

Multi-Agent Coordination with Event-based Communication

Pedro V. Teixeira, Dimos V. Dimarogonas, Karl H. Johansson and João Sousa

Abstract—The problem of driving a set of vehicles (agents) to a desired target configuration under event-based communication and measurement constraints is analyzed. We start by studying the single agent problem, where we propose a waypoint based solution, along with two alternative control strategies. After deriving their properties and proving some relevant results, we proceed to study the two agent case. We generalize the results obtained for this network to the multi agent network. Our strategy is able to position each agent within a given distance of its target while satisfying the constraints. We provide some numerical examples for relevant scenarios.

I. INTRODUCTION

The deployment of a formation of several vehicles has, in some applications, several advantages over the use of just one vehicle. A far from complete account of formation control designs includes [1]–[6], to name a few. In the particular case of maritime applications, an autonomous underwater vehicle (AUV) formation will take less time to cover a wider area. Also, if the sampled property has a low spatial rate of change, then the larger number of samples will result in increased data redundancy. These advantages come at a cost, namely, the complexity that arises from the coordination of the agents involved.

In multi-agent problems a common issue is that of limited communication range, which is usually taken into account as a restriction on the inter-agent distance. However, in underwater applications there are some additional limitations. Underwater communication, for one, is typically severely constrained both in range and in bandwidth (1200 bps is a typical figure [7]). Moreover, acoustic modems are typically power hungry and expensive [8]. Underwater positioning is also quite challenging (GPS does not work underwater) and good navigation instruments are very expensive. This is why in some applications low cost vehicles have to surface periodically to get GPS fixes.

We take advantage of the fact that the agent has to surface to get a position fix to limit communication to these intervals, where it can use more efficient wireless modules to communicate with other agents. At the same time, since these are the only instants where we are able to have position

feedback (*i.e.*, each agent can only measure its position at these instants), the computation of control signals will also share this constraint. Motivated by the above observations, the goal of this paper is to design a control strategy that drives a formation of AUVs from the initial to the target set of positions, subject to the given event constraints and under the effect of external disturbances such as ocean currents.

The problem presented here is related to some extent to those usually posed in the context of event-driven control (EDC). Our approach shares some aspects with what is presented in [9], particularly, that the control signal is only updated when the error norm exceeds a certain threshold. Of interest is also a comparative analysis of the time and event-driven paradigms presented in [10]. EDC has also been extended to networked control systems. In [11], the event triggering scheme proposed in [9] is used on a wireless network based distributed control system. The applications of EDC to both formation control and communication-constrained problems are also of interest to our problem. In [12] the authors present both centralized and distributed approaches to an agreement problem, which is considered as a simplification of the formation control problem presented in [13].

In this article we use a boundedness assumption regarding the disturbance set to obtain an event-triggered control strategy for the single agent problem that generates a waypoint so that the agent can measure its position and update the control signal before the position uncertainty exceeds a threshold value. This waypoint-based solution is later generalized and proven to work for a formation. The conditions for this generalization are also stated. Two provably correct alternative control laws that satisfy the event constraints are derived under the same assumption, which is then used to obtain the corresponding sufficient conditions.

This note is organized as follows: in Section II we present the problem definition along with the vehicle model and some relevant background concepts. The single-agent version of the problem is studied in Section III, where we introduce the waypoint-based approach and the proposed control strategies, which are shown to be provably correct. Moreover, we will also derive sufficient conditions for target reachability for both strategies, making room for a short comparative analysis. Section IV deals with the application of our solution to a generic multi-agent network. Some relevant numerical examples are shown and discussed in Section V, followed by a summary of our results and open problems in Section VI.

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II. PROBLEM STATEMENT

Our problem can be defined as driving a formation of M planar ($x \in \mathbb{R}^2$) agents $A = \{a_1, a_2, \dots, a_M\}$ from the set of initial positions $X_0 = \{x_0^1, x_0^2, \dots, x_0^M\}$ to a set of target positions $X_T = \{x_T^1, x_T^2, \dots, x_T^M\}$ within a specified time t_T . We denote the position of agent k (a_k) at time t_i by $x^k(t_i)$ or x_i^k interchangeably.

