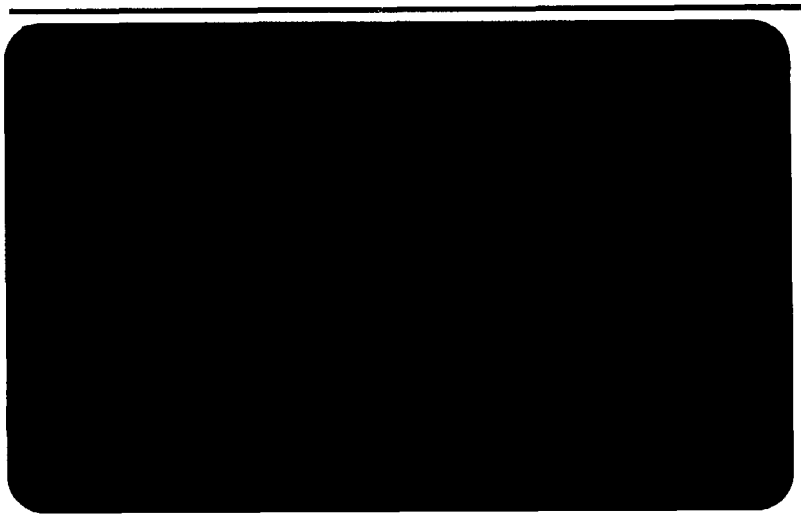

TU Delft
Delft University of Technology

Faculty of Aerospace Engineering

Memorandum M-618

MULTI-LEVEL OPTIMIZATION OF AIRCRAFT SHELL STRUCTURES

A. Rothwell

Faculty of Aerospace Engineering
Delft University of Technology

December 1989

Abstract

Various aspects of multi-level optimization are reviewed, and the application to problems typical of aircraft wing and fuselage structures is discussed. A three-level optimization is proposed, since this relates in a convenient way to the design of a shell structure in which stiffened panels are assembled into individual cross-sections which are in turn linked together to form the complete structure. Furthermore the different tasks which the designer has to perform can then retain their separate identity, and a considerable degree of 'local' optimization can be carried out. The suitability of multi-level optimization in more complex design problems is tested on a structure representative of a wing box in composite material, with buckling limitations in each panel, and another problem in which aeroelastic requirements are included. Proposals are made for a simplified approach to the sensitivity data used in multi-level optimization.

1. INTRODUCTION

The aim of the work described here is the development of a preliminary design program for stiffened shell structures typical of aircraft wings and fuselages, based on multi-level optimization (1). The optimization of large structural systems is currently receiving much attention in the literature, and various strategies have been proposed to reduce the size of the optimization problem (see for example refs. 2 and 3). Multi-level optimization enables a large problem to be broken down into a number of smaller ones, at different levels according to the type of problem being solved. The nature of an aircraft shell structure makes multi-level optimization highly practical, not only in terms of reducing the computing cost but also because the individual tasks in the traditional design process are then preserved. For example, the design of stringer-skin panels and shear webs in a wing is a detail activity which can best be carried out as far as possible in isolation from other aspects of the design. Nevertheless their design has a direct influence on the stiffness and therefore on the aeroelastic characteristics of the wing as a whole, and to some extent on the load distribution within the wing. The rib pitch, on the other hand, is a parameter belonging to the overall design of the wing which clearly has a major influence on the design of compression panels in the wing. In a multi-level optimization these interactions are taken into account in a relatively simple way, without attempting a one-step optimization of the complete wing. Even at the detail level, with composite materials for example, the design freedom may still be too great for a conventional, intuitive approach to the design. Resort to more formal optimization methods becomes essential if the full advantage is to be gained from the use of these materials. It is emphasized that the aim of the present work is an optimization program for use in the early stages of the design process, for comparison of alternative designs and for an efficient initial sizing of the structure. Therefore analytical or semi-analytical methods for structural behaviour are preferred to purely numerical methods, especially at the level of detail design. Inevitably, though, the optimization process itself remains firmly rooted in numerical techniques.

2. MULTI-LEVEL APPROACH

A simple example will be used to illustrate some features of the optimization problem, and as a basis for a multi-level approach. Consider a rectangular box structure (such as the one shown inset in Fig. 4) under a bending moment resulting in a loading intensity p = 1000 N/mm in the upper and lower panels. The efficiency formula (4) for a compression panel, based on simultaneous flexural and local buckling, gives a design stress:

$$\sigma = \eta \sqrt{\frac{pE}{L}} \, . \qquad (1)$$

Assume a rib pitch L = 1000 mm and Young's modulus E = 72 000 N/mm^2. With a maximum efficiency η_{max} = 0.955 for a panel with Z-section stringers, the optimum equivalent (i.e. smeared) thickness of the compression panel $\bar{t}$ = 3.90 mm and the actual skin thickness t = 1.56 mm. (See Fig. 1: this figure has been reconstructed to show specifically the dependence of η on $\bar{t}/t$.). For the tension panel take, somewhat arbitrarily, a ratio $\bar{t}_1/t_1 = 2$. If the allowable tensile stress is 400 N/mm^2, the equivalent thickness of the tension panel $\bar{t}_1$ = 2.5 mm and the actual skin thickness t_1 = 1.25 mm.

Suppose now that in addition there is a torsional stiffness requirement equivalent to a uniform thickness of the box of 2.0 mm. The redesign will be done in three steps as follows:

Step 1. Simply increase the skin thickness of the tension and compression panels to 2.0 mm, with no change in the stringers. The total equivalent thickness ($\bar{t} + \bar{t}_1$) becomes 7.59 mm (see Table 1). Neither the compression panel nor the tension panel is now fully-stressed.

Step 2. Take advantage of the increased skin thickness t of the compression panel to reduce the size of the stringers. (Note: it is assumed that the stringers on the tension panel are already of minimum size and cannot be reduced further). At an efficiency η now slightly less than the maximum, due to a lower $\bar{t}/t$, the total equivalent thickness ($\bar{t} + \bar{t}_1$) becomes 7.21 mm, a reduction of 5%.

Step 3. Since the tension panel is still thicker than necessary for its tensile loading, the thickness of the compression skin can be increased at the expense of the tension skin (maintaining the same torsional stiffness) to obtain a further reduction in the stringers. This implies an optimization to find the best ratio t/t_1. The result is given in Table 1, with $(t/t_1)_{opt} = 1.6$ and a net reduction in total equivalent thickness $(\bar{t} + \bar{t}_1)$ of 7%.

For the purpose of this paper, the important step is this last one (the percentage reduction obtained depends on the required torsional stiffness and on the loading intensity, and is not in itself important, of course). Redistribution of material between the tension and compression panels has resulted in an overall weight saving, even though the efficiency of the compression panel itself is further reduced and the thickness distribution is not the optimum for torsional stiffness. Note that the tension skin does not reduce to its original thickness, i.e. it is not fully-stressed. Note also that optimization of the compression panel (implied by use of Fig. 1) remains independent of those aspects of the design involving the whole cross-section (in this example variation of the t/t_1 ratio). This is characteristic of the multi-level approach - more formally stated it is then the sensitivity of the optimum for the compression panel that is carried through to the level of the complete cross-section.

