z$ in space and time, the following difference equation is obtained

$$\frac{1}{\Delta t}({}_{k+1}D^{\phi x}_{l+1/2,m} - {}_{k}D^{\phi x}_{l+1/2,m}) =$$

$$-\frac{1}{\Delta y}\Big(\sum_{i=m-m_2}^{m+m_1} a(i)_{k+1/2}\, H^{\phi y}_{l+1/2,i+1/2}$$

$$+\sum_{i=m-m_4}^{m+m_3} b(i)_{k+1/2}\, H^{\psi y}_{l+1/2,i+1/2}\Big) \quad , \qquad (1)$$

$$\frac{1}{\Delta t}\left({}_{k+1}D^{\psi x}_{l+1/2,m} - {}_{k}D^{\psi x}_{l+1/2,m}\right) =$$

$$-\frac{1}{\Delta y}\Big(\sum_{i=m-m_4}^{m+m_3} b(i)_{k+1/2}\, H^{\phi y}_{l+1/2,i+1/2}$$

$$+\sum_{i=m-m_6}^{m+m_5} c(i)_{k+1/2}\, H^{\psi y}_{l+1/2,i+1/2}\Big) \quad , \qquad (2)$$

where ${}_kD^{\xi x}_{l,m}$ and ${}_kH^{\xi y}_{l,m}$ with $\xi=\phi$ (scaling),ψ (wavelets) are the coefficients for the electric and magnetic field expansions. The indices l, m and k are the discrete space and time indices, which are related to the space and time coordinates via $x = l\Delta x$,$z = m\Delta z$ and $t = k\Delta t$, where Δx,Δz are the space discretization intervals in x- and z-direction and Δt is the time discretization interval. The coefficients $a(i), b(i), c(i)$ are derived and given in [2]. For an accuracy of 0.1% the values $m_1 = m_5 = 8$,$m_2 = m_3 = m_4 = m_6 = 9$ have been used.

For open structures, the perfectly matched layer (PML) technique can be applied by assuming that the conductivity is given in terms of scaling and wavelet functions instead of pulse functions with respect to space [4]. The spatial distribution of the conductivity for the absorbing layers is modelled by assuming that the amplitudes of the scaling functions have a parabolic distribution. The MRTD mesh is terminated by a perfect electric conductor (PEC) at the end of the PML region. Usually, 8-16 cells of PML medium with σ^E_{max}=0.4S/m provide reflection coefficients smaller than -90 dB.

In order to use a pulse excitation at $z = m\Delta z$ with respect to space and to obtain an excitation identical to an FDTD excitation, we decompose the pulse in terms of scaling and wavelet functions

$$ {}_kE^{pulse}_m \approx E_F(0, k\Delta t)$$

$$\Big(\sum_{i=-4}^{+4} c_\phi(i)\phi_{m+i} + \sum_{i=-4}^{+4} c_\psi(i)\psi_{m+i}\Big) \qquad (3)$$

where the coefficients $c_\phi(i), c_\psi(i)$ are given in Table 1 for $i \geq 0$. For $i < 0$ it is $c_\phi(-i) = c_\phi(i)$ and $c_\psi(i) = c_\psi(1-i)$. $E_F(0, k\Delta t)$ is the time dependence of the excitation. For $|i| \leq 4$, the above excitation components are superimposed to the field values obtained by the MRTD algorithm. For example, the total $E^\phi_{k,m+i}$ will be given by

$$E^\phi_{k,m+i}\Big|_{total} = E_F(0, k\Delta t)\, c_\phi(i) + E^\phi_{k,m+i}\,(1 - c_\phi(i))$$

Due to the nature of the Battle-Lemarie expansion functions, the total field is a summation of the contributions from the non-localized scaling and wavelet functions. For example, the total electric field $E_x(x_o, z_o, t_o)$ with $(k - 1/2)\,\Delta t < t_o < (k + 1/2)\,\Delta t$ is calculated in the same way with [2, 3] by

$$E_x(x_o, z_o, t_o) = \sum_{l',m'=-l_1}^{l_1} {}_kE^{\phi x}_{l'+1/2,m'}\, \phi_{l'+1/2}(x_o)\, \phi_{m'}(z_o)$$

$$+\sum_i \sum_{l',m'=-l_{2,i}}^{l_{2,i}} {}_kE^{\psi_0 x}_{l'+1/2,m'}\, \phi_{l'+1/2}(x_o)\, \psi_{0,m'}(z_o)$$

where $\phi_m(x) = \phi(\frac{x}{\Delta x} - m)$ and $\psi_{i,m}(x) = \psi_i(\frac{x}{\Delta x} - m)$ represent the Battle-Lemarie scaling and i-resolution wavelet function respectively. For an accuracy of 0.1% the values $l_1 = l_{2,i} = 4$ have been used.

There are many different ways to take advantage of the capability of the MRTD technique to provide space and time adaptive gridding. In DSP, thresholding of the wavelet coefficients over a specific time- and space- window (5-10 points) contribute significant memory economy, but increase the implementation complexity and the execution time. The simplest way is to threshold the wavelet components to a fraction (usually $\leq$ 0.1%) of the scaling function at the same cell for each time-step. All components below this threshold are eliminated from the subsequent calculations. This is the simplest thresholding algorithm. It doesn't add any significant overhead in execution time, but it offers only a moderate (pessimistic) economy in memory (factor close to 2). Also, this algorithm allows for the dynamic memory allocation in its programming implementation.

III Applications of 2D-MRTD

The 2D-MRTD scheme is applied to the analysis of the partially-loaded parallel-plate waveguide of (Fig.1) for the frequency range 0-30GHz. For the analysis based on Yee's FDTD scheme, a 16×800 mesh is used resulting in a total number of 14400 grid points. When the structure is analyzed with the 2D-MRTD scheme, a mesh 2×100 (200 grid points) is chosen ($dx = 0.24\lambda_o$, $dz = 0.4\lambda_o$ for $f = 30GHz$). This size is based on the number of the scaling functions, since the wavelets are used only when and where necessary. The time discretization interval is selected to be identical for both schemes and equal to the 1/10 of the 2D-MRTD maximum Δt. For the analysis we use 8,000 time-steps. The waveguide is excited with a Gabor function 0-30GHz along a vertical line for the FDTD simulation and for a rectangular region for the MRTD simulations. In all cases, the front and back open planes are terminated with a PML region of 16 cells and σ^E_{max}=0.4S/m. The longitudinal distance between the excitation and the dielectric interface is chosen such that no reflections would appear before the Gabor function is complete.

The capability of the MRTD technique to provide space and time adaptive gridding is verified by thresholding the wavelet components to the 0.1% of the value of the scaling function at the same cell for each time-step. It has been observed that the accuracy by using only a small number of wavelets is equal to what would be achieved if wavelets were used everywhere. Though this number is varying in time, its maximum value is 22 out of a total of 100 to the z-direction (economy in memory by a factor of 28-30). In addition, execution time is reduced by a factor 4-5. For larger thresholds, the ringing effect due to the elimination of the wavelets deteriorates the performance of the algorithm. For example, using a threshold of 1% (6 out of a 100 wavelets to the z-direction) increases the error by a factor of 2.5.

The normal electric field is probed at a distance 10 cells away from the source and is plotted in (Fig.2) in time-domain. Comparable accuracy can be observed for the FDTD and the MRTD meshes. In addition, the reflection coefficient S_{11} is calculated by separating the incident and the reflected part of the probed field and taking the Fourier transform of their ratio (Fig.3). The results for 5 GHz (TEM propagation) are validated by comparison to the theoretical value obtained applying ideal transmission line theory [6] and are plotted at Table 2. The time- and space-adaptive character of the gridding is exploited in (Figs.4,5) which show that the wavelets follow the propagating pulses before and after the incidence to the dielectric interfaces and have negligible values elsewhere. The location and the number of the wavelet coefficients with significant values are different for each time-step, something that creates a dense mesh in regions of strong field variations, while maintaining a much coarser mesh for the other cells.

IV Conclusion

A space- and time- adaptive gridding algorithm based on a multiresolution time-domain scheme in two dimensions has been proposed and has been applied to the numerical analysis of a waveguide problem. The field pattern and the reflection coefficient have been calculated and verified by comparison to reference data. In comparison to Yee's conventional FDTD scheme, the proposed scheme offers memory savings by a factor of 5-6 per dimension maintaining a similar accuracy. The above algorithm can be effectively extended to three-dimension problems.

V Acknowledgments

This work has been funded by NSF.