We assume that the external disturbances are additive. This way, vehicle motion can be described by

$$\dot{x}(t) = u(t) + \omega(t) \quad (1)$$

where the external disturbance $\omega(t)$ will lie in the disturbance set $\Omega \subset \mathbb{R}^2$. As it is likely that this set is unknown, we will make the least number of assumptions regarding it. The control signal $u(t)$ takes values in the admissible control set \mathcal{U} , which is expressed as an upper bound on the control signal norm, arising from the vehicle's maximum linear speed:

$$\mathcal{U} = \{u \in \mathbb{R}^2 : \|u\| \leq u_{max}\} \quad (2)$$

Although this may seem an overly simplified model, our initial assumption allows for the superposition principle to be applied, so the vehicle dynamics can be disregarded to some extent - even more so if we recall that our interest is in path planning and not attitude control.

As we have mentioned, position measurement can only take place at certain time instants t_i . Communication can also take place at these instants if the agents between which it occurs are connected. Here, we use a fairly simple communication model to construct the network graph. Given a communication range r , the network's adjacency matrix A at time t_i is such that

$$a_{j,k}(t_i) = \begin{cases} 1, & \text{if } \|x^j(t_i) - x^k(t_i)\| \leq r \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

We are thus interested in an event-based control strategy which is able to keep the formation connected while driving it to the target.

III. THE SINGLE AGENT PROBLEM

Let us start by analyzing the motion of a single agent. We are particularly interested in the position at the instants t_i when the agent is stopped, and at which the control $u(t)$ can be updated. For this reason, it is natural to assume that $u(t)$ is an admissible piecewise continuous signal $u(t) = \bigcup_{i=0}^{N-1} u_i(t)$, such that $u_i : [t_i, t_{i+1}] \rightarrow \mathbb{R}^2, \forall i \in \{0, 1, \dots, N-1\}$. The solution to (1) is thus

$$x(t_i) = x_0 + \sum_{k=0}^{i-1} \left(\int_{t_k}^{t_{k+1}} u_k(\tau) d\tau + \delta_{k+1} \right) \quad (4)$$

where δ_{i+1} is the position drift from t_i to t_{i+1} :

$$\delta_{i+1} = \int_{t_i}^{t_{i+1}} \omega(t) dt \quad (5)$$

As we will see, the particular case where all the u_i are constant will be of interest to us, and for which we can rewrite (4) as

$$x(t_N) = x_0 + \sum_{i=0}^{N-1} u_i(t_{i+1} - t_i) + \sum_{i=1}^N \delta_i \quad (6)$$

Before devising a control strategy that is able to drive the agent from its initial position to the target within the specified time, we should first ask if the target is *reachable*, that is, if it can be reached in the specified time using an admissible control signal. To answer this question, we can compute the system's reachable set at time t_T which, for the undisturbed system, is simply a disc of radius $u_{max}(t_T - t_0)$ centered at $x(t_0)$. Thus, if the target position is inside this disc it will be reachable. We state this condition as follows:

Lemma 1 (Target reachability under no disturbances):

Consider the system described by (1) with $u(t)$ in (2) and $\omega(t) = 0$ for $t \in [t_0, t_T]$. x_T is reachable within t_T departing from x_0 at t_0 if and only if $\|x_T - x_0\| (t_T - t_0)^{-1} \leq u_{max}$, and can be reached by setting $u(t) = (x_T - x_0)(t_T - t_0)^{-1}$. The control signal defined in lemma 1 is energy-wise optimal, in the sense that it has the smallest possible $\|u(t)\|$ (linear speed). This way, if the target isn't reachable using that control signal, then it isn't reachable.

