

# Multi-hop Routing is Order-optimal in Underwater Extended Networks

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**Abstract**—Capacity scaling laws are analyzed in an underwater acoustic network with  $n$  regularly located nodes. A narrow-band model is assumed where the carrier frequency is allowed to scale as a function of  $n$ . In the network, we characterize an attenuation parameter that depends on the frequency scaling as well as the transmission distance. A cut-set upper bound on the throughput scaling is then derived in extended networks. Our result indicates that the upper bound is inversely proportional to the attenuation parameter, thus resulting in a highly power-limited network. Furthermore, we describe an achievable scheme based on the simple nearest-neighbor multi-hop (MH) transmission. It is shown under extended networks that the MH scheme is order-optimal as the attenuation parameter scales exponentially with  $\sqrt{n}$  (or faster). Finally, these scaling results are extended to a random network realization.

## I. INTRODUCTION

A pioneering work of [1], introduced by Gupta and Kumar, characterized the sum throughput scaling in a large wireless radio network. They showed that the total throughput scales as  $\Theta(\sqrt{n/\log n})$  when a multi-hop (MH) routing strategy is used for  $n$  source-destination (S–D) pairs randomly distributed in a unit area.<sup>1</sup> MH schemes are then further developed and analyzed in [2], [3]. A recent result [4] has shown that an almost linear throughput in the radio network, i.e.,  $\Theta(n^{1-\epsilon})$  for an arbitrarily small  $\epsilon > 0$ , is achievable by using a hierarchical cooperation strategy.

Along with the studies in terrestrial radio networks, the interest in study of underwater networks has been growing with recent advances in acoustic communication technology [5], [6]. In underwater acoustic communication systems, both bandwidth and power are severely limited due to the exponential (rather than polynomial) path-loss attenuation with

propagation distance and frequency-dependent attenuation. This is a main feature that distinguishes underwater systems from wireless radio links. Based on these characteristics, network coding schemes [6]–[8] have been presented for underwater acoustic channels. One natural question is what are the fundamental capabilities of underwater networks in supporting multiple S–D pairs over an acoustic channel. To answer this question, the throughput scaling for underwater networks of unit area was first studied [9], where  $n$  nodes were arbitrarily located in a planar disk of unit area and the carrier frequency was set to a constant independent of  $n$ . That work showed an upper bound on the throughput of each node based on the physical model assumption in [1]. This upper bound scales as  $n^{-1/\alpha} e^{-W_0(\Theta(n^{-1/\alpha}))}$ , where  $\alpha$  corresponds to the spreading factor of the underwater channel, and  $W_0$  represents the branch zero of the Lambert function [10].<sup>2</sup> Since the spreading factor typically has values in the range  $1 \leq \alpha \leq 2$  [9], the throughput per node decreases almost as  $O(n^{-1/\alpha})$  for large enough  $n$ , which is considerably faster than the  $\Theta(\sqrt{n})$  scaling characterized for wireless radio settings [1].

In this paper, a capacity scaling law for underwater networks is analyzed in extended networks of unit node density. Especially, we are interested in the case where the carrier frequency scales as a certain function of  $n$  in a narrow-band model. Such an assumption changes the scaling behavior significantly due to the attenuation characteristics. We aim to study both an information-theoretic upper bound and achievable scaling rate.

We explicitly characterize an *attenuation parameter* that depends on the transmission distance and also on the carrier frequency. For networks with  $n$  regularly distributed nodes, we derive an upper bound on the total throughput scaling using the cut-set bound. Our upper bound is based on the characteristics of power-limited regimes shown in [4]. In extended networks, it is shown that the upper bound is inversely proportional to the attenuation parameter. This leads to a highly power-limited network for all the operating regimes. Interestingly, it is seen that unlike the case of wireless radio networks, our upper bound heavily depends on the attenuation parameter but not on the spreading factor (corresponding to the path-loss exponent

