# Multi-objective facility layout problems using BBO, NSBBO and NSGA-II metaheuristic algorithms 

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ABSTRACT

Quantitative and qualitative objectives are both significant to solve any facility layout problem (FLP), which is called as multi-objective FLP. Generally, quantitative factors are considered as material handling cost, time, etc., and qualitative factors are considered as closeness rating, hazardous movement between the facilities, etc. For solving and optimizing two or more objectives, two methods are available. First is weight approach method and second is nondominated sorting method. In the former method, suitable weights are given to each objective and combined in a single objective function; while in later method, the objectives are defined separately and by making comparison of the solutions on the non-dominance criteria, best Pareto-optimal solutions are obtained. In this paper, equal area multi-objective FLPs which are formulated as quadratic assignment problem (QAP) are considered and optimized using biogeography based optimization (BBO) algorithm and non-dominated sorting BBO (NSBBO) algorithm. BBO is one of the efficient metaheuristic techniques, developed to solve complex optimization problems. Computational results of BBO algorithm using weight approach illustrate its better performance compared to other methods while solving multi-objective FLPs. Furthermore to obtain Pareto optimal solutions, NSBBO algorithm is implemented.

## 1. Introduction

In any manufacturing industry, the material handling cost (MHC) is the key factor for obtaining the optimal layout. The MHC is termed as quantitative factor and is considered as a single objective function to minimize. However, while solving any facility layout problem (FLP), only considering quantitative factor does not guarantee the optimal layout, therefore qualitative factors such as closeness relation between the departments should be considered. In the past literature, for solving the FLPs, more reliable factors such as material handling time, distance requirement in case of hazardous movement between the facilities, input/output stations, orientation of facilities, etc. are also considered. Various types of FLPs are extensively studied by researchers in the past literature. These are solved by formulating several mathematical models of facility layout. The FLPs are categorised as equal area, unequal area, static, dynamic, multi-objective, multi-floor, etc. In this paper, equal area FLPs are considered in which all the

[^0]facilities in the layout are of equal area and identical shape and formulated as QAP. Each facility is assigned to a square block in the layout. The problems are solved using multi-objective optimization. Singh and Singh (2010) proposed an approach to solve multi-objective FLPs by using weight method. In this work, the same problems are solved and optimized using BBO algorithm with weight approach and non-dominated sorting biogeography-based optimization (NSBBO) algorithm with Pareto approach. In this paper, BBO and NSBBO algorithms are proposed because it is observed that BBO and NSBBO algorithms are applicable in wide variety of complex optimization problems. BBO and NSBBO are innovative non-traditional optimization techniques and till date, they are not applied to solve multiobjective FLPs. The results obtained using BBO are better than previous employed techniques. The performance of NSBBO is compared with non-dominated sorting genetic algorithm (NSGA-II). The paper further consists of Section 2: Literature review; Section 3: QAP function for multi-objective FLP; Section 4: BBO and NSBBO algorithm; Section 5: Performance evaluation of BBO and NSBBO algorithms for multi-objective FLPs and Section 6: Conclusion.

## 2. Literature Review

The facility layout of equal and unequal area with single objective function is discussed and solved by many researchers; however for obtaining more improved results, layout planner should consider more than one objective while solving FLP. The present literature review focussed on multi-objective FLPs and various heuristic and meta-heuristic techniques used to optimize them. The literature review categorised as follows:

### 2.1 Multi-objective FLPs using weight approach

Weight method is implemented by many researchers while solving multi-objective FLPs to normalise the different objective functions and obtain the optimal solution. Rosenblatt (1979) first attempted the concept of multi-goal approach for FLPs. Dutta and Sahu (1982) implemented a pairwise exchange heuristic to solve multi-objective FLP considering material handling cost and closeness rating. Fortenberry and Cox (1985) considered material flow as quantitative objective to minimize and closeness rating as qualitative objective to maximize. The objectives are combined using weight approach. Khare et al. (1988) proposed a combined computer-aided approach for multi-objective FLP. They obtained the solutions by summation of objective values at different weightage. Harmonosky and Tothero (1992) formulated a methodology to solve multi-factor FLP by combining the quantitative and qualitative factors with suitable weights. Chen and Sha (2001) proposed a new approach by considering suboptimal solutions for large size multi-objective layout problems. Chen and Sha (2005) considered four objectives to solve equal area FLP (namely material handling cost, adjacency requirement, material handling time and hazardous movement between the facilities).

While designing a facility layout, the effect of workflow interference is studied (Chiang et al., 2006) with linear and non-linear formulations of the problem. Ye and Zhou (2007) developed a hybrid algorithm using genetic algorithm (GA) and tabu search to solve multi-objective FLPs having material handling cost and adjacency requirement. They considered unequal area facilities with fixed aisle structure. A multi-goal layout approach is presented by Peer and Sharma (2008) and the objective function consists of closeness relationship and workflow. Material flow factor cost (MFFC), shape ratio factor (SRF) and area utilisation factor (AUF) are considered by Ku et al. (2011) to determine total layout cost by conducting a weighted summation of MFFC, SRF and AUF and optimized using parallel GA. Total MHC (quantitative) and the total closeness rating (qualitative) are considered to solve FLP by using weight assignment method for each objective by Sahin (2011). Matai (2015) proposed modified simulated annealing (SA) algorithm to solve multi-objective FLP using the weight assignment method introduced by Singh and Singh (2010). This method is independent of decision maker and thus makes the layout design process easy.

### 2.2 Multi-objective FLPs using Pareto-optimal approach

This section gives the idea about the Pareto-optimal criteria used to solve multi-objective FLPs in previous literature. Aiello et al. (2006) applied a multi-objective constrained GA to find Pareto-optimal solutions of unequal area FLP and the optimal solution is selected using multi-criteria decision-making procedure i.e. Electre. They considered four objectives namely, MHC, aspect ratio, closeness request and distance request between the departments. A dynamic FLP with unequal shape facilities and pickup-drop-off points explained by Jolai et al. (2012) using multi-objective particle swarm optimization (PSO). The objectives considered are adjacency, distance request, orientation and rearrangement cost of moving or reorienting departments. A multi objective GA is proposed for solving unequal area FLP with more than one objective function and obtained Pareto-optimal solutions (Aiello et al. 2012).