Having a control that is able to drive the agent to the target position, we move on to the scenario where disturbances are present. To compute the system's reachable set for this case would require a complete knowledge of the disturbance set Ω , which is something that might not be available. Assuming Ω is bounded, we can consider its over approximation $\Omega_{over} \supseteq \Omega$ and use it to determine the *uncertainty set* - the set of all positions the agent can reach given a control $u(t)$. The motivation for this is that while underwater, the vehicle will be running open-loop, so results on the vehicle's position uncertainty become necessary. We define the *upper bound on the external disturbance*, γ :

$$\gamma \geq \max_{\omega \in \Omega} (\|\omega\|) \quad (7)$$

where γ , which can be interpreted as an upper bound on the position uncertainty growth rate, defines the *over-approximating disturbance set*: $\Omega_{over} = \{\omega : \|\omega\| \leq \gamma\}$. Using the superposition principle, we can split (1) into two systems and consider its effects separately: $\dot{y}(t) = u(t)$ and $\dot{z}(t) = \omega(t)$, with $u(t) \in \mathcal{U}$ and $\omega(t) \in \Omega$ respectively. The uncertainty set at time t_1 , $\Delta(t_1)$, will thus be the latter system's reachable set at time t_1 , centered around $y(t_1)$. Again, as we do not know Ω , we use Ω_{over} to obtain an over approximation of the reachable set, yielding $(t_1 - t_0)\Omega_{over}$ - a scaling relative to the origin of Ω_{over} by a factor of $(t_1 - t_0)$. So, at a given time t , the over-approximating uncertainty set $\Delta_{over}(t)$ will be a disc of radius t centered around the agent's ideal position at that time instant (figure 1).

A. A waypoint based approach

Definition (7) gives us an upper bound on the uncertainty as a function of time. Assuming that we do not want the

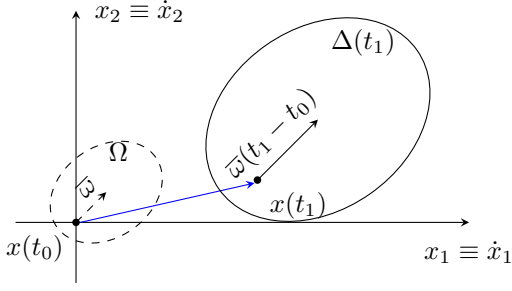


Fig. 1. Disturbance and uncertainty sets for system (1), given a control $u(t)$ that would take the system from $x(t_0)$ to $x(t_1)$, were there no disturbances.

agent to drift more than ϵ meters from its ideal path, we can use it to compute the instants at which the agent should stop to surface and get a position fix: $t_\epsilon = \epsilon\gamma^{-1}$, where ϵ is the *maximum position uncertainty*. Notice that with this equation we have implicitly assumed that our upper bound γ holds for the interval $[t_0, t_T]$. The agent will thus stop every t_ϵ seconds, in a total of $N = \lfloor (t_T - t_0)t_\epsilon^{-1} \rfloor$ stops. We define

$$t_i = t_0 + i \cdot t_\epsilon, i \in \{0, 1, \dots, N\} \quad (8)$$

as the i^{th} stopping time. Consequently, we have that $t_T = t_N + t_f$, with $t_f \in [0, t_\epsilon)$. As we will see, we will try to reach the target at $t = t_N$, leaving the remaining time (t_f) to adjust our position.

Noting that $t_{i+1} = t_i + t_\epsilon$, and taking the norm of equation (5) yields

$$\begin{aligned} \|\delta_{i+1}\| &= \left\| \int_{t_i}^{t_i+t_\epsilon} \omega(t) dt \right\| \leq \int_{t_i}^{t_i+t_\epsilon} \|\omega(t)\| dt \\ &\leq \gamma \cdot t_\epsilon \leq \epsilon, \forall i \in \{0, 1, \dots, N-1\} \end{aligned} \quad (9)$$

Thus, the agent will never be more than ϵ meters from its estimated (ideal) position.

As we have mentioned earlier, we assume we have a control strategy $u = h(\cdot)$ that is able to drive the agent from $x(t_i)$ to $x(t_T) = x_T$ under no disturbances. This being true, we can use the given control strategy in the case where there are disturbances, knowing from equation (9) that we will never be more than ϵ meters from where we intended to be. So, at time t_i , the agent will compute u_i , the control signal it will use in the time interval $[t_i, t_{i+1}]$, driving it from x_i to a point W_{i+1} given by $W_{i+1} = x_i + \int_{t_i}^{t_{i+1}} u_i(t) dt$, which we call the $i+1^{\text{th}}$ waypoint. Since there are disturbances, however, then the agent will most likely stop at a position different from W_{i+1} , x_{i+1} given by

$$\begin{aligned} x_{i+1} &= x_i + \int_{t_i}^{t_{i+1}} (u_i(t) + \omega(t)) dt \\ &= W_{i+1} + \delta_{i+1} \end{aligned} \quad (10)$$

with the distance between the two points given by eq. (9).

