Open access • Journal Article • DOI:10.1109/49.44588

Multi-H phase-coded modulations with asymmetric modulation indexes

— Source link

H. K. Hwang, Lin-Shan Lee, Sin-Horng Chen

Institutions: National Chiao Tung University, National Taiwan University

Published on: 01 Jan 1989 - IEEE Journal on Selected Areas in Communications (Institute of Electrical and Electronics Engineers Inc.)

Topics: Phase modulation, Continuous phase modulation, Coding gain and Modulation (music)

Multi-*H* Phase-Coded Modulations with Asymmetric Modulation Indexes

HONG-KUANG HWANG, STUDENT MEMBER, IEEE, LIN-SHAN LEE, AND SIN-HORNG CHEN, MEMBER, IEEE

Abstract—**Multi-*h* phase-coded modulation (MHPM) is a bandwidth efficient modulation scheme which offers substantial coding gain over conventional digital modulations. In this paper, a new concept of MHPM with asymmetric modulation indices corresponding to the bipolar data +1 and −1 is considered, and numerical results of the minimum Euclidean distances for such asymmetric binary multi-*h* schemes are provided. It is shown that performance improvements on the error probability are gained over conventional MHPM with essentially the same bandwidth and a very slight modification in implementation. The upper bounds of error probabilities as functions of observation intervals and received E_b/N_o are also investigated in detail.**

I. Introduction

A NUMBER of power and bandwidth efficient modulation techniques have received wide attention during recent years. Constant envelope continuous phase modulation (CPM) techniques [2], [3] provide a class of signaling schemes for the transmission of digital information over bandlimited channels with higher efficiency in using bandwidth than phase shift keying (PSK) or frequency shift keying (FSK) formats. The constant envelope makes these techniques fairly immune to nonlinear channel effects and useful in satellite and terrestrial radio links, while the approach of combined modulation and coding provides good potential for further power and bandwidth efficiencies at the price of increased complexity [9]. Multi-*h* phase-coded modulation (MHPM), described in detail by Anderson and Taylor [1], represents one trend in this area toward the development of efficient signaling schemes for the transmission of digital data as compared to techniques such as minimum shift keying (MSK) or quaternary phase shift keying (QPSK) [4]–[8]. In the MHPM schemes, cyclically varying modulation indexes are used in a prescribed manner, such that the transmitted signal has phase slope variation changing from one symbol interval to the next in response to the data symbols being transmitted. As a special class of CPM systems, the information-carrying phase in MHPM signals is always continuous at the data transitions, which provides very attractive features including spectral behaviors. Since the phase function is altered in such a manner that unequal phase changes can result from transmission of the same data symbol in different contiguous intervals, the phase change that occurred in the first interval cannot be undone during the second. The delays in the merge of neighboring phase trellis paths will thus result in longer minimum Euclidean distances for MHPM schemes than those for MSK and hence provide the coding gain [10].

Although the modulation indexes for MHPM could be any value smaller than 1, practically the nonrational indices will lead to extremely complicated phase trellis and very difficult maximum likelihood decoding due to the lack of well defined periodicity in the phase trellis. This is why the modulation indexes for MHPM are always restricted to be multiples of $1/q$ where q is an integer; finitely many phase states can therefore be used to demodulate the data in the receiver. In conventional MHPM schemes, the phase states at the transition times are multiples of π/q, but not all possible phase states are used, i.e., only even or odd multiples of π/q are used at any transition time $t = nT$. However, if all the phase states of multiples of π/q can be used, we could have more flexibility in finding interesting signaling schemes even if the constraint length [1] in which the neighboring phase trellis paths merge may remain unchanged, as will be clear later in examples. In this paper, we propose to use asymmetric modulation indexes corresponding to the bipolar data +1 and −1, as compared to the symmetric indexes used in conventional MHPM schemes. In this new approach, the modulation indexes h_{+i} for the data +1 and h_{-i} for the data −1 are not necessarily equal; better flexibilities are therefore available for the designers to optimize the system performance, and longer minimum Euclidean distance and hence further performance improvements are thus possible. It is also shown in this paper that essentially the same bandwidth and only a slight modification in implementation will result. The coding gain improvement over MHPM with symmetric modulation indexes using a full response linear phase function for binary multi-*h* schemes are investigated in great detail.

The basic concept for MHPM with a symmetric indexes together with examples of two simple types of such schemes for 3-*h* and 4-*h* cases are described in Section II. The results of an extensive search and calculation for the minimum Euclidean distances of these asymmetric

Manuscript received October 11, 1988.

H.-K. Hwang is with the Institute of Electronics, National Chiao Tung University, Hsinchu, Taiwan, Republic of China.

L.-S. Lee is with the Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, Republic of China.

S.-H. Chen is with the Institute of Communication Engineering, National Chiao Tung University, Hsinchu, Taiwan, Republic of China.

IEEE Log Number 8928297.

MHPM schemes with the best combinations of modulation indexes are then presented in Section III. The analysis in Section IV which pertains to the implementation of such asymmetric MHPM then reveals that only a slight modification to the modulator and demodulator of conventional MHPM is necessary. For coherent detection of binary multi-h schemes, the upper bounds on error probability as a function of observation interval and received E_b/N_o are further investigated and the optimum modulation indices are determined in Section V. Simulation methods are then employed in Section VI to find the power spectrum of an asymmetric MHPM signal. Finally, some concluding remarks are given in Section VII.

II. MHPM with Asymmetric Modulation Indexes

MHPM is a class of digital modulation schemes with constant envelope and continuous phase. The general form for an MHPM signal is

$$S(t, \boldsymbol{\alpha}) = \sqrt{2E/T}\cos\left(\omega_c t + \varphi(t, \boldsymbol{\alpha}) + \varphi_0\right) \quad (1)$$

where E is the symbol energy, T is the duration of a symbol, ω_c is the carrier angular frequency and φ_0 is an arbitrary carrier phase which can be set to be zero. The information-carrying phase function $\varphi(t, \boldsymbol{\alpha})$ can be expressed as

$$\varphi(t, \boldsymbol{\alpha}) = 2\pi \sum_{i=-\infty}^{\infty} a_i h_i f(t - (i-1)T) \qquad -\infty \le t \le \infty \quad (2)$$

where $\boldsymbol{\alpha} = \{\cdots, a_{-2}, a_{-1}, a_0, a_1, a_2, \cdots\}$ represents the sequence of data symbols, h_i is the cyclically varying modulation index corresponding to the ith symbol taken from a chosen set as described below, and $f(t)$ is a phase pulse function. In practice, the modulation indexes are often obtained from a set of rational values of the form $\{l_0/q, l_1/q, \cdots, l_{K-1}/q\}$ where $l_i < q$ for $0 \le i \le K - 1$, l_i and q are all integers, and K is the number of different modulation indexes. These indexes are used cyclically so that

$$h_{nk+j} = l_j/q, \quad 0 \le j \le K-1 \quad \text{and} \quad n = 0, 1, 2, \cdots.$$

The phase pulse function can be expressed as

$$f(t) = \int_{-\infty}^{t} g(\tau)\, d\tau \quad (3)$$

where $g(t)$ is a frequency pulse shape function with duration LT, i.e., it is zero for $t < 0$ and $t > LT$ where L is an integer. $L = 1$ yields a full response signal, while $L > 1$ corresponds to a partial response signal. As an example, for a full response linear phase function scheme, $f(t)$ is

$$f(t) = \begin{cases} 0 & t < 0 \\ t/2T & 0 \le t \le T. \\ 1/2 & t \ge T \end{cases} \quad (4)$$

This phase pulse function will be used in all the following numerical results in this paper.

An instructive way to appreciate MHPM schemes is in terms of the phase trellis which indicates the possible paths followed by the process of the modulation. An example of phase trellis with paths emanating from phase = 0 using the full response linear phase function for 3-h scheme with code $\{6/8, 4/8, 5/8\}$ is shown in Fig. 1. It should be noted that similar paths can emanate from any of the 16 phases shown. With proper choice of the modulation indices, no pair of phase trellis paths for different symbol sequences will merge before some interval defined by an integer called the constraint length v [1], which is dependent on the number of different modulation indexes K and the chosen index set. It can be easily seen that the constraint length is 4 for the phase trellis in Fig. 1. An important result is that the "separation" between two possible paths in the trellis for MHPM is greater than that for MSK, and consequently a receiver that bases its decision on a longer observation inverval can yield improved performance.

