Multi-Antenna Based Spectrum Sensing for Cognitive Radios: A GLRT Approach

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***Abstract*—In this letter, we propose multi-antenna based spectrum sensing methods for cognitive radios (CRs) using the generalized likelihood ratio test (GLRT) paradigm. The proposed methods utilize the eigenvalues of the sample covariance matrix of the received signal vector from multiple antennas, taking advantage of the fact that in practice, the primary user signals to be detected will either occupy a subspace of dimension strictly smaller than the dimension of the observation space, or have a non-white spatial spectrum. These methods do not require prior knowledge of the primary user signals, or the channels from the primary users to the CR. By making different assumptions on the availability of the white noise power value at the CR receiver, we derive two algorithms that are shown to outperform the standard energy detector.**

***Index Terms*—Antenna array, blind detection, cognitive radio, generalized likelihood ratio test (GLRT), spectrum sensing.**

I. Introduction

SPECTRUM sensing to detect the presence of primary user transmissions is a crucial task for a cognitive radio (CR) system, which opportunistically accesses the spectrum of interest. To date many spectrum sensing algorithms have been proposed in the literature, including, e.g., energy detection [1], [2], matched-filter detection [3]-[5], and cyclostationarity-based detection [6], [7]. All of these techniques may be applied at an individual CR sensor or multiple CR sensors in a collaborative manner such as those discussed in [8]–[12]. In prior works on spectrum sensing for CRs, it is usually assumed that there is full or partial knowledge available at the CR of the primary user signal characteristics, the channel from the primary user to the CR, and/or the noise power level at the CR receiver. Such assumptions may limit the applicability of these algorithms in realistic CR environments.

In this letter, we propose for *multi-antenna* CR sensors a class of spectrum sensing methods that require no information about the primary users or the channels from the primary to the secondary users. The proposed methods utilize the eigenvalues of the sample covariance matrix of the received signal vector from multiple antennas, taking advantage of the fact that in practice, the primary signals to be detected will either occupy a subspace of dimension strictly smaller than the number of antennas, or have a non-white spectrum in space. Using the *generalized likelihood ratio test* (GLRT) paradigm [13], we derive two new algorithms for spectrum sensing under different assumptions on the availability of the white noise power value at the CR receiver.

The use of antenna arrays for signal detection is well known in radar applications [14], [15], but these methods are based on knowing the waveform of the signal to be detected, and therefore cannot be used for CRs, in which the primary user transmits a random signal that cannot be known *a priori*. Blind signal detection for multi-antenna sensors with no knowledge about the signal to be detected and the sensor noise level has been studied in [16] based on information-theoretic criteria rather than the GLRT principle. Furthermore, heuristic algorithms for eigenvalue-based spectrum sensing for CRs has been proposed in [17]. In this letter, by applying the GLRT principle we derive new eigenvalue-based spectrum sensing algorithms that are different from those in [17]. Finally, multi-antenna transceivers have also been studied in [18] for transmit beamforming instead of spectrum sensing for CRs.

II. Signal Model

Consider a CR terminal performing spectrum sensing based on a set of N discrete-time vector observations $\mathbf{x}[n]$, $n = 0, \ldots, N-1$, to decide if the primary signal is present. The i-th component of $\mathbf{x}[n]$, denoted as $x_i[n]$, $i = 0, \ldots, M-1$, is the output of the i-th antenna, where M is the number of antennas at the CR terminal.[1] For notational convenience, an aggregate observation matrix is defined as $\mathbf{X} = [\mathbf{x}[0], \ldots, \mathbf{x}[N-1]]$. The hypothesis testing problem of interest is expressed as

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{x}[n] = \mathbf{w}[n], \quad n = 0, \ldots, N-1 \\ \mathcal{H}_1 &: \mathbf{x}[n] = \mathbf{s}[n] + \mathbf{w}[n], \quad n = 0, \ldots, N-1 \end{aligned} \tag{1}$$

where $\mathbf{w}[n]$ is the additive noise at the CR receiver, modeled as an independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) vector with zero mean and the covariance matrix $\sigma^2\mathbf{I}$, with $\mathbf{I}$ denoting an identity matrix, while $\mathbf{s}[n]$ is the received primary signal to be detected. In the absence of any prior knowledge of the form of $\mathbf{s}[n]$, or any attempt to estimate it, the signal $\mathbf{s}[n]$ is suitably