B. Control strategies

Now that we have devised a waypoint-based mechanism that gives us an upper bound on the distance to the ideal

trajectory, we still need to devise a control strategy that satisfies our requirements. One idea is to use the energy-wise optimal control strategy introduced in the ideal case (lemma 1), by re-applying it at each stopping point. We call this strategy $h_1(\cdot)$ and the main results for it are given in the following paragraphs.

Theorem 1 (The $h_1(\cdot)$ control strategy): Consider the system described by (1) and controlled by

$$\begin{aligned} u_i(t) &= h_1(t_i, x_i, t_T, x_T) \\ &= (x_T - x_i) (t_N - t_i)^{-1} \end{aligned} \quad (11)$$

for all $i \in \{0, 1, \dots, N-1\}$. Under the specified control law the following properties hold: (i) $x(t_N) = x_T + \delta_N$, (ii) $\|x_T - x(t_N)\| \leq \epsilon$. A sufficient condition for this theorem to hold is $\|x_T - x_0\| (t_N - t_0)^{-1} + \gamma \cdot H_{N-1} \leq u_{max}$, where $H_k = \sum_{i=1}^k \frac{1}{i}$ is the k^{th} harmonic number.

Proof: We begin by proving properties (i) and (ii). Writing the expressions for u_i and W_{i+1} for the first iterations will lead to the following expressions

$$u_i = (x_i - x_0) (t_N - t_0)^{-1} - \sum_{k=1}^i \delta_k (t_N - t_k)^{-1} \quad (12)$$

$$W_{i+1} = x_0 + \frac{i+1}{N} (x_T - x_0) + \sum_{k=1}^i \frac{N - (i+1)}{N - k} \delta_k \quad (13)$$

To obtain the agent's position at time t_N , we use equation (6):

$$x(t_N) = x_0 + \sum_{i=0}^{N-1} u_i t_\epsilon + \sum_{i=1}^N \delta_i.$$

Using equations (12) and (8), the second term on the right hand side can be rewritten as

$$\sum_{i=0}^{N-1} u_i t_\epsilon = x_T - x_0 - \sum_{i=1}^{N-1} \delta_i$$

yielding $x_N = x_T + \delta_N$, and from (9) we have that $\|x_N - x_T\| \leq \epsilon$. As for the condition, it can be obtained by considering the worst-case scenario, where the disturbance velocity is constant and opposite to the direction of motion. Let $\delta_i = \delta^*$, for all $i \in \{1, 2, \dots, N-1\}$, where

$$\delta^* = -(x_T - x_0) \frac{\epsilon}{\|x_T - x_0\|} \quad (14)$$

Under this assumption, the control signal norm can be written as $\|u_i\| = \|x_T - x_0\| (t_N - t_0)^{-1} + \gamma H_i$. This expression is monotonically increasing with respect to i , and will achieve its maximum for $i = N-1$: $\|u_i\| = \|x_T - x_0\| (t_N - t_0)^{-1} + \gamma H_{N-1}$. ■

Looking at the expression for the control signal obtained using this control strategy (12), we can see that if ω does not have a zero mean and, more specifically, if it is opposite to the direction of motion, then u_i will become monotonically increasing, which may become a problem given our admissible control set. This is also easy to see if we notice that $h_1(\cdot)$ uses $[t_i, t_N]$ to compensate for δ_i . If, instead, we compensate for δ_i during $[t_i, t_{i+1}]$, we will obtain the following results:

Theorem 2 (The $h_2(\cdot)$ control strategy): Consider the system described by (1) and controlled by

$$\begin{aligned} u_i(t) &= h_2(\delta_i, t_T, x_T) \\ &= (x_T - x_0)(t_N - t_0)^{-1} - \delta_i t_\epsilon^{-1} \end{aligned} \quad (15)$$

for all $i \in \{0, 1, \dots, N-1\}$. Under the specified control law the following properties hold: (i) $x(t_N) = x_T + \delta_N$, (ii) $\|x_T - x(t_N)\| \leq \epsilon$. A sufficient condition for this theorem to hold is $\|x_T - x_0\|(t_N - t_0)^{-1} + \gamma \leq u_{max}$.