If the box structure is intended to represent an aircraft wing, then a series of such cross-sections along the span must be considered, with an increasing bending moment from section to section. Provided that the required torsional stiffness can be treated as an average over the length of the structure, an optimum distribution of material must be found not only within the cross-section but also along the span. (At this stage the optimization becomes more than a simple hand calculation and the numerical example will not be continued further!) It may well be found that towards the wing root the necessary strength of the structure leads to a torsional stiffness greater than the average required, while at less heavily loaded sections redistribution of material is still possible. If the wing is tapered, theory dictates that purely for torsional stiffness the optimum has an increased thickness towards the tip, in contrast to the requirements for strength. Further, it should be pointed out that in this example redistribution of material is the result of a stiffness

constraint, but can also be due to redundancy in the structure. A three-spar wing, for example, introduces a major redundancy in the cross-section, and redistribution of material can then take place between the three spar webs. The considerable modification of the stress distribution near the root of a swept wing is evidence of redundancy of a different kind in the type of structures considered in this paper.

Obviously, the design of a real wing structure is much more complicated than the example discussed here due to its shape, geometric limitations, discontinuities in the structure, and also because of the influence of discrete sheet thicknesses, standard stringer sections, rib pitch limitations and so on. Nevertheless, the example illustrates the essential compromises in the optimization process. In particular, if each component of the structure were designed individually on the basis of the loads imposed on it (in the manner of a fully-stressed design) and with some prescribed contribution to the total stiffness, an optimum design is unlikely to be achieved. An optimum distribution of material to different parts of the structure (or, to be more precise, an optimum stiffness distribution) could be achieved by optimization of the structure as a whole but more efficient - and matching better the traditional design process - is the multi-level approach.

The different levels of such an approach are already recognizable in the example. The design of stringer-skin panels, shear webs and ribs are very conveniently treated as individual problems (as has always been the case in practice) and this is identified as 'level 1' (lowest level). The necessary interaction between the design of these will be treated at 'level 2' (intermediate level) which is the assembly of such panels into a single cross-section. Although not strictly part of the cross-section, the assembly of ribs into the structure is also placed in level 2, as will be discussed later. At 'level 3' (highest, or system level) the individual cross-sections are linked together to form albeit an idealisation of the complete wing. The theoretical basis for multi-level optimization, following closely ref. 1, will be reviewed in the following section, after which attention will be turned to specific aspects of the application to wing design.

3. THEORY OF MULTI-LEVEL OPTIMIZATION

For simplicity consider first a two-level optimization - the complete structure at the upper level is built up of a number of sub-structures at the lower level. The design variables of sub-structure (element) e are:

$$y^e = \{y_i^e\} .$$

Sufficient quantities:

$$X^e = \{X_p^e\}$$

to define the stiffness of the sub-structure will be expressed:

$$X_p^e = f_{1_p}^e (y^e)$$

while its mass M^e must also be expressible as:

$$M^e = f_2^e (X^e) .$$

The X^e might be simple quantities such as cross-sectional area, or more complex ones such as second moment of area, depending on the nature of the problem. Together the variables:

$$X = \{X^e\}$$

for all the sub-structures become the design variables of the complete structure, defining in fact the mass and stiffness distribution throughout the structure. The intention is, of course, that by suitable choice of the quantities X^e the number of upper level variables can be minimised. In effect each variable X_p^e transfers one design variable from a sub-structure at the lower level to the complete structure at the upper level.

Just as the majority of the design variables can be confined to individual sub-structures, so can the constraints. Constraints g^s are those generally

relatively few constraints (such as limitations on deflection) which relate to the complete structure:

$$g_q^s = f_{3_q}(X)$$

while constraints:

$$g_j^e = f_{4_j}(y^e, Q^e)$$

are confined to the sub-structure e. The numerical value of a constraint is the difference between the actual value of a stress, deflection or some other quantity and the allowable value of that quantity, i.e. a constraint is defined here as zero or negative when the condition is satisfied. In the above formulae Q^e are the forces on the sub-structure, dependent on the upper level variables X if the structure is redundant (or if the weight of the structure has a significant effect on its loading). Through Q^e, therefore, the constraints g^e depend on the stiffness distribution in the structure as a whole, as do the constraints g^s.

In a two-level optimization, instead of minimising the mass of each sub-structure individually (which would lead to a purely local optimization) the design variables y^e are chosen to minimise the extent of constraint violation in the sub-structure while keeping X^e (and therefore Q^e) constant. Constraint violation is measured by a 'cumulative constraint' C^e, for example the Kresselmeir-Steinhauser function:

$$C^e = \frac{1}{\rho} \ln \left[\sum_j \exp(\rho g_j^e)\right] \qquad (2)$$

where the constant ρ is chosen by the user. At the lower level then the optimization problem is:

$$\min_{y^e} [C^e(g^e)]$$

subject to constraints:

$$X^e - f_1^e (y^e) = 0$$

$$y_l^e \leq y^e \leq y_u^e$$

where y_l^e, y_u^e are lower and upper limits on the design variables. The equality constraints above imply that the number of design variables y^e should exceed the number of quantities X^e if optimization at the lower level is to be possible.

The mass of the complete structure is minimised at the upper level:

$$\min_X [\sum_e M^e]$$

subject to constraints:

$$g^s (X) \leq 0 ,$$

$$C^e (X^e, Q^e) \leq 0 ,$$

$$y_l^e \leq y^e \leq y_u^e ,$$

$$X_l \leq X \leq X_u .$$

where upper and lower limits on X may be necessary to avoid physically impossible demands at the lower level. For an efficient procedure at the upper level the cumulative constraints C^e (one for each sub-structure) are linearised in terms of X^e and Q^e, i.e. the sensitivity of each lower level optimum C^e to these parameters ($\partial C^e/\partial X^e$ and $\partial C^e/\partial Q^e$) must be determined. Move limits are introduced to reduce linearisation errors. Further, upper and lower limits on the variables y^e are included to avoid violation of these limits during optimization at the upper level. This requires additional sensitivity data, i.e. the derivatives of the optimum y^e with respect to X^e and Q^e ($\partial y^e/\partial X^e$ and $\partial y^e/\partial Q^e$). For small problems this might be achieved by repeated optimization at the lower level. A more efficient procedure (nevertheless computationally expensive) is referred to ref. 1.

The two-level process involves optimization of each sub-structure followed by optimization of the complete structure, this being repeated until convergence is obtained. The upper level supplies values of X^e to each sub-structure at the lower level together with the loads Q^e, while the lower level supplies values of the cumulative constraint C^e and its derivatives to the upper level. In a multi-level process, the sub-structures of the two-level case are further divided into smaller structural components, and so on. Cumulative constraints are minimised at each sub-level, while the mass of the complete structure is minimised at the highest (system) level. This is further elaborated for a three-level process in the following section, which also serves as an illustration of the type of structure suitable for multi-level optimization.