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<sup>1</sup>We use the following notations: i)  $f(x) = O(g(x))$  means that there exist constants  $C$  and  $c$  such that  $f(x) \leq Cg(x)$  for all  $x > c$ . ii)  $f(x) = o(g(x))$  means that  $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$ . iii)  $f(x) = \Omega(g(x))$  if  $g(x) = O(f(x))$ . iv)  $f(x) = \omega(g(x))$  if  $g(x) = o(f(x))$ . v)  $f(x) = \Theta(g(x))$  if  $f(x) = O(g(x))$  and  $g(x) = O(f(x))$ .

<sup>2</sup>The Lambert W function is defined to be the inverse of the function  $z = W(z)e^{W(z)}$  and the branch satisfying  $W(z) \geq -1$  is denoted by  $W_0(z)$ .

in wireless networks). In addition, to constructively show our achievability result for extended regular networks, we describe the conventional nearest-neighbor MH transmission [1] with a slight modification, and analyze its achievable throughput. It is shown under extended networks that the achievable rate based on the MH routing scheme matches the upper bound within a factor of  $n$  with arbitrarily small exponent as long as the attenuation parameter increases exponentially with respect to  $\sqrt{n}$  (or faster). Furthermore, a random network scenario is also discussed.

The rest of this paper is organized as follows. Section II describes our system and channel models. In Section III, the cut-set upper bound on the throughput is derived. In Section IV, achievable throughput scaling is analyzed. These results are extended to the random network case in Section V. Finally, Section VI summarizes the paper with some concluding remarks. We refer to the full paper [11] for the detailed description and all the proofs.

## II. SYSTEM AND CHANNEL MODELS

We consider a two-dimensional underwater network that consists of  $n$  nodes on a square with unit node density such that two neighboring nodes are 1 unit of distance apart from each other in an extended network, i.e., a regular network [12], [13]. We randomly pick a matching of S–D pairs, so that each node is the destination of exactly one source. We assume frequency-flat channel of bandwidth  $W$  Hz around carrier frequency  $f$ , which satisfies  $f \gg W$ , i.e., narrow-band model. This is a highly simplified model, but nonetheless one that suffices to demonstrate the fundamental mechanisms that govern capacity scaling. Assuming that all the nodes have perfectly directional transmissions, we also disregard multipath propagation. Each node has an average transmit power constraint  $P$  (constant), and we assume that the channel state information is available at all receivers, but not at the transmitters. It is assumed that each node transmits at a rate  $T(n)/n$ , where  $T(n)$  denotes the total throughput of the network.

Now let us turn to channel modeling. An underwater acoustic channel is characterized by an attenuation that depends on both the distance  $r_{ki}$  between nodes  $i$  and  $k$  ( $i, k \in \{1, \dots, n\}$ ) and the signal frequency  $f$ , and is given by

$$A(r_{ki}, f) = c_0 r_{ki}^\alpha a(f)^{r_{ki}} \quad (1)$$

for some constant  $c_0 > 0$  independent of  $n$ , where  $\alpha$  is the spreading factor and  $a(f) > 1$  is the absorption coefficient [5]. The spreading factor describes the geometry of propagation and is typically  $1 \leq \alpha \leq 2$ . Note that existing models of wireless networks typically correspond to the case for which  $a(f) = 1$  (or a positive constant independent of  $n$ ) and  $\alpha > 2$ .

A common empirical model gives  $a(f)$  in dB/km for  $f$  in kHz as [5]:

$$10 \log a(f) = a_0 + a_1 f^2 + a_2 \frac{f^2}{b_1 + f^2} + a_3 \frac{f^2}{b_2 + f^2},$$

where  $\{a_0, \dots, a_3, b_1, b_2\}$  are positive constants independent of  $n$ . As mentioned earlier, we will allow the carrier frequency  $f$  to scale with  $n$ . Especially, we consider the case where the frequency scales at arbitrarily increasing rates relative to  $n$ . The absorption  $a(f)$  is then an increasing function of  $f$  such that

$$a(f) = \Theta(e^{c_1 f^2}) \quad (2)$$

with respect to  $f$  for some constant  $c_1 > 0$  independent of  $n$ .