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Table 1: Excitation Decomposition Coeffs

i	0	1	2	3	4
$c_\phi(i)$	0.915	0.038	0.010	-0.009	0.005
$c_\psi(i)$	-0.103	-0.103	0.121	-0.030	0.015

Table 2: S_{11} calculated by 2D-MRTD

	S_{11} (Ω)	Relative error
Analyt. Value [6]	0.4298	0.0%
16x800 FDTD	0.4283	-0.3%
2x100 MRTD	0.4360	+1.4%

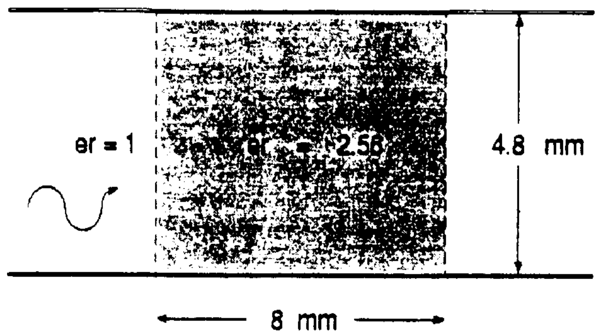

Figure 1: Dielectric-loaded Waveguide.

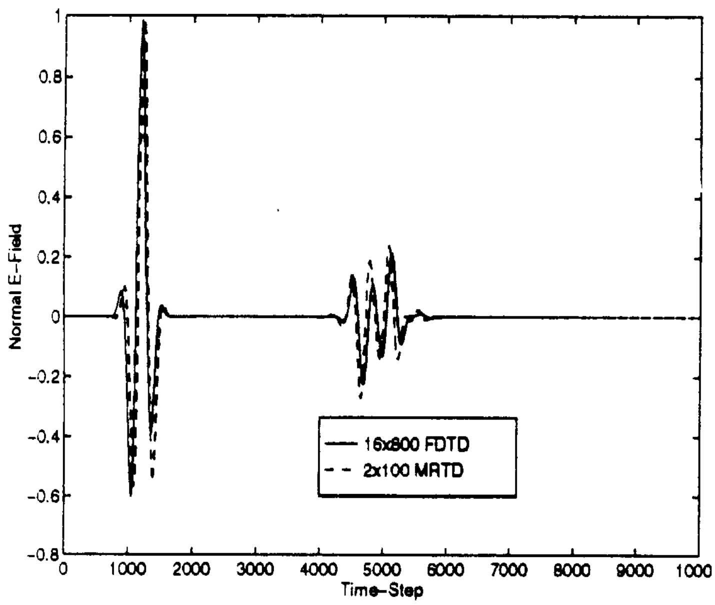

Figure 2: Normal E-field (Time-Domain).

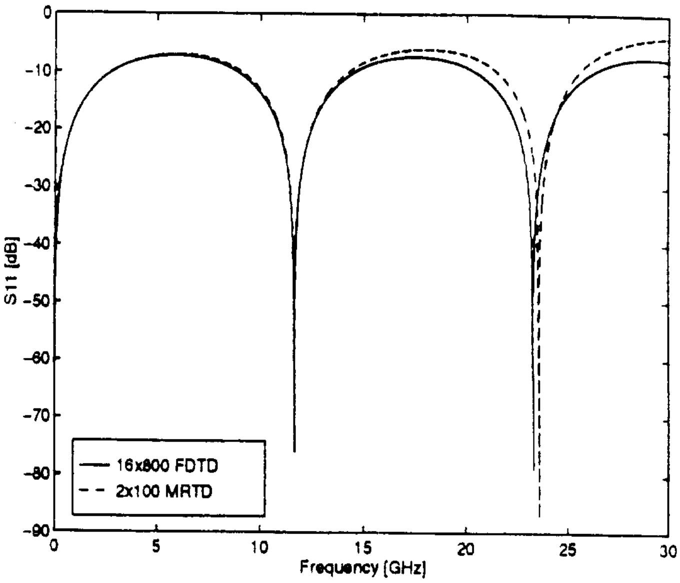

Figure 3: S_{11} values (Frequency-Domain).

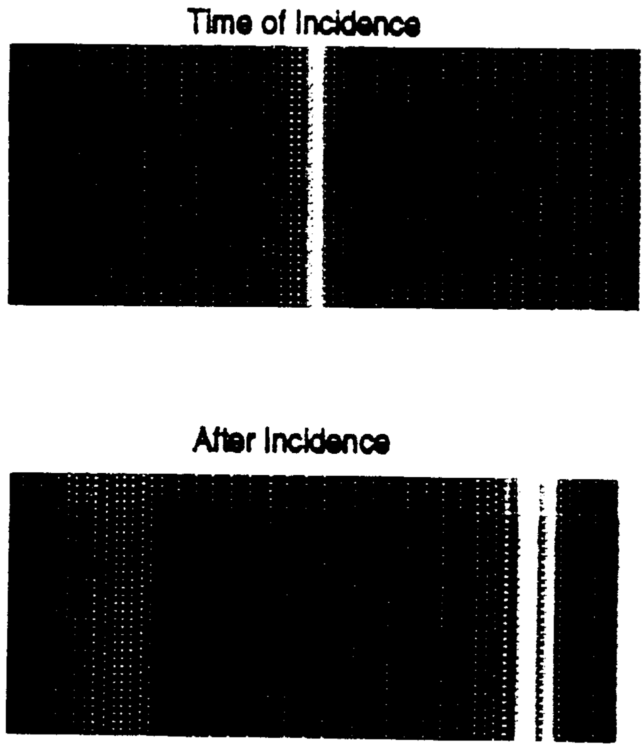

Figure 4: Adaptive Grid Demonstration.

APPENDIX C

Space/Time Adaptive Meshing and Multiresolution Time Domain Method (MRTD)

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I Introduction

Recently the principles of the Multiresolution Analysis have been successfully applied [1, 2] to the time-domain numerical techniques used for the analysis of a variety of microwave problems. New techniques have been derived by the use of scaling and wavelet functions for the discretization of the time-domain Maxwell's equations. The multiresolution time domain technique (MRTD) based on Battle Lemarie functions has been used for the simulations of planar circuits and resonating structures. The conventional FDTD absorbers (e.g. PML) have been generalized in order to analyze open planar structures. MRTD has demonstrated unparalleled savings in execution time and memory requirements (2 orders of magnitude for 3D problems). In addition to time and memory, MRTD technique can provide space- and time- adaptive meshing without the problems that the conventional FDTD variable grids are encountering (e.g. reflections between dense-coarse regions). This unique feature stems from the use of two separate sets of basis functions, the scaling and wavelets. Due to the excellent conditioning of the formulated mathematical problem, MRTD offers the capability to threshold the wavelet field coefficients. This advantage of the MRTD Technique is demonstrated herein by performing a space-/time-adaptive meshing.

In this paper, a space-/time- adaptive meshing algorithm based on the MRTD scheme is proposed and validated for a specific waveguide problem. Wavelets up to the second resolution are placed only at locations where the EM fields have significant values. These locations are changing with the time as the pulse is propagating inside the waveguide and with the space as the pulse is approaching regions of discontinuities. The proposed algorithm offers the opportunity of a space-/time- adaptive mesh with variable resolution of the field representation. In this way, significant memory and execution time savings can be achieved in comparison to the conventional variable-mesh FDTD algorithms.

II MRTD Formulation

Without loss of generality, the 2D-MRTD scheme for the TM_z modes will be described herein. To derive the scheme equations, the field components are represented by a series of cubic spline Battle-Lemarie scaling and 1-order wavelet functions along the z-direction, while pulses are used for the time representation. Wavelets of higher-order can be included in a similar way. After inserting these series expansions in Maxwell's equations and sampling them with pulse functions in time and scaling/wavelet functions in space domain, we derive the following equations for the electric field:

$$\frac{1}{\Delta t}({}_{k+1}D^{\phi x}_{l+1/2,m} - {}_{k}D^{\phi x}_{l+1/2,m}) = -\frac{1}{\Delta z}(\sum_{i=m-m_2}^{m+m_1} a(i)_{k+1/2}\, H^{\phi y}_{l+1/2,i+1/2} + \sum_{i=m-m_4}^{m+m_3} b(i)_{k+1/2}\, H^{\psi y}_{l+1/2,i+1/2})$$

$$\frac{1}{\Delta t}({}_{k+1}D^{\psi x}_{l+1/2,m} - {}_{k}D^{\psi x}_{l+1/2,m}) = -\frac{1}{\Delta z}(\sum_{i=m-m_4}^{m+m_3} b(i)_{k+1/2}\, H^{\phi y}_{l+1/2,i+1/2} + \sum_{i=m-m_6}^{m+m_5} c(i)_{k+1/2}\, H^{\psi y}_{l+1/2,i+1/2})$$

$$\frac{1}{\Delta t}({}_{k+1}D^{\phi z}_{l,m+1/2} - {}_{k}D^{\phi z}_{l,m+1/2}) = \frac{1}{\Delta x}(\sum_{i=l-l_2}^{l+l_1} a(i)_{k+1/2}\, H^{\phi y}_{i+1/2,m+1/2}) \quad ,$$

$$\frac{1}{\Delta t}({}_{k+1}D^{\psi z}_{l,m+1/2} - {}_{k}D^{\psi z}_{l,m+1/2}) = \frac{1}{\Delta x}(\sum_{i=l-l_4}^{l+l_3} c(i)_{k+1/2}\, H^{\psi y}_{i+1/2,l+1/2}) \quad ,$$

where ${}_{k}D^{\xi x}_{l,m}$, ${}_{k}E^{\xi x}_{l,m}$ and ${}_{k}H^{\xi y}_{l,m}$ with $\xi=\phi$ (scaling),ψ (wavelets) are the coefficients for the electric flux, electric and magnetic field expansions. The indices l, m and k are the discrete space and time indices, which are related to the space and time coordinates via $x = l\Delta x$,$z = m\Delta z$ and $t = k\Delta t$, where Δx,Δz are the space discretization intervals in x- and z-direction and Δt is the time discretization interval. The coefficients $a(i), b(i), c(i)$ are derived and given in [1]. For an accuracy of 0.1% the values $m_1 = m_5 = 8$, $m_2 = m_3 = m_4 = m_6 = 9$ have been used. The indices l_i have to take similar values to achieve tha same accuracy in the summations.