4. APPLICATION TO WING DESIGN

A three-level process for the application of multi-level optimization to wing structures was already suggested, and this can now be discussed in more detail for each level in turn.

Level 3. At the highest level a chosen number of cross-sections are linked together to form a model of the wing as a whole. The design variables are the masses m_i allocated to each section of the wing and the torsional stiffnesses k_i of each section. This appears to represent adequately the conflicting requirements in the design, as discussed in section 2. Out of the mass allocation to each section must be found the material necessary to satisfy strength and buckling requirements at the lowest level. The torsional stiffness acts as a constraint on the design at the intermediate level. The total mass of the wing to be minimised is, of course, the sum of the masses m_i. Constraints at this level, apart from the cumulative constraints supplied by each section of the wing at the intermediate level and a torsional stiffness requirement for the wing as a whole, can also include more sophisticated design requirements such as a flutter speed limitation (see section 5.2). Strictly speaking the rib pitch should also be considered a design variable at this level, but in fact will be treated as a continuous variable at the next level. At a suitable stage in the design the user can choose a rib pitch which fits in with known fixed rib positions. Differences from the optimum rib spacing are then compensated for elsewhere in the design, as is the effect of stringer pitch which must have a continuity along the wing. (The need for an interactive program, in which the user can control the progress of the design, becomes self-evident.) At the present stage it is assumed that the geometry of the wing (wing profile, spar positions, etc.) is fixed, the purpose of the program under development being to arrive at a suitable detail design within that fixed geometry.

Level 2. The intermediate level is the design of individual cross-sections representative of the surrounding wing structure, while maintaining the required torsional stiffness (in effect this is what is taking place in the numerical part of the example in section 2). Design variables are the mass of material allocated to skin panels, shear webs and ribs (for a given material the equivalent thickness $\bar{t}_i$) the shear stiffness (for a metal structure the actual

skin or web thickness t_i) and the rib pitch L. For a composite structure, the shear stiffness itself must be retained (see section 5.1). Equality constraints imposed from the level above are the mass m allocated to that section of wing, and the torsional stiffness k of the section. Note that m and k are directly expressible in terms of $\bar{t}_i$ and L, and t_i respectively. Since the only function of this level is to keep separate the individual detail design tasks at the lowest level (where the greatest number of design variables and constraints are found) there are no other constraints apart from the cumulative constraints supplied by each component at that level. These are directly formed into a cumulative constraint at the intermediate level.

Level 1. The lowest level is the detail design of the individual panels, webs and ribs making up the structure, and is characterised by discrete design variables for sheet thicknesses, standard stringer sections, and other limitations. These can have a significant effect on the efficiency of the structure as a whole and it is therefore preferred, even at the preliminary design stage, to take discrete variables into account. While every effort is made to keep the number of design variables small, the number of constraints may nevertheless be quite large - including stresses in the structure, various buckling modes, local skin deflections, and so on. The aim is for a flexible approach, so that other requirements such as fatigue and damage tolerance can be included as necessary. Allowance can also be made for the weight of joints in the structure. Due to the discrete design variables, minimisation of the cumulative constraint (subject to the equality constraints from the level above) becomes more of a 'sorting' process between a limited number of discrete values than a mathematical optimization. The individual design tasks at this level are now small enough that it is considered more effective to use a zero-order optimization, in which the variables are forced to take discrete values at every stage, than to use a higher order method followed by a search for the appropriate discrete values in the neighbourhood of the optimum. More important, the effect of discrete design variables would be to cause a discontinuous behaviour of the cumulative constraint. To avoid upsetting convergence there may then be an advantage in the use of analytic approximations to the sensitivity of the cumulative constraint (in which discrete values are ignored) to steer the design towards an optimum, as discussed in the following section.

4.1. Simplified Approach to Sensitivity Data

Various modifications of the multi-level procedure have been put forward in the literature, for example in ref. 5. The method of ref. 1 is preferred here for its theoretical basis and the close relationship between the numerical procedure and the structure itself. However, some simplifications are introduced which, while remaining close to the original concept of ref. 1, appear to have advantages in the type of problem considered in this paper. These are summarised as follows:

(i) Upper and lower limits on design variables are included in the cumulative constraint at their own level, and not carried through as constraints to the next level. This avoids the need to compute the sensitivity of optimum values of these variables ($\partial y^e/\partial X^e$ and $\partial y^e/\partial Q^e$) the use of which is in any case questionable if some of the y^e are discrete variables. However, this also affects linearisation of the cumulative constraint, so that tighter move limits may be required.

(ii) Use of analytic approximations to the sensitivity of the cumulative constraints ($\partial C^e/\partial X^e$ and $\partial C^e/\partial Q^e$) at the lowest level, on the basis of optimality criteria for the different components of the structure at this level, is still being studied. This is intended in particular for use with discrete variables which would otherwise cause a discontinuous behaviour of the cumulative constraint.

Point (ii) above needs some further explanation. The required sensitivities are readily obtained from eqn. (2) if $\partial g_j^e/\partial X^e$ and $\partial g_j^e/\partial Q^e$ can be found. Consider, for example, the design of a compression panel in the structure. If the individual constraints can be replaced by a constraint based on the design stress σ given by eqn. (1) then as well as the direct effect of $\bar{t}$ on the applied stress in the panel there is an indirect effect of both $\bar{t}$ and t on the efficiency η of the panel. Furthermore, to allow for yielding of the material, Young's modulus E can be replaced by the tangent modulus E_t. Substitution of $p = \sigma\bar{t}$ in eqn. (1) for the loading intensity that can actually be carried by the panel (as opposed to the applied loading intensity) gives

$$\sigma = \eta^2 \frac{\bar{t}E_t}{L}$$

from which

$$\frac{\partial\sigma}{\partial t} = -\frac{\sigma}{t}\left[\frac{2\dfrac{(\bar{t}/t)}{\eta}\cdot\dfrac{d\eta}{d(\bar{t}/t)}}{1-\dfrac{\sigma}{E_t}\cdot\dfrac{dE_t}{d\sigma}}\right]$$

and

$$\frac{\partial\sigma}{\partial\bar{t}} = \frac{\sigma}{\bar{t}}\left[\frac{1+2\dfrac{(\bar{t}/t)}{\eta}\cdot\dfrac{d\eta}{d(\bar{t}/t)}}{1-\dfrac{\sigma}{E_t}\cdot\dfrac{dE_t}{d\sigma}}\right]$$

Note that eqn. (1) refers to an optimized panel, i.e. with the required equivalent thickness $\bar{t}$ and actual skin thickness t the remaining dimensions of the panel are chosen to give maximum loading intensity p. The efficiency η is defined as a function of $\bar{t}/t$ in Fig. 1. Since $\bar{t}$ and t are design variables at the next level the required $\partial g_j^e/\partial\bar{t}^e$ and $\partial g_j^e/\partial t^e$, and therefore an approximation to the sensitivity of the cumulative constraint, can be obtained from the above formulae.