For conventional binary MHPM schemes, the modulation indexes h_i have only one value for the ith symbol no matter whether it is $+1$ or -1, so that the phase trellis of the binary MHPM is always symmetric. The phase differences between any two possible phase states at the symbol transition instants $t = nT$ are multiples of $2\pi/q$ rad. Let $\{h_a, h_b, h_c\}$ be the modulation index set for such a binary 3-h scheme where h_a, h_b, and h_c are multiples of $1/q$ and smaller than 1 , and h_{+i}, h_{-i} represent the indexes h_i for the ith symbol being $+1$ and -1, respectively. The modulation indexes can then be written as

$$\begin{array}{llllllll} i: & 1 & 2 & 3 & 4 & 5 & 6 & \cdots \\ h_{+i}: & h_a, & h_b, & h_c, & h_a, & h_b, & h_c, & \cdots. \\ h_{-i}: & h_a, & h_b, & h_c, & h_a, & h_b, & h_c, & \cdots \end{array} \quad (5)$$

Assume two phase trajectories diverge from a given state at time $t = 0$; the phase difference at time $t = T$ will be $\pi(h_{+1} + h_{-1}) = \pi(2h_a)$ and $q(2h_a)$ will be an even number. In this paper, a new concept of MHPM with asymmetric modulation indexes corresponding to the bipolar data $+1$ and -1 is proposed, i.e., h_{+i} and h_{-i} are not necessarily equal. This new asymmetric MHPM concept can provide an additional degree of freedom in choosing indices with better performance, because the separations among phase trajectories and the resulting minimum Euclidean distances depend on the values h_{+i}, h_{-i} and $(h_{+i} + h_{-i})$. For conventional MHPM, although $q(h_{+i})$ and $q(h_{-i})$ could be any chosen number, $q(h_{+i} + h_{-i})$ will always be an even number, and this constraint will limit the possibilities to maximize the minimum Euclidean distance. This is the basic idea for the asymmetric MHPM concept.

A simple approach to realize the asymmetric MHPM concept described above is to use the same set of the modulation indices for both h_{+i} and h_{-i}. One way to accomplish this based on the 3-h scheme described in (5) is sim-

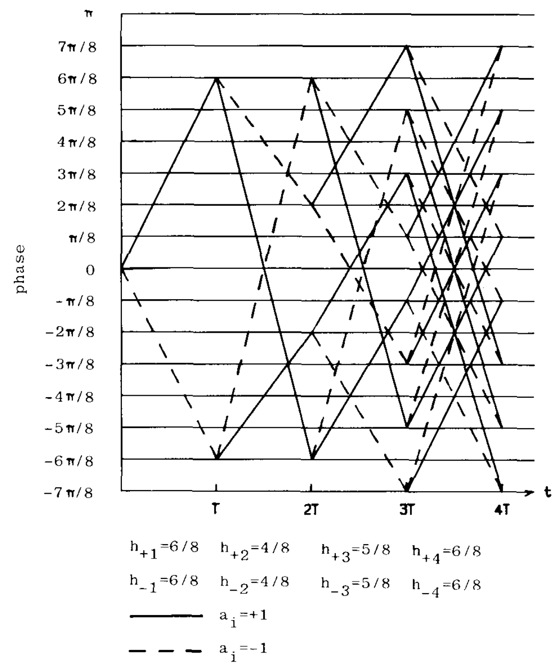


Fig. 1. Phase trellis of S-type MHPM with code $\{6/8, 4/8, 5/8\}$.

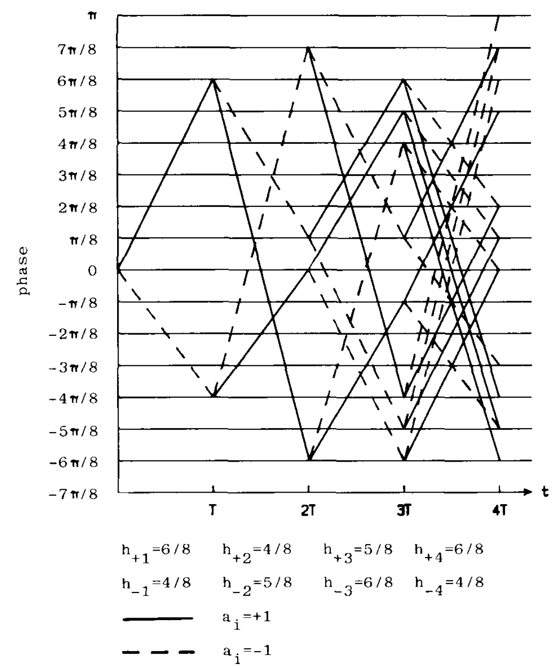


Fig. 2. Phase trellis of A-type MHPM with code $\{6/8, 4/8, 5/8\}$.

ply to shift h_{-i} with respect to h_{+i} by one symbol interval T. We can represent the modulation indexes in this case as

$$\begin{array}{llllllll} i: & 1 & 2 & 3 & 4 & 5 & 6 & \cdots \\ h_{+i}: & h_a, & h_b, & h_c, & h_a, & h_b, & h_c, & \cdots \\ h_{-i}: & h_b, & h_c, & h_a, & h_b, & h_c, & h_a, & \cdots \end{array} \tag{6}$$

i.e., $h_{-} = h_{+(i+1)}$. Let the MHPM scheme with modulation indexes shown in (6) be referred to as A-type and that with modulation indexes shown in (5) as S-type. An example of phase trellis of such an A-type MHPM scheme for the same set of modulation indexes and linear phase function as in Fig. 1, i.e., $(h_a, h_b, h_c) = (6/8, 4/8, 5/8)$, is shown in Fig. 2. We can find from this figure that the constraint length of this A-type MHPM is equal to that of S-type in Fig. 1; however, for A-type MHPM scheme, since $q(h_a + h_b)$ is not necessarily an even number, the phase values at any symbol transition time $t = nT$ will be in general a multiple of π/q as shown in Fig. 2. This gives more flexibility for A-type MHPM than S-type, and hence provides better opportunities to make the minimum Euclidean distance closer to the upperbound [11].

To shift h_{-i} with respect to h_{+i} is attractive for 3-h asymmetric MHPM, but this approach cannot be directly applied to 2-h schemes. In such an A-type MHPM schemes with $K = 2$, there are two different values for the modulation indices only, and $h_{-1} = h_{+2}$, $h_{-2} = h_{+3} = h_{+1}$. The sum of the two possible modulation indexes in the first interval is $h_{+1} + h_{-1} = h_{-2} + h_{+2}$, which is equal to that of the second interval, and this will apparently degenerate the 2-h scheme back to an 1-h scheme.

For 4-h schemes, on the other hand, to shift all h_{-i} with respect to h_{+i} by one symbol interval T will limit the constraint length to 4, because the phase trajectories for two data sequences $(+1, -1, +1, -1)$ and $(-1, +1, -1, +1)$ will merge after 4 symbol periods no matter what the values of the modulation indexes are. Similarly, to shift all h_{-i} with respect to h_{+i} by $2T$ and $3T$ for 4-h schemes will also limit the constraint length to 4. This is in fact a phenomenon for a more general situation, i.e., to shift h_{-i} with respect to h_{+i} by one symbol interval T is not very helpful for K-index asymmetric MHPM when K is an even number, because it will always divide the values of $\{h_{+i} + h_{-i}\}$ into two identical groups and limit the constraint length to K.

Despite the above observation, there do exist simple approaches to design asymmetric MHPM for even K. Consider a special type of 4-h asymmetric MHPM represented as

$$\begin{array}{llllllllll} i: & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & \cdots \\ h_{+i}: & h_a, & h_b, & h_c, & h_d, & h_a, & h_b, & h_c, & h_d, & \cdots \\ h_{-i}: & h_a, & h_c, & h_d, & h_b, & h_a, & h_c, & h_d, & h_b, & \cdots \end{array} \tag{7}$$

and let this type be referred to as G-type MHPM. Here h_a, h_b, h_c, and h_d are four different multiples of $1/q$, and in a cycle one h_{-i} is equal to h_{+i} and the other three h_{-i}'s are shifted cyclically by one symbol interval. Such an arrangement cannot only extend the constraint length to $K + 1$ for G-type MHPM with even K, i.e., 5 in this 4-h case, but it also leads to better free distance.

III. Minimum Euclidean Distances

The error probability of a coherent receiver for MHPM schemes is complicated to analyze. One common approach to observe the error rate behavior for such systems is to calculate the minimum Euclidean distance ($D^2_{\min}$) for all possible cyclic shifts of h_i values over several (N) symbol intervals instead where $D^2_{\min}$ is the minimum of

$$D^2 = \sum_{i=n}^{n+N-1} \int_{iT}^{(i+1)T} [S(t, \boldsymbol{\alpha}) - S(t, \boldsymbol{\beta})]^2 \, dt$$

for two arbitrary but different data sequences $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ where $\boldsymbol{\alpha} = \{\cdots, a_{-2}, a_{-1}, a_0, a_1, a_2, \cdots\}$, $\boldsymbol{\beta} = \{\cdots, b_{-2}, b_{-1}, b_0, b_1, b_2, \cdots\}$ are data sequences as used in (1), (2), $a_n \neq b_n$, and $S(t, \boldsymbol{\alpha})$, $S(t, \boldsymbol{\beta})$ are the signals as in (1). For high signal-to-noise ratios, the probability of error (P_e) is dominated by the minimum Euclidean distance, i.e., if the MHPM signal is contaminated by white Gaussian noise of spectral density $N_o/2$, then

$$P_e \sim Q(\sqrt{D^2_{\min}/2N_o})$$

for high signal-to-noise ratios where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-z^2/2} \, dz.$$

The minimum Euclidean distances for best multi-h codes with $K = 2$, 3, and 4 have been calculated for conventional S-type MHPM [1], and the results for $K = 2$ are listed in column 1 of Table I. By noting the fact that the minimum Euclidean distances for MSK and QPSK are four, the average coding gain with respect to the MSK and QPSK modulations for q varying from 4 to 13 are also calculated and listed in this column. Although A-type MHPM is not helpful for 2-h schemes, the remove of the restriction of using the same set of modulation indexes for h_{-i} and h_{+i} will make the minimum Euclidean distances larger. We have calculated and listed the minimum Euclidean distances of such asymmetric 2-h schemes in the second column of Table I. From this table, we can find that there is some coding gain when asymmetric modulation indices are used for 2-h schemes.