The use of non-localized basis functions in the 2D-MRTD scheme causes significant effects. Localized boundary conditions are impossible to be implemented, so the perfect electric boundary conditions are modelled by use of the image principle in a generic way. The implementation of the image theory is performed automatically for any number of PEC, PMC boundaries. The material discontinuities are represented in terms of scaling and wavelet functions resulting into a linear matrix equation as explained in [1, 3] where this technique was used in the modeling of anisotropic dielectric media. In addition, the total value of a field component at a specific point of the mesh is a summation of the contributions from the neighbooring non-localized scaling and wavelet functions. The field values at the neighbooring cells can be combined appropriately by adjusting the scaling and wavelet function values and by applying the image principle.

The demand for the simulation of open structures led to the generalization of the perfectly matched layer (PML) technique [4], so as it can be used in the MRTD simulations. The conductivity is expanded in terms of scaling functions instead of pulse functions with respect to space. The amplitudes of the expansion scaling functions follow the PML spatial conductivity distribution. In our simulations, the parabolic distribution was used, though the realization of other distributions (linear, cubic, ...) is straightforward. For example, if we assume that the PML absorbing material $(\epsilon, \mu, \sigma^E)$ extends to the z-direction, substituting

$$D^{(i)x,z}(x,z,t) = \tilde{D}^{(i)x,z}(x,z,t)e^{-\sigma^E_{(z)}t/\epsilon} \tag{1}$$

and

$$H^{(i)y}(x,z,t) = \tilde{H}^{(i)y}(x,z,t)e^{-\sigma^H_{(z)}t/\mu} \tag{2}$$

for i=ϕ, ψ, leads to the following equation:

$$\frac{\partial \tilde{D}^x}{\partial t} = -\frac{\partial \tilde{H}^y}{\partial y} \tag{3}$$

Following a procedure similar to the one used for the derivation of the non-PML region equations, we get for D_x components

$$\begin{aligned} {}_{k+1}D^{\phi x}_{l+1/2,m} &= e^{-\sigma^E_{(m\Delta z)}\Delta t/\epsilon}\,{}_kD^{\phi x}_{l+1/2,m} \\ &- \frac{\Delta t}{\Delta z}e^{-\sigma^E_{(m\Delta z)}0.5\Delta t/\epsilon}\Big(\sum_{i=m-m_2}^{m+m_1} a(i)_{k+1/2}\,H^{\phi y}_{l+1/2,i+1/2} + \sum_{i=m-m_4}^{m+m_3} b(i)_{k+1/2}\,H^{\psi y}_{l+1/2,i+1/2}\Big) \end{aligned},$$

$$\begin{aligned} {}_{k+1}D^{\psi x}_{l+1/2,m} &= e^{-\sigma^E_{(m\Delta z)}\Delta t/\epsilon}\,{}_kD^{\psi x}_{l+1/2,m} \\ &- \frac{\Delta t}{\Delta z}e^{-\sigma^E_{(m\Delta z)}0.5\Delta t/\epsilon}\Big(\sum_{i=m-m_4}^{m+m_3} b(i)_{k+1/2}\,H^{\phi y}_{l+1/2,i+1/2} + \sum_{i=m-m_6}^{m+m_5} c(i)_{k+1/2}\,H^{\psi y}_{l+1/2,i+1/2}\Big) \end{aligned},$$

The finite-difference equations for $D^{(\phi,\psi)z}$ and $H^{(\phi,\psi)y}$ are similar. For all simulations, a parabolic distribution of the conductivity σ is used in the PML region (N cells):

$$\sigma^{E,H}_{(m\Delta z)} = \sigma^{E,H}_{max}\left(\frac{m}{N}\right)^2 \qquad \text{for m=0,1,..,N,} \tag{4}$$

with $\sigma^{E,H}_{max}$ the maximum conductivity at the end of the absorbing layer. As in [5], the "magnetic" conductivity σ^H is given by:

$$\frac{\sigma^E_{(m\Delta z)}}{\epsilon} = \frac{\sigma^H_{(m\Delta z)}}{\mu} \qquad \text{for m=0,1,..,N,} \tag{5}$$

and the MRTD mesh is terminated by a perfect electric conductor (PEC) at the end of the PML region. This PEC is modelled by applying the image theory.

III Space/Time Adaptive Meshing

The wavelet components' amplitudes have negligible values away from the discontinuities or at regions where the excitation pulse has not propagated yet. There are numerous ways of taking advantage of the above feature. The simplest one is to threshold the wavelet components to a fraction (usually $\leq 0.1\%$) of the scaling component at the same cell (space adaptivity) for each time-step. All components below this threshold are eliminated from the subsequent calculations for the same time-step (time adaptivity). This procedure offers only a moderate economy in memory (factor close to 2). Also, this algorithm allows for the dynamic memory allocation in its programming implementation, while maintaining a low complexity.

The above space-/time- adaptive meshing scheme is applied to the analysis of the partially-loaded parallel-plate waveguide of (Fig.1) for the frequency range 0-22.5GHz. The waveguide is half-filled with air and half-filled with dielectric with $\epsilon_r = 2.56$. An FDTD 16×640 (10240 cells) mesh and an MRTD 2×80 (160 cells) mesh (160 grid points with $dx = 0.18\lambda_o$, $dz = 0.3\lambda_o$ - close to the Nyquist Limit for $f = 22.5GHz$) are used for the Time-Domain simulations (3,000 time-steps). The 160 grid points of the MRTD mesh express the number of the used scaling functions. The number of the wavelets is varying with time and depends on the predefined threshold. For consistency, the time step for both schemes is chosen to be equal to the 1/8 of the FDTD maximum Δt.

The waveguide is excited with a Gabor function 0-22.5GHz along a vertical line for the FDTD simulation and for a rectangular region of 12 cells to the longitudinal direction (due to the non-localized character of the Battle-Lemarie scaling and wavelet functions) for the MRTD simulations. Other excitations (e.g.Gaussian) can be applied i a straightforward way. For both cases, a PML region of 16 cells and σ^E_{max}=0.4S/m absorbs the waves in the front and back open planes. The capability of the MRTD technique to provide space- and time- adaptive gridding is verified by thresholding the wavelet components to the 0.1% of the value of the scaling function at the same cell for each time-step. The accuracy achieved by using only the wavelets with values above the threshold is equal to what would be if wavelets were used everywhere. Though this number is varying in time , its maximum value is 36 out of a total of 160 to the z-direction (economy in memory by a factor of 52 instead of 32). In addition, execution time is reduced by a factor 4-5. For larger thresholds, the ringing effect due to the elimination of the wavelets deteriorates the performance of the algorithm. For example, using a threshold of 1% (13 out of a 160 wavelets to the z-direction) increases the error by a factor of 2.1.

The results for the Reflection Coefficient for 10 GHz are validated by comparison to the theoretical value $|R| = 0.231$ (=($\sqrt{2.56}$-1.0)/($\sqrt{2.56}$+1.0)). MRTD gives the value 0.2296 and FDTD gives 0.2304 (similar accuracy). The normal electric field is probed at a distance 10 cells away from the source and is plotted in (Fig.2) in time-domain. Similar accuracy can be observed for the FDTD and the MRTD meshes.

Fig.3 demonstrates the space- and time-adaptive character of the meshing algorithm. It is clearly shownw that the wavelets follow the propagating exciation pulse before and after the incidence to the dielectric interface and can be omitted elsewhere. The location and the number of the wavelet coefficients with values above the threshold ("effective wavelets") are different for each time-step, something that creates a mesh with high resolution ("dense") in regions of strong field variations, while maintaining a much lower resolution ("coarse") for the rest cells.

IV Conclusion

A simple space- and time- adaptive meshing algorithm based on an MRTD scheme has been proposed and has been validated for a parallel-plate waveguide problem. The electric field value and the reflection coefficient have been calculated and verified by comparison to reference data. The proposed scheme exhibits memory savings by a factor of 52 in 2D, as well as execution time savings by a factor of 4-5, while maintaining a similar accuracy with Yee's conventional FDTD scheme. In addition, this algorithm doesn't increase the programming complexity and can be effectively extended to 3D problems.

V Acknowledgments

This work has been partially funded by ARO and by NSF.