5. NUMERICAL RESULTS

Two test problems have been selected, the results of which will be reviewed briefly here. These are intended primarily to demonstrate convergence of the multi-level procedure. The first problem is a composite box structure, with some features of an aircraft wing. This was chosen because the design of the composite panels at the lower level is perhaps sufficiently complicated to provide a pointer to real design problems. At the upper level there is a single torsional stiffness constraint. The second problem is a simplified wing structure with aeroelastic constraints. In this case the highest level in a three-level procedure is much more complex, while the lowest level is based on a more practical approach to the wing cross-section.

5.1 Composite Wing Box

The problem chosen (6) is a rectangular box structure with a fixed number of hat-section stringers in the upper and lower panels, and similar stiffeners placed transversely on the front and rear shear webs (see inset in Fig. 4). The box is divided lengthwise into 3 bays (by ribs not included in the optimization) so that in total the structure consists of 12 separate panels. As well as the 3 design variables defining the shape of the stringers (see inset in Fig. 3) 4 additional design variables define the thickness of the laminate and its lay-up in each panel, so that in total the problem has (3+4)x12 = 84 design variables. The box is loaded by a bending moment at its free end, with a minimum torsional stiffness requirement imposed on the box as a whole.

The optimization is treated as a two-level problem, the lower level being the design of each of the 12 panels, and the upper level the assembly of these panels into the box structure. At the lower level, constraints are the stresses in each layer, a strain criterion for the laminate, and panel buckling. At the upper level the design variables are chosen to be the cross-sectional area of each panel and (being a composite structure) its shear stiffness. In total this is 24 upper level design variables. The single constraint at the upper level is the torsional stiffness. For this problem, to adhere closely to the theoretical procedure of ref. 1, it was chosen to follow the optimization of each panel at the lower level by a calculation of the sensitivity of the panel optima to the

upper level variables. Maximum and minimum values of the lower level variables are therefore included in the upper level constraints (and not incorporated in the cumulative constraint as proposed in section 4.1). At this stage, discrete layer thicknesses of the composite were disregarded. A 5% move limit was imposed on the upper level variables, after investigation of the linearity of the cumulative constraint, although it is thought that some relaxation of this move limit could be allowed.

With suitable starting values for the upper level variables, each of the 12 panels is optimized in turn, i.e. subject to two equality constraints imposed by the upper level variables, the cumulative constraint C^e is minimised. Optimization at both levels is performed by sequential quadratic programming. A typical reduction in C^e for one of the compression panels is shown in Fig. 2. It is seen that the cumulative constraint remains well behaved throughout the optimization, in spite of large changes in the geometric design variables of the stringer (see Fig. 3) and similar changes in thickness and lay-up (not shown). Some design variables reach prescribed maximum or minimum values during the optimization (as is clear in Fig. 3) but this is reflected in only minor kinks in the graph of C^e. It is concluded that the cumulative constraint in eqn. (2) is a suitable representation of how closely a panel is approaching its constraints, and is highly tolerant of changes within the panel itself. The result of minimisation of the cumulative constraint is a panel in which the constraints tend to be equalised, i.e. as far as the design freedom of the panel permits it approaches a uniform margin of safety in all modes.

This lower level optimization is followed by optimization at the upper level. Figure 4 shows the iteration history, and the reduction in volume of the structure. It so happens in this example that the reduction in each of the upper level variables reaches the 5% move limit, so that the maximum reduction in volume is also 5%. Saving of material is primarily due to improvement in shear stiffness of the front and rear webs by change in lay-up, and improvement in the buckling efficiency of the upper and lower panels by re-shaping the stringers. It is found that redistribution of shear stiffness continues in the later iterations, even though there is then little further reduction in volume. Another round of optimization at the lower level follows after the 16th iteration; this can primarily be seen as updating the sensitivity coefficients.

This is followed by a second optimization at the upper level, with a further 2.4% weight reduction. The initial increase in volume is the result of constraint violations detected at the lower level. A small further weight reduction would be possible in a third optimization at the upper level.

5.2 Wing with Aeroelastic Constraints

The second problem (7) is a simplified form of wing, in this case of conventional metal construction, now with flutter and divergence constraints included. Optimization is treated as a three-level problem, in a manner similar to that described in section 4. At the lowest level the design of individual stiffened panels uses an optimality criterion approach rather than a formal optimization, based on a previously developed program (8) for wing cross-sections. This was considered necessary at this stage because of the complexity of the calculation at the highest level; furthermore sensitivity data is then easily obtained by finite difference (none of the design variables was treated as discrete). Optimization at the intermediate and highest levels uses Rosen's gradient projection method. Maximum and minimum values of the design variables are included in the cumulative constraint at the appropriate level (as in section 4.1). In the test problem there are no constraints on the strength of the structure so that, apart from the above, flutter and divergence are the only constraints.

By far the heaviest burden of analysis in this problem comes at the highest level, where flutter and divergence constraints must be evaluated. The analysis is based on a simple bending and twisting deformation, with instationary aerodynamic forces on the wing based on two-dimensional strip theory. The mass distribution consists of non-variable masses together with the mass of the wing structure itself - the latter being the mass to be minimised. The resulting equations of motion lead to a form of eigenvalue problem requiring a numerical procedure to extract the critical flutter speed. The same equations are used for divergence. Although the aeroelastic model is too simple to obtain reliable flutter and divergence speeds for an actual wing, it is considered satisfactory for the present purpose - which is to test the multi-level procedure on this kind of problem. Nevertheless the calculation of this model, and of the gradients of the flutter and divergence constraints with respect to the highest

level variables (obtained analytically by differentiation of the eigenvalue equations) was carried out to a good degree of accuracy, which is thought to account for the highly satisfactory behaviour of these constraints in the optimization as a whole.

Figure 5 shows the reduction in the flutter and divergence constraints with number of cycles at the highest level. One cycle implies here optimization at the highest level, followed by re-optimization and evaluation of sensitivity data at the intermediate and lowest levels (the intermediate level being visited in both directions). It is seen that the flutter constraint becomes active at the 59th cycle, and remains critical for the remainder of the optimization. (Note that in Fig. 5 constraints are defined positive when satisfied, and that each is normalised by dividing by the initial value of the constraint.)
The corresponding reduction in mass of the structure (relative to the mass of the initial design) is shown in Fig. 6, with a 60% reduction up to the 59th cycle, followed by a small further reduction up to about the 80th cycle. This shows that redistribution of material continues after the flutter condition has become active. In fact examination of the behaviour of individual design variables, and the cumulative constraints at the intermediate and lowest levels, shows that considerable internal variation is taking place up to and beyond the 80th cycle. This may be partly the result of lack of strength constraints on individual components of the structure, coupled with a flutter constraint which is relatively insensitive to the individual design variables. Nevertheless these lower level variations are not reflected at the highest level, which shows a highly stable behaviour, so that the multi-level process is considered very satisfactory in this problem. Less satisfactory is the number of cycles necessary to reach convergence, but it is believed that this has more to do with the optimization algorithm used (and the move limits imposed) than with the multi-level procedure itself. Figure 7 shows the change in mass allocated to each of the five sections into which the wing is divided, as obtained at the 80th cycle, and the change in torsional stiffness. (These are normalised by the appropriate values for the root section in the initial design.) It is evident from Fig. 7 that, for flutter alone, the optimum has its maximum torsional stiffness some distance away from the root, demanding a generally similar mass distribution. At the highest level the mass of each section and its torsional stiffness are the only design variables. This implies that flexural stiffness is

not directly controlled during optimization. While flexural stiffness is fairly closely related to the mass distribution, it is clear that the mass allocated to ribs, for example, does not contribute to the flexural stiffness. Both flexural and torsional stiffness play an essential role in the flutter calculation, but as far as can be judged this reduction in the number of highest level variables has no adverse effect on convergence.