For binary 3-h schemes, the minimum distances of S-type MHPM are listed in the first column of Table II. We can see from this column that the minimum distance for $q = 9$ is much smaller than those for the other values of q. The minimum distances of the A-type MHPM schemes using the best code of the S-type have been calculated and listed in the second column of Table II. It is very clear that, with the best code of the S-type, the minimum Euclidean distances of the A-type are always higher than those of the S-type for all values of q ranging from 8 to 17. The minimum Euclidean distance for $q = 9$ is 7.53, which is substantially higher than that of S-type and much closer to those for other values of q when A-type MHPM schemes are used. Since the error probability depends essentially on $D^2_{\min}$, the increment in $D^2_{\min}$ roughly indicates the improvement in error rate performance. There is an average 0.4 dB coding gain over conventional MHPM for q varying from 8 to 17.

Although the minimum Euclidean distances have been improved in A-type MHPM as compared to those of the S-type when we use the best codes of S-type, these codes are still not necessarily optimal for A-type. An extensive search for the best combinations of modulation indexes for A-type MHPM is therefore performed. The results of the minimum Euclidean distances with the best combi-

TABLE I
Minimum Euclidean Distances and Coding Gains Over MSK for Multi-h Phase-Coded Modulations with $K = 2$

q	Conv. MHPM	Asym. MHPM
4	5.58(1.44dB)	7.11(2.49dB)
5	6.14(1.86dB)	7.25(2.58dB)
6	6.90(2.42dB)	7.36(2.65dB)
7	6.65(2.21dB)	7.45(2.70dB)
8	7.10(2.49dB)	7.51(2.74dB)
9	6.92(2.38dB)	7.57(2.77dB)
10	7.25(2.58dB)	7.61(2.79dB)
11	7.14(2.52dB)	7.64(2.81dB)
12	7.36(2.65dB)	7.67(2.83dB)
13	7.28(2.60dB)	7.70(2.84dB)
Average coding gain	2.32dB	2.72dB

TABLE II
Minimum Euclidean Distances and Coding Gains Over MSK for Multi-h Phase-Coded Modulations with $K = 3$

q	Minimum Euclidean distances (coding gains)			
	S-type	A-type (best code of S-type)	A-type (best code)	Maximum achievable distance
8	7.58(2.77dB)	8.22(3.13dB)	8.22(3.13dB)	8.68(3.36dB)
9	5.52(1.40dB)	7.53(2.75dB)	8.11(3.07dB)	8.11(3.07dB)
10	7.63(2.81dB)	7.91(2.96dB)	8.85(3.45dB)	8.85(3.45dB)
11	7.46(2.71dB)	8.68(3.36dB)	8.68(3.36dB)	8.77(3.41dB)
12	7.63(2.81dB)	7.77(2.89dB)	8.85(3.45dB)	8.85(3.45dB)
13	8.34(3.19dB)	9.18(3.61dB)	9.18(3.61dB)	9.18(3.61dB)
14	8.23(3.13dB)	8.82(3.43dB)	8.85(3.45dB)	8.85(3.45dB)
15	7.76(2.88dB)	8.35(3.20dB)	9.28(3.65dB)	9.28(3.65dB)
16	8.68(3.36dB)	9.15(3.59dB)	9.15(3.59dB)	9.15(3.59dB)
17	8.46(3.25dB)	9.23(3.63dB)	9.23(3.63dB)	9.23(3.63dB)
Average coding gain	2.86 dB	3.27 dB	3.44 dB	3.47 dB

nations of modulation indexes are shown in the third column of Table II. Comparing to the values shown in column 2, we find that, for $q = 9$, 10, 12, 14, and 15, there do exist better codes for A-type other than the best codes of the S-type as far as the minimum Euclidean distances are concerned. Also, the coding gains in this column are more than 3 dB better than MSK for all values of q varying from 8 to 17, and the gains are actually significantly higher than 3 dB for most values of q. For $q = 9$, it is clear that A-type MHPM provides 1.67 dB gain over S-type MHPM. In summary, A-type MHPM with the best combination of codes has on average a coding gain of 3.4 dB with respect to MSK, which is about 0.6 dB better than the average coding gain of S-type.

We can further relax the restriction of using the same set of modulation indexes for h_{-i} and h_{+i}, and in this way even better performance can be achieved. This means any multiples of $1/q$ can be chosen for h_{-i} and h_{+i}. The maximum achievable minimum Euclidean distances for binary 3-h schemes with q varying from 8 to 17 for this case are calculated and shown in the fourth column of Table II. We find that the maximum achievable minimum distances obtained in this way are in general equal to the minimum Euclidean distances in column 3 except for $q = 8$ and $q = 11$, and the improvements for $q = 8$ and $q = 11$ are in fact not significant. It can thus be concluded that A-type MHPM is still a reasonably good approach to design MHPM with asymmetric modulation indices for 3-h systems.

For binary 4-h schemes, we also calculated the minimum Euclidean distances of the A-type MHPM, G-type MHPM and the maximum achievable minimum Euclidean distances by choosing arbitrary set of modulation indexes as done previously for q varying from 16 to 22, and the results are listed in Table III. As shown in the second column of Table III, the 4-h A-type MHPM can get larger minimum Euclidean distances than those of S-type except for $q = 20$ and $q = 22$, with an averaging coding gain 3.7 dB, although the A-type will produce the MHPM schemes with a constraint length equal to 4 for $K = 4$ as discussed above.

We can also find from column 3 of Table III that the G-type MHPM has a coding gain of 4.0 dB on average, which is also about 0.6 dB higher than the average coding gain of S-type codes. Furthermore, by a closer look at the numbers in this column, it is revealed that the coding gains of G-type over MSK range from 3.89 to 4.11 dB for q varying from 16 to 22 while the gains for S-type range from 3.02 to 3.88 dB. In other words, the gains for G-type codes not only are higher than those for S-type, but have a much smaller variation with respect to the value of q, i.e., the dependence of the minimum Euclidean distance on the choice of q is much less when asymmetric modulation indices are used. This phenomenon is in fact even clearer if we look at the numbers in column 2 of Table III for A-type schemes. The fourth column of Table III lists the maximum achievable minimum Euclidean distances with arbitrary index set as discussed above. It can be seen that in this case improvements are achievable as compared to G-type except for $q = 18$ and $q = 22$. In fact, these maximum achievable minimum distances are all obtained for the modulation indexes with $q(\Sigma_{i=1}^{K} (h_{+i} + h_{-i}))$ being an odd number except for $q = 18$ and $q = 22$, and it is impossible to obtain such best achievable performance if the same set of indexes are used for $\{h_{+i}\}$ and $\{h_{-i}\}$. On the other hand, it was found that G-type MHPM is still an optimum choice if h_{+i} and h_{-i} are restricted to use the same set of modulation indexes.

TABLE III
MINIMUM EUCLIDEAN DISTANCES AND CODING GAINS OVER MSK FOR MULTI-h PHASE-CODED MODULATIONS WITH $K = 4$

q	Minimum Euclidean distances (coding gains)			
	S-type	A-type	G-type	Maximum achievable distance
16	9.29(3.66dB)	9.34(3.68dB)	9.80(3.89dB)	10.32(4.12dB)
17	8.02(3.02dB)	9.34(3.68dB)	10.03(3.99dB)	10.21(4.07dB)
18	8.12(3.07dB)	9.38(3.70dB)	10.21(4.07dB)	10.21(4.07dB)
19	8.83(3.44dB)	9.38(3.70dB)	9.99(3.97dB)	10.13(4.04dB)
20	9.55(3.78dB)	9.42(3.72dB)	10.21(4.07dB)	10.35(4.13dB)
21	8.54(3.29dB)	9.42(3.72dB)	10.07(4.01dB)	10.38(4.14dB)
22	9.78(3.88dB)	9.45(3.73dB)	10.31(4.11dB)	10.31(4.11dB)
Average coding gain	3.45 dB	3.70dB	4.02 dB	4.10 dB

IV. IMPLEMENTATION COMPLEXITY OF ASYMMETRIC MHPM

In binary continuous phase frequency shift keying (CPFSK), one modulation index is used and the transmitted frequencies are $(f_c \pm h/2T)$ where f_c is the nominal center frequency. MHPM can be conceived similarly except that the modulation index is one of K predetermined values and the indexes are used consecutively and cyclically, one in each interval. The transmitted frequencies in each interval are $(f_c \pm h_i/2T)$ where $i = 1, 2, \cdots, K$. Using the same approach, the generation procedure of an asymmetric MHPM can be represented as in Fig. 3. The two sideband tones are selected by the switch depending on the data a_i. We need two sets of shift registers for the generation of asymmetric MHPM instead of one for conventional MHPM. For the generation of A-type MHPM signals, one can get h_{-i} from h_{+i} simply by a delay unit; in that case, one set of shift registers is enough.