The noise  $n_i$  at node  $i \in \{1, \dots, n\}$  in an acoustic channel can be modeled through four basic sources: turbulence, shipping, waves, and thermal noise [5]. We assume that  $n_i$  is the circularly symmetric complex additive colored Gaussian noise with zero mean and power spectral density (psd)  $N(f)$ , and thus the noise is frequency-dependent. The overall psd of four sources decays linearly on the logarithmic scale in the frequency region 100 Hz – 100 kHz, which is the operating regime used by the majority of acoustic systems, and thus is approximately given by [5]

$$\log N(f) = a_4 - a_5 \log f$$

for some positive constants  $a_4$  and  $a_5$  independent of  $n$ . This means that  $N(f) = O(1)$  since

$$N(f) = \Theta\left(\frac{1}{f^{a_5}}\right) \quad (3)$$

in terms of  $f$  increasing with  $n$ .

The received signal  $y_k$  at node  $k \in \{1, \dots, n\}$  at a given time instance is given by

$$y_k = \sum_{i \in I} h_{ki} x_i + n_k,$$

where

$$h_{ki} = \frac{e^{j\theta_{ki}}}{\sqrt{A(r_{ki}, f)}}, \quad (4)$$

represents the channel,  $x_i \in \mathbb{C}$  is the signal transmitted by node  $i$  and  $I \subset \{1, \dots, n\}$  is the set of simultaneously transmitting nodes. The random phases  $e^{j\theta_{ki}}$  are uniformly distributed over  $[0, 2\pi)$  and independent for different  $i, k$ , and time. We thus assume a narrow-band time-varying channel, whose gain changes to a new independent value for every symbol.

Based on the above channel characteristics, operating regimes of the network are identified according to the following physical parameters: the absorption  $a(f)$  and the noise psd  $N(f)$  which are exploited here by choosing the frequency  $f$  based on the number  $n$  of nodes. In other words, if the relationship between  $f$  and  $n$  is specified, then  $a(f)$  and  $N(f)$  can be given by a certain scaling function of  $n$  from (2) and (3), respectively.

## III. CUT-SET UPPER BOUND

To access the fundamental limits of an underwater network, a cut-set upper bound on the total throughput scaling is analyzed from an information-theoretic perspective [14]. Specifically, an upper bound based on the power transfer argument [4]

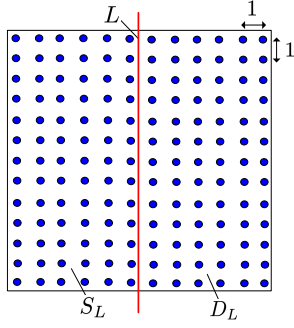


Fig. 1. The cut  $L$  in a two-dimensional extended regular network.  $S_L$  and  $D_L$  represent the sets of source and destination nodes, respectively.

is established for extended networks. Note, however, that the present problem is not equivalent to the conventional extended network framework [4] due to different channel characteristics. Our interest is particularly in the operating regimes for which the upper bound is tight.

Consider a given cut  $L$  dividing the network area into two halves as in [4], [15] (see Fig. 1). Let  $S_L$  and  $D_L$  denote the sets of sources and destinations, respectively, for the cut  $L$  in the network. More precisely, under  $L$ , source nodes  $S_L$  are on the left, while all nodes on the right are destinations  $D_L$ . In this case, we have an  $\Theta(n) \times \Theta(n)$  multiple-input multiple-output (MIMO) channel between the two sets of nodes separated by the cut.