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[1] Alternatively, we may apply the model to collaborative sensing, in which each single-antenna node senses the spectrum and then amplifies and forwards via orthogonal channels the observations to a fusion center where the detection decision is made using one of the methods to be described in this letter.

assumed to be an i.i.d. CSCG random vector[2] with zero mean and the covariance matrix denoted by $\mathbf{R}_s = E[\mathbf{s}[n]\mathbf{s}^H[n]]$, where $(\cdot)^H$ denotes the conjugate transpose. We also write $\mathbf{R}_x = E[\mathbf{x}[n]\mathbf{x}^H[n]]$. We define the empirical or sample covariance matrix of $\mathbf{x}[n]$ to be

$$\hat{\mathbf{R}}_x = \frac{1}{N}\sum_{n=0}^{N-1}\mathbf{x}[n]\mathbf{x}^H[n] \tag{2}$$

and assume that it is feasible to perform an eigen-decomposition so that the unitary eigenvector matrix $\mathbf{U}_x$ and diagonal eigenvalue matrix $\mathbf{\Lambda}_x$ in $\hat{\mathbf{R}}_x = \mathbf{U}_x\mathbf{\Lambda}_x\mathbf{U}_x^H$ are known in each block of N observations.

The algorithms to be proposed rely on one of the following conditions to hold:

1) $\mathbf{R}_s$ is *rank-deficient*: In other words, $\text{rank}(\mathbf{R}_s) = N_s < M$, the dimension of the received signal space.[3] In this case, the smallest $M - N_s$ eigenvalues of $\hat{\mathbf{R}}_x$ will be approximately equal to the noise variance σ^2, while the N_s largest eigenvalues of $\hat{\mathbf{R}}_x$ will be approximately the sum of an eigenvalue of $\mathbf{R}_s$ and σ^2. These approximations become exact in the limit $N \to \infty$.
2) $\mathbf{R}_s$ is *full-rank* but $\mathbf{R}_s \neq \alpha\mathbf{I}$, where α is an arbitrary positive constant: In this case, $\text{rank}(\mathbf{R}_s) = M$, and each eigenvalue of $\hat{\mathbf{R}}_x$ will be approximately the sum of an eigenvalue of $\mathbf{R}_s$ and σ^2. Under this condition, the eigenvalues of $\mathbf{R}_s$ are unequal,[4] and hence so are those of $\hat{\mathbf{R}}_x$.

Note that when the primary signal is not present, $\hat{\mathbf{R}}_x \to \sigma^2\mathbf{I}$ as $N \to \infty$, i.e., $\hat{\mathbf{R}}_x$ is a full-rank diagonal matrix with equal eigenvalues, which is different from $\hat{\mathbf{R}}_x$ when the primary signal is present provided that one of the above two conditions is satisfied. Thus, $\hat{\mathbf{R}}_x$ can be used to detect the existence of a primary signal, as will be more rigorously addressed next.

III. Detection Algorithms

If the primary signal covariance matrix, $\mathbf{R}_s$, and the noise variance, σ^2, are both known, the Neyman-Pearson approach [13] leads to the following *estimator-correlator* detector that is optimal (in the sense of maximizing detection probability, P_D, at a given probability of false alarm, P_{FA}) for the hypothesis testing problem in (1):

$$T_{\mathsf{EC}}(\mathbf{X}) = \sum_{n=0}^{N-1}\mathbf{x}^H[n]\mathbf{R}_s(\mathbf{R}_s + \sigma^2\mathbf{I})^{-1}\mathbf{x}[n] \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma \tag{3}$$

where $(\cdot)^{-1}$ denotes the matrix inverse, while γ is set to provide a desired P_{FA}. In the special case that $\mathbf{R}_s = \alpha\mathbf{I}$, simplification of (3) (after discarding irrelevant constant terms) reveals the *energy detector* to be optimal:

$$T_{\mathsf{ED}}(\mathbf{X}) = \sum_{n=0}^{N-1}\|\mathbf{x}[n]\|^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma \tag{4}$$

where $\|\cdot\|$ denotes the Euclidean norm of a complex vector.

