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Multi Attribute Decision Making Strategy on Projection and Bidirectional Projection Measures of Interval Rough Neutrosophic Sets

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Abstract. In this paper, we define projection and bidirectional projection measures between interval rough neutrosophic sets and prove their basic properties. Then two new multi attribute decision making strategies are proposed based on interval rough neutrosophic projection and bidirectional projection measures respectively. Then the proposed methods are applied for solving multi attribute decision making problems. Finally, a numerical example is solved to show the feasibility, applicability and effectiveness of the proposed strategies.

Keywords: Projection measure, Bidirectional projection measure, Interval rough neutrosophic set, MADM problem.

1 Introduction

The concept of neutrosophic set[1, 2, 3, 4, 5] introduced by Smarandache is a generalization of crisp set[6], fuzzy set[7] and intuitionistic fuzzy set[8]. To use neutrosophic set in real fields, Wang et al. extended it to single valued neutrosophic set[9].

Broumi et al. introduced rough neutrosophic set[10, 11] by combining the concept of rough set[12] and neutrosophic set.

Broumi and Smarandache defined interval rough neutrosophic set[13] by combining the concept of rough set and interval neutrosophic set theory[14].

Projection measure is a very useful for solving decision making problems because it takes into account the distance as well as the included angle between points. Yue [15] studied projection based MADM problem in crisp environment.Yue also[16] presented a projection method to obtain weights of the experts in a group decision making problem. Xu and Da [17] and Xu [18] studied projection method for decision making in uncertain environment with preference information. Yang et al. [19] develop projection method for material selection in fuzzy environment. Xu and Hu [20] developed two projection based models for MADM in intuitionistic fuzzy and interval valued intuitionistic fuzzy environment. Zeng et al. [21] provided weighted projection algorithm for intuitionistic fuzzy MADM problems and interval-valued intuitionistic fuzzy MADM problems. Chen and Ye [22] developed the projection based model for solving MADM problem and applied it to select clay-bricks in construction field.

To overcome the shortcomings of the general projection measure Ye [23] introduced a bidirectional projection measure between single valued neutrosophic numbers and developed MADM method for selecting problems of mechanical design schemes under a single valued neutrosophic environment. Ye [24] also presented the bidirectional projection method for multiple attribute group decision making with neutrosophic numbers. Dey et al. [25] defined weighted projection measure with interval neutrosophic environment and applied it to solve MADM problems with interval valued neutrosophic information. Yue [26] proposed a projection based approach for partner selection in a group decision making problem with linguistic value and intuitionistic fuzzy information.Dey et al. [27] defined projection, bidirectional projection and hybrid projection measures between bipolar neutrosophic sets and presented bipolar neutrosophic projection based models for MADM problems. Pramanik et al. [28] defined projection and bidirectional projection measure between rough neutrosophic sets and proposed the decision making methods based on them.

Research gap MADM strategy using projection and bidirectional projection measures under interval rough neutrosophic environment.

Research questions

(i) Is it possible to define two new projection and bidirectional projection measure between interval rough neutrosophic sets?

(ii) Is it possible to develop two new MADM strategies based on the proposed measures in interval rough neutrosophic environment?

The objectives of the paper are

(i) To define two new projection and bidirectional projection measure between interval rough neutrosophic sets.

(ii) To develop two new MADM strategies based on the proposed measures in interval rough neutrosophic environment.

Contributions

(i) In this paper, we propose projection and bidirectional projection measures under interval rough neutrosophic environment.

(ii) In this paper, we develop two new MADM strategies based on the proposed measures in interval rough neutrosophic environment.

(iii) We also present numerical example to show the effectiveness and applicability of the proposed measures.

Rest of the paper is organized as follows: Section 2 describes preliminaries of neutrosophic number, SVNS, RNS and IRNS. Section 3 presents definitions and properties of proposed projection and bidirectional projection measure between IRNSs. Section 4 describes the MADM methods based on projection and bidirectional projection measures of IRNSs. In section 5 we describe a numerical example. Finally, section 6 presents the conclusion.

2 Preliminaries

In this Section, we provide some basic definitions regarding SVNSs, IRNSs which are useful in the paper.

2.1 Neutrosophic set:

In 1999, Smarandache gave the following definition of neutrosophic set(NS) [1].

Definition 2.1.1. Let X be a space of points (objects) with generic element in X denoted by x. A NS A in X is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A. The functions T_A , I_A and F_A are real standard or non-standard subsets of $(^-0,1^+)$ that is $T_A:X \to (^-0, 1^+)$, $I_A:X \to (^-0, 1^+)$ and $F_A:X \to (^-0, 1^+)$. It should be noted that there is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ i.e. $^-0 \le T_A(X) + I_A(X) + F_A(X) \le 3^+$

Definition 2.1.2: (complement)

The complement of a neutrosophic set A is denoted by C(A) and is defined by $T_{c(A)}(x) = \{1^+\} - T_A(x), I_{c(A)}(x) = \{1^+\} - I_A(x), F_{c(A)}(x) = \{1^+\} - F_A(x)$.