6. CONCLUSION

The results obtained demonstrate the practicality of multi-level optimization in the design of aircraft shell structures. Formulation of the problem is such that a clear distinction between levels can be drawn, with the result that the majority of design variables and constraints are confined to individual, relatively small design tasks at the lowest level. This gives encouragement to continue the development of a more general program for use at the preliminary design stage. Any conclusion on the reduction in computing cost as a result of multi-level optimization in real design problems would be premature, because the problems described here are in fact too small for this purpose. Nevertheless this remains a critical issue, since the purpose of such a design program is to enable a rapid sizing of the structure to be carried out, and to make effective comparisons between alternative designs. From the various test problems investigated it becomes clear that the choice of optimization algorithm at each level, as well as the definition of design variables and constraints, plays a major role in the efficiency of the whole process. The use of analytical approximations to the sensitivity data appears promising, and work on this and on a satisfactory treatment of discrete design variables is continuing.

Acknowledgement

Much of the program development referred to in this paper was done by P.G. van Bladel (8), H.J.W.M. Koelman (6) and P. Arendsen (7) (the last two partly while completing their study at the National Aerospace Laboratory NLR). As well as the programming, they have made many other contributions to the work, and the author gratefully acknowledges their assistance. The author also wishes to thank W. Spee for his painstaking work on the figures.

References

1. Sobieszczanski-Sobieski, J., James, B. and Dovi, A. Structural optimization by multilevel decomposition. AIAA Paper No. 83-0832, AIAA/ASME/ASCE/AHS 24th Structures, Structural Dynamics and Materials Conference, 1983.

2. Bartholomew, P. and Wellen, H. Computer-aided structural optimization of aircraft structures.
ICAS-88-1.11.3, 16th Congress of the International Council of the Aeronautical Sciences, 1988.

3. Berkes, U.-L. and Wiedemann, J. Efficient procedures for the optimization of aircraft structures with a large number of design variables.
ICAS-88-1.10.3, 16th Congress of the International Council of the Aeronautical Sciences, 1988.

4. Farrar, D.J. The design of compression structures for minimum weight. Journal of the Royal Aeronautical Society, 53 (1949) 1041-1052.

5. Vanderplaats, G.N., Yang, Y.J. and Kim, D.S. An efficient multilevel optimization method for engineering design. AIAA Paper No. 88-2226, AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics and Materials Conference, 1988.

6. Koelman, H.J.W.M. Multi-level optimization of composite structures (in Dutch). ir. thesis, Delft University of Technology, Faculty of Aerospace Engineering, 1987.

7. Arendsen, P. Multilevel optimization with aeroelastic constraints (in Dutch). ir. thesis, Delft University of Technology, Faculty of Aerospace Engineering, 1988.

8. Van Bladel, P.G. A Fortran program for the design of a wing cross-section (in Dutch). Delft University of Technology, Faculty of Aerospace Engineering, 1986.

Table 1

	compression		tension			
step	$\bar{t}$ mm	t mm	$\bar{t}_1$ mm	t_1 mm	n	$(\bar{t} + \bar{t}_1)$ mm
1	4.34	2.0	3.25	2.0	--	7.59
2	3.96	2.0	3.25	2.0	0.941	7.21
3	4.17	2.6	2.88	1.63	0.893	7.05

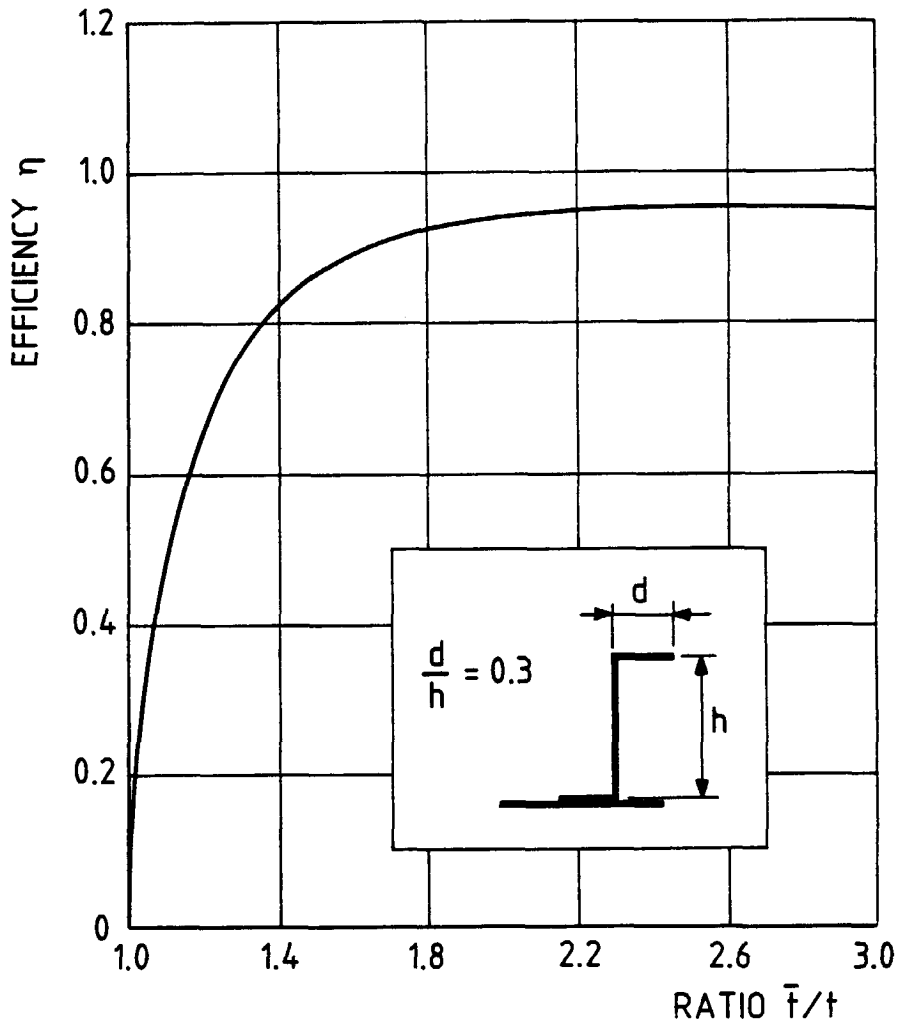

Fig. 1. Efficiency of a compression panel with Z-section stringers as a function of the ratio: equivalent thickness/actual skin thickness.