Hathhorn et al. (2013) considered two objective functions of minimising material handling and facility building costs and formulated using mixed integer programming (MIP); further they proposed a lexicographic ordering technique to handle multiple objectives. A multi-objective FLP having unequal area facilities with slicing tree representation is approached in two steps by Aiello et al (2013). The first step is to determine Pareto-optimal solutions by using multi-objective GA and the second step is to select the optimal solution by means of the multi-criteria decision-making procedure. A modified SA based approach is presented for solving equal area multi-objective FLP by Matai et al. (2013). An evolutionary approach is investigated by Ripon et al. (2013) for solving unequal area multi-objective FLP using variable neighbourhood search (VNS) with an adaptive scheme that presents the final layouts as a set of Pareto-optimal solutions. A mathematical model presented for equal area FLP considering two objectives (i.e. MHC and adjacency requirement) and optimized using a new developed heuristic technique (Chen \& Lo, 2014).

### 2.3 Application of BBO and NSBBO from literature

The literature survey states that a number of meta-heuristic techniques such as GA, PSO, SA, etc., have been implemented to solve multi-objective FLPs, but till now, BBO algorithm is not attempted to solve the FLPs. However, an application of BBO algorithm for machine layout design problems to minimize the material handling distance is described by Sooncharoen et al. (2015). A hybrid technique is proposed combining BBO and TS algorithms to solve QAP and obtained better results within reasonable computational time by Lim et al. (2016). Regarding multi-objective BBO, some of its applications are depicted here. Ma et al. (2012) introduced biogeography-based multi-objective optimization (BBMO) to solve several test functions and compared with NSGA-II developed by Deb et al. (2002). Further, Simon (2013) explained multi-objective BBO and NSBBO algorithms to find Pareto optimal solutions of multiobjective problems. Chutima and Naruemitwong (2014) implemented Pareto-BBO algorithm to solve mixed-model sequencing problems on a two-sided assembly line by considering three conflicting objectives. Ma et al. (2015) proposed an ensemble multi-objective BBO (EMBBO) algorithm to solve the automated warehouse scheduling problem. The EMBBO algorithm constitutes vector evaluated BBO (VEBBO), NSBBO and niched Pareto BBO (NPBBO). A community detection problem in dynamic networks with two objectives is solved using multi-objective BBO algorithm with decomposition mechanism (Zhou et al., 2015).

BBO is a nature-inspired optimization technique which is developed by Simon in 2008 and is based on the migration characteristic of species from one island to another. Simon (2008) successfully optimized all the continuous functions by using BBO algorithm. Therefore, by observing the efficiency of BBO and NSBBO algorithm, it is implemented to solve multi-objective FLP which is capable of providing better solutions in combinatorial optimization problems. The comparison of results obtained from BBO algorithm and the previous literature's results are shown with the best obtained layouts. In weight approach, the solutions are dominated towards one of the objectives whose data is more than the other.

Therefore, NSBBO algorithm is implemented to solve multi-objective FLPs to obtain non-dominated solutions and to test the performance of NSBBO algorithm.

## 3. QAP function for multi-objective FLP

In this paper, two objective functions (i.e. MHC and closeness rating) are considered. The multi-objective FLP is formulated as QAP (Singh \& Singh, 2010).

$$
\begin{align*}
& \min f_{1}=\sum_{i=1}^{n} \sum_{j=1}^{n} f_{i j} d_{i j}  \tag{1}\\
& \max f_{2}=\sum_{i=1}^{n} \sum_{j=1}^{n} c_{i j} \tag{2}
\end{align*}
$$

To compare the results with previous methods, weight approach is considered. The objective functions are converted into single-objective function using suitable weights given as below:

$$
\begin{equation*}
\min F_{1}=w_{1} f_{1}-w_{2} f_{2} \tag{3}
\end{equation*}
$$

where
$f_{l}$-Objective function to minimise the material work flow or MHC
$f_{2}$ - Objective function to maximise the closeness rating (CR)
$n$ - Number of facility
$f_{i j}$-Work flow from facility $i$ to $j$
$d_{i j}$ - Distance from facility $i$ to $j$
$c_{i j}$ - Closeness rating between facility $i$ and facility $j$
$w$ - Objective weight for each objective
$F_{1}$ - Combined objective function
It is observed that Singh and Singh (2010) and Matai (2015) considered the sum of closeness rating as a second objective which is constant for every solution and it is not having contribution towards the improvement of the layout. However, as mentioned by Urban (1987) and Khare et al. (1988), closeness rating is multiplied with the distance between the departments which will provide the actual combined solution of multi-objective FLP. Therefore, second objective is considered as given in Eq. (4) and the combined objective function is shown in Eq. (5).

$$
\begin{align*}
& \min f_{2}=\sum_{i=1}^{n} \sum_{j=1}^{n} c_{i j} d_{i j}  \tag{4}\\
& \min F=w_{1} f_{1}+w_{2} f_{2} \tag{5}
\end{align*}
$$

The multi-objective FLPs from (Singh \& Singh, 2010) are solved using the objective function considered by them for making comparison. The same problems are solved using the function in Eq. (5) to observe the difference between the results. Using weighted sum method, each objective function should be given proper weight to obtain the best solution. All factors are normalized before solving the final objective function, as the value of each factor is different from other. For instance, the values of material flow can vary from zero to very large amount while the value of closeness rating can be in the range of 0 to 5 . Singh and Singh (2010) described a capable method to calculate the weights of each objective. For detail procedure, please refer (Singh \& Singh, 2010). To calculate weights for each objective following steps are given:

Normalize the matrices of material work flow and closeness rating. The first step is summation of all the elements of each matrix and the second step is dividing each element of the matrix with the sum of all the elements of the matrix.

## Stage 2: Objective weight calculation

In this stage, suitable weight of each objective is calculated using four methods proposed by (Singh \& Singh, 2010). These methods are mean weight method (MWM), geometric mean weight method (GMWM), standard deviation weight method (SDWM) and critical importance through inter criteria correlation method (CRITICM). The weights considered in this paper for solving multi-objective FLPs are directly considered from weights calculated by Singh and Singh (2010).