In an extended network, we take into account an approach based on the amount of power transferred across the network according to different operating regimes. As pointed out in [4], the information transfer from  $S_L$  to  $D_L$  is highly power-limited since all the nodes in the set  $D_L$  are ill-connected to the left-half network in terms of power. This implies that the information transfer is bounded by the total received power transfer, rather than the cardinality of the set  $D_L$ . For the cut  $L$ , the total throughput  $T(n)$  for sources on the left is bounded by the (ergodic) capacity of the MIMO channel between  $S_L$  and  $D_L$  under time-varying channel assumption, and thus is given by

$$T(n) \leq \max_{\mathbf{Q}_L \geq 0} E \left[ \log \det \left( \mathbf{I}_{\Theta(n)} + \frac{1}{N(f)} \mathbf{H}_L \mathbf{Q}_L \mathbf{H}_L^H \right) \right], \quad (5)$$

where  $\mathbf{H}_L$  is the matrix with entries  $[\mathbf{H}_L]_{ki}$  for  $i \in S_L, k \in D_L$ , and  $\mathbf{Q}_L \in \mathbb{C}^{\Theta(n) \times \Theta(n)}$  is the positive semi-definite input signal covariance matrix whose  $k$ -th diagonal element satisfies  $[\mathbf{Q}_L]_{kk} \leq P$  for  $k \in S_L$ .

The relationship in (5) will be further specified in Theorem 1. Before that, we first apply the techniques of [4], [16] to obtain the total power transfer of the set  $D_L$ . These techniques involves the relaxation of the individual power constraints to a total weighted power constraint, where the weight assigned to each source corresponds to the total received power on the other side of the cut. To be specific, each column  $i$  of the matrix  $\mathbf{H}_L$  is normalized by the square root of the total received power on the other side of the cut from source  $i \in S_L$ . From (1) and (4), the total power  $P_L^{(i)}$  received from the signal

sent by the source  $i$  is given by

$$P_L^{(i)} = P d_L^{(i)}, \quad (6)$$

where

$$d_L^{(i)} = \frac{1}{c_0} \sum_{k \in D_L} r_{ki}^{-\alpha} a(f)^{-r_{ki}} \quad (7)$$

for some constant  $c_0 > 0$  independent of  $n$ . For convenience, we now index the node positions such that the source and destination nodes under the cut  $L$  are located at positions  $(-i_x + 1, i_y)$  and  $(k_x, k_y)$ , respectively, for  $i_x, k_x = 1, \dots, \sqrt{n}/2$  and  $i_y, k_y = 1, \dots, \sqrt{n}$ . The scaling result of  $d_L^{(i)}$  defined in (7) can be derived as follows.

*Lemma 1:* In an extended network, the term  $d_L^{(i)}$  in (7) is

$$d_L^{(i)} = \Theta \left( i_x^{1-\alpha} a(f)^{-i_x} \right),$$

where  $-i_x + 1$  represents the horizontal coordinate of node  $i \in S_L$  for  $i_x = 1, \dots, \sqrt{n}/2$ .

The proof of this lemma is obtained by finding upper and lower bounds on  $d_L^{(i)}$  from layering techniques. The expression (5) is then rewritten as

$$\max_{\tilde{\mathbf{Q}}_L \geq 0} E \left[ \log \det \left( \mathbf{I}_{\Theta(n)} + \frac{1}{N(f)} \mathbf{F}_L \tilde{\mathbf{Q}}_L \mathbf{F}_L^H \right) \right], \quad (8)$$

where  $\mathbf{F}_L$  is the matrix with entries  $[\mathbf{F}_L]_{ki} = \frac{1}{\sqrt{d_L^{(i)}}} [\mathbf{H}_L]_{ki}$ ,

which are obtained from (7), for  $i \in S_L, k \in D_L$ . Here,  $\tilde{\mathbf{Q}}_L$  is the matrix satisfying

$$[\tilde{\mathbf{Q}}_L]_{ki} = \sqrt{d_L^{(k)} d_L^{(i)}} [\mathbf{Q}_L]_{ki},$$

which means  $\text{tr}(\tilde{\mathbf{Q}}_L) \leq \sum_{i \in S_L} P_L^{(i)}$ .