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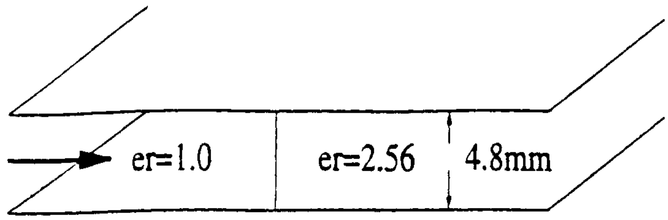

Figure 1: Dielectric-loaded Waveguide.

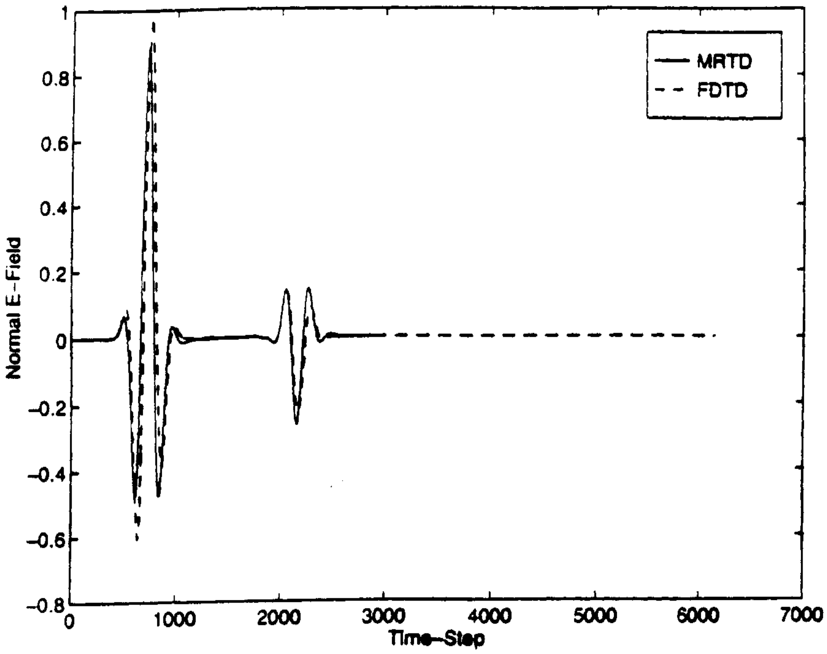

Figure 2: Normal E-field Time Evolution.

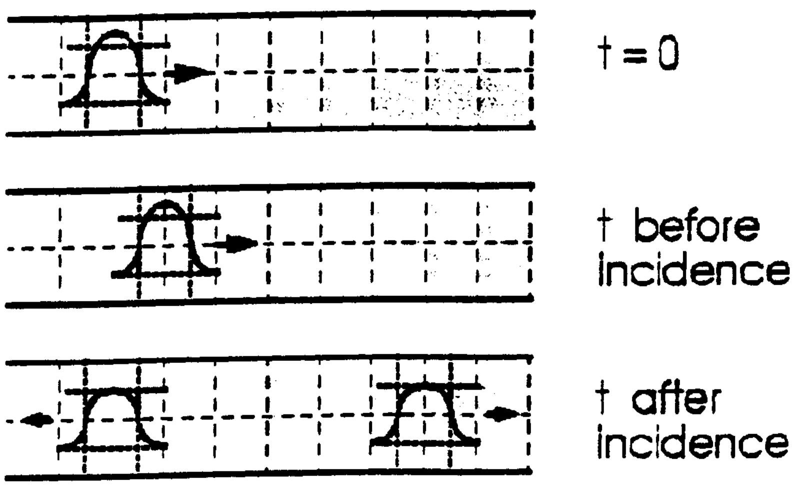

Figure 3: Space-/Time- Adaptive Meshing Demonstration.

APPENDIX D

Revised Version of Paper Previously Submitted to the IEEE AP-S 1997

Time Adaptive Time-Domain Techniques for the Design of Microwave Circuits

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Abstract

The recently developed MRTD schemes are used for the development of a time adaptive time-domain technique for circuit design. The new technique exhibits considerable savings in memory and computational times in comparison to the conventional FDTD scheme.

I Introduction

Significant attention is being devoted now-a-days to the analysis and design of various types of microwave circuits. The finite-difference-time-domain (FDTD) scheme is one of the most powerful numerical techniques used for numerical simulations. However, despite its simplicity and modeling versatility, the FDTD scheme suffers from serious limitations due to the substantial computer resources required to model electromagnetic problems with medium or large computational volumes. In addition, the FDTD scheme cannot provide the accuracy required for computer simulations of time-dependent electromagnetic interactions in electrically long regions or in regions which contain non-linear materials. Such simulations are very important for integrated device modelling, especially in relation to the design of non-linear photonic devices. To alleviate these problems hybrid combinations of FDTD with other numerical techniques and higher order FDTD schemes based on Yee's grid have been proposed. MRTD (MultiResolution Time Domain Method) [1, 2] has shown unparalled properties in comparison to Yee's FDTD. MRTD is not a new methodology. It is a correct and accurate generalization of the conventional discretization approaches. It provides the correct mathematical frame for solving problems in time domain and allows for the development of time/space adaptive grids.

II Introduction to MRTD

It is well known that the method of moments provides a mathematically correct approach for the discretization of integral and partial differential equations. Since it allows for the use of any complete and orthonormal set, the choice of an appropriate expansion set may lead to different time domain schemes. For example, the expansion of the unknown fields using pulse

functions leads to Yee's FDTD scheme. In a MRTD scheme the fields are represented by a two-fold expansion in scaling and wavelet functions with respect to time/space. Scaling functions guarantee a correct modelling of smoothly-varying fields. In regions characterized by strong field variations or field singularities, higher resolution is enhanced by incorporating wavelets in the field expansions. Wavelets are introduced only at specific locations, allowing for a time/space adaptive grid capability.
MRTD schemes based on cubic spline Battle-Lemarie scaling and wavelet functions (Fig.1) have been successfully applied to the simulation of 2D and 3D open and shielded problems [1, 2, 3, 4]. The functions of this family do not have compact support, thus the MRTD schemes have to be truncated with respect to space. Localized boundary conditions (PECs, PMCs etc.) and material properties are modelled by use of the image principle and of matrix equations respectively. However, this disadvantage is offset by the low-pass (scaling) and band-pass (wavelets) characteristics in spectral domain, allowing for an a priori estimate of the number of resolution levels necessary for a correct field modelling. In addition, the evaluation of the moment method integrals during the discretization of Maxwell's PDEs is simplified due to the existence of closed form expressions in spectral domain and simple representations in space domain. Dispersion analysis of this MRTD scheme shows the capability of excellent accuracy with up to 2 points/wavelength (Nyquist Limit). However, specific circuit problems may require the use of functions with compact support. For that reason, Haar basis functions have been utilized and have led to [5]. As an extension to this approach, intervalic wavelets of higher order may be incorporated into the solution of SPICE-type circuits. Results from that new technique will be shown at the Conference.

III Time Adaptive MRTD Scheme

The major advantage of the use of Mutiresolution analysis to time domain is the capability to develop time and space adaptive schemes. This is due to the property of the wavelet expansion functions to interact weakly and allow for a spatial sparsity that may vary with time through a thresholding process. The adaptive character of this technique is extremely important for the accurate modelling of sharp field variations of the type encountered in beam focusing in nonlinear optics, etc. The use of the principles of the multiresolution analysis for adaptive grid computations for PDEs has been suggested by Perrier and Basdevant [6]. To understand the fundamental steps of such an adaptive scheme for Maxwell's hyperbolic system, let's consider Maxwell's equations in 2D (1 for space and 1 for time):

$$\frac{\partial \hat{u}}{\partial t} = A\hat{u} = \begin{bmatrix} 0 & -\epsilon(z)^{-1}\frac{\partial}{\partial z} \\ -\mu(z)^{-1}\frac{\partial}{\partial z} & 0 \end{bmatrix} \hat{u}, \quad \hat{u} = (E(z,t), H(z,t))^T \quad , \tag{1}$$

After manipulation, the above equation can be written as

$$\mathbf{M}\hat{u} = \begin{bmatrix} \epsilon T_h^\dagger D_t & T_h^\dagger D_z \\ Z_h^\dagger D_z & \mu Z_h^\dagger D_t \end{bmatrix} \hat{u} = 0 \tag{2}$$

where Z_h, T_h are half shift operators for space and time coordinates z,t and $Z_h^\dagger, T_h^\dagger$ are their Hermitian conjugates. D_t, D_z are difference operators given by:

$$D_t = \frac{1}{\Delta t}(\sum_{i=-9}^{8} a_{\phi t}(i)T^{-i} + \sum_{i=-9}^{9} a_{\psi t}(i)T^{-i}), \quad D_z = \frac{1}{\Delta z}(\sum_{i=-9}^{8} a_{\phi z}(i)Z^{-i} + \sum_{i=-9}^{9} a_{\psi z}(i)Z^{-i}) \quad . \tag{3}$$

where a_ϕ, a_ψ are the coefficients associated with the scalar and the wavelet functions respectively. At each time step we keep both the wavelet field values that are larger than a given threshold as well as the adjacent values. An adjacent wavelet field value is defined on the basis of the wavelet resolution level(s) incorporated in the solution. Recently, an efficient space/time adaptive meshing prosedure was proposed [7] for Battle-Lemarie expansion functions. In this paper, intervalic wavelets are used for the expansion of the fields (Fig.2). The adaptive mesh will be applied to a variety of circuit problems and results will be discussed during the presentation.