Proof: Just as we did in the previous proof, we begin by proving properties (i) and (ii) and deriving the expressions for u_i and W_{i+1} by writing down the corresponding expressions for the first iterations, which will lead us to

$$u_i = (x_T - x_0)(t_N - t_0)^{-1} - \delta_i t_\epsilon^{-1} \quad (16)$$

$$W_{i+1} = x_0 + \frac{i+1}{N}(x_T - x_0) \quad (17)$$

Replacing u_i in equation (6) by equation (16) will yield $x_N = x_T + \delta_N$, and again, from (9) we have $\|x_N - x_T\| \leq \epsilon$. To derive the condition we again take the worst case approach of considering $\delta_i = \delta^*$, as in equation (14) for all $i \in \{1, 2, \dots, N-1\}$. This way the control signal norm becomes $\|u_i\| = \|x_T - x_0\|(t_N - t_0)^{-1} + \gamma$. Notice that as this norm is not dependent on i , it will correspond to the upper bound on the vehicle's speed. ■

The $h_2(\cdot)$ control strategy was devised with the purpose of improving $h_1(\cdot)$'s performance. Although we were able to relax the (sufficient) target reachability condition, this does not necessarily mean that it is a better performing control strategy, as the conditions were obtained under the particular assumption of *adverse disturbances*. Still, looking at expressions (13) and (17), we can see that using $h_2(\cdot)$ the waypoints will always lie in the straight line connecting x_0 to x_T - which may not always happen for $h_1(\cdot)$.

As both control strategies will try to reach the target at $t = t_N$, in some cases there will be some time left, more precisely t_f seconds, which can be used for finer positioning:

Corollary 1 (The final approach): Consider the system described by (1) such that $x(t_N) = x_T + \delta_N$. Setting, at $t = t_N$, $u_f = -\delta_N \cdot t_f^{-1}$, the following properties hold: (i) $x_f = x_T + \delta_f$, (ii) $\|x_f - x_T\| \leq \gamma \cdot t_f$.

IV. THE MULTI-AGENT PROBLEM

Having devised a strategy for the single agent version of the original problem, we move on to the multi-agent problem. We begin extending our results to the two-agent network.

A. The two-agent network

Consider a simple, two-agent network, where we let one of them (the leader, a_L) "behave" as in the single agent scenario, with the difference that it must, at every instant t_i , inform the other agent about where it is going to go next - the next waypoint - as well as when it plans to get there - the travel time.

The follower then uses this information to determine its next waypoint. However, there are two requirements that must be met in order for the follower to have access to the

leader's information - at all stopping instants t_i , the follower must: (i) be synchronized with the leader (more specifically the departure and arrival times are required to be the same as the leader's), and (ii) be within the leader's communication range. Failing to meet either one of these requirements will likely result in agent loss.

We define the initial and target sets by $X_0 = \{x_0^L, x_0^F\}$ and $X_T = \{x_T^L, x_T^F\}$ respectively. Since the network is also a formation, it is natural to assume that $x_0^F = x_0^L + c_{L,F}$ and $x_T^F = x_T^L + c_{L,F}$, where $c_{L,F}$ denotes the desired relative position between the leader and the follower. In fact, as it is desirable to maintain the inter-agent relative position, we use this to determine the follower's waypoint:

$$W_{i+1}^F = W_{i+1}^L + c_{L,F} \quad (18)$$

Assuming our requirements are satisfied - we will later derive the conditions for which this is true - we want to know the follower's position at time t_N , $x^F(t_N) = x_N^F$. We start by rewriting equation (6) for the follower:

$$x^F(t_N) = x_0^F + \sum_{i=0}^{N-1} u_i^F \cdot t_\epsilon + \sum_{i=1}^N \delta_i^F \quad (19)$$

