Multi Axle Order Tracking with the Vold-Kalman Tracking Filter

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The Vold-Kalman filter allows for the high performance simultaneous tracking of orders in systems with multiple independent axles. The interactions associated with close and crossing orders give rise to beating and complex psychoacoustic phenomena. With this new filter and using multiple tachometer references, waveforms as well as amplitude and phase may be extracted without the beating interactions that are associated with conventional methods. The Vold-Kalman filter provides several filter shapes for optimum resolution and stopband suppression. Orders extracted as waveforms have no phase bias and may, therefore, be used in synthesis and tailoring.

Vold and Leuridan[1] in 1993 introduced an algorithm for high resolution, slew rate independent order tracking based on the concepts of Kalman filters.[9,10] The algorithm has been highly successful as implemented in a commercial software system in solving data analysis problems previously intractable with other analysis methods. At the same time certain deficiencies have surfaced, prompting the development of an improved formulation. This article presents the new Vold-Kalman algorithm and some of its applications in the Brüel & Kjær PULSE, Multi-Analyzer system.[5]

Order tracking is the art and science of extracting the sinusoidal content of measurements from acousto-mechanical systems under periodic loading. Order tracking is used for troubleshooting, design and synthesis.

Each periodic loading produces sinusoidal overtones, or orders, or harmonics, at frequencies that are multiples of that for the fundamental tone. The orders may be regarded as amplitude modulated carrier waves that frequency hop. Many practical systems have multiple axles that may run coherently through locked up transmissions, may be partially related through belt slippage and control loops, or may run independently, such as when a cooling fan cycles in an engine compartment.

The Vold-Kalman algorithm allows for the simultaneous estimation of multiple orders, effectively decoupling close and crossing orders. This is especially important for acoustics applications, where order crossings cause transient beating events. The new algorithm allows for a much wider range of filter shapes, such that signals with sideband modulations are processed with high fidelity. Finally, systems subject to radical RPM changes, such as transmissions, are also tracked through the transient events associated with abrupt changes in inertia and boundary conditions. The order functions are extracted without phase error, and may then be used in synthesis applications for sound quality and laboratory simulations.

Multi Axle Order Tracking

Typical machinery with rotating components will have multiple axles that interact in different modes. Axles may have coherent motion through locked up transmissions and crankshafts or they may act independently, as in the case of cooling fans running in an engine compartment with an operating combustion engine. There is also the weakly coherent motion where transmission ratios change – examples would be torque converters at low speeds and conical belt drive transmissions.

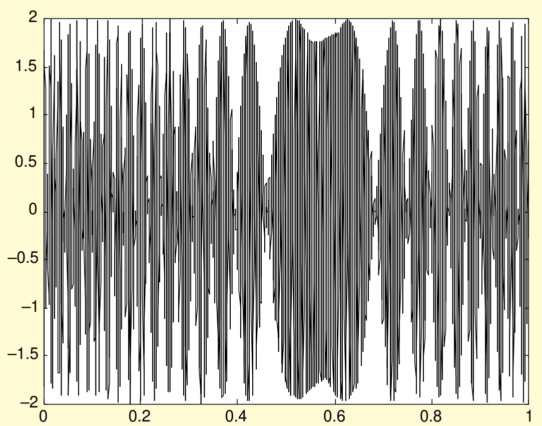

Figure 1. Two constant amplitude sine waves crossing in frequency.

Beating Phenomena

Sine waves that are close in frequency interact in a pattern called beating, as is easily seen in the trigonometric identity:

$$\sin(x) + \cos(y) \equiv 2\cos\left(\frac{x-y}{2}\right)\sin\left(\frac{x+y}{2}\right) \tag{1}$$

which says that the sum of two sine waves can be looked upon as a slowly modulated high frequency wave. A typical pattern of two constant amplitude sine waves which are crossing in frequency is shown in Figure 1. The modulation in this example is rapid until the sines are almost identical in frequency, the waves stay in phase around 0.6 sec then start oscillating again.