IV Conclusion

A Time Adaptive Time-Domain Technique based on intervalic wavelets has been proposed and applied to various types of circuits problems with lumped and distributed elements. This scheme exhibits significant savings in execution time and memory requirements while maintaining a similar accuracy with conventional circuit simulators.

V Acknowledgments

This work has been partially funded by NSF.

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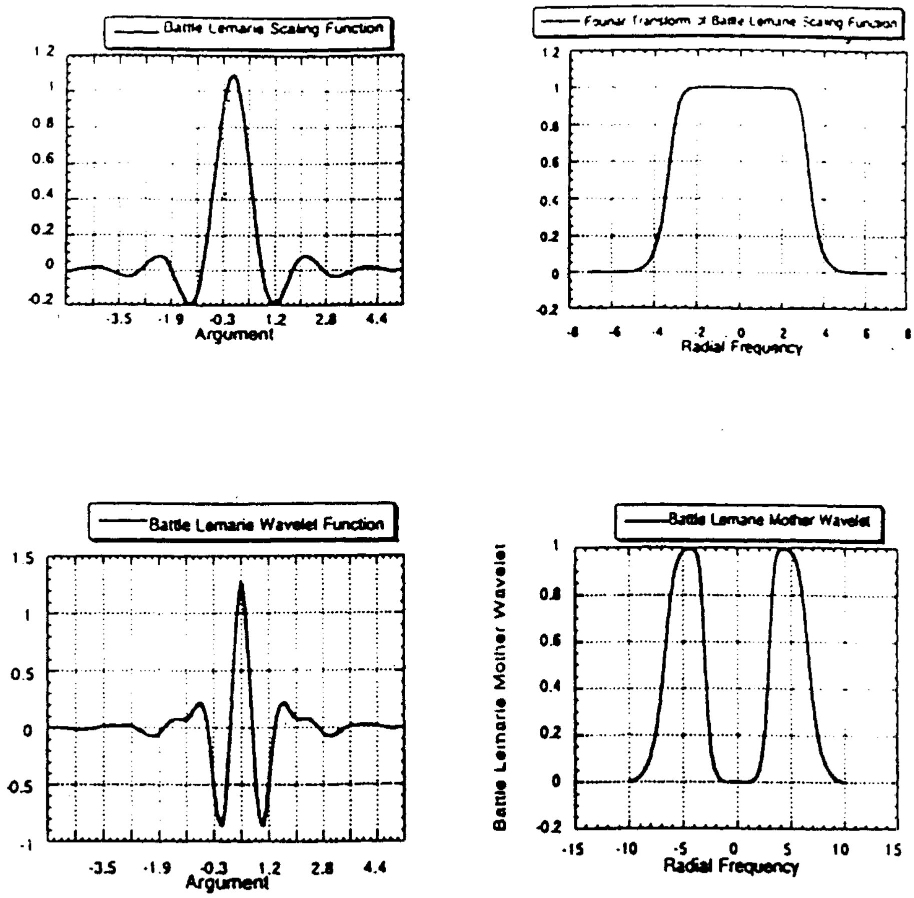

Figure 1: Battle-Lemarie Scaling and Wavelet Functions.

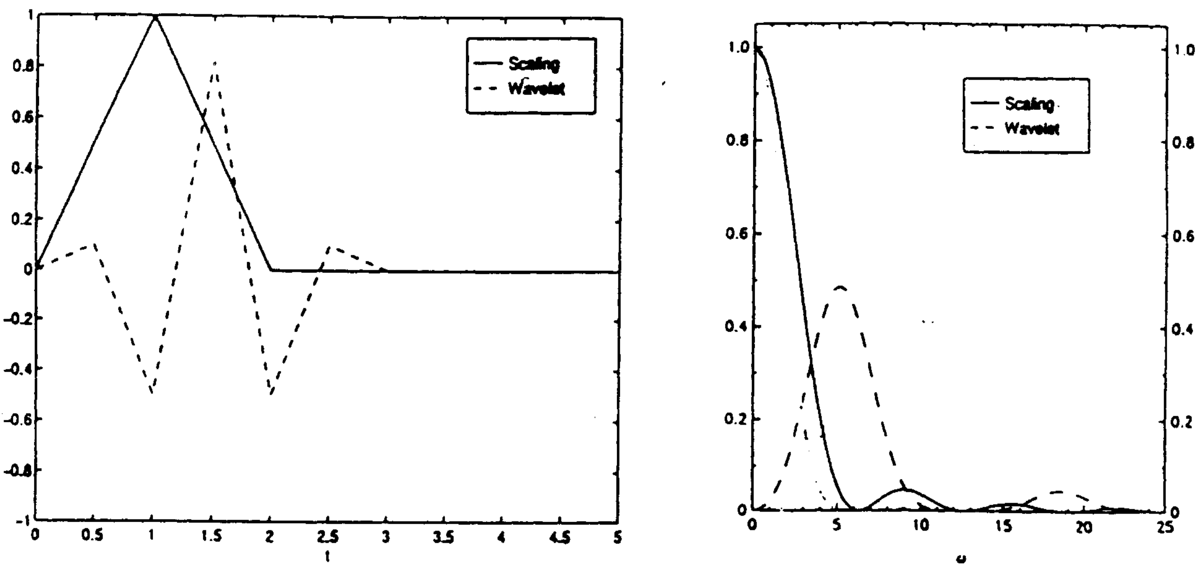

Figure 2: Intervalic Wavelets (Linear - Order 1).

APPENDIX E

Modeling of Membrane Patch Antennas Using MRTD Analysis

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Abstract

The Multiresolution Time-Domain (MRTD) scheme with perfectly matched layer (PML) absorbing boundaries is applied to the analysis of a membrane patch antenna. The results are compared to those obtained by use of the conventional FDTD technique and substantial reductions in memory requirements are observed.

I Introduction

Recently Multiresolution Time–Domain Analysis has been successfully applied to simulate a variety of microwave structures [1]. MRTD has performed complete analysis of both planar circuits [2] and resonating structures [3]. Additionally conventional FDTD absorbers, such as PML [4] have been generalized in order to analyze open planar structures [5]. In all cases, MRTD has demonstrated a high degree of savings in execution time and memory requirements with respect to FDTD.

In this paper the techniques described are applied to the simulation of a membrane patch antenna with a center frequency of 9 GHz. Full 3D MRTD analysis with PML along three coordinate directions is used to simulate the antenna. The MRTD scheme is applied to the calculation of S-parameters for the membrane antenna and is compared to conventional FDTD.

II Application of PML to the 3D MRTD scheme

To derive the 3D MRTD scheme, the field components in Maxwell's E-curl and H-curl equations are represented by a series of cubic spline Battle–Lemarie scaling functions in space and pulse functions in time. These equations are sampled with pulse functions in time– and scaling functions in space–domain, as detailed in [1]. As an example, consider the discretization of:

$$\epsilon\frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}. \tag{1}$$

For a homogeneous medium with the permittivity ϵ, expanding and sampling $\partial E_x/\partial t$, $\partial H_z/\partial y$ and $\partial H_y/\partial z$ with scaling and pulse functions in space and time gives

$$\frac{\varepsilon}{\Delta t}\left({}_{k+1}E^{\phi x}_{l+1/2,m,n} - {}_{k}E^{\phi x}_{l+1/2,m,n}\right) = \frac{1}{\Delta y}\sum_{i=m-9}^{m+8} a(i)\ {}_{k+1/2}H^{\phi z}_{l+1/2,i+1/2,n} - \frac{1}{\Delta z}\sum_{i=n-9}^{n+8} a(i)\ {}_{k+1/2}H^{\phi y}_{l+1/2,m,i+1/2} \tag{2}$$

where ${}_kE^{\phi x}_{l,m,n}$, ${}_kH^{\phi z}_{l,m,n}$ and ${}_kH^{\phi y}_{l,m,n}$ are the coefficients for the electric and magnetic field expansions. The indices l, m, n and k are the discrete space and time indices related to the space and time coordinates via $x = l\Delta x$, $y = m\Delta y$, $z = n\Delta z$ and $t = k\Delta t$, where Δx, Δy, Δz are the space discretization intervals in x–, y– and z–directions and Δt is the time discretization interval. The coefficients $a(i)$ are given in [1].