Stage 3: Solve multi-objective FLP considering above weights using BBO algorithm
Each problem is solved considering the weight values of all the above mentioned methods and optimized using BBO algorithm. Amongst all the methods, whichever is providing minimum objective function value will be considered as best method.

## 4. Biogeography-based optimization (BBO)

In this section, the working of BBO and NSBBO algorithms are described in details. BBO algorithm is implemented using weight approach to compare with previous results and NSBBO is implemented to obtain Pareto-optimal solutions.

### 4.1 Introduction to BBO algorithm

BBO is an evolutionary, population-based algorithm (Simon, 2008). This algorithm is based on the principle of island biogeography in which geographical distribution of biological species is explained (Alroomi et al., 2013). BBO algorithm follows migration and mutation operations to reach global minimum solution. The mathematical formulation of BBO algorithm is developed by considering the migration behaviour of species from one place to another. On an island or habitat, if the living conditions are appropriate for species then that habitat have high suitability index (HSI), because this habitat have better features than other habitats. The variables that characterize habitability of an island are called as suitability index variables (SIVs) (Simon, 2008). Habitats having high HSI consist of more number of species than that of low HSI habitats. Therefore, the species on the high HSI habitats can emigrate to other habitat and have low species immigration rate as they are already full with species. Similarly, the low HSI habitats have high immigration rate of species because of their low population.

### 4.1.1 BBO Algorithm

BBO algorithm is established on the concept of distribution of species on islands which converts into a general problem function solution. Each island is considered as one member or solution. By considering the emigration $(\mu)$ and immigration $(\lambda)$ rates of each member, the information between the habitats is shared probabilistically. High-HSI habitat represents a good solution and low-HSI habitat represents poor solution. The good solution means the island has lots of good features such as trees, food, rainfall, temperature, humidity, etc.; therefore this island or habitat has high HSI. Each feature is indicated as SIV, which denotes the independent variable of the problem function or facilities in case of FLP. Good solution features emigrate from high-HSI islands to low-HSI islands (Rahmati \& Zandieh, 2012). A poor solution accepts features and information from good solution and further improves the solutions in algorithm. In BBO algorithm, there is an elitism strategy, which preserves best member from the
population in each iteration. If the algorithm traps in local minima, then the elitist solutions will remain intact and gives near-optimal solutions.

### 4.1.2 Algorithm Steps of BBO

Step 1. Initialisation of BBO parameters
$k$ - Number of islands or number of layouts, each island represents the permutation of facilities from $n=1,2, \ldots . ., k$
$G$ - Generations of algorithm
$I$ - maximum immigration rate (Fig. 2)
$E$ - maximum emigration rate (Fig. 2)
$S_{\text {max }}$ - Maximum number of species on an island
Step 2. Defining the objective function and finding the objective function value (OFV) of each island or layout
Step 2. Migration of species from one island to another with a probability
Step 3. Mutation is performed to update the islands
Step 4. Calculation of OFV of updated islands
Step 5. Repeat steps 2-4 until best value is obtained or maximum iterations are reached.

### 4.2 Non-dominated sorting biogeography-based optimization (NSBBO)

NSBBO algorithm is described by Simon (2013). The working of NSBBO algorithm is the same as that of NSGA-II. To overcome the problem of assigning the proper weights to the objective, NSGA-II is developed by Deb et al. (2002) for multi-objective optimization problems of continuous functions. The working of NSGA-II is based on the sorting of non-dominated or Pareto-optimum solutions, calculation of crowding distance and sharing fitness between the solutions. A Pareto frontier is generated when a set of optimal solutions are connected together.

Pareto optimal solutions are called as non-dominated solutions. For understanding, two objective functions $F_{i}$ and $F_{j}$ are considered with two solutions, $P_{1}$ and $P_{2}$. If $P_{1}$ dominates $P_{2}$ in terms of both objectives, $P_{1}$ is dominant over $P_{2}$. If $P_{1}$ is not dominated by the other solution, $P_{1}$ is called the nondominant solution. The set of non-dominant solutions is called the Pareto front or frontier. A nondominated solution can be considered as explained here. Suppose, $x^{*}$ represents a feasible solution while $x$ represents some other feasible solution. For a minimization problem, if $x^{*}$ satisfy following two conditions

> (1) $F_{i}\left(x^{*}\right)<F_{i}(x)$ and
> (2) $F_{j}\left(x^{*}\right) \leq F_{j}(x)(j \neq i)$,

Then $x^{*}$ is called the non-dominated solution. Therefore, when $x *$ is a non-dominated solution, its objective function value can be equal to or less than other feasible solutions (Chen \& Lo, 2014). The non-dominated sorting procedure of NSGA-II is implemented in NSBBO. The crossover and mutation operators of NSGA-II are replaced by migration and mutation operators of BBO algorithm.

### 4.2.1 NSBBO procedure steps

## 1. Initialisation

Initial solutions of arbitrary islands are generated. Each island solution is compared with other solutions to find the non-dominant solutions. These island solutions are sorted using non-dominance approach and numbers of Pareto-fronts of final objective values are obtained as discussed in section 4.2. According to non-domination level, each solution is allotted a fitness or rank. The first front of subpopulation of islands is given rank 1. This subpopulation is assigned a dummy fitness $F_{1}$ to this front set.

## 2. Crowding distance calculation

After obtaining non-dominated solutions, crowding distance is calculated; this is the distance between the islands. The distance is calculated to get an evaluation of the concentration of solutions nearby a particular point in the population of islands. The crowding distance provides an estimate of the density of solutions around each solution belonging to the same front. Suppose, $n$ number of solutions belonging to the same feasible front, so these solutions are sorted in descending order, with respect to an objective $K$. The crowding distance is shown in Fig. 1.

$$
\begin{gather*}
d_{i}^{K}=\frac{f_{(i+1)}^{K}-f_{(i-1)}^{K}}{f_{\max }^{K}-f_{\min }^{K}}  \tag{6}\\
(i=2,3, \ldots, n-1) \text { and }(K=1,2, \ldots ., k)
\end{gather*}
$$

where $f_{\max }^{K}$ and $f_{\min }^{K}$ are the maximum and minimum values of the objective function $K$ of the solutions belonging to the front, respectively. The overall crowding distance of the solution $i$, considering all the $k$ objectives, is

$$
\begin{equation*}
d_{i}=\sum_{K=1}^{k} d_{i}^{K} \tag{7}
\end{equation*}
$$



Fig. 1 Crowding distance (Aiello et al., 2006)