Order tracking methods all have resolution characteristics. So, they all have a point at which close sine waves are not distinguishable unless explicit provisions are made to account for multiple signal sources. The general principle for separating close and crossing orders is to perform a simultaneous estimation of the orders, supported by constraints to distribute the total signal energy between the orders. This has been accomplished in the new Vold-Kalman tracking filter[3,4] and also in a new method based on a time variant zoom transform.[7]

Fundamental Notions and Equations

Mechanical systems under periodic loading, such as those with one or more rotating shafts will respond in measurements with a superposition of sine waves whose frequencies are integer multiples of the fundamental excitation frequencies. Notice that this also applies to the limit cycle response of nonlinear systems. As the periodic loadings change their speed, the responses will change their frequencies accordingly. Since mechanical systems normally have transfer characteristics dependent upon frequency, the amplitude and phase of these sine waves will typically also change as the periodic loadings change their speed. The sine waves whose frequencies are constant multiples of an underlying periodic loading are said to be harmonics or orders of that loading, and also when the multiple is fractional due to gears or belt drives.

It is often helpful to visualize an order as an amplitude modulated radio signal. The underlying sine wave whose frequency is a multiple of the fundamental periodic phenomenon would be the carrier wave, while the slowly varying amplitude and phase function that modulates the carrier wave is the radio program. A radio receiver demodulates the signal by removing the carrier wave and playing the modulation function which is also known as the (complex) envelope. Now, in mechanical systems, the carrier wave may continually change its frequency, making the orders similar to amplitude modulated spread spectrum radio signals.

The goal of order tracking is to extract selected orders in terms of amplitude and phase or as real time series. These entities will in general be estimated as functions of time to allow for any pattern of speed or axle RPM variations.

First Order Phasors and Complex Envelopes. The generic carrier wave of an order is called a first order complex phasor, and is a complex oscillator with an instantaneous frequency proportional to a constant multiple of the underlying axle speed, as in the equation:

$$\Theta_k(t) = \exp\left(2\pi k i \int_0^t f(u)\,du\right) \tag{2}$$

where the integral of frequency gives the angle traveled by the axle up to the current time. Note that the phasor is always on the complex unit circle.

The amplitude modulated complex order is now given as the product $A_k(t)\Theta_k(t)$, where $A_k(t)$ is the complex envelope. These complex orders must occur in complex conjugate pairs to sum to a real signal, such that the total superposition $X(t)$ of orders relative to an axle can be written as:

$$X(t) = \sum_k A_k(t)\Theta_k(t) \tag{3}$$

where k runs over all positive and negative multiples of the underlying axle speed.

Time Variant Zoom. The expression for the phasor Equation (2) implies the functional relationship

$$\Theta_k(t)\Theta_j(t) = \Theta_{k+j}(t) \tag{4}$$

In particular, inspection of Equation (2) shows that $\Theta_0(t) \equiv 1$ When the axle speed has been estimated as a function of time, Equations (3) and (4) show that we can center a designated order at DC (zero frequency) by the *time variant zoom transform*

$$\Theta_{-j}(t)\,X(t) = A_j(t) + \sum_{k \neq j} A_k(t)\,\Theta_{k-j}(t) \tag{5}$$

The low frequency complex envelope $A_j(t)$ has now been straightened and can, for example, be extracted in the time domain by any suitable lowpass filter. This super-heterodyning process is similar to the tracking of an amplitude modulated spread spectrum radio source.

Vold-Kalman Filter

The basic idea behind the Vold-Kalman filter is to define local constraints which state that the unknown complex envelopes are smooth and that the sum of the orders should approximate the total measured signal. The smoothness condition is called the *structural equation*, and the relationship with the measured data is called the *data equation*. The somewhat ambiguous notation $X(n\Delta t) \equiv X(n)$, where Δt is the sampling time increment, is used here to simplify the mathematical exposition.