The general form for a binary multi-h signal in one symbol interval is

$$\begin{aligned} s(t) &= \sqrt{2E/T} \cos \Big\{ \omega_c t + \pi\big(t - (i-1)T\big) \\ &\quad \cdot \big[a_i(h_{+i} + h_{-i}) + h_{+i} - h_{-i}\big]/2T + \varphi_i \Big\} \\ &= \sqrt{2E/T} \cos \Big\{ \omega_c t + \pi t\big[(a_i + 1)h_{+i} \\ &\quad + (a_i - 1)h_{-i}\big]/2T + \phi_i \Big\} \\ &\qquad (i-1)T \le t \le iT \end{aligned} \tag{8}$$

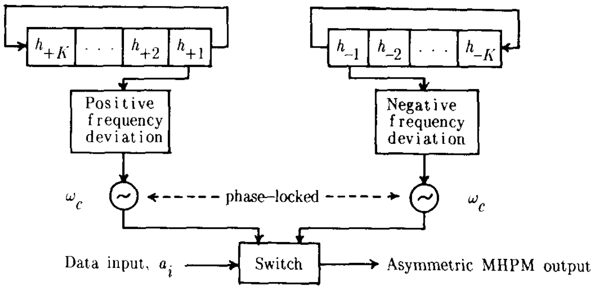


Fig. 3. The concept of asymmetric MHPM generator.

where φ_i is the excess phase at time $t = (i - 1)T$ due to previous information data and $\phi_i = \varphi_i - \pi(i - 1)[(a_i + 1)h_{+i} + (a_i - 1)h_{-i}]/2$. By standard trigonometric identities, (8) becomes

$$\begin{aligned} s(t) = {} & \sqrt{2E/T}(\cos\phi_i)[(1 + a_i)/2] \\ & \cdot \cos(\omega_c t + \pi h_{+i} t/T) \\ & - \sqrt{2E/T}(\sin\phi_i)[(1 + a_i)/2] \\ & \cdot \sin(\omega_c t + \pi h_{+i} t/T) \\ & + \sqrt{2E/T}(\cos\phi_i)[(1 - a_i)/2] \\ & \cdot \cos(\omega_c t - \pi h_{-i} t/T) \\ & - \sqrt{2E/T}(\sin\phi_i)[(1 - a_i)/2] \\ & \cdot \sin(\omega_c t - \pi h_{-i} t/T). \end{aligned} \tag{9}$$

Although the first two terms and the second two terms in (9) form orthogonal pairs individually, terms in one pair are not orthogonal to terms in the other pair. It would be convenient to transform (9) into an orthonormal expansion. By the Gram-Schmidt orthogonalization procedure [17], the set of orthonormal functions can be assigned as

$$\psi_1(t) = \sqrt{2/T}\cos(\omega_c t + \pi h_{+i} t/T) \tag{10}$$

$$\psi_2(t) = \sqrt{2/T}\sin(\omega_c t + \pi h_{+i} t/T) \tag{11}$$

$$\begin{aligned} \psi_3(t) = {} & [\sqrt{2/T}\cos(\omega_c t - \pi h_{-i} t/T) \\ & - C_{1,i}\psi_1(t) - C_{2,i}\psi_2(t)]/D_i \end{aligned} \tag{12}$$

$$\begin{aligned} \psi_4(t) = {} & [\sqrt{2/T}\sin(\omega_c t - \pi h_{-i} t/T) \\ & + C_{2,i}\psi_1(t) - C_{1,i}\psi_2(t)]/D_i \end{aligned} \tag{13}$$

where

$$\begin{aligned} C_{1,i} &= \sin[\pi(h_{+i} + h_{-i})]/[\pi(h_{+i} + h_{-i})] \\ C_{2,i} &= \{1 - \cos[\pi(h_{+i} + h_{-i})]\}/[\pi(h_{+i} + h_{-i})] \\ D_i^2 &= 1 - C_{1,i}^2 - C_{2,i}^2. \end{aligned}$$

Substitution and rearrangement of terms in (9) gives

$$\begin{aligned} s(t) = {} & \sqrt{E}[A_{1,i}\psi_1(t) + A_{2,i}\psi_2(t) \\ & + A_{3,i}\psi_3(t) + A_{4,i}\psi_4(t)] \end{aligned} \tag{14}$$

where

$$\begin{aligned} A_{1,i} = {} & (\cos\phi_i)[(1 + a_i)/2] + C_{1,i}(\cos\phi_i) \\ & \cdot [(1 - a_i)/2] + C_{2,i}(\sin\phi_i)[(1 - a_i)/2] \\ A_{2,i} = {} & -(\sin\phi_i)[(1 + a_i)/2] + C_{2,i}(\cos\phi_i) \\ & \cdot [(1 - a_i)/2] - C_{1,i}(\sin\phi_i)[(1 - a_i)/2] \\ A_{3,i} = {} & (\cos\phi_i)[(1 - a_i)/2]D_i \\ A_{4,i} = {} & -(\sin\phi_i)[(1 - a_i)/2]D_i. \end{aligned}$$

The demodulator based upon the use of the above orthonormal functions is shown in Fig. 4. Any received multi-*h* signal can be expressed as a linear combination of these basis functions. The first signal path uses $\psi_1(t)$ as reference and the resulting integrator output is $A_{1,i}$. Similarly, the multiplier/integrator output for the second path is $A_{2,i}$. $A_{3,i}$ is then derived by combining paths 1, 2, and 3 with appropriate weightings, because $\psi_3(t)$ is not specifically used as a input reference for the third path, and therefore an equivalent operation as shown in (12) is required. Similar situation occurs for $A_{4,i}$. These outputs A_{1i}, A_{2i}, A_{3i}, A_{4i} can be used together to calculate the branch metrics in a Viterbi processor to determine the transmitted data [18]. The reference carriers at the signaling frequencies $f_c \pm h_i/2T$ are required for the receiver to produce in-phase and quadrature baseband components. Comparing the above receiver structure to that for conventional MHPM signals [18], the only differences are the values of $C_{1,i}$, $C_{2,i}$ and the fact that the reference signals are dependent on both h_{+i} and h_{-i} instead of h_i only. Therefore, the implementation complexity for the asymmetric MHPM is essentially the same as that for the conventional MHPM systems, except that some modifications are needed.

There are three levels of synchronization required in the demodulation of MHPM signals. These are 1) carrier phase synchronization, 2) symbol timing, and 3) interval synchronization for modulo *K*. The technique for acquiring these in conventional MHPM systems is to pass the signal through a *q*th power-law device [12], [13], which can be similarly used directly for the synchronization of asymmetric MHPM signals.

V. Performance Bounds for Coherent Detection

The optimum coherent receiver can observe the received MHPM signal with additive white Gaussian noise over several bit intervals and then make a decision on the first bit in this interval. The error probability performance of such a receiver is in fact very complicated to analyze, but there are some bounds which can be used to determine optimum signal parameters [14], [15]. Although the minimum Euclidean distance itself can be used to determine the bit error probability at high SNR as discussed previously, it is really not sufficient to describe the overall system performance. This is because in the calculation of minimum Euclidean distance as in Section III, we always calculate the minimum distance of two paths merging at

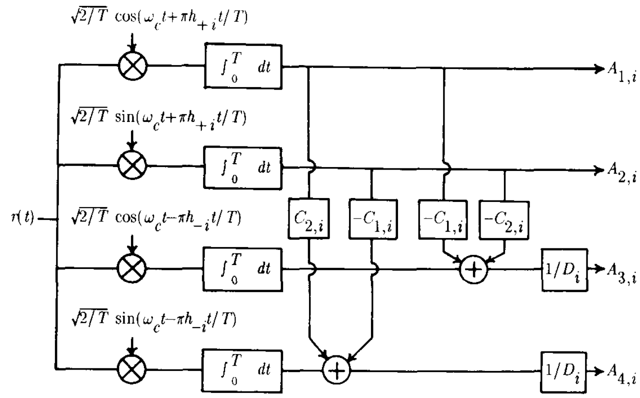


Fig. 4. The determination of the signal components in the orthonormal basis for demodulation of asymmetric MHPM.

the constraint length, but there do exist, for all the cases examined previously, some phase paths which are not yet merged at the constraint length but have smaller distances between them than the minimum Euclidean distances found. If the length of observation interval at the demodulator is not long enough, these smaller distances will actually dominate the error probability.