## 3. Sharing function and niche count

A proportionate variety is maintained using the sharing function with suitable setting of all the associated parameters. A sharing parameter $\sigma_{\text {share }}$ is involved in sharing function method which sets the level of sharing preferred in a problem. The parameter indicates the largest value of that distance within which any two solutions share each other's fitness. This parameter is usually set by the user (Deb et al., 2002). The sharing function is as follows:

$$
\operatorname{Sh}\left(d_{i}\right)=\left\{\begin{array}{l}
1-\left(\frac{d_{i}}{\sigma_{\text {share }}}\right)^{2}, \text { if } d_{i}<\sigma_{\text {share }}  \tag{8}\\
0, \text { otherwise }
\end{array}\right.
$$

where, $\sigma_{\text {share }}$ is the maximum distance allowed between any two island solutions to be the members of a niche. The $\sigma_{\text {share }}$ value is to be chosen properly. A niche count $\left(n c_{i}\right)$ that provides an estimate of the extent of crowding near a chromosome is calculated using following equation:

$$
\begin{equation*}
n c_{i}=\sum_{i=1}^{n} \operatorname{Sh}\left(d_{i}\right) \tag{9}
\end{equation*}
$$

The shared fitness values are calculated by dividing the dummy fitness values by the niche count, which is given by

$$
\begin{equation*}
F_{i}^{\prime}=\frac{F_{i}}{n c_{i}} \tag{10}
\end{equation*}
$$

This procedure is continued till the shared fitness values are calculated for all the fronts.

## 4. Selection of the habitats for migration

In migration operator, the good features and information are shared between habitats or islands. This depends on emigration rate $\mu$ and immigration rate $\lambda$ of each solution. Migration is the interchanging of facilities between the population members or habitats. We determine the objective function value or HSI of each island. In FLP, the objective function is to minimize the MHC/total layout cost; therefore lowHSI is a good solution. In Fig. 2, immigration curve is indicated as $I$, which follows when there are no species on the island. The maximum number of species that an island can maintain is $S_{\text {max }}$, at this point the immigration rate becomes zero. Now, considering the emigration curve indicated as $E$. If no species present on an island then the emigration rate will be zero. When an island holds the largest number of species, then the maximum emigration rate is $E$. The emigration rate $\mu_{s}$ and immigration rate $\lambda_{s}$ at the presence of $S$ species in that island are calculated from Eq. (11) and Eq. (12) respectively to start the migration operator.

$$
\begin{align*}
& \mu_{S}=\frac{E}{S_{\max }} S  \tag{11}\\
& \lambda_{s}=I\left(1-\frac{S}{S_{\max }}\right) \tag{12}
\end{align*}
$$



Fig. 2 Immigration and Emigration rates of species (Alroomi et al., 2013)
Now the probability of presence of $S$ species in the island is calculated, which is denoted by Ps. This probability is obtained from Eq. (13), as the number of species varies from time $t$ to $t+\Delta t$.

$$
\begin{equation*}
P_{S}(t+\Delta t)=P_{S}(t)\left(1-\lambda_{S} \Delta t-\mu_{S} \Delta t\right)+P_{S-1} \lambda_{S-1} \Delta t+P_{S+1} \mu_{S+1} \Delta \tag{13}
\end{equation*}
$$

From Eq. (13), one of the following three conditions should satisfy to contain $S$ species on an island at time $(t+\Delta t)$ :

1. $S$ species at time $t$ and no immigration or emigration took place during the interval $\Delta t$;
2. $(S-1)$ species at time $t$ and one species immigrated;
3. $(S+1)$ species at time $t$ and one species emigrated

For finding $P_{s}(t)$ in steady state, Eq. (14) is as given:

$$
\begin{equation*}
P_{S}=\frac{v}{\sum_{i=1}^{S_{\max }+1} v_{i}} \tag{14}
\end{equation*}
$$

where $v$ and $v_{\mathrm{i}}$ can be calculated from following Eq. (15) and Eq. (16)
$v=\left[v_{1}, v_{2}, \ldots ., v_{s_{\text {max }}+1}\right]$
$v_{i}=\frac{S_{\text {max }}!}{\left(S_{\max }+1-i\right)!(i-1)!} \quad\left(i=1, \ldots, S_{\max }+1\right)$

## Migration loop of NSBBO algorithm:

```
Select an island \(H_{i}\) in which the species will immigrate using probability \(\lambda_{i}(i=1,2, \ldots . ., k)\)
    If rand \(<\lambda_{i}\)
        for \(j=1\) to \(k\)
    Select an island \(H_{j}\) from which the species will emigrate using probability \(\mu_{j}\)
        If \(H_{j}\) is selected
                Randomly select an SIV or department from \(H_{j}\)
                Replace a random SIV or department in \(H_{i}\) with that of \(H_{j}\)
        end If
            end for
    end If
    go to next SIV or department
    go to next Island
```

After selecting immigrating and emigrating islands from initial population, the migration is done using the probabilities.

## 5. Mutation

In island principle, the number of species present at equilibrium state can be differed due to some peripheral happenings such as diseases, tsunamis, volcanoes or earthquakes which cause decrease in total number of species. If there are other suitable events which provide good features to an island, they improve the solution (Alroomi et al., 2013). The mutation operation is used to increase the diversity of the population members to obtain the good solutions. In BBO algorithm, mutation is done based on probability of species $P_{S}$ and is used for modifying SIV or facility which is randomly selected. The mutation rate can be obtained from Eq. (17).

$$
\begin{equation*}
m=m_{\max }\left(1-\frac{P_{s}}{P_{\max }}\right) \tag{17}
\end{equation*}
$$

$m_{\max }$ is a user-defined maximum mutation rate that $m$ can reach, and $P_{\max }$ is the maximum species probability.