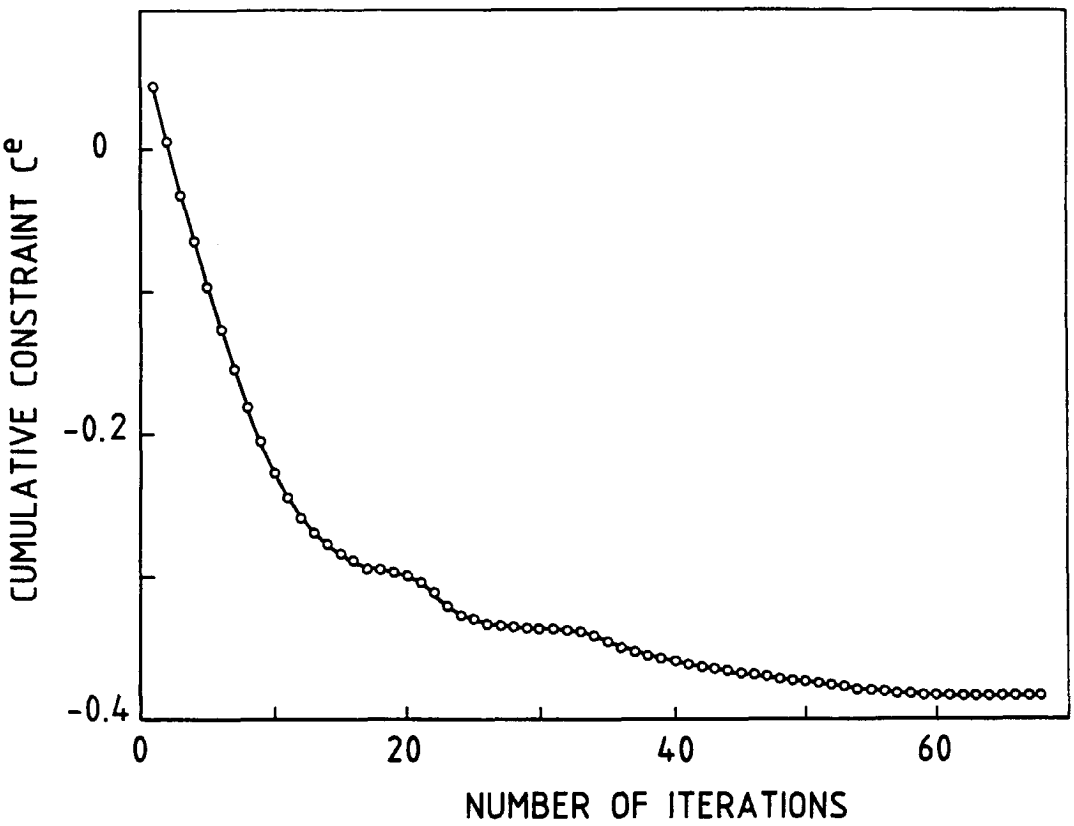

Fig. 2. Reduction in cumulative constraint during optimization of a composite compression panel.

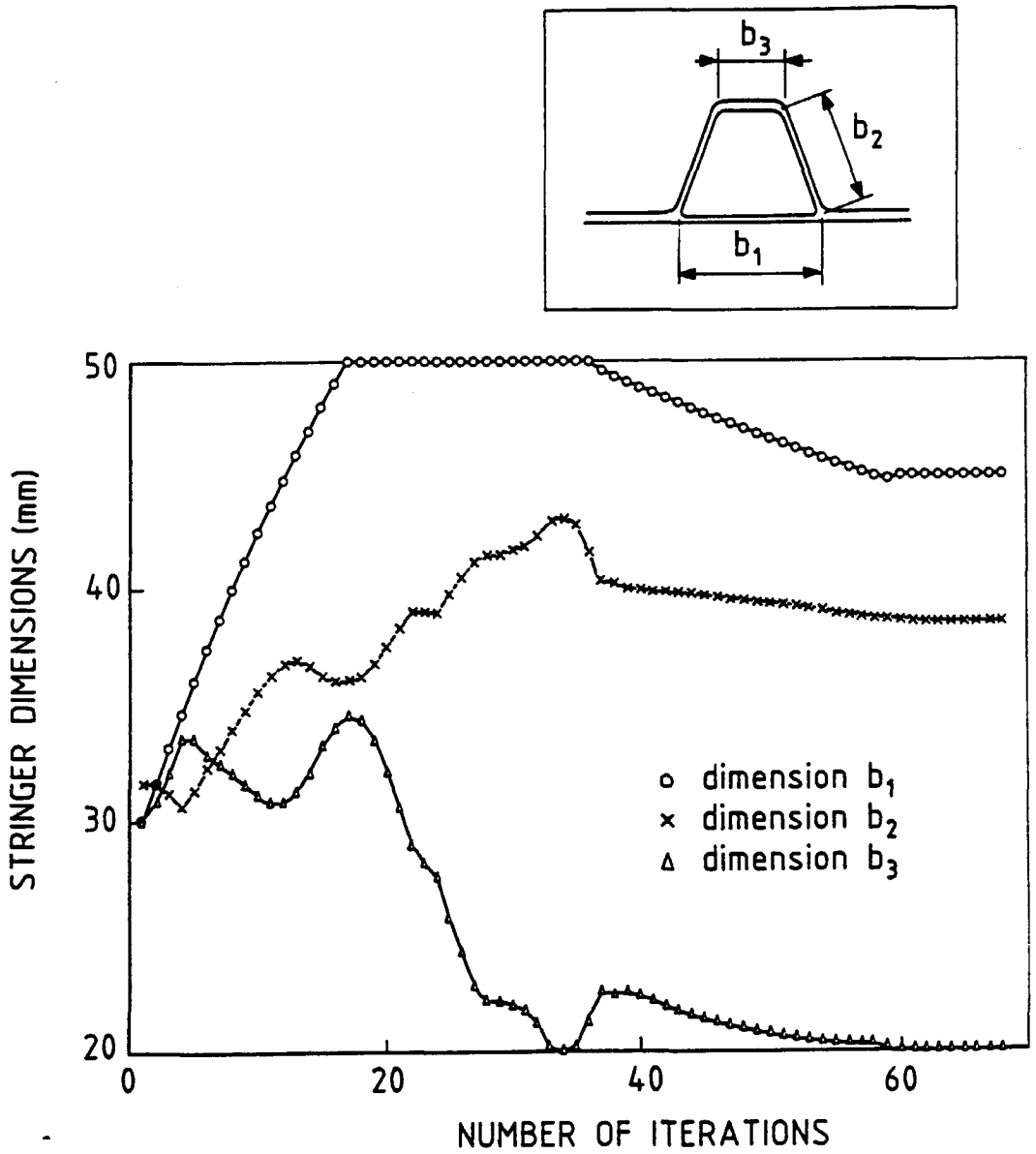

Fig. 3. Variation in stringer dimensions during optimization of a composite compression panel.