We can use equations (18) and (10) to obtain $u_i^F = u_i^L + (\delta_i^L - \delta_i^F) t_\epsilon^{-1}$. Replacing in (19) will yield

$$x^F(t_N) = x_0^F + \sum_{i=0}^{N-1} u_i^L t_\epsilon + \sum_{i=1}^{N-1} \delta_i^L + \delta_N^F \quad (20)$$

so in order to find x_N^F we have to check this equation for each control strategy. As we know from the previous section, both control strategies provide constant control signals, so equation (6) holds. Using this equation together with theorems 1 and 2 we can write, regardless of the leader's control strategy:

$$x_0^L + \sum_{i=0}^{N-1} u_i^L t_\epsilon + \sum_{i=1}^N \delta_i^L = x_T^L + \delta_N^L$$

Plugging this into equation (20) will result in $x^F(t_N) = x_T^F + \delta_N^F$ and $\|x^F(t_N) - x_T^F\| \leq \epsilon$, which means we have for the follower the same results we have for the leader.

Now that we have the results on the follower's position, we have to deal with our initial requirements. Assuming synchronization is possible if the agents are within each other's range, we only need to care about the communication requirement. Since communication only takes place at the stopping instants, we will be interested in the inter-agent distance at those instants t_i , which can be expressed as $\|x^L(t_i) - x^F(t_i)\| = \|\delta_i^L - \delta_i^F - c_{L,F}\|$. In order to obtain the upper bound on this distance, we assume Ω is the same for both agents and let the two drift apart from each other ($\delta_i^L = -\delta_i^F = -\frac{\epsilon}{\|c_{L,F}\|} c_{L,F}$) to obtain $\|x_i^L - x_i^F\| \leq 2\epsilon + \|c_{L,F}\|$. So, for the two agents to be able to communicate with each other, the following condition should hold: $\epsilon \leq \frac{1}{2}(r - \|c_{L,F}\|)$. To obtain this condition we took the expression for the inter agent distance at time t_i and let the two agents drift in opposite directions, away from

each other. If instead we let them drift towards each other, there will be an instant at which $x^L(t) = x^F(t)$ and the two agents will collide. In this worst case perspective, this will take $t_\epsilon = \frac{1}{2}\|c_{L,F}\|\gamma^{-1}$ time units to happen, assuming the initial relative position between the two agents is equal to $c_{L,F}$. Since $t_\epsilon = \epsilon\gamma^{-1}$, the corresponding condition on ϵ will be $\epsilon < \frac{1}{2}\|c_{L,F}\|$. We have thus derived sufficient conditions on ϵ for our initial requirements to hold.

B. The multi-agent network

We can now move on to our original problem, where we have a set of agents $A = \{a_1, a_2, \dots, a_M\}$ as well as the initial and target sets, $X_0 = \{x_0^1, x_0^2, \dots, x_0^M\}$ and $X_T = \{x_T^1, x_T^2, \dots, x_T^M\}$, and a matrix $C \in \mathbb{R}^{M \times M \times 2}$ expressing the desired relative positions between the agents ($c_{i,j} \in \mathbb{R}^2$ is the desired relative position between agents a_i and a_j). We will keep some aspects of the two-agent network, such as the presence of a leader, and the use of the formation property of the network to define waypoints.

We assign one agent, a_1 , the task of leading the network to the target set, which it does by computing its next waypoint and then transmitting it (broadcast) to the rest of the network. Even though it is very likely that not all agents are within the leader's communication range, the followers can act as repeaters, so we have for agent j :

$$W_{i+1}^j = W_{i+1}^L + c_{L,j} \quad (21)$$

Looking at the equation above we see that if we consider the leader and any other agent separately from the rest of the network, we will be dealing with the same two-agent network as before. Doing this for all agents in the network, it is easy to see that we are able to extend the results we obtained for the two-agent network.

Theorem 3 (The $h_1(\cdot)$ control strategy (multi-agent)):

Consider a set of M agents, $A = \{a_1, a_2, \dots, a_M\}$ where each agent is described by (1), and let the leader (a_1) be controlled by

$$\begin{aligned} u_i^L(t) &= h_1(x_T^L, x_i^L, t_N, t_i) \\ &= (x_T^L - x_i^L)(t_N - t_i)^{-1} \end{aligned} \quad (22)$$

for all $i \in \{0, 1, \dots, N-1\}$. Under this control law the following properties hold: (i) $x^k(t_N) = x_T^k + \delta_N^k$, (ii) $\|x_T^k - x^k(t_N)\| \leq \epsilon$ for all $k \in \{1, 2, \dots, M\}$.