Structural Equations. The complex envelope is the low frequency modulation of the carrier wave. Low frequency entails smoothness, and one sufficient condition for smoothness is that the function locally can be represented by a low order polynomial. This condition can be expressed in continuous time using multiple differentiation as

$$\frac{d^s A_j(t)}{dt^s} = \varepsilon(t) \tag{6}$$

where $\varepsilon(t)$ represents higher order terms. The same constraint can be applied to sampled data by using the difference operator such that

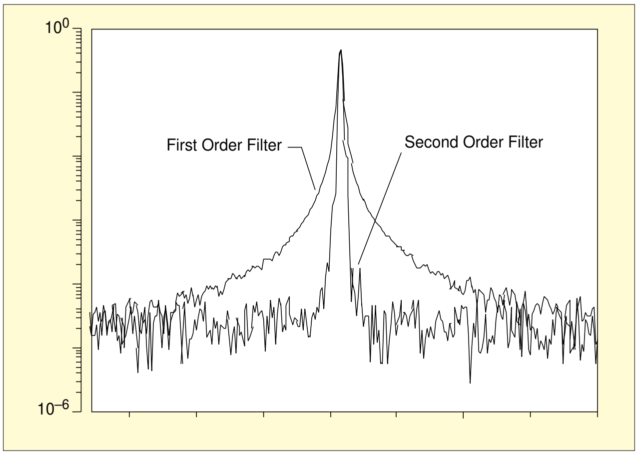

Figure 2. Comparison of constant RPM first and second order Vold-Kalman filters on white noise.

$$\nabla^s A_j(n) = \tilde{\varepsilon}(n) \tag{7}$$

Note that the difference operator of a given order annihilates all polynomials of one order less. Equation (7) is called the *structural equation* when $\tilde{\varepsilon}(n)$ is regarded as noise or error. This complex equation will be satisfied in an approximate sense for all discrete time points in the data set. To see the effect of filter order on the frequency response of the Vold-Kalman filter, refer to Figure 2, where a constant frequency filter was applied to white noise.

The structural difference equation in the first generation algorithm is a real second order equation for the real modulated waveform

$$X_j(n) = 2\,\mathrm{Re}(A_j(n)\Theta_j(n)) \tag{8}$$

that requires a demodulation through the Vandermonde equations to find the amplitude and phase, see Reference 1. This operator also only annihilates the constant polynomial.

Data Equation. The structural equation only enforces the smoothness conditions on the estimates of the complex envelopes and, therefore, we need an equation that relates the estimates to the measured data. The simplest such condition is to state that the sum of the orders differs only by an error term from the measured data as expressed in the equation

$$X(n) - \sum_{j \in J} A_j(n)\Theta_j(n) = \tilde{\delta}(n) \tag{9}$$

where the summation is for a desired subset of orders.

The above Equation (9) is called the *data equation* when the right hand side $\tilde{\delta}(n)$ is regarded as noise or error. This equation will also be enforced in an approximate sense for all sampled data points.

Decoupling. When several orders are estimated simultaneously, Equation (9) ensures that the total signal energy will be distributed between these orders. Together with the smoothness conditions of the structural Equation (7) this enforces a decoupling of close and crossing orders as is demonstrated in the examples. The mathematics of this procedure are quite analogous to the repeated root problem in modal analysis, see Reference 8. When orders are coinciding in frequency over an extended time segment, the allocation of energy to such orders is poorly defined and numerical ill conditioning may ensue. Widening the filter bandwidth is one possible remedy in this case.