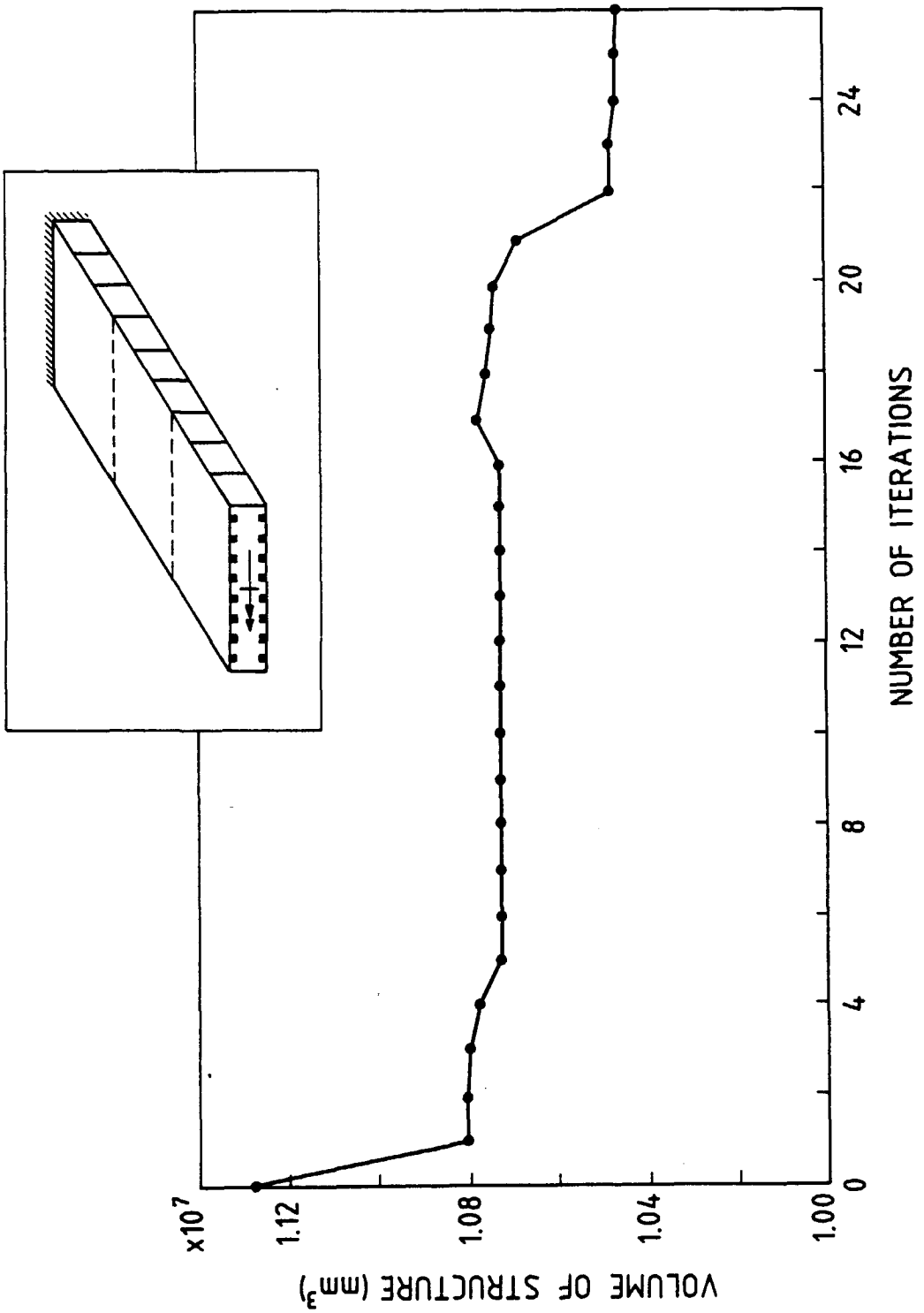

Fig. 4. Reduction in volume of the composite wing box.

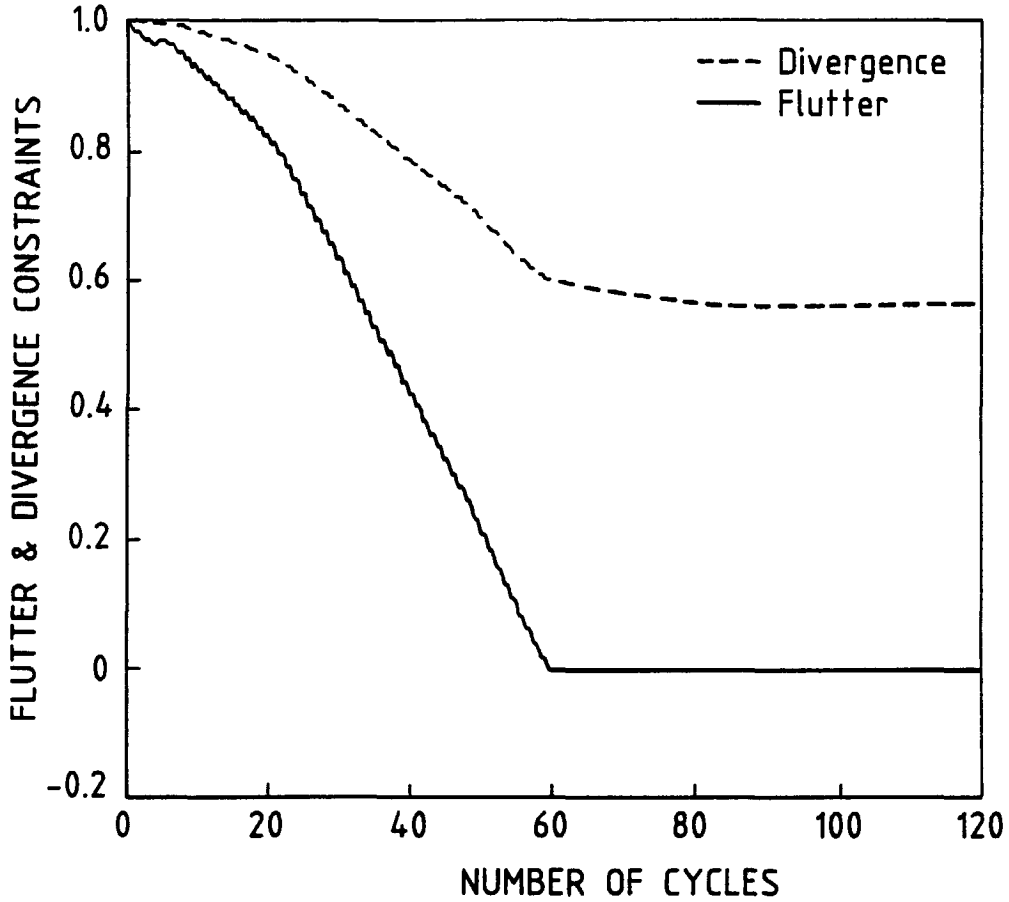

Fig. 5. Reduction in flutter and divergence constraints.
(Note: constraints are positive when satisfied.)