For large values of E_b/N_o, the upper bound of the probability of error for coherent reception of MHPM signals for an n-bit observation interval is given by [14]

$$P_e < \frac{1}{K2^{2n-1}} \sum_{m=1}^{2^{n-1}} \sum_{j=1}^{2^{n-1}} \sum_{\bar{h}} Q\left\{\left[\frac{nE_b}{N_o}\left(1 - \rho_{\bar{h}}(m, j)\right)\right]^{1/2}\right\} \tag{15}$$

where K is the number of different modulation indices, $\bar{h}$ denotes the sequence of modulation indexes over the n-bit interval, the summation $\Sigma_{\bar{h}}$ is over all K possible shifts of $\bar{h}$, n is the length of the observation interval and the sequence correlation function $\rho_{\bar{h}}(m, j)$ is evaluated using

$$\rho_{\bar{h}}(m, j) = \frac{1}{nE_b} \int_0^{nT} S^{(\bar{h})}(t, -1, A_m)\, S^{(\bar{h})}(t, +1, A_j)\, dt \tag{16}$$

where $S^{(\bar{h})}(t, -1, A_m)$ is the signal as in (1) containing a datum -1 in the decision bit interval and A_m is the $(n-1)$-tuple $(a_2, \cdots, a_n)$. From (15), we can find that the error probability bound is a function of the received E_b/N_o, the modulation index set $\{h_i;\ i = 1, \cdots, K\}$, and the length of observation interval n. For a given phase function, given observation interval and suitably high E_b/N_o, the modulation index set which should be chosen is quite obviously the one which minimizes the error probability bound in (15), and this set could be different from the optimum modulation indexes obtained for the minimum Euclidean distance as calculated in Section III.

The upper bounds (15) have been searched extensively for a large number of combinations of modulation indexes, and the optimum modulation indexes in the sense of error bounds were found for E_b/N_o varying from 6 to 12 dB and observation intervals $5T$, $8T$, $11T$ for 3-h schemes and $6T$, $9T$, $12T$ for 4-h schemes. In the calculation of the upper bounds of the error probability, shifting h_{-i} with respect to h_{+i} by one symbol interval T and two symbol intervals $2T$ will give different distances for two paths which do not merge within the observation length. It is therefore necessary to calculate and compare the error probability bounds for these two shifts. Some numerical results for the logarithm of error probability bounds and the optimum modulation indexes are listed in Tables IV and V.

From these two tables, we find that the optimum modulation indexes $\{h_i\}$ are functions of received E_b/N_o and the length of observation interval n. Since $Q(.)$ is not a linear function and the probability distributions of the distances between phase paths are different for different sets of modulation indexes, the optimum modulation index set may be different for different E_b/N_o or different observation length n with given q. For example, for the S-type 3-h scheme with $q = 12$ and observation length $5T$, the optimum codes for $E_b/N_o = 6$ dB and 8 dB are different from those for 10 dB and 12 dB. Similarly, for the S-type 4-h scheme with $q = 17$ and $E_b/N_o = 6$ dB, the optimum codes for observation intervals $6T$, $9T$, and $12T$ are different. It is also interesting to note from Table IV that for 3-h schemes with $q = 8$, the error probability bounds are equal for A-type and S-type systems when observation intervals are $8T$ and $11T$. This means the observation interval is not long enough for A-type MHPM.

In Figs. 5–8, the upper bounds of error probability performance versus E_b/N_o are plotted with $q = 9$ and $q = 15$ for 3-h schemes and with $q = 16$ and $q = 21$ for 4-h schemes. From Fig. 5, we find that although the performance of A-type MHPM are equal to that of S-type when $n = 5$, A-type MHPM with observation interval $8T$ has in fact a performance better than that of S-type with an $11T$

TABLE IV
THE UPPER BOUNDS OF ERROR PROBABILITY FOR 3-h MHPM SYSTEMS WITH OBSERVATION INTERVALS $5T$, $8T$, AND $11T$ ($n = 5, 8, 11$), AND E_b/N_o = 6–12 dB WITH OPTIMUM CODE $\{h_1, h_2, h_3\} = \{l_1/q, l_2/q, l_3/q\}$. (a) S-TYPE. (b) A-TYPE

(*a*). *S*–type

q	n	Logarithm of error probability bound and (l_1, l_2, l_3) E_b/N_0=6 dB	8 dB	10 dB	12 dB
8	5	−3.507 (4, 6, 5)	−5.141 (4, 6, 5)	−7.456 (4, 6, 5)	−10.964 (4, 5, 6)
	8	−4.145 (4, 6, 5)	−6.270 (4, 6, 5)	−9.352 (4, 6, 5)	−14.014 (4, 5, 6)
	11	−4.497 (4, 6, 5)	−6.809 (4, 6, 5)	−10.124 (4, 6, 5)	−15.135 (4, 5, 6)
9	5	−3.618 (5, 7, 6)	−5.411 (5, 7, 6)	−8.007 (5, 7, 6)	−11.919 (5. 7. 6)
	8	−3.905 (5. 7, 6)	−5.655 (5, 7, 6)	−8.160 (5, 7, 6)	−11.966 (5, 7, 6)
	11	−3.978 (5, 7, 6)	−5.686 (5, 7, 6)	−8.165 (5, 7, 6)	−11.997 (5, 7, 6)
12	5	−3.560 (6, 9, 8)	−5.309 (6, 9, 8)	−7.903 (7, 10, 8)	−11.919 (7, 10, 8)
	8	−4.003 (6, 9, 8)	−5.976 (6, 9, 8)	−8.841 (6, 9, 8)	−13.180 (7, 10, 8)
	11	−4.273 (6, 9, 8)	−6.454 (6, 9, 8)	−9.650 (6, 9, 8)	−14.501 (6, 9, 8)
14	5	−3.581 (8, 11, 10)	−5.314 (8, 11 10)	−7.805 (8, 11 10)	−11.617 (7, 9, 11)
	8	−4.062 (7, 10, 9)	−6.009 (7, 10, 9)	−8.878 (7, 9, 11)	−13.128 (7, 9, 11)
	11	−4.320 (7, 10, 9)	−6.419 (7, 10, 9)	−9.489 (7, 9, 11)	−14.157 (7, 9, 11)
15	5	−3.601 (9, 12, 11)	−5.371 (8, 12, 10)	−7.940 (8, 12, 10)	−11.784 (8, 12 10)
	8	−3.981 (8, 11, 9)	−5.837 (8, 11, 10)	−8.514 (8, 11, 10)	−12.554 (8, 11, 10)
	11	−4.225 (8, 11, 9)	−6.183 (8, 11, 10)	−9.080 (8, 11, 10)	−13.461 (8, 11, 10)
17	5	−3.629 (9, 13, 11)	−5.489 (10, 14, 12)	−8.302 (10, 14, 12)	−12.590 (10, 14, 12)
	8	−4.107 (9, 13, 11)	−6.141 (9, 13, 11)	−9.083 (10, 14, 12)	−13.550 (10, 12, 14)
	11	−4.337 (9, 13, 11)	−6.470 (9, 13, 11)	−9.535 (9, 13, 11)	−14.191 (9, 13, 11)