Tachometer Processing

Any method with high resolution needs proper controls. For the Vold-Kalman filter this means a very accurate estimation of the instantaneous RPM such that the tracking filter will follow the peak of the order functions instead of extracting data from the foothills. The methodology which has been chosen for the Vold-Kalman filter is that of fitting cubic splines in a

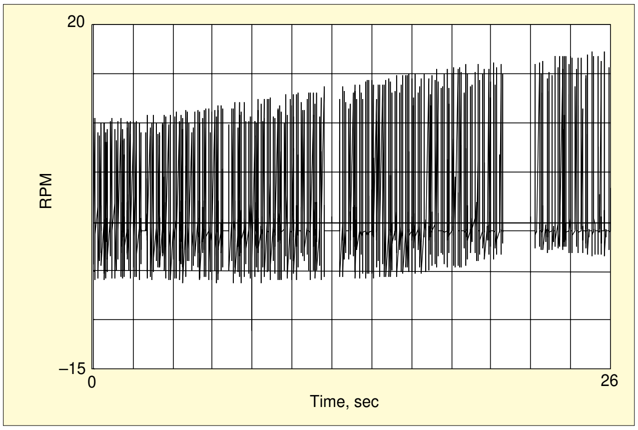

Figure 3. Corrupted tachometer waveform.

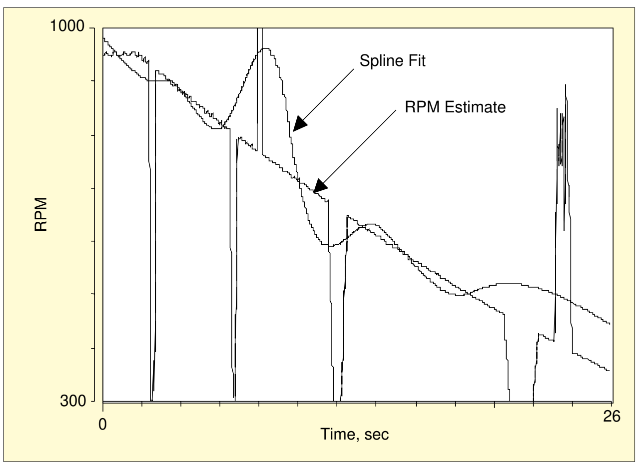

Figure 4. Raw data and spline fit.

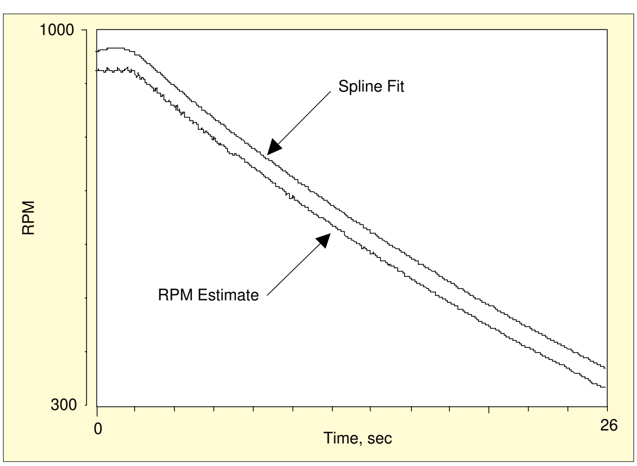

Figure 5. Censored data with corrected spline fit (curves offset for ease of comparison).

least squares sense to the table of level crossings from a tachometer waveform.

The spline fit also allows for an automatic rejection of outlier data points with a subsequent refit on a censored table of level crossings. Note also that this procedure allows for an analytic expression of the RPM as a continuous function of time, with a true tracking of the shaft rotation angle for phasing fidelity. There is also the option of specifying hinge points in the spline fit, such that sudden changes in inertial properties can be tracked, as in the case of clutching and gear shifts.