## Mutation loop of NSBBO algorithm:

```
        For i=1:k (k is the number of islands)
            Calculate mutation rate (m) using Eq. (17)
        Select an island Hi with probability PS for mutating SSV
            If H}\mp@subsup{H}{i}{}\mathrm{ is selected for mutation
            Replace selected SIV of island with a randomly generated SIV
        end if
    end for
```


## 5. Application of NSBBO and BBO algorithms to multi-objective FLPs

The problems considered are having $6,8,12$ and 15 departments. To make comparison with previous results, BBO algorithm is applied to solve multi-objective FLPs. The data related to material flow and closeness rating is provided by Singh and Singh (2010) and shown in Appendix B. The demonstration
steps of NSBBO algorithm are shown in Appendix A. 2 with 8-department FLP. The proposed BBO and NSBBO algorithms are programmed in MATLAB (2010) for all the problems and run on Microsoft Windows 8, Intel core processor with 4GB RAM. The detail procedure of implementation of BBO algorithm to equal area FLP is given in Appendix A. 1 with an illustrative pseudo example of 5 departments.

### 5.1 Performance evaluation of BBO algorithm

The specifications of FLPs are shown in Table 1. The problem size, objectives, fine-tuned parameter values of BBO algorithm and computational time required to run one trial of algorithm presented in Table 1. The parameters are finalised by taking number of trials to reach at optimum solutions. Table 2 shows the objective weights of all the problems determined by MWM, GMWM, SDWM and CRITICM methods explained in (Singh \& Singh, 2010).

Table 1
Problem specification and BBO parameters setting

| Problem | Objectives | $k$ | $G$ | Mutation rate | Computational Time (s) |
| :---: | :--- | :---: | :---: | :---: | :---: |
| $\mathrm{N}=6$ | material handling cost and closeness rating | 100 | 100 | 0.3 | 14.75 |
| $\mathrm{~N}=8$ | material handling cost and closeness rating | 200 | 200 | 0.2 | 36.75 |
| $\mathrm{~N}=12$ | material handling cost and closeness rating | 1000 | 100 | 0.2 | 153.24 |
| $\mathrm{~N}=15$ | material handling cost and closeness rating | 1000 | 500 | 0.2 | 324.14 |

Table 2
Objective weights obtained using MWM, GMWM, SDWM and CRITICM

| Problem | MWM | GMWM | SDWM | CRITICM |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{N}=6$ (Rosenblatt, 1979) | $\mathrm{w}_{1}=0.5899$ | $\mathrm{w}_{1}=0.5036$ | $\mathrm{w}_{1}=0.5051$ | $\mathrm{w}_{1}=0.5051$ |
| $\mathrm{~N}=8$ (Fortenberry and Cox, 1985) | $\mathrm{w}_{2}=0.4101$ | $\mathrm{w}_{2}=0.4964$ | $\mathrm{w}_{2}=0.4949$ | $\mathrm{w}_{2}=0.4949$ |
|  | $\mathrm{w}_{2}=0.4949$ | $\mathrm{w}_{1}=0.4703$ | $\mathrm{w}_{1}=0.5991$ | $\mathrm{w}_{1}=0.5991$ |
| $\mathrm{~N}=12$ (Dutta and Sahu, 1982) | $\mathrm{w}_{1}=0.6945$ | $\mathrm{w}_{2}=0.5297$ | $\mathrm{w}_{2}=0.4009$ | $\mathrm{w}_{2}=0.4009$ |
|  | $\mathrm{w}_{2}=0.3055$ | $\mathrm{w}_{2}=0.5303$ | $\mathrm{w}_{1}=0.5096$ | $\mathrm{w}_{1}=0.5096$ |
| $\mathrm{~N}=15$ (Chen and Sha, 1999) | $\mathrm{w}_{1}=0.7448$ | na | $\mathrm{w}_{2}=0.4904$ | $\mathrm{w}_{2}=0.4904$ |
|  | $\mathrm{w}_{2}=0.2552$ |  | $\mathrm{w}_{2}=0.5466$ | $\mathrm{w}_{1}=0.4566$ |
|  |  |  |  | $\mathrm{w}_{2}=0.5434$ |

The objective function value (OFV) using each weight method and the layout obtained using BBO algorithm is shown in Table 3. The best solutions are displayed in bold face. For N=8, BBO obtained two layouts using GMWM method which are shown in Table 3.


Fig. 3 Convergence of BBO for $\mathrm{N}=6 \mathrm{FLP}$
From Table 3, it is seen that for problem size of 6,8 and 12 the results obtained using BBO algorithm are same as obtained by Matai (2015) and for size $15, \mathrm{BBO}$ algorithm is giving better solutions in all the
three methods. In 8-department FLP, two layouts are obtained for the same OFV. In Figs. 3-6, the graphs are plotted between generations and OFV which show the convergence nature of BBO algorithm. The algorithm is converging to near optimal solutions in minimum number of generations. This shows the capability of BBO algorithm to solve FLPs.


Fig. 4 Convergence of BBO for $\mathrm{N}=8 \mathrm{FLP}$
Table 3
Comparison of multi-objective FLPs with previous results