Definition 2.1.3: (Containment)

A neutrosophic set A is contained in the other neutrosophic set B, denoted by $A \subseteq B$ iff

$$\inf T_A(x) \le \inf T_B(x), \sup T_A(x) \le \sup T_B(x),$$
$$\inf I_A(x) \ge \inf I_B(x), \sup I_A(x) \ge \sup I_B(x),$$
$$\inf F_A(x) \ge \inf F_B(x), \sup F_A(x) \ge \sup F_B(x) \ \forall x \in X$$

Definition 2.1.4: (Single-valued neutrosophic set).

Let X be a universal space of points (objects) with a generic element of X denoted by x. A single valued neutrosophic set A is characterized by a truth membership function $T_A(x)$, a falsity membership function $F_A(x)$ and indeterminacy function $I_A(x)$ with

$$T_A(x), I_A(x) \text{ and } F_A(x) \in [0,1] \ \forall x \text{ in } X$$

When X is continuous, a SNVS S can be written as follows

$$A = \int_x < T_A(x), F_A(x), I_A(x) > /x \forall x \in X$$

and when X is discrete, a SVNS S can be written as follows

$$A = \sum < T_A(x), F_A(x), I_A(x) > /x \forall x \in X$$

For a SVNS S, $0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$.

Definition2.1.5:

The complement of a single valued neutrosophic set A is denoted by c(A) and is defined by $T_{c(A)}(x) = F_A(x)$, $I_{c(A)}(x) = 1 - I_A(x)$, $F_{c(A)}(x) = T_A(x)$.

Definition 2.1.6: A SVNS A is contained in the other SVNS B, denoted as $A \subseteq B$ iff,

$$T_A(x) \le T_B(x), I_A(x) \ge I_B(x)$$

and $F_A(x) \ge F_B(x) \forall x \in X$.

2.2Rough neutrosophic set

Rough neutrosophic sets [10, 11] are the generalization of rough fuzzy sets [29, 30] and rough intuitionistic fuzzy sets [31].

Definition 2.2.1:

Let Y be a non-null set and R be an equivalence relation on Y. Let P be neutrosophic set in Y with the membership function T_P , indeterminacy function I_P and non-membership function F_P . The lower and the upper approximations of P in the approximation (Y, R) denoted by are respectively defined as:

$\underline{N}(P) = << x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) > /$
$y \in [x]_R, x \in Y >$

and

$\overline{N}(P) = << x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x) > /$
$y \in [x]_R, x \in Y >$

where,

$T_{\underline{N}(P)}(x) = \wedge z \in [x]_R\, T_P(Y),$
$I_{\underline{N}(P)}(x) = \wedge z \in [x]_R\, I_P(Y),$
$F_{\underline{N}(P)}(x) = \wedge z \in [x]_R\, F_P(Y)$

and

$T_{\overline{N}(P)}(x) = \vee z \in [x]_R\, T_P(Y),$
$I_{\overline{N}(P)}(x) = \vee z \in [x]_R\, I_P(Y),$.
$F_{\overline{N}(P)}(x) = \vee z \in [x]_R\, F_P(Y)$

So,

$0 \le T_{\underline{N}(P)}(x) + I_{\underline{N}(P)}(x) + F_{\underline{N}(P)}(x) \le 3$

and

$0 \le T_{\overline{N}(P)}(x) + I_{\overline{N}(P)}(x) + F_{\overline{N}(P)}(x) \le 3$

Here $\vee$ and $\wedge$ denote "max" and "min" operators respectively, $T_P(y)$, $I_P(y)$ and $F_P(y)$ are the membership, indeterminacy and non-membership of Y with respect to P.

Thus NS mapping,

$\underline{N}, \overline{N} : N(Y) \to N(Y)$ are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set in (Y, R).

Definition 2.2.2 If $N(P) = (\underline{N}(P), \overline{N}(P))$ is a rough neutrosophic set in (Y, R), the rough complement of N(P) is the rough neutrosophic set denoted by

$\sim N(P) = ((\overline{N}(P))^C, (\underline{N}(P))^C)$

,where

$(\underline{N}(P))^C$ and $(\overline{N}(P))^C$

are the complements of neutrosophic sets $\underline{N}(P)$ and $\overline{N}(P)$ respectively.

2.3 Interval rough neutrosophic set

Interval neutrosophic rough set is the hybrid structure of rough sets and interval neutrosophic sets. According to Broumi and Smarandache interval neutrosophic roughset is the generalizations of interval valued intuitionistic fuzzy rough set.