To derive the perfectly matched layer (PML) technique [4] along one coordinate direction it is assumed that the conductivity is given in terms of scaling functions with respect to space. The spatial distribution of the conductivity for the absorbing layers is simulated by assuming that the amplitudes of the scaling functions have a parabolic distribution [5]. For the PML absorbing material in the y-direction with ϵ, μ and conductivity σ^E, the term $\sigma^E_{(y)} E_x$ must be added to the left side of eq.(1). Then, substituting the following into eq.(1):

$$E_i(x,y,z,t) = \tilde{E}_i(x,y,z,t) e^{-\sigma^E_{(y)} t/\epsilon} \tag{3}$$

$$H_i(x,y,z,t) = \tilde{H}_i(x,y,z,t) e^{-\sigma^H_{(y)} t/\mu}, \tag{4}$$

and assuming that the PML is only along the y-direction leads to the following equation:

$$\begin{aligned} {}_{k+1}E^{\phi x}_{l+1/2,m,n} &= e^{-\sigma^E_{(m\Delta y)}\Delta t/\epsilon} {}_k E^{\phi x}_{l+1/2,m,n} + \frac{\Delta t}{\epsilon} e^{-\sigma^E_{(m\Delta y)}(\Delta t/2)/\epsilon} \Big(\frac{1}{\Delta y} \sum_{i=m-9}^{m+8} a(i)_{k+1/2} H^{\phi z}_{l+1/2,i+1/2,n} + \\ &+ \frac{1}{\Delta z} \sum_{i=m-9}^{m+8} a(i)_{k+1/2} H^{\phi y}_{l+1/2,m,i+1/2} \Big) \end{aligned} \tag{5}$$

For all simulations, a parabolic distribution of the conductivity σ is used in the PML region (N cells):

$$\sigma^{E,H}_{(m\Delta y)} = \sigma^{E,H}_{max} \left(\frac{m}{N}\right)^2 \qquad \text{for m=0,1,..,N,} \tag{6}$$

where $\sigma^{E,H}_{max}$ is the maximum conductivity at the end of the absorbing layer. As in [4], the magnetic conductivity σ^H has to be chosen as:

$$\frac{\sigma^E_{(m\Delta y)}}{\epsilon} = \frac{\sigma^H_{(m\Delta y)}}{\mu} \qquad \text{for m=0,1,..,N,} \tag{7}$$

for a perfect absorption of the outgoing waves. The MRTD mesh is terminated by a perfect electric conductor (PEC) at the end of the PML region, modelled by applying the image theory.

While the above derivation is adequate for a structure which only needs to be terminated with PML along one direction, such as a shielded thru-line, structures such as patch antennas need PML termination in all three coordinate directions. In this case, the derivation discussed above needs to be extended to three dimensions. The procedure is straightforward and results in the following equation:

$$\begin{aligned} {}_{k+1}E^{\phi x}_{l+1/2,m,n} &= e^{-\sigma^E_{(l\Delta x)}\Delta t/\epsilon} e^{-\sigma^E_{(m\Delta y)}\Delta t/\epsilon} e^{-\sigma^E_{(n\Delta z)}\Delta t/\epsilon} {}_k E^{\phi x}_{l+1/2,m,n} + \\ &+ \frac{\Delta t}{\epsilon} e^{-\sigma^E_{(l\Delta x)}(\Delta t/2)/\epsilon} e^{-\sigma^E_{(m\Delta y)}(\Delta t/2)/\epsilon} e^{-\sigma^E_{(n\Delta z)}(\Delta t/2)/\epsilon} \Big(\frac{1}{\Delta y} \sum_{i=m-9}^{m+8} a(i)_{k+1/2} H^{\phi z}_{l+1/2,i+1/2,n} + \\ &+ \frac{1}{\Delta z} \sum_{i=m-9}^{m+8} a(i)_{k+1/2} H^{\phi y}_{l+1/2,m,i+1/2} \Big) \end{aligned} \tag{8}$$

As in eq.(6) a parabolic distribution for the conductivity is applied in the x-, y- and z-directions.

III Applications of the 3D-MRTD scheme

The object of this paper is to apply the 3D MRTD scheme to the analysis of membrane patch antennas. However, in order to test the application of PML to the 3D MRTD scheme, a microwave thru-line is analyzed using MRTD and FDTD. The thru-line has a width of 0.4 mm and length of 10.0 cm and is placed in the center of a cavity with dimensions $1.6mm \times 10.0cm \times 1.6mm$. A Gaussian pulse is used to excite the thru-line with $f_{max} = 50GHz$ [6] and is placed in the middle of the center conductor 9 mm from the PML layer along the y-axis. The FDTD analysis uses $16 \times 100 \times 16$ mesh while the MRTD analysis uses a $8 \times 20 \times 8$ mesh. Additionally six cells of PML are used along the y-direction at either end of the thru-line with a $\sigma_{max}^{Ey} = 3.0$. Therefore the total discretization of the thru line is $16 \times 112 \times 16$ for FDTD and $8 \times 32 \times 8$ for the MRTD scheme, resulting in a factor of 14.0 savings in memory. The time discretization interval for the MRTD scheme is $\Delta t = 3.92 \cdot 10^{-14}s$, while the FDTD scheme has $\Delta t = 6.335 \cdot 10^{-14}s$. In both cases, the simulation is performed for 6000 time steps. A comparison plot of time vs. Ez-field amplitude is shown in Figure 1. Note that the amplitude of the Gaussian has been normalized and the time-steps multiplied by a constant factor in order to compare the two plots more easily. The initial Gaussian pulse has been completely absorbed by the PML layer along the y-direction.

The membrane.patch antenna shown in Figure 2 is simulated using 3D MRTD and FDTD. A full description of the parameters of the antenna can be found in [7]. A PML layer of six cells is used along the $\pm x$, $\pm y$ and $+z$ directions, resulting in an FDTD mesh of $72 \times 112 \times 28$ and a MRTD mesh of $42 \times 62 \times 12$, a factor of 7.22 savings in memory. In the PML layers $\sigma_{max}^{Ex} = \sigma_{max}^{Ey} = \sigma_{max}^{Ez} = 3.0$ for FDTD and MRTD. The time discretization interval used for the MRTD scheme is $\Delta t = 1.6008 \cdot 10^{-13}s$ while the FDTD time discretization interval is $\Delta t = 1.3297 \cdot 10^{-13}s$. In both cases the simulation is performed for 7000 time steps. The antenna feed line is 20 mm long and the Gaussian pulse is sent from a point y=4 mm from the edge of the PML layer in the FDTD and MRTD simulations. Figure 3 shows a plot of Ez field values vs. time for MRTD and FDTD. Measurement of the intial and reflected normalized Gaussian pulses occurred at y = 14 mm from the edge of the PML layer. Figure 4 shows a plot of the calculated S_{11} [7] for the membrane patch antenna. Note that excellent correlation is achieved between FDTD and MRTD results.

IV Conclusion

A membrane patch antenna is successfully simulated using the 3D MRTD scheme with PML absorber along the x-, y- and z-directions. With respect to calculated S_{11} results MRTD shows excellent correlation with FDTD while exhibiting a memory savings with a factor of 7.22.

V Acknowledgements

This work has been partially funded by NSF and ARL.

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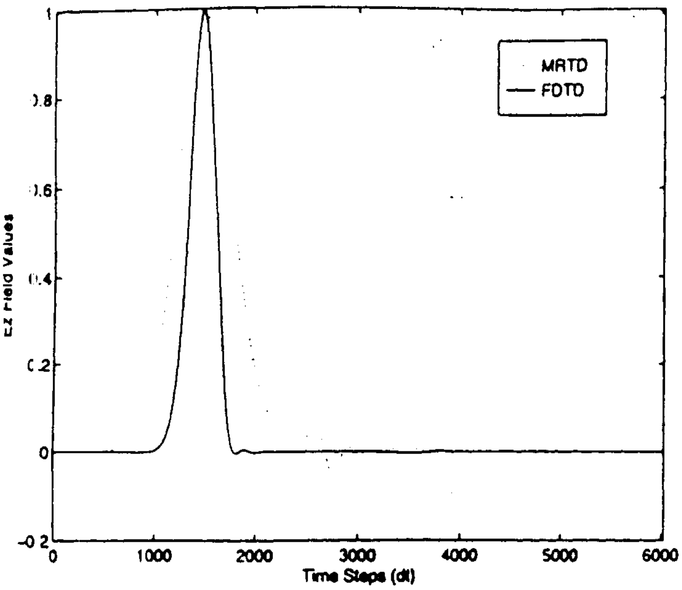

Figure 1: Time–Domain Analysis of a Microstrip Thru–Line.

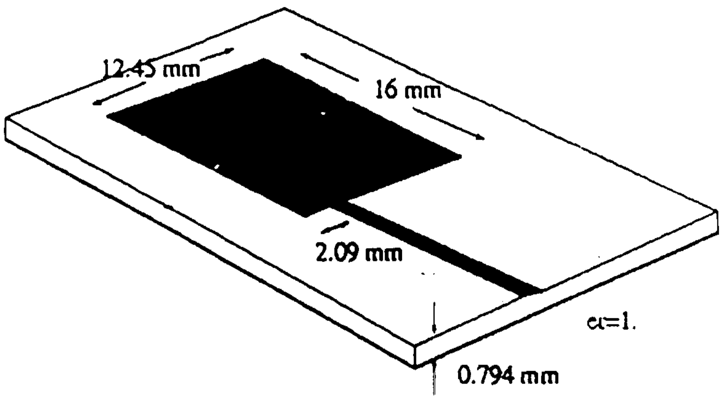

Figure 2: Membrane Patch Antenna.