The spline fit and censoring process capability is illustrated by the corrupted tachometer signal as shown in Figure 3, with the raw level crossing table and spline fit in Figure 4. The fitted data are then compared with the raw table, the extreme deviants are censored and a refit is performed, with the clean results depicted in Figure 5. This is a fully automatic process,

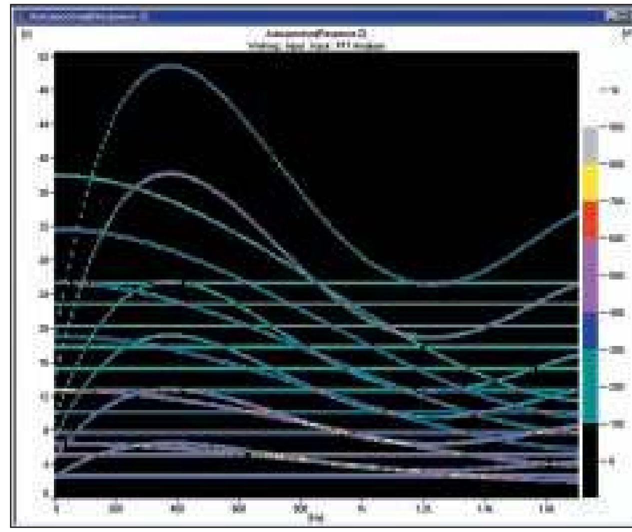

Figure 6. Spectrogram of synthetic multi axle data.

Figure 7. First order function extracted without decoupling and single pole Vold-Kalman filter.

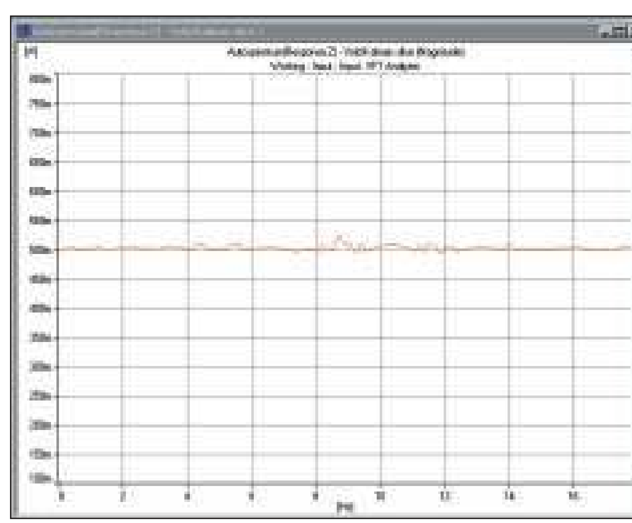

Figure 8. First order function extracted with decoupling and single pole Vold-Kalman filter.

often obviating the need for manual tachometer repair.

Example

To illustrate the power of the Vold-Kalman filter with decoupling of close and crossing orders, we constructed a numerical system with three independent axles, one was running at constant speed and the other two were slewing independently. Of particular interest to us in this example is the axle that monotonically loses speed over the analysis range. All the orders were generated with constant amplitude. An FFT based spectrogram of this data is shown in Figure 6. Here it can be clearly seen how the orders, especially in the low frequency range interact strongly. This system also has a more severe coupling pattern than most realistic automotive systems.

The first order of the axle that loses speed was extracted using a single pole Vold-Kalman filter without the decoupling. As can be seen from Figure 7, the estimated order function is severely contaminated.

When the three tachometers were used in a simultaneous estimation, but with the same filter parameters as in the single order estimation, we achieved a dramatic improvement in the quality of estimation, Figure 8. There are still warts in the order function, but this is due to the extreme closeness and crossings of the many orders in the neighborhood of the target order.

A Complete Order Tracking Solution

The Vold-Kalman tracking filter completes the set of tools needed to process and inspect data from rotating machinery measurements by providing ultra high resolution, beat free order decoupling and the capability of extracting both complex envelopes and modulated waveforms at the original sample rate.