(*b*) *A*–type

q	n	Logarithm of error probability bound and (l_1, l_2, l_3) E_b/N_0=6 dB	8 dB	10 dB	12 dB
8	5	−3.588 (4, 6, 7)	−5.362 (4, 6, 7)	−7.873 (4, 6, 7)	−11.544 (4, 6, 7)
	8	−4.145 (3, 5, 7)	−6.270 (3, 5, 7)	−9.352 (3, 5, 7)	−14.014 (3, 7, 5)
	11	−4.497 (3, 5, 7)	−6.809 (3, 5, 7)	−10.124 (3, 5, 7)	−15.135 (3, 7, 5)
9	5	−3.618 (4, 6, 8)	−5.411 (4, 6, 8)	−8.007 (4, 6, 8)	−11.919 (4, 6, 8)
	8	−3.994 (3, 6, 8)	−5.942 (3, 6, 8)	−8.709 (3, 6, 8)	−12.895 (3, 6, 8)
	11	−4.352 (3, 6, 8)	−6.639 (3, 6, 8)	−9.961 (3, 6, 8)	−15.060 (3, 8, 6)
12	5	−3.614 (6, 8, 11)	−5.416 (5, 8, 10)	−8.067 (5, 8, 10)	−12.099 (6, 9, 11)
	8	−4.115 (5, 8, 10)	−6.157 (5, 8, 10)	−9.096 (5, 8, 10)	−13.514 (5, 7, 10)
	11	−4.393 (5, 7, 10)	−6.661 (5, 7, 10)	−9.950 (5, 7, 10)	−14.960 (5, 7, 10)
14	5	−3.638 (6, 9, 12)	−5.507 (7, 10, 13)	−8.317 (7, 10, 13)	−12.579 (7, 10, 13)
	8	−4.154 (6, 9, 12)	−6.273 (6, 9, 12)	−9.378 (6, 9, 12)	−14.070 (6, 9, 12)
	11	−4.423 (6, 9, 12)	−6.700 (6, 9, 12)	−10.011 (6, 9, 12)	−15.053 (6, 9, 12)
15	5	−3.616 (6, 10, 13)	−5.446 (6, 10, 13)	−8.162 (6, 10, 13)	−12.308 (7, 11 14)
	8	−4.170 (6, 9, 13)	−6.337 (6, 9, 13)	−9.535 (6, 9, 13)	−14.444 (6, 9, 13)
	11	−4.499 (6, 9, 13)	−6.888 (6, 9, 13)	−10.313 (6, 9, 13)	−15.468 (6, 9, 13)
17	5	−3.629 (7, 11, 15)	−5.489 (8, 12, 16)	−8.302 (8, 12, 16)	−12.590 (8, 12, 16)
	8	−4.170 (7, 11, 14)	−6.283 (7, 11, 14)	−9.328 (7, 10, 15)	−13.970 (7, 10, 15)
	11	−4.468 (7, 11, 14)	−6.776 (7, 11, 14)	−10.156 (7, 10, 15)	−15.298 (7, 10, 15)

TABLE V
THE UPPER BOUNDS OF ERROR PROBABILITY FOR 4-h MHPM SYSTEMS WITH OBSERVATION INTERVALS $6T$, $9T$, AND $12T$ ($n = 6, 9, 12$), AND E_b/N_o = 6–12 dB WITH OPTIMUM CODE $\{h_1, h_2, h_3, h_4\} = \{l_1/q, l_2/q, l_3/q, l_4/q\}$. (a) S-TYPE. (b) G-TYPE

(*a*). *S*–type

q	n	Logarithm of error probability bound and (l_1,l_2,l_3,l_4) E_b/N_0=6 dB	8 dB	10 dB	12 dB
16	6	−3.838 (9,12,10,13)	−5.826 (9,12,10,13)	−8.660 (9,12,10,13)	−12.863 (9,12,10,13)
	9	−4.099 (9,12,10,13)	−6.104 (8,13,10,12)	−8.984 (8,13,10,12)	−13.244 (8,13,10,12)
	12	−4.286 (8,13,10,12)	−6.427 (8,13,10,12)	−9.490 (8,13,10,12)	−14.044 (8,13,10,12)
17	6	−3.825 (9,12,10,13)	−5.818 (9,14,10,13)	−8.704 (9,14,10,13)	−13.008 (9,14,10,13)
	9	−4.138 (9,12,10,14)	−6.268 (8,11,9,13)	−9.355 (8,11,9,12)	−13.975 (8,11,9,12)
	12	−4.347 (8,11,9,13)	−6.596 (8,11,9,13)	−9.819 (8,11,9,13)	−14.625 (8,11,9,13)
18	6	−3.850 (10,13,11,14)	−5.885 (9,12,10,13)	−8.842 (9,12,10,13)	−13.170 (9,12,10,13)
	9	−4.174 (9,12,10,13)	−6.292 (9,12,10,13)	−9.259 (9,12,10,13)	−13.686 (9,12,10,13)
	12	−4.337 (10,13,11,14)	−6.443 (9,12,14,11)	−9.598 (8,12,10,13)	−14.359 (8,12,10,13)
20	6	−3.868 (11,15,12,16)	−5.855 (11,15,12,16)	−8.789 (11,15,12,16)	−13.289 (10,13,11,14)
	9	−4.196 (10,13,11,15)	−6.384 (10,13,11,14)	−9.504 (10,13,11,14)	−14.232 (10,17,11,15)
	12	−4.408 (10,13,11,15)	−6.609 (10,13,11,15)	−9.659 (10,17,11,15)	−14.330 (10,17,11,15)
21	6	−3.834 (12,16,13,17)	−5.848 (10,14,11,15)	−8.821 (10,14,11,15)	−13.251 (10,14,11,15)
	9	−4.186 (12,15,13,17)	−6.285 (10,14,11,15)	−9.361 (10,14,11,15)	−13.912 (10,14,11,15)
	12	−4.366 (12,15,13,17)	−6.444 (11,14,12,15)	−9.495 (10,14,11,15)	−14.055 (10,14,11,15)

(*b*). *G*–type

q	n	Logarithm of error probability bound and (l_1,l_2,l_3,l_4) E_b/N_0=6 dB	8 dB	10 dB	12 dB
16	6	−3.862 (9,15,8,11)	−5.915 (9,15,8,11)	−8.864 (9,15,8,11)	−13.211 (9,15,8,11)
	9	−4.218 (8,14,7,10)	−6.443 (8,14,7,10)	−9.578 (8,14,7,10)	−14.161 (8,14,7,10)
	12	−4.416 (8,14,7,10)	−6.703 (8,14,7,10)	−9.999 (8,14,7,10)	−14.918 (8,14,7,10)
17	6	−3.855 (14,7,12,13)	−5.919 (14,7,12,13)	−8.905 (14,7,12,13)	−13.351 (9,15,10,7)
	9	−4.252 (9,15,10,7)	−6.565 (9,15,10,7)	−9.938 (9,15,10,7)	−15.021 (9,15,10,7)
	12	−4.459 (9,15,10,7)	−6.806 (9,15,10,7)	−10.246 (9,15,10,7)	−15.526 (9,15,10,7)
18	6	−3.872 (10,17,9,12)	−5.918 (10,17,9,12)	−8.862 (15,6,14,13)	−13.414 (15,6,14,13)
	9	−4.229 (9,15,8,11)	−6.533 (9,15,8,11)	−9.912 (9,15,8,11)	−15.000 (9,15,8,11)
	12	−4.439 (9,15,8,11)	−6.774 (9,15,8,11)	−10.193 (9,15,8,11)	−15.431 (9,15,8,11)
20	6	−3.873 (12,19,13,10)	−5.911 (12,19,13,10)	−8.898 (10,17,9,12)	−13.469 (10,17,9,12)
	9	−4.234 (10,17,9,12)	−6.520 (10,17,9,12)	−9.843 (10,17,9,12)	−14.821 (10,17,9,12)
	12	−4.440 (11,17,12,9)	−6.720 (10,17,9,12)	−10.093 (10,17,9,12)	−15.234 (10,17,9,12)
21	6	−3.877 (12,20,11,14)	−5.927 (12,20,11,14)	−8.927 (11,19,12,9)	−13.375 (11,19,12,9)
	9	−4.228 (11,19,13,8)	−6.513 (11,19,13,8)	−9.798 (11,19,13,8)	−14.640 (11,19,13,8)
	12	−4.440 (11,19,13,8)	−6.766 (11,19,13,8)	−10.158 (11,19,13,8)	−15.352 (11,19,13,8)

interval. Also, it can be observed on the same figure that for S-type MHPM the difference in coding gain between $n = 8$ and $n = 11$ is very small, so the increment of observation interval does not improve the performance very much, but this difference in the error probability for A-type MHPM is apparently significant, which implies the incre-

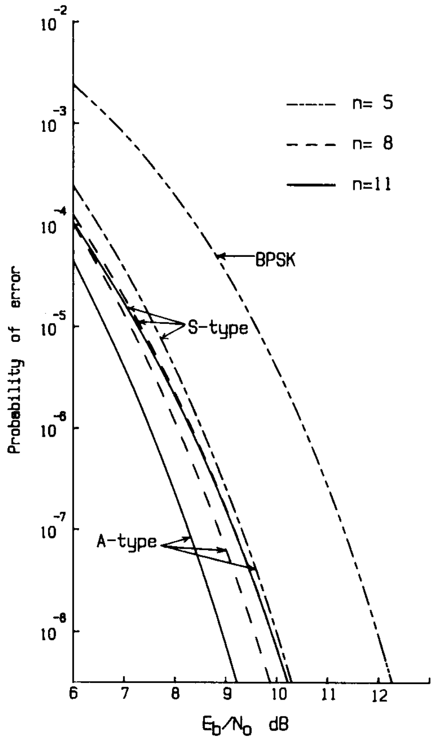


Fig. 5. Error probability bounds for 3-h MHPM with $q = 9$ and $n = 5$, 8, and 11.