If the secondary sensor does not know $\mathbf{R}_s$ and/or σ^2 prior to spectrum sensing, the detection problem becomes hypothesis testing in the presence of uncertain parameters, which is generally known as "composite" hypothesis testing. One useful solution for this type of problems is the GLRT [13], which first obtains the maximum likelihood estimate (MLE) of the unknown parameters under $\mathcal{H}_0$ and $\mathcal{H}_1$:

$$\hat{\theta}_0 = \arg\max_{\theta_0} p(\mathbf{X}|\mathcal{H}_0, \theta_0), \quad \hat{\theta}_1 = \arg\max_{\theta_1} p(\mathbf{X}|\mathcal{H}_1, \theta_1)$$

where θ_0 (θ_1) is the set of parameters unknown under $\mathcal{H}_0$ ($\mathcal{H}_1$), and then forms the GLRT statistic

$$L_G(\mathbf{X}) = \frac{p(\mathbf{X}|\hat{\theta}_1, \mathcal{H}_1)}{p(\mathbf{X}|\hat{\theta}_0, \mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma. \tag{5}$$

In the next section, we will present two new spectrum sensing algorithms designed using the GLRT principle, one for the case where both $\mathbf{R}_s$ and σ^2 are unknown, and the other for the case where σ^2 is known, but $\mathbf{R}_s$ is unknown.

A. Both $\mathbf{R}_s$ and σ^2 Unknown

In this case, the log-likelihood function (LLF) under $\mathcal{H}_0$ of the unknown parameter σ^2 can be expressed as

$$\ln p(\mathbf{X}|\mathcal{H}_0, \sigma^2) = -\frac{MN}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{n=0}^{N-1}\|\mathbf{x}[n]\|^2. \tag{6}$$

The MLE of σ^2 under $\mathcal{H}_0$ minimizes (6), and is given by

$$\widehat{\sigma_0^2} = \frac{1}{MN}\sum_{n=0}^{N-1}\|\mathbf{x}[n]\|^2, \tag{7}$$

which upon substitution into (6) yields

$$\ln p(\mathbf{X}|\mathcal{H}_0, \widehat{\sigma_0^2}) = -\frac{MN}{2}\left[\ln\left(\frac{2\pi}{MN}\sum_{n=0}^{N-1}\|\mathbf{x}[n]\|^2\right) + 1\right]. \tag{8}$$

Similarly, the LLF under $\mathcal{H}_1$ of both unknowns $\mathbf{R}_s$ and σ^2, or equivalently, $\mathbf{R}_x$, can be expressed as

$$\begin{aligned}\ln p(\mathbf{X}|\mathcal{H}_1, \mathbf{R}_x) &= -\frac{MN}{2}\ln(2\pi) - \frac{N}{2}\ln\left(\det(\mathbf{R}_x)\right) \\ &- \frac{1}{2}\sum_{n=0}^{N-1}\mathbf{x}^H[n]\mathbf{R}_x^{-1}\mathbf{x}[n]\end{aligned} \tag{9}$$

where $\det(\cdot)$ denotes the matrix determinant. The MLE of $\mathbf{R}_x$ under $\mathcal{H}_1$ can be derived as follows. First, define $\mathbf{A} = \mathbf{R}_x^{-1}$. Thus, (9) can be written as

$$f(\mathbf{A}) = -\frac{MN}{2}\ln(2\pi) + \frac{N}{2}\ln\left(\det(\mathbf{A})\right) - \frac{1}{2}\sum_{n=0}^{N-1}\mathbf{x}^H[n]\mathbf{A}\mathbf{x}[n]. \tag{10}$$

Since $\mathbf{R}_x \succeq 0$, i.e., $\mathbf{R}_x$ is positive semi-definite, so is $\mathbf{A}$. It is then easy to verify that $f(\mathbf{A})$ is a concave function of $\mathbf{A}$.