Definition 2.3.1

Let R be an equivalence relation on the universal set U.Then the pair (U, R) is called a Pawlak approximationspace. An equivalence class of R containing x will bedenoted by $[x]_R$ for X $\in$ U, the lower and upper approximationof X with respect to (U, R) are denoted by respectively,

$\underline{R}X$ and $\overline{R}X$ and are defined by

$\underline{R}X = \{x \in U : [x]_R \subseteq X\},$

$\overline{R}X = \{x \in U : [x]_R \cap X \neq \emptyset\}.$

Now if $\underline{R}X = \overline{R}X$, then X is called definable; otherwise Xis called a rough set.

Definition 2.3.2

Let U be a universe and X, a rough set in U. An intuitionistic fuzzy rough set A in U is characterized by a membership function μA:U→ [0, 1] and non-membership functionν_A: U→ [0, 1] such that $\mu_A(\underline{R}X)$=1and $\nu_A(\underline{R}X) = 0$

ie, $[\mu_A(x), \nu_A(x)] = [1,0]$ if $x \in (\underline{R}X)$ and $\mu_A(U - \overline{R}X) = 0$, $\nu_A(U - \overline{R}X) = 1$

ie,

$0 \le \mu_A(\overline{R}X - \underline{R}X) + \nu_A(\overline{R}X - \underline{R}X) \le 1$

Definition 2.3.3

Assume that, (U, R) be a Pawlak approximation space, for an interval neutrosophic set

$A = \{<x, [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)]> : x \in U\}$

The lower approximation $\underline{A}_R$ and the upper approximation $\overline{A}_R$ of A in the Pawlak approximation space (U, R) are expressed as follows:

$\underline{A}_R = \{<x, [\wedge_{y\in[x]_R} \{T_A^L(y)\}, {}_{y\ [x]} \{T_A(y)\}],$
$[\vee_{y\in[x]_R} \{I_A^L(y)\}, {}_{y\ [x]} \{I_A(y)\}],$
$[\vee_{y\in[x]_R} \{F_A^L(y)\}, \vee_{y\ [x]} \{F_A(y)\}]> : x \in U\}$

$\overline{A}_R = \{<x, [\vee_{y\in[x]_R} \{T_A^L(y)\}, {}_{y\ [x]} \{T_A(y)\}],$
$[\wedge_{y\in[x]_R} \{I_A^L(y)\}, {}_{y\ [x]} \{I_A(y)\}],$
$[\wedge_{y\in[x]_R} \{F_A^L(y)\}, \wedge_{y\ [x]} \{F_A(y)\}]> : x \in U\}$

The symbols $\wedge$ and $\vee$ indicate "min" and "max" operators respectively. R denotes an equivalence relation for interval neutrosophic set A. Here $[x]_R$ is the equivalence class of the element x. It is obvious that

$[\wedge_{y\in[x]_R} \{T_A(y)\}, \wedge_{y\ [x]} \{T_A^L(y)\}] \subset [0,1],$
$[\vee_{y\in[x]_R} \{I_A^U(y)\}, \vee_{y\ [x]} \{I_A(y)\}] \subset [0,1],$
$[\vee_{y\in[x]_R} \{F_A^U(y)\}, \vee_{y\ [x]} \{F_A(y)\}] \subset [0,1].$

and $0 \le \wedge_{y\in[x]_R} \{T_A^U(y)\} + \vee_{y\in[x]_R} \{I_A^U(y)\} + \vee_{y\in[x]_R} \{F_A^U(y)\} \le 3$

Then $\underline{A}_R$ is an interval neutrosophic set (INS)

Similarly, we have

$$[\vee_{y\in[x]_R}\{T_A^L(y)\},\vee_{y\in[x]_R}\{T_A^U(y)\}]\subset[0,1],$$
$$[\wedge_{y\in[x]_R}\{I_A^L(y)\},\wedge_{y\in[x]_R}\{I_A^U(y)\}]\subset[0,1],$$
$$[\wedge_{y\in[x]_R}\{F_A^L(y)\},\wedge_{y\in[x]_R}\{F_A^U(y)\}]\subset[0,1]$$

and

$$0\le\ \vee_{y\in[x]_R}\{T_A^U(y)\}+\wedge_{y\in[x]_R}\{I_A^U(y)\}+\wedge_{y\in[x]_R}\{F_A^U(y)\}]\le 3$$

Then $\underline{A}_R$ is an interval neutrosophic set.

If $\underline{A}_R = \overline{A}_R$ then A is a definable set, otherwise A is an interval valued neutrosophic rough set. Here, $\underline{A}_R$ and $\overline{A}_R$ are called the lower and upper approximations of interval neutrosophic set with respect to approximation space (U,R) respectively. $\underline{A}_R$ and $\overline{A}_R$ are simply denoted by $\underline{A}$ and $\overline{A}$ respectively.