A complete order tracking system requires both a real-time instrument-like capability (real-time is here meant in the sense that the data are being analyzed while being produced) as well as computer oriented post processing tools. The real-time order tracking is used for monitoring the order signatures during data acquisition, and provides for high bandwidth feedback of troubleshooting and design information as well as quality control of the experiment and instrumentation. Fourier transform-based visualization tools, such as waterfalls, spectrograms, Campbell diagrams and color contour plots and thereof extracted slices, are employed for data inspection, diagnosis as well as synthesis and modeling. The complex envelopes have coherent phase between channels and may be used for animation of operating deflection shapes, visualizing the geometry of structural and acoustic fields as a function of order and axle speed. The time domain modulated waveforms are also used in transient animations, but more importantly, in signal tailoring and synthesis for the subjective design and evaluation of product performance.

Analysis Techniques for Run-Up/Down Tests

One of the main applications of order tracking is analysis of run-ups and run-downs. Investigation of the system responses and dynamic behavior at the various rotational speeds is a key element in design, troubleshooting, product testing and quality control. Depending on the information required from the test, different analysis techniques can be applied.

The most simple is determination of the acoustic response in terms of the overall Linear, A, B or C weighted level as a function of RPM. This is often used in product testing for comparison with reference (tolerance) curves. No diagnostic information is obtained.

"Next level" of analysis is the inclusion of spectral information. In acoustic testing, 1/1 octave or 1/3 octave spectra are often used in order to get the spectral content in constant percentage filter bands as a function of rotational speed (constant percentage bandwidth means that the bandwidth is proportional with the frequency). These can reveal frequency regions with annoying resonances and they relate to the human perception of the radiated noise. Information about the individual orders cannot be extracted except for the lowest harmonics with 1/3 octave filters. Figure 9 shows the contour plot of the acoustic response during a run-up of a motor analyzed in 1/3 octaves. The 1st order is identified together with regions of high response levels. Figure 10 shows the overall level as well as some individual 1/3 octave bands as a function of RPM. More information about the lower harmonic order components could

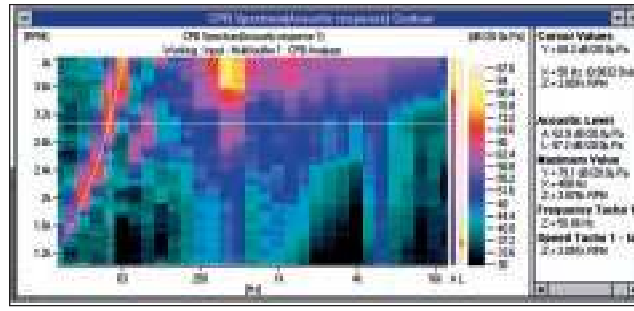

Figure 9. Third octave color contour plot of the acoustic response during run-up.

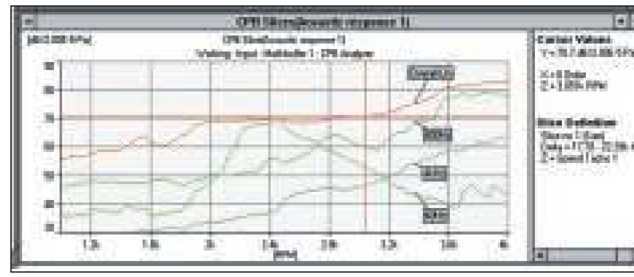

Figure 10. Selected 1/3 octave bands and overall linear level of acoustic response as functions of RPM.

be obtained by going to 1/12 octave filter bandwidth. Real-time 1/12 octave filters however require more DSP (Digital Signal Processing) power in the system.