Theorem 4 (The $h_2(\cdot)$ control strategy (multi-agent)):

Consider a set of M agents, $A = \{a_1, a_2, \dots, a_M\}$ where each agent is described by (1), and let the leader (a_1) be controlled by

$$\begin{aligned} u_i^L(t) &= h_2(x_T^L, \delta_i^L, t_N, t_\epsilon) \\ &= x_T^L \cdot t_N^{-1} - \delta_i^L \cdot t_\epsilon^{-1} \end{aligned} \quad (23)$$

for all $i \in \{0, 1, \dots, N-1\}$. Under this control law the following properties hold: (i) $x^k(t_N) = x_T^k + \delta_N^k$, (ii) $\|x_T^k - x^k(t_N)\| \leq \epsilon$ for all $k \in \{1, 2, \dots, M\}$.

Both theorems can be proved using the two-agent network approach to the $M-1$ leader-follower subnetworks.

When analyzing the two-agent network, we saw that having the leader use any of the control strategies proposed for the single agent case would successfully drive the network close to the target set, if the conditions on the inter-agent distance were met. Having concluded that the multi-agent network can be "reduced" (in some sense) to the two-agent network, the same conditions will apply. Consequently, we will also need to extend these conditions on the inter-agent distance to the n -agent network.

Consider the C matrix, expressing the desired relative positions between the M agents in the network. Defining $d_{i,j} = \|c_{i,j}\|$ as the (desired) distance between agents a_i and a_j , then $i^*, j^* = \arg \min (d_{i,j})_{i \neq j}$ will be the two closest agents in the network. Consequently, their uncertainty sets, Δ_{i^*} and Δ_{j^*} , will be the ones that take the least amount of time to overlap. As this overlap represents a collision possibility between the two agents, we must choose ϵ such that the uncertainty sets do not overlap or, in other words, the position uncertainty does not exceed half of the distance between the two agents:

$$\epsilon < \frac{1}{2}d_{i^*,j^*} \quad (24)$$

Equation (24) defines what we call the *collision avoidance condition*.

As we had seen, an essential requirement for our approach to work with the two agent network was that the two agents had to be within each other's communication range. This also applies to the multi-agent scenario, meaning that the network has to remain connected. Consider agent a_i , for example, in a network where every agent has a communication range equal to r . This agent has a certain set of agents within its communication range - its neighboring set $\mathcal{N}(i)$ to be exact. Assuming we do not want it to lose connectivity to any of these, we write the corresponding condition for agent i (ϵ_i) as $\epsilon_i \leq \frac{1}{2}(r - \max_{j \in \mathcal{N}(i)}(d_{i,j}))$. Notice that this is the same as stating that we want the network's adjacency matrix to remain the same for all t_i . As we want this to be true for all agents, we express the *range condition* as

$$\epsilon \leq \min_{i \in A} \left[\frac{1}{2} \left(r - \max_{j \in \mathcal{N}(i)} (d_{i,j}) \right) \right] \quad (25)$$

where A is the set of all agents in the network.

It should be mentioned that both these conditions have been obtained by considering worst-case scenarios and, consequently, might be overly conservative, so care should be taken when choosing the maximum position uncertainty parameter, ϵ .