| Problem | Approach | MWM | GMWM | SDWM | CRITICM |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \mathrm{N}=6 \\ \text { (Rosenblatt, } \\ \text { 1979) } \end{gathered}$ | $\begin{gathered} \hline \text { Singh and Singh } \\ (2010) \\ \text { Matai }(2015) \end{gathered}$ | 34.1759 | 22.0076 | 22.2191 | 22.2191 |
|  |  | [2-6-5-3-1-4] | [2-6-5-3-1-4] | [2-6-5-3-1-4] | [2-6-5-3-1-4] |
|  |  | 34.1759 | 22.0076 | 22.2191 | 22.2191 |
|  |  | [4-3-1-5-6-2] | [5-6-3-4-2-1] | [4-3-1-5-6-2] | [4-3-1-5-6-2] |
|  | BBO | 34.1759 | 22.0076 | 22.2191 | 22.2191 |
|  |  | [2-6-5-3-1-4] | [3-6-5-1-2-4] | [5-6-2-4-3-1] | [1-2-4-3-6-5] |
| $\mathbf{N}=8$ <br> (Fortenberry and Cox, 1985) | Singh and Singh (2010) <br> Matai (2015) | 128.6624 | 105.78 | 127.256 | 127.256 |
|  |  | [1-5-8-3-2-7-6-4] | [4-6-7-2-3-8-5-1] | [2-7-6-4-1-5-8-3] | [2-7-6-4-1-5-8-3] |
|  |  | 124.6624 | 52.8928 | 127.0816 | 127.0816 |
|  |  | [1-5-8-3-2-7-6-4] | [4-6-7-2-3-8-5-1] | [2-7-6-4-1-5-8-3] | [2-7-6-4-1-5-8-3] |
|  | BBO | 124.6624 | 52.8928 | 127.0816 | 127.0816 |
|  |  | [2-7-6-4-1-5-8-3] | [1-5-8-3-2-7-6-4] | [3-8-5-1-4-6-7-2] | [3-8-5-1-4-6-7-2] |
|  |  |  | [3-8-5-1-4-6-7-2] |  |  |
| $\begin{gathered} \mathrm{N}=12 \text { (Dutta } \\ \text { and Sahu, 1982) } \end{gathered}$ | Singh and Singh (2010) | 1766.99 | 1116.36 [4-6- | 1255.435 [4-6- | 1255.435 [4-6- |
|  |  | $\begin{gathered} {[1-6-7-2-9-10-12-8-4-} \\ 11-5-3] \end{gathered}$ | $\begin{gathered} 5-2-9-10-12-8-1-11-7- \\ 3] \end{gathered}$ | $\begin{gathered} 5-2-9-10-12-8-1-11-7- \\ 3] \end{gathered}$ | $\begin{gathered} 5-2-9-10-12-8-1-11-7- \\ 3] \end{gathered}$ |
|  | Matai (2015) | 1678.2656 | 1050.4075 [3-5- | 1162.7645 [4-9- | 1162.7645 [4-9- |
|  |  | $\begin{gathered} {[3-5-12-2-8-7-10-6-1-} \\ 11-9-4] \end{gathered}$ | $\begin{gathered} 12-2-8-6-7-10-1-11-9- \\ 4] \end{gathered}$ | $\begin{gathered} 11-1-10-7-6-8-2-12-5- \\ 3] \end{gathered}$ | $\begin{gathered} 11-1-10-7-6-8-2-12-5- \\ 3] \end{gathered}$ |
|  | BBO | 1678.2656 | 1050.4000 [2- | 1164.8000 [3-8- | 1164.8000 [3-8- |
|  |  | $\begin{gathered} {[4-9-11-1-10-7-6-8-2-} \\ 12-5-3] \end{gathered}$ | $\begin{gathered} 12-5-3-6-10-7-8-4-9- \\ 11-1] \end{gathered}$ | $\begin{gathered} \text { 11-1-12-5-6-7-2-10-9- } \\ 4] \end{gathered}$ | $\begin{aligned} & 11-1-12-5-6-7-2-10-9- \\ & 4] \end{aligned}$ |
| $\begin{gathered} \text { N = } 15 \text { (Chen } \\ \text { and Sha, 1999) } \end{gathered}$ | Singh and Singh (2010) | 2701.64 [10- | na | 1557.68 | 1557.68 [10- |
|  |  | $\begin{gathered} 12-1-13-6-15-14-8-2- \\ 11-4-7-9-3-5] \end{gathered}$ |  | $\begin{gathered} {[10-12-1-7-4-11-14-2-} \\ 8-15-5-13-9-3-6] \end{gathered}$ | $\begin{gathered} 12-1-7-4-11-14-2-8-15- \\ 5-13-9-3-6] \end{gathered}$ |
|  | Matai (2015) | 2459.5818 [12- | na | 1411.1108 | 1411.1108 [12-7- |
|  |  | $\begin{gathered} 7-14-15-6-13-9-1-8-11- \\ 10-5-3-2-4] \end{gathered}$ |  | $\begin{gathered} {[12-7-14-15-6-13-9-1-} \\ 8-11-10-5-3-2-4] \end{gathered}$ | $\begin{gathered} 14-15-6-13-9-1-8-11- \\ 10-5-3-2-4] \end{gathered}$ |
|  | BBO | 2445.5000 [10- | na | 1402.4000 | 1402.9000 [10- |
|  |  | 5-3-2-4-13-1-14-8-11- |  | [10-5-3-2-4-13-1-14- | $3-1-2-4-13-5-14-8-11-$ |



Fig. 5 Convergence of BBO for $\mathrm{N}=12$ FLP


Fig. 6 Convergence of BBO for $\mathrm{N}=15$ FLP

As we have earlier stated in section 3 about the objective functions; the results of multi-objective FLPs obtained using Eq. (4) and Eq. (5) are shown in Table 4. All the objective functions are evaluated separately to show the difference between the results. The comparison of original OFV obtained in this work with that of Singh and Singh (2010) and Matai (2015) is given in Table 4. It is seen from Table 4 that the new values obtained from remodelled objective functions are different from original values. The results obtained for different objective weights are presented in Table 4 and these results cannot be compared with the previous results due to the difference in second objective i.e. $f_{2}$. The reason to show the difference between the results obtained in this paper and results obtained by Singh and Singh (2010) and Matai (2015), is the correct values of the objective functions which should be considered as best layout by the layout planner.