[2]Such assumptions are made merely for the convenience of analysis, while the sensing methods to be proposed in this letter work even when the primary signals are correlated over samples and/or have a non-Gaussian distribution, as will be shown via simulation results.

[3]This occurs when e.g., the primary transmitter uses N_s antennas in a spatial multiplexing mode, and $N_s < M$; or more generally when there are a total of N_s independent inputs from the primary network, originating from one or more nodes.

[4]This could be due to the independent channels from the primary signal sources to the CR terminal.

By setting the first derivative of $\mathbf{A}$ to the all-zero matrix, the optimal $\mathbf{A}$ that maximizes $f(\mathbf{A})$ can be obtained. Equivalently, the MLE of $\mathbf{R}_x$ that maximizes $p(\mathbf{X}|\mathcal{H}_1, \mathbf{R}_x)$ is obtained as (manipulations omitted)

$$\hat{\mathbf{R}}_x = \frac{1}{N}\sum_{n=0}^{N-1} \mathbf{x}[n]\mathbf{x}^H[n]. \tag{11}$$

Note that the above MLE of $\mathbf{R}_x$ is identical to the sample covariance matrix defined in (2). Substituting (11) into (9) and using the identity $\mathrm{tr}(\mathbf{AB}) = \mathrm{tr}(\mathbf{BA})$ with $\mathrm{tr}(\cdot)$ denoting the matrix trace, we obtain

$$\ln p(\mathbf{X}|\mathcal{H}_1, \hat{\mathbf{R}}_x) = -\frac{MN}{2}\ln(2\pi) - \frac{N}{2}\ln\left(\det\left(\hat{\mathbf{R}}_x\right)\right) - \frac{MN}{2}. \tag{12}$$

Let the M eigenvalues of $\hat{\mathbf{R}}_x$ be denoted by $\boldsymbol{\lambda}_x = [\lambda_{1,x}, \ldots, \lambda_{M,x}]$. Hence, subtracting (8) from (12) gives the log-GLRT statistic

$$\ln L_G(\mathbf{X}) = \frac{MN}{2}\left\{\ln\left(\frac{1}{M}\sum_{m=1}^{M}\lambda_{m,x}\right) - \frac{1}{M}\ln\left(\prod_{m=1}^{M}\lambda_{m,x}\right)\right\}. \tag{13}$$

Finally, removing constant terms and using the monotonicity of the logarithm function gives the GLRT statistic

$$T_{\mathsf{AGM}}(\boldsymbol{\lambda}_x) = \frac{\frac{1}{M}\sum_m \lambda_{m,x}}{\left(\prod_m \lambda_{m,x}\right)^{1/M}} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma. \tag{14}$$

Note that the above test statistic depends only on the eigenvalues of the sample covariance matrix, $\boldsymbol{\lambda}_x$. This test statistic is the ratio of the arithmetic mean (AM) to the geometric mean (GM) of the eigenvalues. We thus call this detection algorithm the AGM (arithmetic to geometric mean) method.

B. σ^2 Known, $\mathbf{R}_s$ Unknown

In this subsection, a detector is obtained by assuming that σ^2 is known,[5] but $\mathbf{R}_s$ is unknown and thus has to be estimated in a GLRT. The LLF under $\mathcal{H}_0$ is given in (6) where σ^2 is now treated as a known parameter, and that under $\mathcal{H}_1$, conditioned on the unknown parameter $\mathbf{R}_s$, is

$$\begin{aligned}\ln p(\mathbf{X}|\mathcal{H}_1, \mathbf{R}_s) &= -\frac{MN}{2}\ln(2\pi) - \frac{N}{2}\ln\left(\det(\mathbf{R}_s + \sigma^2\mathbf{I})\right) \\ &- \frac{1}{2}\sum_{n=0}^{N-1}\mathbf{x}^H[n]\left(\mathbf{R}_s + \sigma^2\mathbf{I}\right)^{-1}\mathbf{x}[n].\end{aligned} \tag{15}$$