3 Projection and Bidirectional projection measure of interval rough neutrosophic sets :

Existing projection and bidirectional projection measure does not deal with interval rough neutrosophic set(IRNS)s. Therefore, a new projection and bidirectional projection measure between IRNSs is proposed.

Assume that there are two IRNSs

$$\underline{M}=\{<x_i,([\underline{T}_{iM}^-,\underline{T}_{iM}^+],[\underline{I}_{iM}^-,\underline{I}_{iM}^+],[\underline{F}_{iM}^-,\underline{F}_{iM}^+],$$
$$[\overline{T}_{iM}^-,\overline{T}_{iM}^+],[\overline{I}_{iM}^-,\overline{I}_{iM}^+],[\overline{F}_{iM}^-,\overline{F}_{iM}^+]>:i=1,2,...,n\}$$

and

$$\underline{N}=\{<x_i,[\underline{T}_{iN}^-,\underline{T}_{iN}^+],[\underline{I}_{iN}^-,\underline{I}_{iN}^+],[\underline{F}_{iN}^-,\underline{F}_{iN}^+],$$
$$[\overline{T}_{iN}^-,\overline{T}_{iN}^+],[\overline{I}_{iN}^-,\overline{I}_{iN}^+],[\overline{F}_{iN}^-,\overline{F}_{iN}^+]>:i=1,2,...,n\}$$

Then the inner product of M and N denoted by M.N can be defined as

$$M.N=\sum_{i=1}^{n}[\underline{T}_{iM}^-.\underline{T}_{iN}^-+\underline{T}_{iM}^+\underline{T}_{iN}^++\underline{I}_{iM}^-\underline{I}_{iN}^-+\underline{I}_{iM}^+\underline{I}_{iN}^+$$
$$+\underline{F}_{iM}^-\underline{F}_{iN}^-+\underline{F}_{iM}^+\underline{F}_{iN}^++\overline{T}_{iM}^-\overline{T}_{iN}^-+\overline{T}_{iM}^+\overline{T}_{iN}^+$$
$$+\overline{I}_{iM}^-\overline{I}_{iN}^-+\overline{I}_{iM}^+\overline{I}_{iN}^++\overline{F}_{iM}^-\overline{F}_{iN}^-+\overline{F}_{iM}^+\overline{F}_{iN}^+]$$

The modulus of M can be defined as

$$\|M\|=\sqrt{\sum_{i=1}^{n}\begin{bmatrix}(\underline{T}_{iM}^-)^2+(\underline{T}_{iM}^+)^2+(\underline{I}_{iM}^-)^2+(\underline{I}_{iM}^+)^2\\+(\underline{F}_{iM}^-)^2+(\underline{F}_{iM}^+)^2+(\overline{T}_{iM}^-)^2+(\overline{T}_{iM}^+)^2\\+(\overline{I}_{iM}^-)^2+(\overline{I}_{iM}^+)^2+(\overline{F}_{iM}^-)^2+(\overline{F}_{iM}^+)\end{bmatrix}}$$

and the modulus of N can be defined as

$$\|N\|=\sqrt{\sum_{i=1}^{n}\begin{bmatrix}(\underline{T}_{iN}^-)^2+(\underline{T}_{iN}^+)^2+(\underline{I}_{iN}^-)^2+(\underline{I}_{iN}^+)^2\\+(\underline{F}_{iN}^-)^2+(\underline{F}_{iN}^+)^2+(\overline{T}_{iN}^-)^2+(\overline{T}_{iN}^+)^2\\+(\overline{I}_{iN}^-)^2+(\overline{I}_{iN}^+)^2+(\overline{F}_{iN}^-)^2+(\overline{F}_{iN}^+)\end{bmatrix}}$$

Definition4.1.The projection of M on N can be defined as

$$\mathrm{Proj}(M)_N=\frac{1}{\|N\|}M.N.$$

Definition4.2.The bidirectional projection measure between the RNSs M and N is defined as

$$\mathrm{BProj}(M,N)=\frac{1}{1+\big|\|M\|-\|N\|\big|M.N}$$
$$=\frac{\|M\|\|N\|}{\|M\|\|N\|+\big|\|M\|-\|N\|\big|M.N}$$

Here also the bidirectional projection measure satisfies the following properties :

(1) BProj(M,N) = BProj(N,M);

(2) $0\le \mathit{BProj(M,N)}\le 1$;

(3) BProj(M,N) = 1, iff M = N.