Table 4
Results of multi-objective FLPs using BBO algorithm

| Problem | MWM | GMWM | SDWM | CRITICM |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{N}=6$ | $\begin{gathered} \mathrm{w}_{1}=0.5899 \\ \mathrm{w}_{2}=0.4101 \\ \text { OFV }=86.26 \\ \text { Layout }=\left[\begin{array}{llll} 5 & 6 & 2 & 4 \end{array}\right] \end{gathered}$ | $\begin{gathered} w_{1}=0.5036 \\ \mathbf{w}_{2}=0.4964 \\ \text { OFV }=85.05 \\ \text { Layout }=\left[\begin{array}{lllll} 2 & 6 & 5 & 3 & 1 \end{array}\right] \end{gathered}$ | $\begin{gathered} \mathrm{w}_{1}=0.5051 \\ \mathrm{w}_{2}=0.4949 \\ \text { OFV }=85.07 \\ \text { Layout }=\left[\begin{array}{llll} 3 & 2 & 4 & 1 \end{array}\right] \end{gathered}$ | $\begin{gathered} \mathrm{w}_{1}=0.5051 \\ \mathrm{w}_{2}=0.4949 \\ \text { OFV }=85.07 \text { Layout }= \\ {\left[\begin{array}{lllll} 3 & 2 & 4 & 1 & 6 \end{array}\right]} \end{gathered}$ |
| $\mathrm{N}=8$ | $\begin{gathered} \mathrm{w}_{1}=0.5949 \\ \mathrm{w}_{2}=0.4051 \\ \text { OFV }=196.57 \text { Layout }=[2 \\ 1437568] \end{gathered}$ | $\begin{gathered} \mathrm{w}_{1}=0.4703 \\ \mathrm{w}_{2}=0.5297 \\ \text { OFV }=195.82 \text { Layout }= \\ {[2143756} \end{gathered}$ | $\begin{gathered} w_{1}=0.5991 \\ w_{2}=0.4009 \\ \text { OFV }=188.22 \text { Layout= } \\ {[15832764]} \end{gathered}$ | $\begin{gathered} w_{1}=0.5991 \\ w_{2}=0.4009 \\ \text { OFV }=188.22 \\ \text { Layout }=[1583276 \\ 4 \mid \end{gathered}$ |
| $\mathrm{N}=12$ | $\begin{gathered} \mathrm{w}_{1}=0.6945 \\ \mathrm{w}_{2}=0.3055 \\ \text { OFV } 996.72 \text { Layout }=[3 \\ 14281169571210] \end{gathered}$ | $\begin{gathered} w_{1}=0.4693 \\ w_{2}=0.5307 \\ \text { OFV }=754.46 \\ \text { Layout }=\left[\begin{array}{ll} 186425 \end{array}\right. \\ 121131179] \end{gathered}$ | $\begin{gathered} \mathrm{w}_{1}=0.5096 \\ \mathrm{w}_{2}=0.4904 \\ \text { OFV }=794.15 \text { Layout }= \\ {\left[\begin{array}{lll} 1 & 114938672512 \\ \text { 10 } \\ \text { 10 } \end{array}\right]} \end{gathered}$ | $\begin{gathered} w_{1}=0.5096 \\ w_{2}=0.4904 \\ \text { OFV }=794.15 \\ \text { Layout }=\left[\begin{array}{llll} 1 & 11 & 49 & 9 \end{array} 8\right. \\ 672 \end{gathered}$ |
| $\mathrm{N}=15$ | $\begin{gathered} \mathrm{w}_{1}=0.7448 \\ \mathrm{w}_{2}=0.2552 \\ \text { OF } V=1182.4 \\ \text { Layout }=[1210213597 \\ 15161138144] \end{gathered}$ | Na | $\begin{gathered} w_{1}=0.4566 \\ w_{2}=0.5434 \\ \text { OFV }=859.41 \\ \text { Layout }=[41213514 \\ 15831067 \\ \hline 11912] \end{gathered}$ | $\begin{gathered} \mathrm{w}_{1}=0.4566 \\ \mathrm{w}_{2}=0.5434 \\ \text { OFV }=868.14 \\ \text { Layout }=[12521310 \\ 1118154614739 \end{gathered}$ |

### 5.2 Performance evaluation of NSBBO algorithm

The multi-objective FLPs considered in this paper are not solved using Pareto-optimality method or nondominance criteria in previous literature. In this section, NSBBO algorithm is applied to solve multiobjective FLPs. As we cannot compare the solutions obtained using weight approach with the solutions obtained using Pareto-optimality criteria, therefore both NSBBO and NSGA-II algorithms are implemented. The objective functions are described in previous section. In Pareto-optimality procedure the objective function are kept separate to obtain non-dominated solutions. The multi-objective FLP can be stated as:

$$
\begin{equation*}
\min F=\left\{f_{1}, f_{2}\right\} \tag{18}
\end{equation*}
$$

Eq. (18) indicates the functions of MHC and CR. In $f_{2}$, which represents closeness rating score, is considered as a penalty to be minimized (Sahin \& Turkbey, 2009). The parameters of NSBBO and NSGA-II such as population or island size ( P ), generations ( G ), mutation rate and sharing fitness are decided by taking number of trials and kept same for both the algorithms. The values of these parameters are mentioned in Table 5.

Table 5
NSBBO and NSGA-II parameters setting

| Problem | P | G | Mutation rate | Sharing fitness |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{N}=6$ | 100 | 100 | 0.15 | 1.3 |
| $\mathrm{~N}=8$ | 100 | 150 | 0.15 | 1.2 |
| $\mathrm{~N}=12$ | 200 | 500 | 0.20 | 1.2 |
| $\mathrm{~N}=15$ | 200 | 1000 | 0.20 | 1.3 |

Note: MHC-Material handling cost, CR-Closeness rating, P-Population and G-Generation

Table 6
Results of 6-department FLP

| Algorithm | Layouts | MHC | CR |
| :---: | :---: | :---: | :---: |
|  | $3-1-2-4-5-6$ | 192 | 76 |
| NSBBO | $1-2-3-6-5-4$ | 208 | 72 |
|  | $1-5-6-3-2-4$ | 212 | 71 |
|  | $3-1-4-2-6-5$ | 184 | 78 |
|  | $4-2-5-3-1-6$ | 204 | 75 |
|  | $3-2-1-4-6-5$ | 196 | 75 |
|  | $3-1-4-2-6-5$ | 184 | 78 |
|  | $5-1-4-6-2-3$ | 196 | 75 |
|  | $2-5-6-3-1-4$ | 212 | 71 |
|  | $5-2-4-6-1-3$ | 204 | 75 |

The results of 6-department FLP using NSBBO and NSGA-II are shown in Table 6. This problem is small in size; therefore best solutions are obtained from both the algorithms. Each method has provided 6 solutions with different layouts. Therefore, the layout planner can decide the layout according to his/her preference. Pareto solutions and efficient frontier of 6-department FLP for NSBBO and NSGA-II are shown in Fig. 7 and Fig. 8, respectively. It is seen that both the frontier are same.