The MLE of $\mathbf{R}_s$ under $\mathcal{H}_1$ can be obtained as follows. First, like in the previous subsection, we introduce $\mathbf{A} = \mathbf{R}_x^{-1} = (\mathbf{R}_s + \sigma^2\mathbf{I})^{-1}$ so that $\ln p(\mathbf{X}|\mathcal{H}_1, \mathbf{R}_s)$ can be rewritten as $f(\mathbf{A})$ in (10). Since $\mathbf{R}_s \succeq 0$, it follows that $\mathbf{R}_x \succeq \sigma^2\mathbf{I}$ and hence $\mathbf{A} \preceq \frac{1}{\sigma^2}\mathbf{I}$. The MLE of $\mathbf{R}_s$ can be obtained from the MLE of $\mathbf{A}$ by solving the following *constrained* optimization problem over $\mathbf{A}$:

$$\begin{aligned}\text{Maximize} \quad & f(\mathbf{A}) && (16)\\ \text{Subject to} \quad & \mathbf{A} \succeq 0 && (17)\\ & \mathbf{A} \preceq \frac{1}{\sigma^2}\mathbf{I}. && (18)\end{aligned}$$

[5]Since this is a function of the receiver implementation at each sensor, it may be estimated separately from the sensing operation.

TABLE I
SUMMARY OF SENSING ALGORITHMS

Name	Test Statistic	Equation
Estimator-Correlator (E-C)	T_{EC}	(3)
Energy Detector (ED)	T_{ED}	(4)
AM/GM (AGM)	T_{AGM}	(14)
Signal-Subspace E-Values (SSE)	T_{SSE}	(21)

Since it has been earlier shown that $f(\mathbf{A})$ is a concave function of $\mathbf{A}$, and furthermore, the constraints in (17) and (18) specify a convex set of $\mathbf{A}$, it follows that the above optimization problem is convex. In the appendix, by applying the Karush-Kuhn-Tucker (KKT) optimality conditions [19], we show that the optimal $\mathbf{A}$ for the above problem can be obtained as

$$\mathbf{A}^* = \mathbf{U}_x \mathrm{Diag}\left(\min\left(\frac{1}{\lambda_{1,x}}, \frac{1}{\sigma^2}\right) \ldots \min\left(\frac{1}{\lambda_{M,x}}, \frac{1}{\sigma^2}\right)\right)\mathbf{U}_x^H \tag{19}$$

where $\mathrm{Diag}(\mathbf{x})$ denotes a diagonal matrix with the diagonal elements expressed in $\mathbf{x}$, while $\lambda_{m,x}$'s and $\mathbf{U}_x$ are obtained from the eigen-decomposition of $\hat{\mathbf{R}}_x$. Without loss of generality, we also assume from this point that the eigenvalues are ordered from largest to smallest, i.e., $\lambda_{1,x} \geq \lambda_{2,x} \geq \cdots \geq \lambda_{M,x}$. The MLE of $\mathbf{R}_s$ can then be obtained as

$$\hat{\mathbf{R}}_s = \mathbf{U}_x \mathrm{Diag}\left(\left(\lambda_{1,x} - \sigma^2\right)^+, \ldots, \left(\lambda_{M,x} - \sigma^2\right)^+\right)\mathbf{U}_x^H \tag{20}$$

where $(x)^+ = \max(x, 0)$. Substituting (20) into (15) and subtracting (6) (with known σ^2) from it yields the log-GLRT statistic as

$$T_{\mathsf{SSE}}(\boldsymbol{\lambda}_x) = \frac{Nm'}{2}\left[\frac{\mathsf{AM}(\boldsymbol{\lambda}_x^s)}{\sigma^2} - \ln\left(\frac{\mathsf{GM}(\boldsymbol{\lambda}_x^s)}{\sigma^2}\right) - 1\right] \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma \tag{21}$$

where m' corresponds to the largest m such that $\lambda_{m,x} > \sigma^2$, $\boldsymbol{\lambda}_x^s$ denotes the vector of signal-subspace eigenvalues of $\hat{\mathbf{R}}_x$, i.e., $\boldsymbol{\lambda}_x^s = [\lambda_{1,x}, \ldots, \lambda_{m',x}]$, and $\mathsf{AM}(\mathbf{x})$ and $\mathsf{GM}(\mathbf{x})$ denote the arithmetic mean and the geometric mean over the elements in a vector $\mathbf{x}$, respectively. We call this algorithm the SSE (signal-subspace eigenvalues) method.