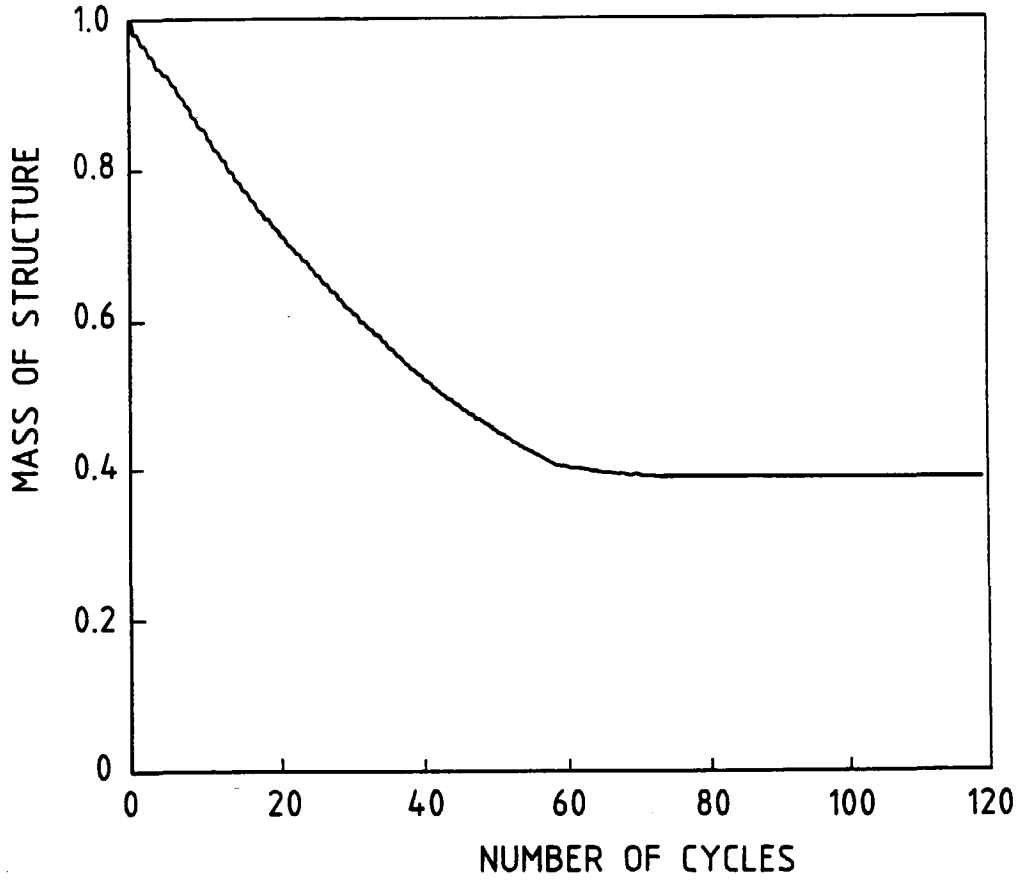

Fig. 6. Reduction in mass of wing structure
(flutter constraint active).

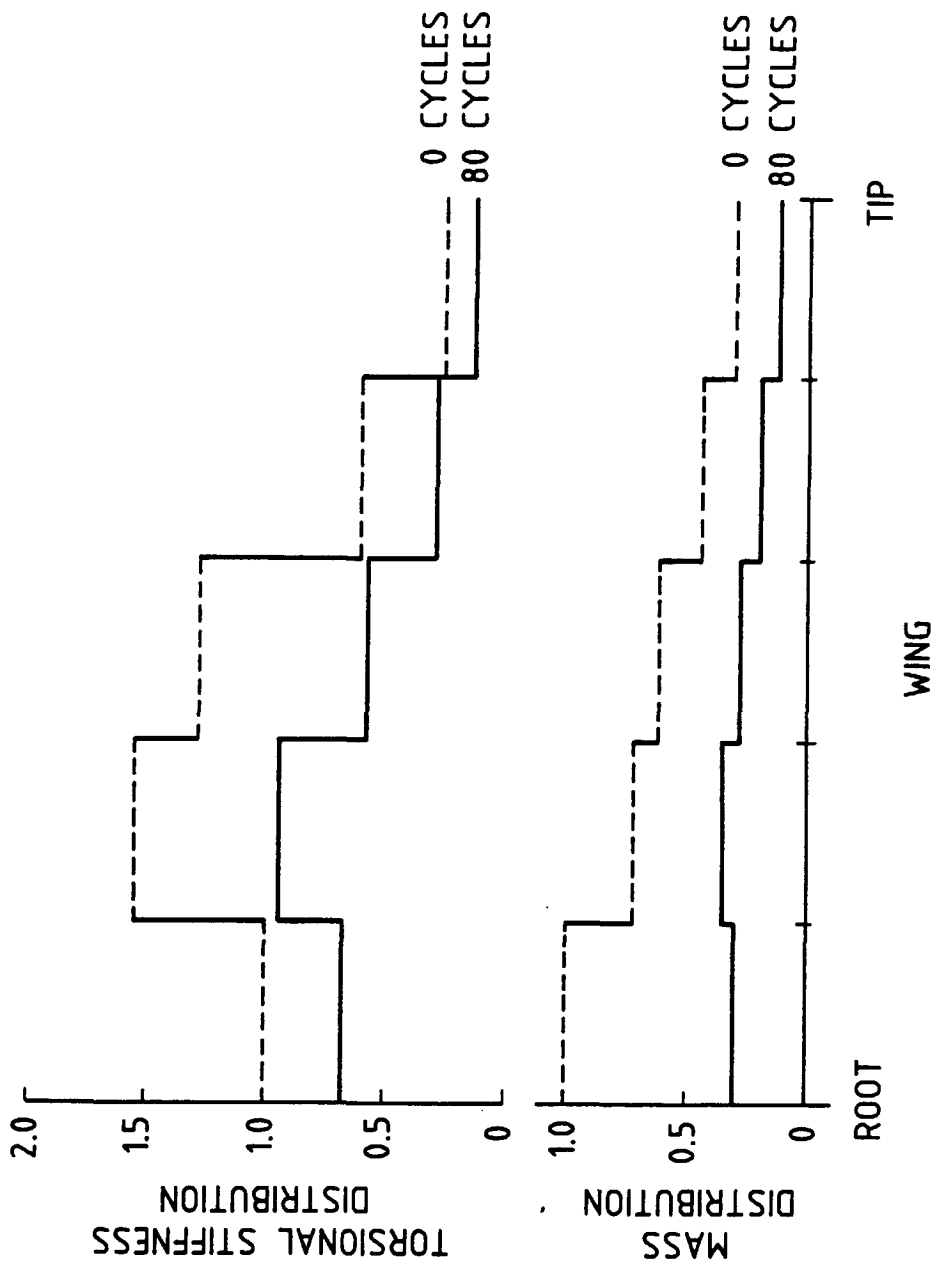

Fig. 7. Optimum mass and torsional stiffness distribution to each section of the wing (flutter constraint active).