

**Multi-moment Asset Allocation
and Pricing Models**

Edited by

Emmanuel Jurczenko and Bertrand Maillet



John Wiley & Sons, Ltd

Contents

About the Contributors	xiii
Foreword	xvii
Preface	xxi
1 Theoretical Foundations of Asset Allocation and Pricing Models with Higher-order Moments	1
<i>Emmanuel Jurczenko and Bertrand Maillet</i>	
1.1 Introduction	1
1.2 Expected utility and higher-order moments	3
1.3 Expected utility as an exact function of the first four moments	10
1.4 Expected utility as an approximating function of the first four moments	16
1.5 Conclusion	22
Appendix A	23
Appendix B	24
Appendix C	25
Appendix D	27
Appendix E	28
Appendix F	30
Acknowledgements	31
References	32
2 On Certain Geometric Aspects of Portfolio Optimisation with Higher Moments	37
<i>Gustavo M. de Athayde and Renato G. Flôres Jr</i>	
2.1 Introduction	37
2.2 Minimal variances and kurtoses subject to the first two odd moments	38
2.2.1 Homothetic properties of the minimum variance set	39
2.2.2 The minimum kurtosis case	41

2.3	Generalising for higher even moments	44
2.4	Further properties and extensions	46
2.5	Concluding remarks	48
	Appendix: The matrix notation for the higher-moments arrays	48
	Acknowledgements	50
	References	50
3	Hedge Fund Portfolio Selection with Higher-order Moments: A Nonparametric Mean–Variance–Skewness–Kurtosis Efficient Frontier	51
	<i>Emmanuel Jurczenko, Bertrand Maillet and Paul Merlin</i>	
3.1	Introduction	51
3.2	Portfolio selection with higher-order moments	53
3.3	The shortage function and the mean–variance–skewness–kurtosis efficient frontier	55
3.4	Data and empirical results	58
3.5	Conclusion	63
	Appendix	64
	Acknowledgements	65
	References	65
4	Higher-order Moments and Beyond	67
	<i>Luisa Tibiletti</i>	
4.1	Introduction	67
4.2	Higher-order moments and simple algebra	68
4.3	Higher moments: Noncoherent risk measures	71
4.4	One-sided higher moments	72
	4.4.1 Portfolio left-sided moment bounds	73
	4.4.2 Properties of the upper bound $U^p(S_-)$	74
4.5	Preservation of marginal ordering under portfolios	75
	4.5.1 Drawbacks in using higher moments	75
	4.5.2 Advantages in using left-sided higher moments	75
4.6	Conclusion	76
	Appendix	77
	References	77
5	Gram–Charlier Expansions and Portfolio Selection in Non-Gaussian Universes	79
	<i>François Desmoulins-Lebeault</i>	
5.1	Introduction	79
5.2	Attempts to extend the CAPM	80
	5.2.1 Extensions based on preferences	80
	5.2.2 Extensions based on return distributions	83
5.3	An example of portfolio optimisation	85
	5.3.1 Portfolio description	86
	5.3.2 The various “optimal” portfolios	86