IV. Simulation Results

In this section, we present simulation results to compare the performances of various sensing algorithms considered in this letter (which are summarized in Table I).

First, we consider an ideal case where a CR sensor with $M = 8$ antennas is to detect $Q = 3$ single-antenna primary signal sources, each carrying an equal-power and independent data stream. For each data stream, the transmitted primary signals are i.i.d. CSCG random variables, the same as assumed for the analysis in this letter. An independent Rayleigh flat-fading channel between each transmit-receive antenna pair is assumed. $1,000$ Monte Carlo simulations are carried out with each simulation consisting of $N = 10^4$ independent observation samples. For the ED, we consider that the estimated noise power is $\hat{\sigma}^2 = \alpha\sigma^2$, and α in dB, i.e., $10\log_{10}\alpha$ is uniformly distributed in an interval $[-B, B]$ [2], [3]. More specifically, for ED we test two cases: without noise uncertainty, i.e., $B = 0$, denoted as ED (0dB); and with 0.5 dB noise uncertainty, i.e.,

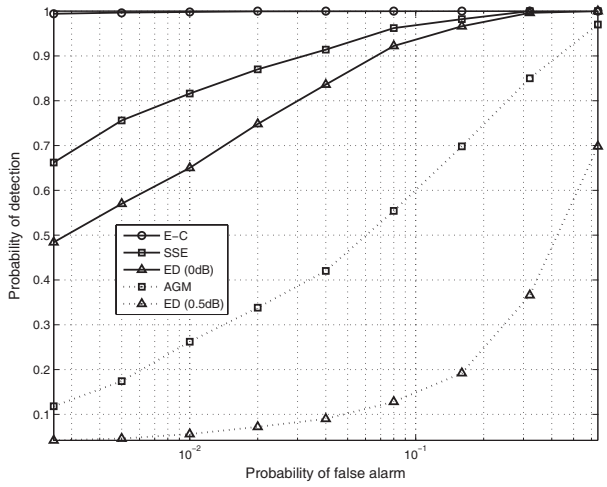

Fig. 1. ROC curves of various sensing algorithms: $M = 8$, $Q = 3$, SNR=-20dB, and $N = 10^4$.

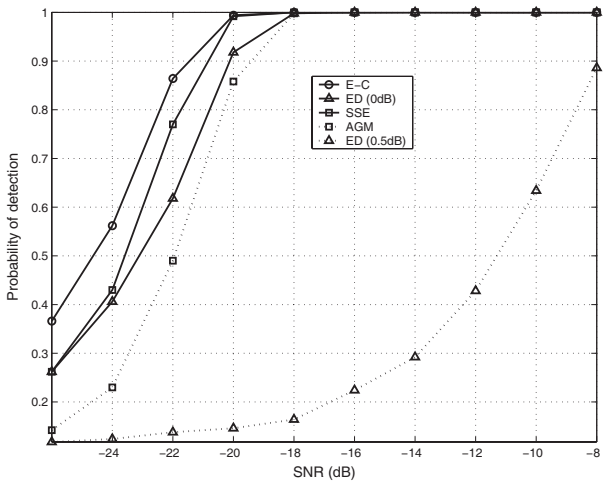

Fig. 2. Probability of detection for wireless microphone primary signals by various sensing algorithms: $P_{FA} = 0.1$, $M = 4$, $Q = 1$, and $N = 10^4$.

$B = 0.5$, denoted as ED (0.5dB). The received average signal-to-noise radio (SNR) per antenna is fixed at -20dB. Fig. 1 shows the Receiver Operating Characteristics (ROC) curves, each of which constitutes all the achievable probability pairs of P_D and P_{FA} for each sensing algorithm. It is observed that the E-C always performs the best since it is the optimal detector assuming perfect knowledge of the received primary signal covariance and the noise variance. If only the noise variance is perfectly known, the SSE performs better than the perfect ED without noise uncertainty. However, if the noise variance is unknown, even with only 0.5dB noise uncertainty, the ED performs substantially worse than the proposed AGM, which operates without knowing the noise variance.