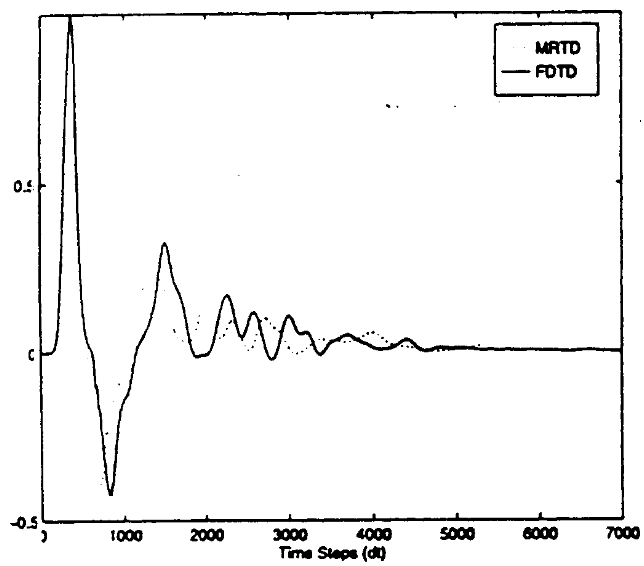

Figure 3: Time Domain Analysis of a Membrane Patch Antenna.

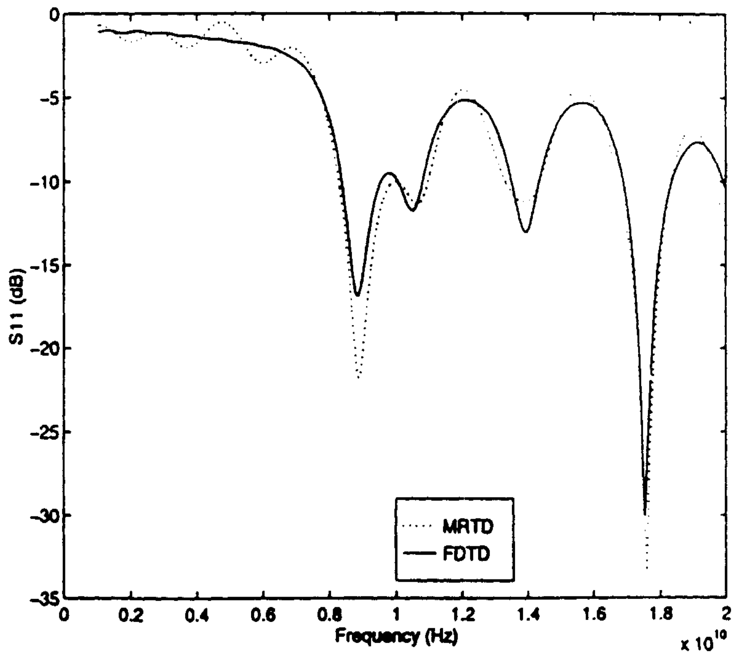

Figure 4: Simulated S11 results for the Patch Antenna.

APPENDIX F

APPLICATIONS OF MULTIRESOLUTION BASED FDTD MULTIGRID

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Abstract- A Multigrid 2-D Finite Difference Time Domain (FDTD) technique based on Multiresolution analysis with Haar wavelets is used to analyze structures such as an empty waveguide and a shielded stripline. The results obtained are compared with those computed using a finer resolution regular FDTD mesh. This comparative study illustrates the benefits of using wavelets in FDTD analysis.

I Introduction

Multiresolution Time Domain (MRTD) Technique is a new approach to solving time domain problems. This technique uses Multiresolution Analysis (MRA) to Discretize Maxwell's equations in time domain and demonstrates excellent capability in solving Electromagnetics problems [1], [2]. Depending on the choice of basis functions, several different schemes result, each one carrying the signature of the basis functions used in MRA. It is also important to note that the design of an MRTD scheme can be accomplished using one's own application-specific basis functions. MRTD technique using Haar scaling functions results in the FDTD technique [3].

Recently, an FDTD multigrid using the Haar wavelet basis has been developed and it has demonstrated that such a scheme exhibits highly linear dispersion characteristics [3]. Motivation for this work stems from the theory of MRA which says that a function which is expanded in terms of scaling functions of a lower resolution level, m1, can be improved to a higher resolution level, m2, by using wavelets of the intermediate levels. In other words, expanding a function using scaling function of resolution level m1 and wavelets upto resolution level m2 gives the same accuracy as expanding the function using just the scaling functions of resolution m2. However, the use of wavelet expansions has major implications in memory savings due to the fact that the wavelet expansion coefficients are significant only in areas of rapid field variations. This allows for the capability to discard wavelet expansion coefficients where they are not significant thereby leading to significant economy in memory. Different resolutions of wavelets can be combined so as to locally improve the accuracy of the approximation of the unknown function. This, combined with the fact that wavelet coefficients are significant only at abrupt field variations and discontinuities allows MRTD to lend itself very naturally to a Multigrid capability.

In this paper, a 2D MRTD scheme based on Haar basis functions (first order resolution) is developed and applied to solve for the Electromagnetic fields in a waveguide and a shielded stripline. The results obtained are compared with those computed using conventional FDTD technique. It will be shown that the wavelet coefficients are significant only at locations with abrupt field variations. This facilitates in obtaining accurate solutions by combining the wavelet and scaling coefficients only in regions where the wavelet coefficients are significant (discontinuities).

II The 2D-MRTD scheme

Consider the following 2-D scalar equation obtained from Maxwell's H-curl equation:

$$\epsilon\frac{\partial E_x}{\partial t} = \frac{\partial H_z}{\partial y} + \beta H_y \quad (1)$$

This equation can be rewritten in a differential operator form as shown below:

$$L_1(f_1(x,y,t)) + L_2(f_2(x,y,t)) = g \quad (2)$$

where L_1 and L_2 are the operators and f_1(x,y,t) and f_2(x,y,t) represent the electric/magnetic fields. We now expand the fields using a Haar based MRA with scaling functions ϕ and wavelet functions ψ [3]. The field expansion can be represented as follows:

$$f(x,y,t) = [A][\phi(x)\phi(y)] + [B][\phi(x)\psi(y)] + [C][\psi(x)\phi(y)] + [D][\psi(x)\psi(y)] \quad (3)$$

where $[\phi(x)\phi(y)]$, $[\phi(x)\psi(y)]$, $[\psi(x)\phi(y)]$ and $[\psi(x)\psi(y)]$ represent matrices whose elements are the corresponding basis functions in the computation domain of interest and [A], [B], [C], [D] represent the matrices of the unknown coefficients which give information about the fields and their derivatives.

Application of Galerkin's technique leads to 4 schemes which can be represented as follows:

$$< [\phi\phi], L_1(f_1) + L_2(f_2) >=< [\phi\phi], g >: \phi\phi Scheme \quad (4)$$
$$< [\phi\psi], L_1(f_1) + L_2(f_2) >=< [\phi\psi], g >: \phi\psi Scheme \quad (5)$$
$$< [\psi\phi], L_1(f_1) + L_2(f_2) >=< [\psi\phi], g >: \psi\phi Scheme \quad (6)$$
$$< [\psi\psi], L_1(f_1) + L_2(f_2) >=< [\psi\psi], g >: \psi\psi Scheme \quad (7)$$

From this system, we obtain a set of simultaneous discretized equations. For the first resolution level of Haar wavelets, the above four schemes decouple and coupling can be achieved only through the excitation term and the boundaries.

The shielded structures analyzed here are terminated at Perfect Electric Conductors (PEC) and the boundary conditions are obtained by applying the natural boundary condition for the electric field on a PEC as shown below:

$$E_t^{\phi\phi}\phi(x)\phi(y) + E_t^{\phi\psi}\phi(x)\psi(y) + E_t^{\psi\phi}\psi(x)\phi(y) + \\ + E_t^{\psi\psi}\psi(x)\psi(y) = 0....At PEC. \quad (8)$$

where $E_t^{\phi\phi}$, $E_t^{\phi\psi}$, $E_t^{\psi\phi}$ and $E_t^{\psi\psi}$ are the scaling and wavelet coefficients of the tangential electric field at the boundary nodes.

The above equations are discretized by the use of Galerkin's method which results in a set of matrix equations of order N = M+1 where M is the order of the considered wavelet resolutions. These equations are solved simultaneously with the discretized Maxwell's equations to numerically apply the correct boundary conditions.

III Applications of 2D FDTD Multigrid and Results

The 2-D MRTD scheme derived above has been applied to analyze the Electromagnetic fields in a waveguide and a shielded stripline.