In order to resolve the various orders in the frequency domain, narrowband frequency spectra (FFT without tracking) could be applied. FFT gives the spectral information in constant bandwidth (same bandwidth at all frequencies) and presented on a linear frequency axis makes identification of harmonic families very easy. This allows for diagnosis and source identification. The advantage of using an FFT without tracking is that it does not cost very much in terms of DSP power. A disadvantage is that the smearing of the individual order components in the frequency spectra will appear if the run-up/down is fast. This is due to the fact that each FFT spectrum represents a certain time window and therefore a certain change of the rotational speed. Another aspect is that rather large Fourier transforms (many lines in the FFT) are required in cases where the test is running over a wide speed (RPM) range and higher harmonics have to be included in the analysis. Example: RPM range from 600 RPM (10 Hz) to 6000 RPM (100 Hz), 16 orders included in the spectra (i.e., min. 16×100 Hz = 1.6 kHz frequency span) and a resolution of 2 Hz (5 FFT lines per order at 600 rpm) will require 800 FFT lines in the analysis (2 Hz resolution with a frequency span of 1.6 kHz).

Figure 11 shows the contour plot of an FFT analysis of the same acoustic response as in Figure 9. The harmonic family of the fundamental periodic loading is easily identified. Resonance frequencies excited by the various harmonic orders appear on vertical lines parallel to the RPM axis. Figure 12 shows the first 4 harmonics and the total level, in the selected frequency span, as a function of RPM. This information is obtained by slicing in the contour plot (calculated in "real-time" during the test).

The above mentioned disadvantages can be overcome by use of tracking. The tracking technique uses the instantaneous RPM values for calculation of the samples referenced to the revolution of the rotating shafts instead of the time clock.[6] Fourier transform of the revolution-based samples results in order spectra instead of frequency spectra and harmonic orders related to the measured RPM remains in fixed lines in the order spectrum. This means that smearing of the order related components is avoided and order components, which might have been smeared out in a frequency spectrum, can be identified. The number of lines required in the order spectra for a certain test is less than the number of lines needed if frequency spectra

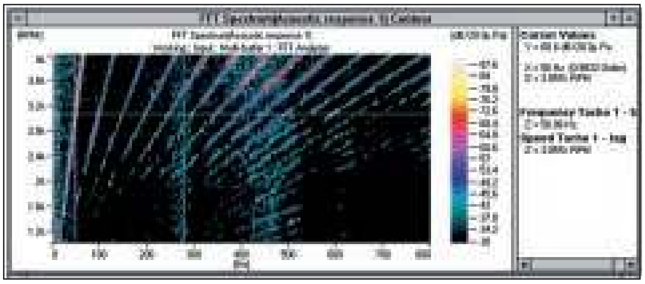

Figure 11. Contour plot of the FFT spectra of the acoustic response as a function of RPM.

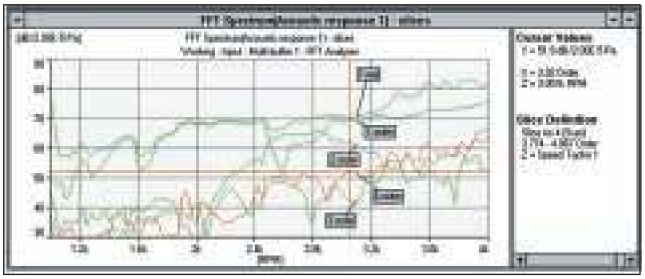

Figure 12. Selected orders and total level (in the frequency span) as functions of RPM.

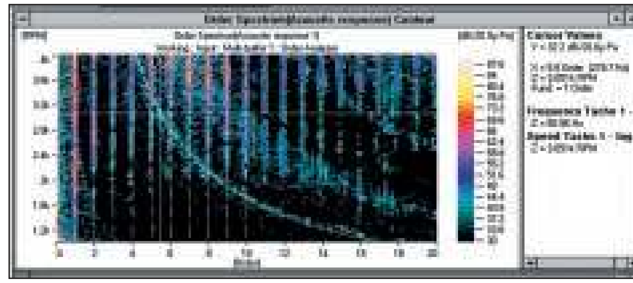

Figure 13. Contour plot of real-time order tracking spectra (resampling technique) of the acoustic response.