Proof:

(i)

$$\mathrm{BProj}(M,N)$$
$$=\frac{1}{1+\big|\|M\|-\|N\|\big|M.N}$$
$$=\frac{1}{1+\big|\|N\|-\|M\|\big|N.M}$$
$$=\mathrm{BProj}(N,M)$$

(ii) As

$$\frac{1}{1+\big|\|M\|-\|N\|\big|M.N}\ge 0$$

and

$$\frac{1}{1+\big|\|M\|-\|N\|\big|M.N}\le 1$$

so, $0\le \mathrm{BProj}(M,N)\le 1$;

(iii) If M=N then

$$\mathrm{BProj}(M,N)$$
$$=\mathrm{BProj}(M,M)$$
$$=\frac{1}{1+\big|\|M\|-\|M\|\big|M.M}$$
$$=1$$

4. Projection And Bidirectional Projection Based Decision Making Methods For MADM Problems With Interval Rough Neutrosophic Information

In this section, we develop projection and bidirectional projection based decision making models to solve MADM problems with interval rough neutrosophic information. Consider $C=\{C_1, ...,C_m\}$ be the set of attributes and $A=\{A_1,......, A_n\}$ be a set of alternatives. Now we provide two algorithms for MADM problems involving interval rough neutrosophic information.

4.1. Algorithm 1.(see Fig 1)

Step 1. The value of alternative A_i(i=1,…..,n) for the attribute C_j(j=1,……,m) is evaluated by the decision maker in terms of IRNSs and the interval rough neutrosophic decision matrix is constructed as:

$$D = \langle z_{ij} \rangle_{n\times m} = \begin{pmatrix} z_{11}\ z_{12} \ldots\ldots\ldots z_{1m} \\ z_{21}\ z_{22} \ldots\ldots\ldots z_{2m} \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ z_{n1}\ z_{n2} \ldots\ldots\ldots z_{nm} \end{pmatrix}$$

where

$$\underline{z_{ij}} = <(\underline{[T^-_{iM}, T^+_{iM}]}, \underline{[I^-_{iM}, I^+_{iM}]}, \underline{[F^-_{iM}, F^+_{iM}]},$$
$$[T^-_{iM}, T^+_{iM}], [I^-_{iM}, I^+_{iM}], [F^-_{iM}, F^+_{iM}])>$$

with

$$0 \le \vee_{y\in[x]_R} \{T^U_A(y)\} + \wedge_{y\in[x]_R} \{I^U_A(y)\} + \wedge_{y\in[x]_R} \{F^U_A(y)\}] \le 3$$

Step 2. Calculate the weighted alternative decision matrix

For the attribute C_j (j=1,……,m) the weight vector of attribute is considered as : $W = (w_1, w_2, \ldots, w_m)$ with

$$w_j \ge 0 \quad \text{and} \quad \sum_{i=1}^{n} w_j = 1$$

On calculating

$$s_{ij} = <(\underline{[w_j T^-_{iM}, w_j T^+_{iM}]}, \underline{[w_j I^-_{iM}, w_j I^+_{iM}]},$$
$$\underline{[w_j F^-_{iM}, w_j F^+_{iM}]}, [w_j T^-_{iM}, w_j T^+_{iM}],$$
$$[w_j I^-_{iM}, w_j I^+_{iM}], [w_j F^-_{iM}, w_j F^+_{iM}])>$$

for i=1, 2, … , n and j=1, 2, … , m , we obtain the weighted alternative decision matrix

$$S = \langle s_{ij} \rangle_{n\times m} = \begin{pmatrix} s_{11}\ s_{12} \ldots\ldots\ldots s_{1m} \\ s_{21}\ s_{22} \ldots\ldots\ldots s_{2m} \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ s_{n1}\ s_{n2} \ldots\ldots\ldots s_{nm} \end{pmatrix}$$

Step 3. Determine the ideal solution S^*.

For benefit type attribute,

$$S^* = \{(\min_i \underline{T_{ij}}, \max_i \underline{I_{ij}}, \max_i \underline{F_{ij}}), (\max_i \overline{T_{ij}}, \min_i \overline{I_{ij}}, \min_i \overline{F_{ij}})\}$$

For cost type attribute,

$$S^* = \{(\max_i \underline{T_{ij}}, \min_i \underline{I_{ij}}, \min_i \underline{F_{ij}}), (\min_i \overline{T_{ij}}, \max_i \overline{I_{ij}}, \max_i \overline{F_{ij}})\}$$

Step 4. Compute the projection measure between S^* and $Z_i = \langle Z_{ij} \rangle_{nxm}$ for all i = 1, ….., n and j = 1, ….., m.

Step 5. Ranking of alternatives is prepared based on the values of projection measure. The highest value reflects the best alternatives.

Step 6. End.