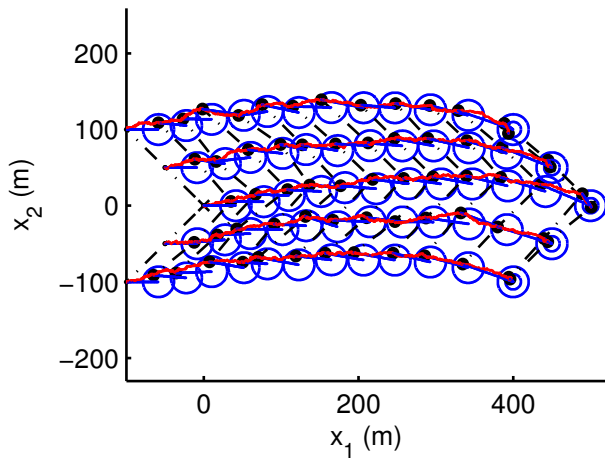
V. NUMERICAL EXAMPLES

Consider a formation $A = \{a_1, a_2, a_3, a_4, a_5\}$ where each agent is described by (1), with $\mathcal{U} = \{u \in \mathbb{R}^2 : \|u\| \leq 10m/s\}$ and $\Omega = \{\omega \in \mathbb{R}^2 : \|\omega - \bar{\omega}\| \leq 1m/s\}$, uniformly distributed with mean $\bar{\omega} = [-1, 1]^T$, for all agents. Each agent has a communication range r equal to 90 meters, and the leader's target is located at $x_T = [500, 0]^T$ with $t_T = 100$ seconds. The formation constraints are $c_{1,2} = c_{2,4} = [-50, 50]^T$ and $c_{1,3} = c_{3,5} = [-50, -50]^T$. We define $\gamma =$

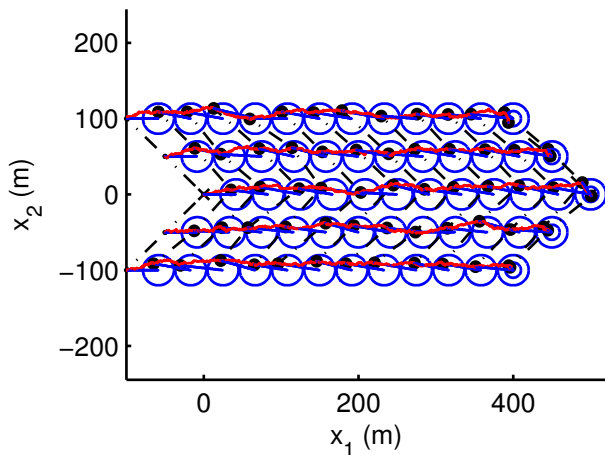
2.5m/s so that (7) holds. As for ϵ , (24) and (25) will lead to $\epsilon \leq 25\sqrt{2}$ and $\epsilon \leq \frac{1}{2}(90 - 50\sqrt{2})$, so for the two conditions to hold we set $\epsilon \leq 10$ meters. These two conditions are somewhat conservative, so we let $\epsilon = 20$, having that $t_\epsilon = 8$ seconds and $N = 12$ stops, leaving $t_f = 4$ seconds for the final approach and thus, a (final) upper bound on the distance to the target of $\gamma \cdot t_f = 10$ meters.

In the following figures, the estimated and true trajectories are shown in blue and red respectively. The stopping points are depicted as black dots and the correspondent over-approximating uncertainty sets as blue circles. The connectivity between agents is represented by a dashed black line.

Looking at figures 2a and 2b the greater sensitivity of the system's trajectory using the $h_1(\cdot)$ control strategy (compared to that of $h_2(\cdot)$) is evident, not only from the trajectory itself, but also from the distance between waypoints. This shows what we had previously pointed out - the use of a larger time frame to compensate for the position drifts can cause some issues in scenarios with an adverse mean disturbance.



(a) Formation trajectory using the $h_1(\cdot)$ control strategy



(b) Formation trajectory using the $h_2(\cdot)$ control strategy

Fig. 2. Simulation results

VI. CONCLUSIONS AND FUTURE WORK

We have proposed an event based approach to solve the multi-agent problem under communication and measurement constraints, which relies on using a mild assumption on the disturbance set: an upper bound on the growth rate of the position uncertainty, γ . This bound, together with the user-defined maximum position uncertainty ϵ is used to determine when the agent should stop. Two alternative control strategies are able to drive the agent to the target while guaranteeing that the agent's distance to its ideal position is never greater than ϵ . All of these results were derived for the single agent problem and later extended to the original multi-agent problem. Sufficient conditions for the main results were also derived.

There are two assumptions we make in the paper that point to future directions for development. Firstly, the disturbance set over-approximation we use to obtain γ may, in some cases, be a very gross approximation. Secondly, we assume γ to be constant over $[t_0, t_T]$. The relaxation of these assumptions is a topic of ongoing efforts. Other important open problems concern extending our solution to the multi-agent network in a distributed fashion (without having just one agent deciding on where should the formation go next), as well as to non-linear agent dynamics.

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