(a) Waveguide: An empty waveguide with cross-section of 12.7 x 25.4 mm is chosen. A coarse 5 x 8 mesh is used to discretize this mesh and 2D MRTD technique was applied to analyze the fields in this geometry. Fig. 1 shows the amplitudes of the wavelet and scaling coefficients of the electric field obtained by using MRTD technique. From this figure it can be seen that only the $\phi\phi$ and $\phi\psi$ coefficients make a significant contribution to the field and that the contribution of $\psi\phi$ and $\psi\psi$ is negligible. From the computed coefficients, the total field is reconstructed using an appropriate combination of the scaling and significant wavelet coefficients. For the waveguide chosen here, elimination of the wavelet coefficients that have no significant contribution leads to 480 unknowns. The reconstructed field obtained by this mesh has the same accuracy as that of a 10 x 16 FDTD mesh with 960 unknowns which is in agreement with the theory of MRA. Fig. 2 shows the results of this comparison and demonstrates that the use of multigrid scheme provides a 50% economy in memory.

(b) **Shielded Stripline** : Next, a stripline of width 1.27mm is considered. It is enclosed in a cavity of area 12.7 x 12.7 mm so that the side walls are sufficiently far away to not affect the propagation. The strip is placed 12.7mm from the ground. A 40 x 40 mesh is used to analyze the fields in this geometry with the 2D MRTD technique. Fig. 3 shows the derived scaling and wavelet coefficients of the fields just below the strip. From the figure, it can be seen that among the wavelet coefficients, only $\psi\phi$ makes a significant contribution close to the vicinity of the strip where the field variation is rather abrupt. Fig. 4 shows the comparision of the total reconstructed field in the 40 x 40 MRTD mesh with that of a 40 x 40 and 80 x 80 FDTD mesh. From the figure it is clear that the field computed by 40 x 40 MRTD mesh using only the significant wavelet coefficients follows the results of the finer 80x80 mesh very closely, demonstrating once again the significant economy in memory as illustrated in Table 1. Fig. 5 shows the Normal Electric field plot of the strip and the variable mesh resulting from MRTD.

Table 1: Comparison of the memory requirements in FDTD and MRTD techniques

Technique	Unknown Coeff.
40x40 FDTD	9600
40x40 MRTD	11328
80x80 FDTD	38400

IV Conclusion

A Haar wavelet based 2D MRTD scheme was developed and applied to analyse the fields in a waveguide and a shielded stripline. The wavelet coefficients obtained are significant only in regions of rapid field variations. Thus the FDTD multigrid capability using MRTD technique has demonstrated significant economy in memory.

V Acknowledgments

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References

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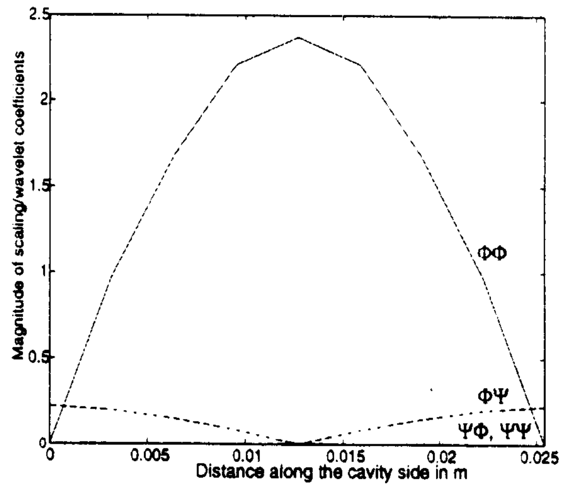

Figure 1: Amplitudes of Scaling and Wavelet Coefficients in a Waveguide.

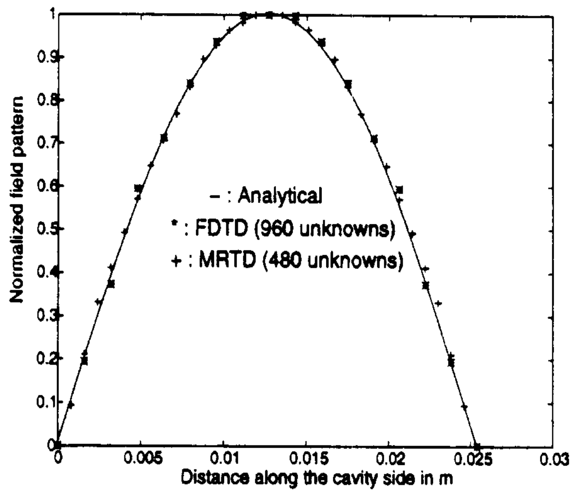

Figure 2: Comparison of MRTD , FDTD and Analytical Fields in a Waveguide.

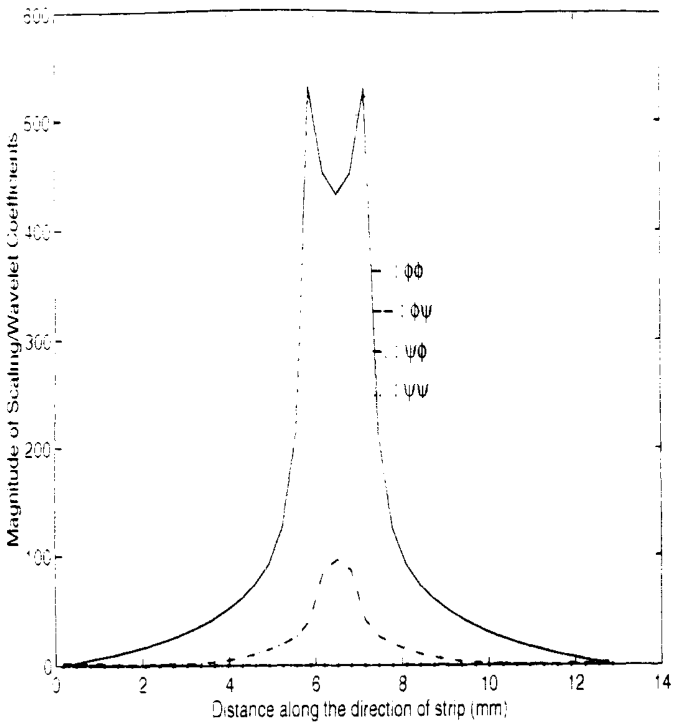

Figure 3: Amplitudes of Scaling and Wavelet Coefficients of a Shielded Stripline.

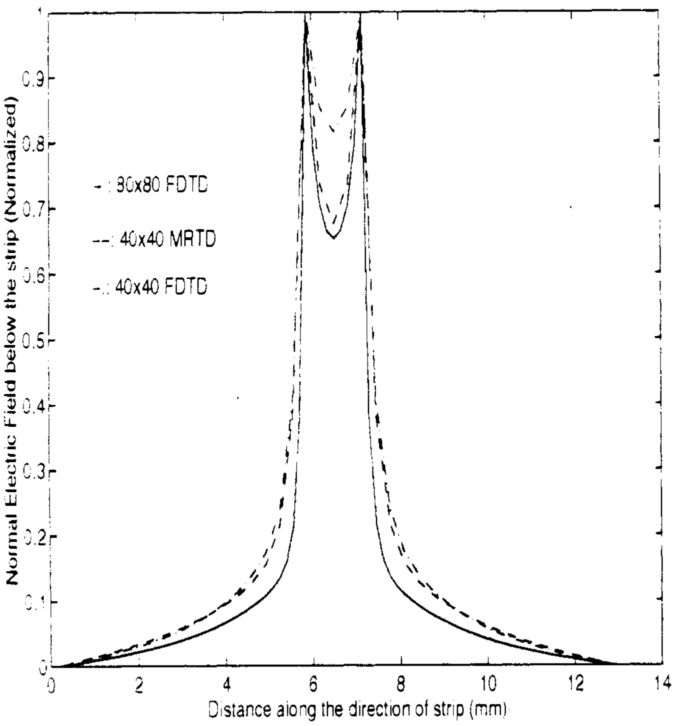

Figure 4: Comparison of Normal Electric Field under a stripline using MRTD and FDTD techniques.

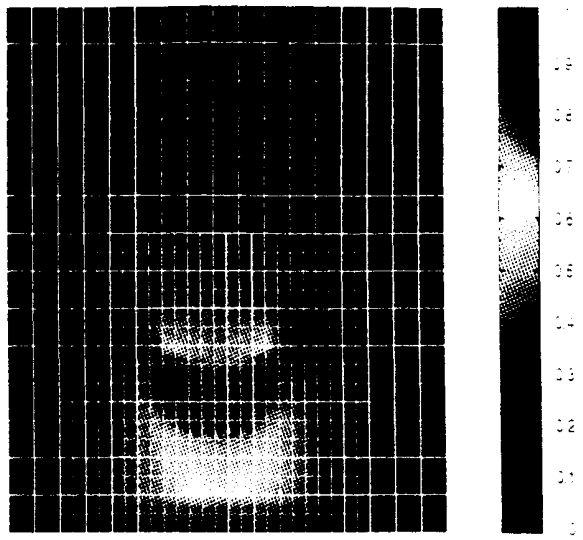

Figure 5: FDTD Multigrid and Field Plot of the Stripline.