Next, we consider a practical CR application scenario in a WRAN network, where a CR sensor with $M = 4$ antennas is to detect a single-source ($Q = 1$) baseband FM modulated wireless microphone signal (soft speaker) [20]. The sampling rate at the receiver is 6MHz (the same as the TV bandwidth in USA). The channel between each transmit-receive antenna pair is a multipath channel with 5 independent, equal-power taps, where each tap coefficient is generated from a CSCG distribution. The test threshold, γ, is chosen to achieve a target P_{FA} of 10%, as required in the IEEE 802.22 standard, and $N = 10^4$. In Fig. 2, P_D is plotted against the average received SNR for each sensing algorithm. Again, it is observed that the proposed eigenvalue-based methods (AGM and SSE) outperform the ED with and without noise variance uncertainty, respectively. It is also observed that the proposed methods work well for realistic channels and non-Gaussian primary signals.

V. Conclusion

This paper applies the well-known GLRT principle to the problem of spectrum sensing in a CR network. Under mild assumptions on the primary signal, the proposed eigenvalue-based algorithms are shown to perform better than the conventional energy detector, with or without noise power uncertainty. The price paid for the performance improvement over energy detection is higher implementation complexity because of the need to estimate the signal covariance matrix, and perform an eigen-decomposition on this estimate. Practically, the proposed algorithms enable shorter sensing intervals for given P_D and P_{FA}, or higher P_D/lower P_{FA} for a given sensing interval; these features are valuable for the successful development of wireless networks based on opportunistic spectrum access.

Appendix

In this appendix, the derivation of (19) is shown in details. By introducing the dual variables $\mathbf{B}$ and $\mathbf{C}$, where $\mathbf{B} \succeq 0$ and $\mathbf{C} \succeq 0$, corresponding to the constraint (17) and (18), respectively, the Lagrangian [19] of this constrained optimization problem can be written as

$$\mathcal{L}(\mathbf{A}, \mathbf{B}, \mathbf{C}) = f(\mathbf{A}) + \mathrm{tr}(\mathbf{BA}) - \mathrm{tr}\left(\mathbf{C}\left(\mathbf{A} - \frac{1}{\sigma^2}\mathbf{I}\right)\right). \quad (22)$$

Let $\mathbf{A}^*$ denote the optimal primal solution of the problem at hand, and $\mathbf{B}^*$ and $\mathbf{C}^*$ the optimal dual solutions for its dual problem. Since the problem at hand is convex and has a feasible solution set, Slater's condition holds and the duality gap is zero [19]. The following KKT conditions [19] must be satisfied by $\mathbf{A}^*$, $\mathbf{B}^*$, and $\mathbf{C}^*$ simultaneously:

$$\frac{\partial \mathcal{L}(\mathbf{A}^*, \mathbf{B}^*, \mathbf{C}^*)}{\partial \mathbf{A}} = f'(\mathbf{A}^*) + \mathbf{B}^* - \mathbf{C}^* = \mathbf{0} \quad (23)$$

$$\mathrm{tr}(\mathbf{B}^*\mathbf{A}^*) = 0 \quad (24)$$

$$\mathrm{tr}\left(\mathbf{C}^*\left(\mathbf{A}^* - \frac{1}{\sigma^2}\mathbf{I}\right)\right) = 0. \quad (25)$$

It can be verified that $\mathbf{A}^*$ given by (19) and the following $\mathbf{B}^*$ and $\mathbf{C}^*$ satisfy the above KKT conditions:

$$\mathbf{B}^* = \mathbf{0} \quad (26)$$

$$\mathbf{C}^* = \frac{N}{2}\mathbf{U}_x \mathrm{Diag}\left(\max\left(\lambda_{1,x}, \sigma^2\right) - \lambda_{1,x}, \ldots, \max\left(\lambda_{M,x}, \sigma^2\right) - \lambda_{M,x}\right)\mathbf{U}_x^H. \quad (27)$$

Since the KKT conditions are both necessary and sufficient for the optimality of the primal and dual solutions of a convex optimization problem, the proof thus follows.

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