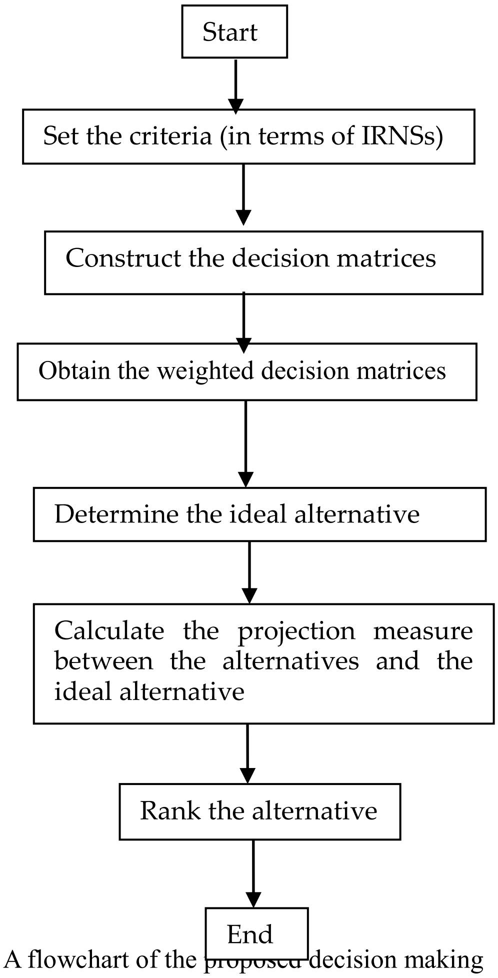

Fig 1. A flowchart of the proposed decision making method

4.2. Algorithm 2. (see Fig 2)

Step 1. The value of alternative A_i(i=1,…..,n) for the attribute C_j(j=1,……,m) is evaluated by the decision maker in terms of IRNSs and the interval rough neutrosophic decision matrix is constructed as:

$$D = \langle z_{ij} \rangle_{n\times m} = \begin{pmatrix} z_{11}\ z_{12} \ldots\ldots\ldots z_{1m} \\ z_{21}\ z_{22} \ldots\ldots\ldots z_{2m} \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ z_{n1}\ z_{n2} \ldots\ldots\ldots z_{nm} \end{pmatrix}$$

where

$$\underline{z_{ij}} = <(\underline{[T^-_{iM}, T^+_{iM}]}, \underline{[I^-_{iM}, I^+_{iM}]}, \underline{[F^-_{iM}, F^+_{iM}]},$$
$$[T^-_{iM}, T^+_{iM}], [I^-_{iM}, I^+_{iM}], [F^-_{iM}, F^+_{iM}])>$$

with

$$0 \le \vee_{y \in [x]_R} \{T_A^U(y)\} + \wedge_{y \in [x]_R} \{I_A^U(y)\} + \wedge_{y \in [x]_R} \{F_A^U(y)\}] \le 3$$

Step 2. Calculate the weighted alternative decision matrix

For the attribute C_j (j=1,……,m) the weight vector of attribute is considered as : $W = (w_1, w_2, \dots, w_m)$ with

$$w_j \ge 0 \quad \text{and} \quad \sum_{i=1}^{n} w_j = 1$$

On calculating

$$s_{ij} = <([w_j \underline{T}_{iM}^-, w_j \underline{T}_{iM}^+], [w_j \underline{I}_{iM}^-, w_j \underline{I}_{iM}^+], [w_j \underline{F}_{iM}^-, w_j \underline{F}_{iM}^+], [w_j \overline{T}_{iM}^-, w_j \overline{T}_{iM}^+], [w_j \overline{I}_{iM}^-, w_j \overline{I}_{iM}^+], [w_j \overline{F}_{iM}^-, w_j \overline{F}_{iM}^+])>$$

for i=1, 2, … , n and j=1, 2, … , m , we obtain the weighted alternative decision matrix

$$S = < s_{ij} >_{n \times m} = \begin{pmatrix} s_{11}\ s_{12} \ldots\ldots\ldots s_{1m} \\ s_{21}\ s_{22} \ldots\ldots\ldots s_{2m} \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots \\ s_{n1}\ s_{n2} \ldots\ldots\ldots s_{nm} \end{pmatrix}$$

Step 3. Determine the ideal solution S^*.

For benefit type attribute,

$$S^* = \{(\min_i \underline{T}_{ij}, \max_i \underline{I}_{ij}, \max_i \underline{F}_{ij}), (\max_i \overline{T}_{ij}, \min_i \overline{I}_{ij}, \min_i \overline{F}_{ij})\}$$

For cost type attribute,

$$S^* = \{(\max_i \underline{T}_{ij}, \min_i \underline{I}_{ij}, \min_i \underline{F}_{ij}), (\min_i \overline{T}_{ij}, \max_i \overline{I}_{ij}, \max_i \overline{F}_{ij})\}$$

.