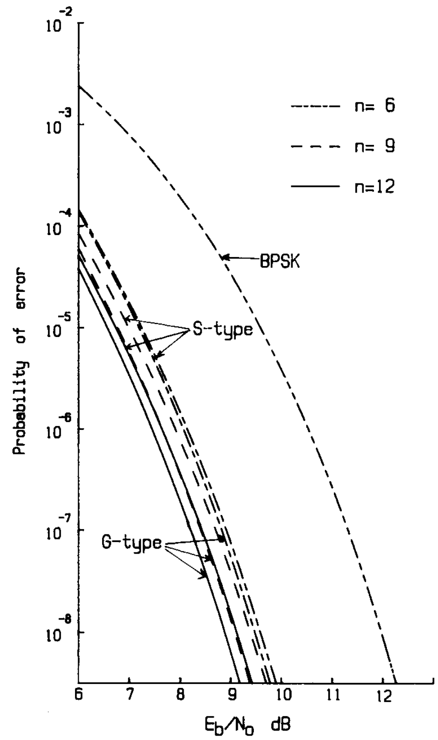


Fig. 7. Error probability bounds for 4-h MHPM with $q = 16$ and $n = 6$, 9, and 12.

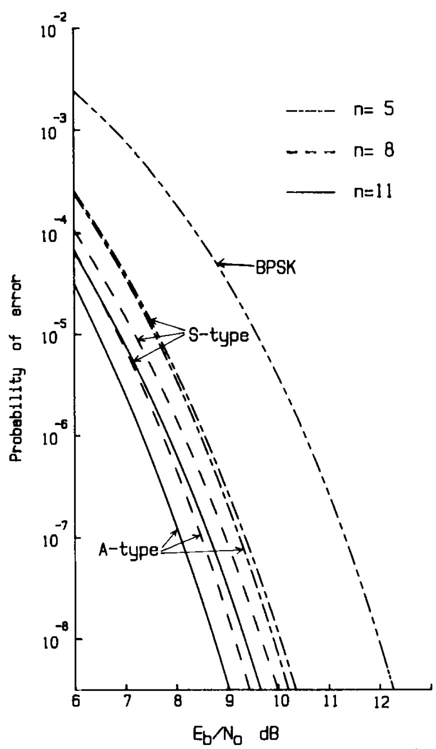


Fig. 6. Error probability bounds for 3-h MHPM with $q = 15$ and $n = 5$, 8, and 11.

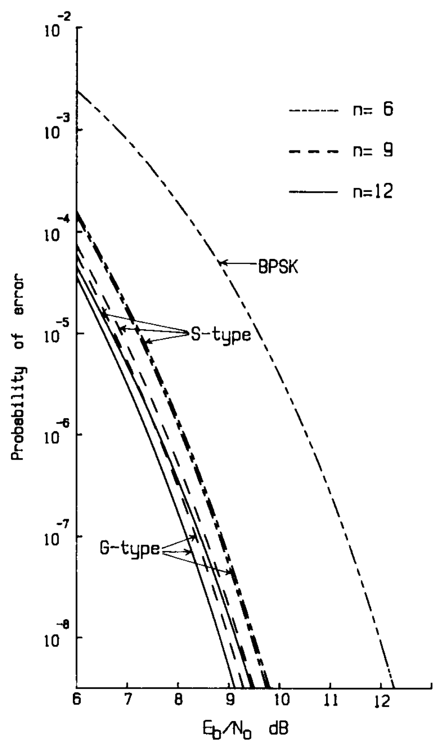


Fig. 8. Error probability bounds for 4-h MHPM with $q = 21$ and $n = 6$, 9, and 12.

ment of observation interval can provide better performance. As another example, for the probability of error 10^{-7}, a gain of about 3.06 dB over BPSK can be found when A-type MHPM is used with observation interval $11T$, as compared to a gain of about 2.16 dB for S-type with the same conditions. In Fig. 6, we can similarly find that the error probability of A-type MHPM with observation interval $8T$ may have performance better than that of S-type MHPM with an $11T$ interval when $E_b/N_o > 6.5$ dB. In other words, we can use the asymmetric MHPM to obtain the performance of conventional MHPM with shorter observation interval. For 4-h schemes in Figs. 7 and 8, the bounds on error probability of asymmetric G-type MHPM with $n = 9$ are again very close to those of conventional S-type MHPM with $n = 12$. Furthermore, a comparison of the curves of the error probability bounds of Figs. 7 and 8 indicates that the differences in upper bounds of error probability are not significant when q is varied from 16 to 22, so we can choose small q to reduce the system complexity. By comparing the performance curves in Figs. 6 and 8, we can also find that although the minimum Euclidean distances for 4-h systems are always higher than those of 3-h systems, if the observation intervals are not long enough, the performances of 3-h systems with $n = 11$ are usually very close to that of 4-h systems with $n = 12$. Therefore, for the coherent detection of asymmetric MHPM with limited observation interval, 3-h schemes can be used for lower complexity. It is thus quite clear that there actually exists a full range of combinations of modulation indexes which can be chosen in the system complexity/SNR tradeoff when asymmetric modulation indexes are used. For example, if the received signal has $E_b/N_o = 8$ dB and the observation interval is $6T$, the G-type 4-h MHPM with $q = 16$ and modulation index set $\{9/16, 15/16, 8/16, 11/16\}$ is a good choice. But when the observation interval is $11T$ and $E_b/N_o = 8$ dB, the A-type 3-h MHPM with $q = 15$ and modulation index set $\{6/15, 9/15, 13/15\}$ is the best.

VI. Power Spectrum

For the asymmetric MHPM schemes proposed in this paper, in most cases the same set of indexes $\{h_i\}$ are used for the modulation of $+1$ and -1, i.e., h_{+i} and h_{-i} have exactly the same set of values except in different sequences. Since we assume that the data $+1$ and -1 are equally probable, apparently in such cases the power spectra of the asymmetric and conventional MHPM will be essentially the same.

Because of the interdependence among the excess phase in different intervals of the MHPM signals, the calculation of the power spectra of MHPM is not as simple as BPSK or MSK. The three principal methods for calculating the power spectra of MHPM signals are 1) simulation, 2) the Markov chain approach, and 3) the direct method [16]. In the simulation method, a pseudorandom data sequence generator is used, and the phase modulation process $\psi(t, \boldsymbol{\alpha})$ is produced. The discrete Fourier transform of the signal envelope $e^{j\varphi(t)}$ is then determined and the

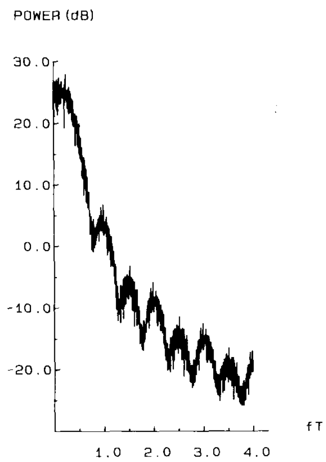


Fig. 9. The power spectral density of S-type MHPM with code $\{4/8, 5/8, 6/8\}$.

spectrum is estimated. In this research, the power spectra for binary S-type and A-type 3-h MHPM with $\{4/8, 5/8, 6/8\}$ are simulated. 256-bit pseudorandom data sequences were used, along with 16 samples of the phase modulation per bit, and the S-type and A-type spectra obtained by averaging the spectra of ten different data sequences are shown in Figs. 9 and 10, respectively. Comparing these two figures, we find that the power spectra for S-type and A-type are very similar as we expected, which means the bandwidth of MHPM with asymmetric modulation indexes will be almost identical to that of conventional MHPM, if the same set of modulation indexes are used for h_{+i} and h_{-i}.

In a different approach, using the simple approximation proposed by Wilson [16] to calculate the multi-h signal spectra as we can obtain rough estimates for the power spectra of asymmetric MHPM signals by a constant-h signal with a modulation index being the average over one cycle of h_i's

$$\bar{h} = \frac{1}{K} \sum_{i=1}^{K} h_i.$$

Since the bandwidth is proportional to the modulation index for constant-h signals, $\bar{h}$ can be used to estimate the bandwidth of the MHPM signals. For the best codes being used to get the distances in Tables I–III, we have calculated the $\bar{h}$ values of conventional and asymmetric MHPM as shown in Table VI. From this table, we can find that the $\bar{h}$ values of asymmetric MHPM are smaller than those of conventional MHPM for 2-h schemes for all q being used. Also, the $\bar{h}$ values of A-type are generally equal to or only slightly larger than those of S-type for 3-h systems with q varying from 8 to 17. For 4-h schemes, the $\bar{h}$ values of G-type are smaller than those of S-type codes for $q = 17$ and 21, which indicates that for 4-h schemes, G-type MHPM will have better noise performance over S-type MHPM even with less bandwidth under these conditions. The average of modulation indices $\bar{h}$ for G-type

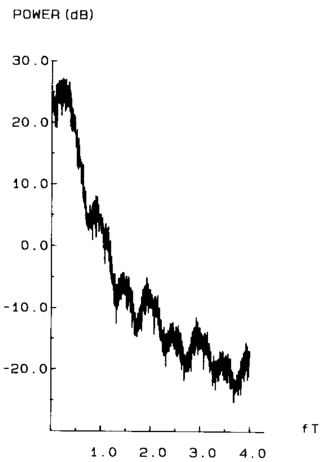


Fig. 10. The power spectral density of A-type MHPM with code $\{4/8, 5/8, 6/8\}$.