5.4	Extension to any form of distribution	89
5.4.1	Obstacles to distribution-based works	89
5.4.2	Generalised Gram–Charlier expansions	90
5.4.3	Convergence of the fourth-order Gram–Charlier expansion	95
5.5	The Distribution of Portfolio Returns	98
5.5.1	Feasible approaches	98
5.5.2	The moments of the portfolio returns' distribution	98
5.5.3	Possible portfolio selection methods	100
5.6	Conclusion	105
	Appendix A: Additional statistics for the example portfolio	105
A.1	Moments and co-moments	105
A.2	Statistical tests of normality	107
	Appendix B: Proofs	108
B.1	Positivity conditions theorem	108
B.2	Approximation of the optimal portfolio density	109
	Acknowledgements	110
	References	110
6	The Four-moment Capital Asset Pricing Model: Between Asset Pricing and Asset Allocation	113
	<i>Emmanuel Jurczenko and Bertrand Maillet</i>	
6.1	Introduction	113
6.2	The four-moment capital asset pricing model	116
6.2.1	Notations and hypotheses	116
6.2.2	Aggregation of the individual asset demands and a two-fund monetary separation theorem	120
6.2.3	The four-moment CAPM fundamental relation and the security market hyperplane	125
6.3	An N risky asset four-moment CAPM extension	130
6.3.1	General properties of the mean–variance–skewness–kurtosis efficient set	131
6.3.2	A zero-beta zero-gamma zero-delta four-moment CAPM	134
6.4	The four-moment CAPM, the cubic market model and the arbitrage asset pricing model	137
6.4.1	The cubic market model and the four-moment CAPM	137
6.4.2	The arbitrage pricing model and the four-moment CAPM	139
6.5	Conclusion	142
	Appendix A	143
	Appendix B	145
	Appendix C	146
	Appendix D	147
	Appendix E	150
	Appendix F	151
	Appendix G	152
	Appendix H	154
	Appendix I	155
	Appendix J	156

Acknowledgements	157
References	157
7 Multi-moment Method for Portfolio Management: Generalised Capital Asset Pricing Model in Homogeneous and Heterogeneous Markets	165
<i>Yannick Malevergne and Didier Sornette</i>	
7.1 Introduction	165
7.2 Measuring large risks of a portfolio	167
7.2.1 Why do higher moments allow us to assess larger risks?	168
7.2.2 Quantifying the fluctuations of an asset	168
7.2.3 Examples	170
7.3 The generalised efficient frontier and some of its properties	172
7.3.1 Efficient frontier without a risk-free asset	173
7.3.2 Efficient frontier with a risk-free asset	175
7.3.3 Two-fund separation theorem	176
7.3.4 Influence of the risk-free interest rate	176
7.4 Classification of the assets and of portfolios	178
7.4.1 The risk-adjustment approach	179
7.4.2 Marginal risk of an asset within a portfolio	181
7.5 A new equilibrium model for asset prices	181
7.5.1 Equilibrium in a homogeneous market	182
7.5.2 Equilibrium in a heterogeneous market	183
7.6 Conclusion	184
Appendix A: Description of the dataset	184
Appendix B: Generalised efficient frontier and two-fund separation theorem	185
B.1 Case of independent assets when the risk is measured by the cumulants	185
B.2 General case	187
Appendix C: Composition of the market portfolio	188
C.1 Homogeneous case	188
C.2 Heterogeneous case	189
Appendix D: Generalised Capital Asset Pricing Model	190
Acknowledgements	191
References	191
8 Modelling the Dynamics of Conditional Dependency Between Financial Series	195
<i>Eric Jondeau and Michael Rockinger</i>	
8.1 Introduction	195
8.2 A model for the marginal distributions	197
8.2.1 Hansen's skewed student- t distribution	197
8.2.2 The cdf of the skewed student- t distribution	199
8.2.3 A GARCH model with time-varying skewness and kurtosis	199
8.3 Copula distribution functions	200
8.3.1 Generalities	200
8.3.2 Construction of the estimated copula functions	201

8.4	Modelling dependency and estimation of the model	205
8.4.1	Conditional dependency	205
8.4.2	Estimation in a copula framework	206
8.5	Empirical Results	207
8.5.1	The data	207
8.5.2	Estimation of the marginal model	209
8.5.3	Estimation of the multivariate model	211
8.6	Further research topics	215
	Acknowledgements	218
	References	219
9	A Test of the Homogeneity of Asset pricing Models	223
	<i>Giovanni Barone-Adesi, Patrick Gagliardini and Giovanni Urga</i>	
9.1	Introduction	223
9.2	The Quadratic Market Model	224
9.3	Empirical Results	225
9.3.1	Data description	225
9.3.2	Results	226
9.4	Conclusion	229
	Acknowledgements	229
	References	229
Index		231