Step 4. Compute the bidirectional projection measure between S^* and $Z_i = <Z_{ij}>_{nxm}$ for all i = 1, ….., n and j = 1, ….., m.

Step 5. Ranking of alternatives is prepared based on the values of bidirectional projection measure. The highest value reflects the best alternatives.

Step 6. End.

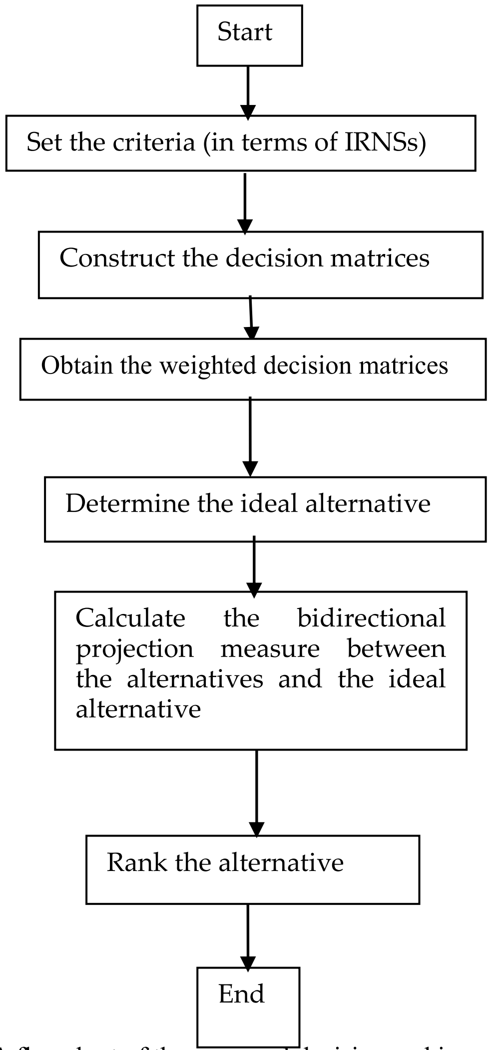

Fig 2. A flowchart of the proposed decision making method

5. A Numerical Example:

Assume that a decision maker intends to select the most suitable laptop for random use from the three initially chosen laptops (A_1, A_2, A_3) by considering four attributes namely: features C_1, reasonable price C_2, customer care C_3, risk factor C_4. Based on the proposed approach discussed in section 5, the considered problem is solved by the following steps:

Step1: Construct the decision matrix with interval rough neutrosophic number

The decision maker construct the decision matrix with respect to the three alternatives and four attributes in terms of interval rough neutrosophic number.

	C_1	C_2	C_3	C_4
A_1	<([.6, .7], [.3, .5], [.3, .4]), ([.8, .9], [.1, .3], [.1, .2])>	<([.5, .7], [.3, .4], [.1, .2]), ([.7, .9], [.3, .5], [.3, .4])>	<([.5, .6], [.4, .5], [.4, .6]), ([.7, .8], [.2, .4], [.3, .4])>	<([.8, .9], [.3, .4], [.5, .6]), ([.7, .8], [.3, .5], [.3, .5])>
A_2	<([.7, .8], [.2, .3], [.0, .2]), ([.7, .9], [.1, .2], [.1, .2])>	<([.6, .7], [.1, .2], [.0, .2]), ([.6, .7], [.1, .3], [.1, .3])>	<([.5, .7], [.2, .3], [.1, .2]), ([.6, .9], [.3, .5], [.2 .4])>	<([.7, .8], [.3, .5], [.1, .3]), ([.5, .7], [.5, .6], [.2, .3])>
A_3	<([.6, .7], [.3, .4], [.0, .3]), ([.6, .9], [.1, .2], [.1, .2])>	<([.5, .7], [.2, .4], [.2, .4]), ([.6, .8], [.1, .3], [.1, .2])>	<([.6, .8], [.2, .4], [.3, .4]), ([.6, .8], [.2, .5], [.3, .5])>	<([.4, .7], [.2, .4], [.4, .5]), ([.5, .8], [.2, .5], [.0, .2])>

Step 2: The weight vectors considered by the decision maker are 0.35, 0.25, 0.25 and 0.15 respectively. The weighted decision matrix is:

	C_1	C_2	C_3	C_4
S_1	<([0.21,0.245], [0.105,0.175], [0.105,0.14]), ([0.28,0.315], [0.035,0.105], [0.035,0.07])>	<([0.125,0.175], [0.075,0.1], [0.025,0.05]), ([0.175,0.225], [0.075,0.125], [0.075,0.1])>	<([0.125,0.15], [0.1,0.125], [0.1,0.15]), ([0.175,0.2], [0.05,0.1], [0.075,0.1])>	<([0.12,0.135], [0.045,0.06], [0.075,0.09]), ([0.105,0.12], [0.045,0.075], [0.045,0.75])>
S_2	<([0.245,0.28], [0.07,0.105], [0.0,0.07]), ([0.245,0.315], [0.035,0.07], [0.035,0.07])>	<([0.15,0.175], [0.025,0.05], [0.0,0.05]), ([0.15,0.175], [0.025,0.075], [0.025,0.075])>	<([0.125,0.175], [0.05,0.075], [0.025,0.05]), ([0.15,0.225], [0.075,0.125], [0.05,0.1])>	<([0.105,0.12], [0.045,0.75], [0.015,0.045]), ([0.075,0.105], [0.075,0.09], [0.03,0.045])>
S_3	<([0.21,0.245], [0.105,0.14], [0.0,0.105]), ([0.21,0.315], [0.035,0.7], [0.035,0.7])>	<([0.125,0.175], [0.05,0.1], [0.05,0.1]), ([0.15,0.2], [0.025,0.075], [0.025,0.05])>	<([0.15,0.2], [0.05,0.1], [0.075,0.1]), ([0.15,0.2], [0.05,0.125], [0.075,0.125])>	<([0.06,0.105], [0.03,0.06], [0.06,0.075]), ([0.075,0.12], [0.03,0.075], [0.0,0.03])>

Step3: Determine the benefit type attribute and cost type attribute

Here three benefit type attributes C_1, C_2, C_3 and one cost type attribute C_4. We calculate the ideal alternative as follows:

$$S^* = \{<([.21,.245],[.07,.175],[.105,.14]), ([.28,.315],[.035,.07],[.035,.07])>,$$
$$<([.15,.175],[.075,.1],[.05,.1]), ([.175,.225],[.025,.075],[.025,.05])>,$$
$$<([.15,.15],[.1,.1],[.1,.1]), ([.175,.225],[.075,.125],[.075,.125])>,$$
$$<([.12,.135],[.03,.06],[.015,.045]), ([.075,.105],[.075,.09],[.045,.075])>)>\}$$

Step4:Calculate the projection and bidirectional projection measure of the alternatives

$\|S_1\| = 0.918273,$

$\|S_2\| = 0.829533,$

$\|S_3\| = 0.832331.$

$\|S^*\| = 0.818175.$

$S_1.S^* = 0.815425,$

$S_2.S^* = 0.563137,$

$S_3.S^* = 0.7337.$

$\operatorname{Proj}(S_1)_{S^*} = 0.99663886,$

$\operatorname{Proj}(S_2)_{S^*} = 0.68828490,$

$\operatorname{Proj}(S_3)_{S^*} = 0.89675192.$

$\Rightarrow \operatorname{Proj}(S_1)_{S^*} > \operatorname{Proj}(S_3)_{S^*} > \operatorname{Proj}(S_2)_{S^*}.$

$\operatorname{BProj}(S_1, S^*) = 0.92453705,$

$\operatorname{BProj}(S_2, S^*) = 0.99364454,$

$\operatorname{BProj}(S_3, S^*) = 0.98972051.$

$\Rightarrow \operatorname{BProj}(S_2, S^*) > \operatorname{BProj}(S_3, S^*) > \operatorname{BProj}(S_1, S^*).$

Step5: Rank the alternatives

Ranking of alternatives is prepared based on the descending order of projection and bidirectional measures. The highest value reflects the best alternatives.

Hence, according to the projection measure, the laptop A_1 is the best alternative and according to the bidirectional projection measure, the laptop A_2 is the best alternative. As bidirectional projection measure gives better result than projection measure, so A_2 is the best laptop for random use.

6. Comparative study and discussions:

Mondal and Pramanik study the MADM method in interval rough neutrosophic environment using cosine, dice and Jaccard similarity measure [32]. We take the same problem and solve the problem using projection and bidirectional projection measure based decision making method. In the existing methods, S_2 is the best alternatives. But in new method S_1 is the best alternative.

7. Conclusion:

In this paper, we have defined projection measure, weighted projection measure, bidirectional projection measure, weighted bidirectional projection measure between interval rough neutrosophic sets. We have also proved their basic properties. We have developed two new MADM strategies based on the proposed projection and bidirectional projection measures respectively. Finally, we have solved a numerical example to demonstrate the feasiblity, applicability and effectiveness of the proposed strategies. The proposed strategies can be applied to solve different MADM problems such as teacher selection [33, 34, 35], school selection [36], weaver selection [37, 38, 39], brick field selection [40, 41], logistics center location selection [42, 43], data mining [44] etc. The proposed strategies can also be extended for MAGDM in interval rough neutrosophic environment.

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