We next examine the behavior of the largest singular value for the normalized channel matrix  $\mathbf{F}_L$ , and then show how much it affects an upper bound on (8). We first address the case where  $\mathbf{F}_L$  is well-conditioned according to the attenuation parameter  $a(f)$ .

*Lemma 2:* Let  $\mathbf{F}_L$  denote the normalized channel matrix defined by the expression (8). Under the attenuation regimes  $a(f) = \Omega \left( (1 + \epsilon_0)^{\sqrt{n}} \right)$  for an arbitrarily small  $\epsilon_0 > 0$ , we have that

$$E \left[ \|\mathbf{F}_L\|_2^2 \right] \leq c_2 \log n$$

for some constant  $c_2 > 0$  independent of  $n$ .

Note that the matrix  $\mathbf{F}_L$  is well-conditioned as  $a(f)$  scales exponentially with respect to  $\sqrt{n}$  (or faster). Otherwise, i.e., if  $a(f) = o \left( (1 + \epsilon_0)^{\sqrt{n}} \right)$ , the largest singular value of  $\mathbf{F}_L$  scales as a polynomial factor of  $n$ , thus resulting in a loose upper bound on the total throughput. Using Lemma 2, we obtain the following result.

*Lemma 3:* Under  $a(f) = \Omega \left( (1 + \epsilon_0)^{\sqrt{n}} \right)$ , the term (8) is upper-bounded by

$$\frac{n^\epsilon}{N(f)} \sum_{i \in S_L} P_L^{(i)} \quad (9)$$

for arbitrarily small positive constants  $\epsilon_0$  and  $\epsilon$ , where  $P_L^{(i)}$  is given by (6).

Note that (9) represents the total amount of received signal-to-noise ratio from the set  $S_L$  of sources to the set  $D_L$  of destinations for a given cut  $L$ . We are now ready to show the cut-set upper bound in extended networks.

*Theorem 1:* For an underwater regular network of unit node density, where the absorption coefficient  $a(f)$  scales as  $\Omega\left((1 + \epsilon_0)\sqrt{n}\right)$  for an arbitrarily small  $\epsilon_0 > 0$ , the total throughput  $T(n)$  is upper-bounded by

$$T(n) \leq \frac{c_3 n^{1/2+\epsilon}}{a(f)N(f)}, \quad (10)$$

where  $c_3 > 0$  is some constant independent of  $n$  and  $\epsilon > 0$  is an arbitrarily small constant.

Note that this upper bound is expressed as a function of the absorption  $a(f)$  and the noise psd  $N(f)$  while an upper bound for wireless radio networks depends only on the constant value  $\alpha$  [4].

By using (2) and the regimes  $a(f) = \Omega((1 + \epsilon_0)\sqrt{n})$ , we can also obtain the following condition:

$$f = \Omega(n^{1/4}),$$

which means that if  $f$  scales faster than  $n^{1/4}$ , then the result in (10) is satisfied.

#### IV. ACHIEVABILITY RESULT

In this section, we show that the considered transmission scheme, commonly used in wireless radio networks, is order-optimal in underwater networks. Under a regular network of unit node density, the conventional MH transmission [1] is used and its achievable throughput scaling is analyzed to show its order optimality.

Instead of original (continuous) MH transmissions, a bursty transmission scheme [4], [15], which uses only a fraction  $1/a(f)N(f)$  of the time for actual transmission with instantaneous power  $a(f)N(f)P$  per node, is used to simply apply the analysis for networks with no power limitation to our network model. With this scheme, the received signal power from the desired transmitter, the noise psd, and the total interference power from the set  $I \subset \{1, \dots, n\}$  have the same scaling, i.e.,  $\Theta(N(f))$ , and the received signal-to-interference-and-noise ratio (SINR) is kept at  $\Theta(1)$  under the narrow-band model.