TABLE VI
AVERAGE OF THE BEST SET OF MODULATION INDEXES $\bar{h}$ FOR MULTI-h PHASE-CODED MODULATIONS

	K=2		K=3		K=4	
q	Conv.	Asym.	S-type	A-type	S-type	G-type
4	0.63	0.56				
5	0.70	0.55				
6	0.58	0.54				
7	0.57	0.54				
8	0.56	0.53	0.63	0.63		
9	0.61	0.53	0.63	0.70		
10	0.55	0.53	0.60	0.70		
11	0.55	0.52	0.64	0.64		
12	0.54	0.52	0.58	0.69		
13	0.54	0.52	0.69	0.69		
14			0.64	0.71		
15			0.62	0.69		
16			0.69	0.69	0.64	0.67
17			0.71	0.71	0.69	0.65
18					0.60	0.67
19					0.65	0.67
20					0.65	0.65
21					0.73	0.67
22					0.65	0.67
Average	0.58	0.53	0.64	0.69	0.66	0.66

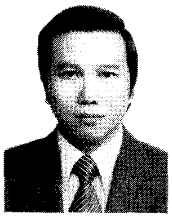
MHPM is 0.65 for $q = 17$, which is the smallest for all values of q being calculated, but this still implies 3.99 dB coding gain over MSK as shown in Table III. Because these asymmetric MHPM schemes can provide significant improvements in error performance with essentially the same bandwidth as shown above, this new class of schemes therefore is very attractive in the sense of bandwidth/power efficiency.

VII. Conclusions

The concept of asymmetric modulation indexes for MHPM has been investigated in this paper and the minimum Euclidean distances and upper bounds of error probability are calculated. Some reasonable improvements in terms of error probability performance over conventional MHPM with symmetric modulation indexes for binary multi-h schemes were found with essentially the same bandwidth and only slight modification in implementation. Also, less dependence of the minimum Euclidean distances on the values of K and q are observed when asymmetric modulation indexes are used. Although further research on the spectral behavior and implementation of this new modulation scheme is still needed, it can be concluded that the concept of asymmetric modulation indices for MHPM is attractive for bandwidth/power efficient modulations.

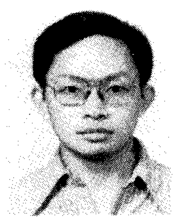
References

[1] J. B. Anderson and D. P. Taylor, "A bandwidth-efficient class of signal-space codes," *IEEE Trans. Inform. Theory*, vol. IT-24, pp. 703–712, Nov. 1978.
[2] T. Aulin and C. E. Sundberg, "Continuous phase modulation—Part I: Full response signaling," *IEEE Trans. Commun.*, vol. COM-29, pp. 196–209, Mar. 1981.
[3] T. Aulin, N. Rydbeck, and C. E. Sundberg, "Continuous phase modulation—Part II: Partial response signaling," *IEEE Trans. Commun.*, vol. COM-29, pp. 210–225, Mar. 1981.
[4] J. B. Anderson and R. de Buda, "Better phase-modulation error performance using trellis phase codes," *Electron. Lett.*, vol. 12, pp. 587–588, Oct. 1976.
[5] J. B. Anderson and D. P. Taylor, "Trellis phase modulation coding: Minimum distance and spectral results," in *Proc. Electron. Aerospace Syst. Conf.*, Arlington, VA, Sept. 25–28, 1977, pp. 29-1A–29-1G.
[6] J. B. Anderson, D. P. Taylor, and A. T. Lereim, "A class of trellis phase modulation codes for coding without bandwidth expansion," in *Proc. Int. Conf. Commun. Conf.*, Toronto, Ont., Canada, June 4–7, 1978, pp. 50.3.1–50.3.5.
[7] J. B. Anderson, C. E. Sundberg, T. Aulin, and N. Rydbeck, "Power bandwidth performance of smoothed phase modulation codes," *IEEE Trans. Commun.*, vol. COM-29, pp. 187–195, Mar. 1981.
[8] J. B. Anderson, "Simulated error performance of multi-*h* phase codes," *IEEE Trans. Inform. Theory*, vol. IT-27, pp. 357–362, May 1981.
[9] G. Ungerboeck, "Channel coding with multilevel/phase signals," *IEEE Trans. Inform. Theory*, vol. IT-28, pp. 55–67, Jan. 1982.
[10] T. Aulin and C. E. Sundberg, "Minimum Euclidean distance and power spectrum for a class of smoothed phase modulation codes with constant envelope," *IEEE Trans. Commun.*, vol. COM-30, pp. 1721–1729, July 1982.
[11] ——, "On the minimum Euclidean distance for a class of signal space codes," *IEEE Trans. Inform. Theory*, vol. IT-28, pp. 43–55, Jan. 1982.
[12] W. U. Lee, "Carrier synchronization of CPFSK signals," in *Proc. Nat. Telecommun. Conf.*, Los Angeles, CA, Dec. 5–7, 1977, pp. 30:2-1–30:2-4.
[13] B. A. Mazur and D. P. Taylor, "Demodulation and carrier synchronization of multi-*h* phase codes," *IEEE Trans. Commun.*, vol. COM-29, pp. 257–266, Mar. 1981.
[14] K. R. Raveendra and R. Srinivasan, "Coherent detection of binary multi-*h* CPM," *Proc. IEE, Part F, Commun., Radar, Signal Processing*, vol. 134, pp. 416–426, July 1987.
[15] S. G. Wilson, J. H. Highfill, III, and C. D. Hsu, "Error bounds for multi-*h* phase codes," *IEEE Trans. Inform. Theory*, vol. IT-28, pp. 660–665, July 1982.
[16] S. G. Wilson and R. C. Gaus, "Power spectra of multi-*h* phase codes," *IEEE Trans. Commun.*, vol. COM-29, pp. 250–256, Mar. 1981.
[17] J. B. Anderson, T. Aulin, and C. E. Sundberg, *Digital Phase Modulation.* New York: Plenum, 1986.
[18] R. E. Ziemer and R. L. Peterson, *Digital Communications and Spread Spectrum Systems.* New York: Macmillan, 1985.



Hong-Kuang Hwang (S'86) was born in Taiwan, Republic of China, in 1951. He received the B.S. degree in communication engineering from National Chiao Tung University, Hsinchu, Taiwan, in 1973, and the M.S. degree in electrical engineering from Northwestern University, Evanston, IL, in 1981. He is currently working toward the Ph.D. degree in electronics engineering at the National Chiao Tung University, Taiwan, Republic of China.

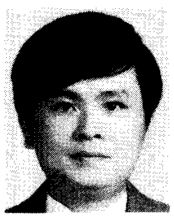
Since 1976 he has been an Engineer at Chung Shan Institute of Science & Technology, Taiwan. His research interests are in the areas of communication systems and digital speech processing.



Lin-Shan Lee received the B.S. degree in electrical engineering from National Taiwan University, Taipei, Taiwan, Republic of China, in 1974, and the M.S. and Ph.D. degrees in electrical engineering from Stanford University, Stanford, CA, in 1975 and 1977, respectively.

He was involved in research of communication systems and satellite systems while a graduate student of Stanford University, and was with EDUTEL Communications and Development, Inc., Palo Alto, CA, from 1977 to 1980 with research interests in various aspects of communication systems, technologies and developments, especially in satellite communications. He became an Associate Professor at the Department of Electrical Engineering, National Taiwan University in September 1979, and a Professor in August 1982. He also became the Acting Chairman of the Department of Computer Science and Information Engineering of the university in September 1982, and the Chairman from August 1983 to July 1987. He is currently doing research in the areas of digital transmissions, satellite communications, and digital speech processing, specially concentrating on the problem of computer input/output techniques using Mandarin Chinese. He authored about 100 technical papers in these areas.

Dr. Lee was the recipient of the Outstanding Young Engineer Award sponsored by the Institute of Chinese Engineers in 1983, the Ten Outstanding Young Men Award of the Republic of China in 1984, the Outstanding Young Scientist Fellowship sponsored by URSI (Union of Radio Science International) in 1984, the Distinguished Research Award sponsored by the National Science Coucil of the Republic of China in 1985 and 1987, the Outstanding Youth Medal of the Republic of China in 1986, and the Distinguished Teaching Award sponsored by the Ministry of Education of the Republic of China in 1988.



Sin-Horng Chen (S'80-M'84) received the B.S. degree in communication engineering and the M.S. degree in electronics engineering from National Chiao Tung University, Hsinchu, Taiwan, Republic of China, in 1976 and 1978, respectively, and the Ph.D. degree in electrical engineering from Texas Tech University, Lubbock, in 1983.

From 1978 to 1980, he was an Assistant Engineer for Telecommunication Labs., Taiwan. He is currently an Associate Professor in the Institute of Communication Engineering at National Chiao Tung University. His research interests include digital signal processing, speech processing, and communication systems.