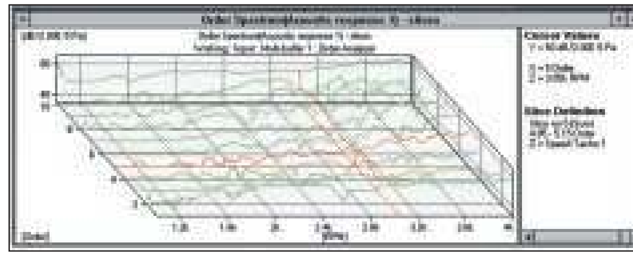

Figure 14. First 10 harmonics and the total level (in the span of 20 orders), from real-time order tracking spectra, as functions of RPM (shown as a waterfall plot).

were used. Example: 20 orders can be analyzed with a resolution of 0.2 order (5 lines per order, giving the possibility to identify inter-harmonic components) by use of 100 line order spectra independent of the speed range. The price is that the real-time tracking algorithms require more DSP power than the "normal" FFT analysis.

Figure 13 shows the contour plot of order spectra of the same acoustic response as in Figure 9 and Figure 11. The orders appear on vertical lines parallel to the RPM axis whereas resonances appear on hyperbolic curves (fixed frequency curves indicated by the cursor). Slices of the first 10 harmonics and the total level as a function of RPM are shown (as a waterfall plot) in Figure 14. The order slice results obtained by the frequency analysis and the order analysis are the same.

In product testing there is often a requirement of having the results in real-time (on-line) containing information of both individual orders, overall levels (Linear, A, B or C weighted) and in some cases 1/1 or 1/3 octave bands as well. This can only be obtained using a combination of the mentioned techniques. The key issue, in order to reduce the test time and have consistency of data in the various analyses, is to perform the analyses simultaneously. This is the case shown in the results illustrated in Figure 9 through Figure 14, where the Brüel and Kjær PULSE – Multi-Analyzer system Type 3560 is used. Four analyzers are used in parallel: A tachometer (for RPM calculations), a 1/3 octave analyzer (1/3 octaves and overall levels), an FFT analyzer (frequency spectra) and an order analyzer (order spectra) giving all the results simultaneously. In this example, only one tachometer signal is used. In multi axle applications the different tachometer signals should be applied to the tachometer. The scaling of the RPM axis and the calculation of the order slices can then be referenced to the different rotation speeds (RPM). If tracking is used (as in Figures 13 and 14) an order analyzer would have to be defined for each reference tachometer signal.

These analysis techniques can be applied in real-time (on-line) which means that the results are obtained during the test. They are based upon conventional frequency (Fourier) analysis that means that the resolution in the time domain is linked to the resolution in the frequency domain and vice versa. This gives a limitation of how accurately phenomena can be identified in both domains in the same analysis. This is also referred to as the uncertainty principle. For FFT analysis the relation is written as $T\Delta f \geq 1$, where T is the record length (resolution in time domain) and Δf is the line spacing in the spectrum (resolution in frequency domain). If the changes in the signals are too rapid, we will not be able to follow (analyze) these changes using these techniques. The real-time order tracking technique, mentioned above, is based on resampling and there is a limitation of how fast the speed (RPM) is allowed to change (also called slew rate limitation).

These limitations are examples of where the Vold-Kalman tracking filter can be used and thus completes the set of analysis tools. It overcomes the resolution problems and has no slew rate limitations.

Conclusions

The class of algorithms based on Kalman filters[1,3,4] shows no data between preselected orders, but does have fine resolution, independent of slew rates and can give both complex envelopes as well as waveforms. The speed may be an issue if a large number of orders is to be extracted. Finally, now the new Vold-Kalman filter has been developed to decouple interacting orders in multi axle systems, and also to provide for higher fidelity extraction thanks to the inclusion of multipole filter shapes.[3]

The Vold-Kalman tracking filter completes the set of analysis tools needed to process and inspect data from rotating machinery measurements.

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