The achievable rate of MH is now shown by quantifying the amount of interference.

*Lemma 4:* Suppose that a regular network of unit node density uses the MH protocol. Then, the total interference power from other simultaneously transmitting nodes, corresponding to the set  $I \subset \{1, \dots, n\}$ , is upper-bounded by  $\Theta(N(f))$ , where  $N(f)$  denotes the psd of noise  $n_i$  at receiver  $i \in \{1, \dots, n\}$ .

The proof of this lemma is obtained by introducing a layering technique for interfering routing cells. Note that the signal power no longer decays polynomially but rather exponentially with propagation distance in our network. This implies that the absorption term  $a(f)$  in (1) will play an important role

in determining the performance. It is also seen that the total interference power does not depend on the spreading factor  $\alpha$ . Using Lemma 4, it is now possible to simply obtain a lower bound on the capacity scaling in the network, and hence the following result presents the achievable rates under the MH protocol.

*Theorem 2:* In an underwater regular network of unit node density,

$$T(n) = \Omega\left(\frac{n^{1/2}}{a(f)N(f)}\right)$$

is achievable.

Based on Theorems 1 and 2, when  $a(f) = \Omega\left((1 + \epsilon_0)\sqrt{n}\right)$ , i.e.,  $f = \Omega(n^{1/4})$ , it is easy to see that the achievable rate and the upper bound are of the same order up to  $n^\epsilon$ , where  $\epsilon$  and  $\epsilon_0$  are vanishingly small positive constants. The MH is therefore order-optimal in regular networks with unit node density under the above attenuation regimes. We also remark that the hierarchical cooperation strategy [4] may not be helpful to improve the achievable throughput due to a long-range MIMO transmission, which severely degrades performance in highly power-limited networks.<sup>3</sup> Even with the random phase model, which may enable us to obtain enough degrees-of-freedom gain, the benefit of randomness cannot be exploited because of the power limitation.

#### V. EXTENSION TO RANDOM NETWORKS

In closing, we would like to mention a random network configuration, where  $n$  S-D pairs are uniformly and independently distributed on a square. We first discuss an upper bound for extended networks. A precise upper bound can be obtained using the binning argument of [4] (refer to Appendix V in [4] for the details). For analytical convenience, we can assume the empty zone  $E_L$ , in which there are no nodes in the network, consisting of a rectangular slab of width  $0 < \bar{c} < \frac{1}{2\sqrt{7}e^{1/4}}$ , independent of  $n$ , immediately to the right of the centerline (cut), as done in [15] (see Fig. 2).<sup>4</sup> If the network area is then divided into  $n$  squares of unit area, then there are fewer than  $\log n$  nodes in each square with high probability regardless of the channel characteristics. Now we take into account the network transformation resulting in a regular network with at most  $\log n$  and  $2 \log n$  nodes, on the left and right, respectively, at each square vertex except for the empty zone (see Fig. 2). Then, the nodes in each square are moved together onto one vertex of the corresponding square. More specifically, under the cut  $L$ , the node displacement is performed in the sense

<sup>3</sup>In wireless radio networks of unit node density, the hierarchical cooperation provides a near-optimal throughput scaling for the operating regimes  $2 < \alpha < 3$ , where  $\alpha$  denotes the path-loss exponent that is greater than 2 [4]. Note that the analysis in [4] is valid under the assumption that  $\alpha$  is kept at the same value on all levels of hierarchy.

<sup>4</sup>Although this assumption does not hold in our random configuration, it is shown in [15] that there exists a vertical cut such that there are no nodes located closer than  $0 < \bar{c} < \frac{1}{2\sqrt{7}e^{1/4}}$  on both sides of this cut when we allow a cut that is not necessarily linear. Such an existence is proved by using percolation theory [2]. This result can be directly applied to our network model since it only relies on the node distribution but not the channel characteristics. Hence, removing the assumption does not cause any change in performance.

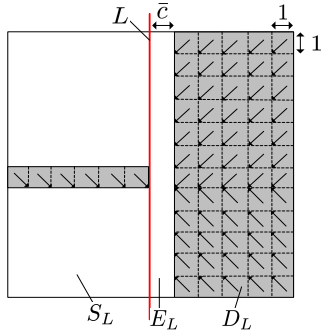


Fig. 2. The node displacement to square vertices, indicated by arrows. The empty zone  $E_L$  with width constant  $\bar{c}$  is assumed for simplicity.

of decreasing the Euclidean distance between source node  $i \in S_L$  and the corresponding destination  $k \in D_L$ , as shown in Fig. 2, which will provide an upper bound on  $d_L^{(i)}$  in (7). It is obviously seen that the amount of power transfer under the transformed regular network is greater than that under another regular network with at most  $\log n$  nodes at each vertex, located at integer lattice positions in a square with area  $n$ . Hence, the upper bound for random networks is boosted by at least a logarithmic factor of  $n$  compared to that of regular networks discussed in Section III.

Now we turn our attention to showing an achievable throughput for extended random networks. In this case, the nearest-neighbor MH protocol [1] can also be utilized since our network is highly power-limited. Then, the area of each routing cell needs to scale with  $2 \log n$  to guarantee at least one node in a cell [1], [3].<sup>5</sup> Each routing cell operates based on 9-time division multiple access to avoid causing large interference to its neighboring cells [1], [3]. For the routing with continuous MH transmissions (i.e., no burstiness), since per-hop distance is given by  $\Theta(\sqrt{\log n})$ , the received signal power from the intended transmitter is expressed as

$$\frac{c_4 P}{(\log n)^{\alpha/2} a(f)^{c_5 \sqrt{\log n}}}$$

for some constants  $c_4 > 0$  and  $c_5 \geq \sqrt{2}$  independent of  $n$ , which thus results in at least a polynomial decrease in the throughput compared to the regular network case shown in Section IV (note that this relies on the fact that  $\log(1+x)$  can be approximated by  $x$  for small  $x > 0$ ).<sup>6</sup> This comes from the fact that the received signal power tends to be mainly limited due to exponential attenuation with transmission distance  $\Theta(\sqrt{\log n})$ . Therefore, we may conclude that the existing MH

<sup>5</sup>When methods from percolation theory are applied to our random network [2], the routing area constructed during the highway phase is a certain positive constant that is less than 1 and independent of  $n$ . The distance in the draining and delivery phases, corresponding to the first and last hops of a packet transmission, is nevertheless given by some constant times  $\log n$ , thereby limiting performance, especially for the condition  $a(f) = \omega(1)$ . Hence, using the protocol in [2] indeed does not perform better than the conventional MH case [1] in random networks.

<sup>6</sup>In terrestrial radio channels, there is a logarithmic gap in the achievable scaling laws between regular and random networks [1], [12].

scheme does not satisfy the order optimality under extended random networks regardless of the attenuation parameter  $a(f)$ .

## VI. CONCLUSION

The attenuation parameter and the capacity scaling laws have been characterized in a narrow-band channel of underwater acoustic networks. Provided that the carrier frequency  $f$  scales at arbitrary rates relative to the number  $n$  of nodes, the information-theoretic upper bound and the achievable throughput were derived as a function of the attenuation parameter  $a(f)$  in extended regular networks. We proved that the nearest-neighbor MH protocol is order-optimal as long as the frequency  $f$  scales faster than  $n^{1/4}$ . Our scaling results were also extended to the random network scenario, where it was shown that the conventional MH scheme does not satisfy the order optimality for all the operating regimes.

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