Fig. 7. Pareto solution front of 6-department FLP using NSBBO


Fig. 8. Pareto solution front of 6-department FLP using NSGA-II

Table 7 shows the results of 8-department problem using NSBBO and NSGA-II methods. NSBBO algorithm has provided five non-dominated solutions while NSGA-II has provided four solutions of MHC and CR.

Table 7
Results of 8-department FLP

| Algorithm | Layouts | MHC | CR |
| :---: | :---: | :---: | :---: |
|  | $7-5-8-4-2-1-6-3$ | 206 | 206 |
| NSBBO | $4-6-5-8-2-7-1-3$ | 210 | 203 |
|  | $3-8-5-1-4-7-6-2$ | 203 | 208 |
|  | $6-7-1-4-8-5-2-3$ | 228 | 182 |
|  | $3-6-8-2-4-7-5-1$ | 202 | 212 |
| NSGA-II | $2-4-1-3-7-6-5-8$ | 208 | 195 |
|  | $2-1-5-7-3-4-6-8$ | 225 | 195 |
|  | $2-1-8-3-7-6-5-4$ | 206 | 206 |
|  | $2-1-7-6-4-3-5-8$ | 227 | 184 |

Fig. 9 and Fig. 10 show that Pareto frontiers of NSBBO and NSGA-II respectively for 8-department problem.


Fig. 9. Pareto solution front of 8-department FLP using NSBBO


Fig. 10. Pareto solution front of 8-department FLP using NSGA-II

In Table 8, results of 12-department problem are presented. NSBBO has provided four non-dominated solutions while NSGA-II has provided three solutions. Efficient Pareto frontier of NSBBO and NSGAII for 12-department MOFLP are shown in Fig. 11 and Fig. 12, respectively.

Table 8
Results of 12-department FLP

| Algorithm | Layouts | MHC | CR |
| :---: | :---: | :---: | :---: |
| NSBBO | $4-10-5-11-9-12-7-8-2-1-6-3$ | 1334 | 307 |
|  | $10-8-7-4-12-5-6-3-9-2-11-1$ | 1380 | 265 |
|  | $1-4-7-6-10-9-12-8-2-11-5-3$ | 1322 | 314 |
| NSGA-II | $2-7-9-4-1-11-8-6-3-12-5-10$ | 1367 | 304 |
|  | $4-1-3-2-6-5-11-8-7-9-12-10$ | 1386 | 270 |
|  | $1-2-3-4-5-12-6-11-8-10-7-9$ | 1380 | 280 |
|  | $1-2-3-8-9-11-5-10-4-7-6-12$ | 1373 | 284 |



Fig. 11 Pareto solution front of 12-department FLP using NSBBO


Fig. 12 Pareto solution front of 12-department FLP using NSGA-II

Table 9 shows the Pareto solutions of 15 -department FLP. NSBBO provided 6 solutions while NSGA-II provided 9 solutions. The Pareto curves of NSBBO and NSGA-II for 15 -department problem are shown in Fig. 13 and Fig.14, respectively.

Table 9
Results of 15-department FLP

| Algorithm | Layouts | MHC | CR |
| :---: | :---: | :---: | :---: |
|  | $13-7-6-2-4-3-5-10-8-1-11-12-14-15-9$ | 1886 | 280 |
| NSBBO | $4-7-12-10-5-9-3-13-2-1-6-15-8-11-14$ | 1624 | 318 |
|  | $2-14-8-4-11-12-1-7-6-9-5-10-13-15-3$ | 1617 | 357 |
|  | $5-10-12-6-11-2-8-14-1-9-3-13-7-15-4$ | 1667 | 302 |
|  | $4-13-1-6-11-12-15-8-2-3-7-9-14-5-10$ | 1693 | 298 |
|  | $6-5-1-4-10-14-11-3-13-9-12-8-2-7-15$ | 1714 | 285 |
| NSGA-II | $14-1-11-8-2-3-4-7-12-5-6-9-13-10-15$ | 1734 | 305 |
|  | $7-4-1-2-3-9-13-8-5-12-6-10-15-14-11$ | 1807 | 289 |
|  | $7-15-1-2-12-4-13-8-3-5-9-10-11-6-14$ | 1820 | 282 |
|  | $5-4-2-15-1-3-10-8-7-6-13-9-12-11-14$ | 1677 | 308 |
|  | $1-2-5-7-3-4-10-11-6-14-9-8-15-13-12$ | 1968 | 277 |
|  | $6-2-1-7-4-3-5-10-8-9-11-13-14-12-15$ | 1893 | 279 |
|  | $5-3-1-4-9-2-14-8-11-7-6-13-15-10-12$ | 1742 | 295 |
|  | $4-1-3-8-2-6-11-15-7-5-14-13-9-12-10$ | 1653 | 329 |
|  | $1-2-3-5-11-4-13-6-7-8-9-10-15-14-12$ | 1892 | 280 |



Fig. 13. Pareto solution front of 15 -department FLP using NSBBO


Fig. 14. Pareto solution front of 15 -department FLP using NSGA-II

All the 15 solutions obtained are non-comparable and considered as equally good. This gives flexibility to the layout planner to select appropriate layout considering other criteria as per requirement. The overall performance of both BBO and NSBBO algorithms show their applicability on complex combinatorial problems of multi-objective FLPs.

## 6. Conclusion

In this paper, BBO meta-heuristic algorithm has been employed to solve complex multi-objective FLPs, which are otherwise very difficult to solve using traditional techniques. Both weight approach and Paretooptimality approach were considered to obtain better results. The results obtained using BBO algorithm (weight approach) for $6,8,12$ and 15 departments FLP are better or equal as compared to previous results. Furthermore, NSBBO algorithm was proposed to find Pareto-optimal solutions of FLPs. The solutions are non-dominated which are not biased to any one objective. The comparison of NSBBO and NSGA-II is proving that NSBBO is performing as good as NSGA-II. The layout planner can decide best layout among number of solutions according to his/her preference.

The computational time required to run BBO algorithm is satisfactory. This makes BBO algorithm efficient to solve combinatorial optimization problems like FLP. An important observation regarding objective functions of FLP is discussed and remodelled functions are evaluated to show the difference between the final solutions. The distance between the departments should be incorporated while considering the objectives related to movement of material. As BBO and NSBBO algorithms are capable to solve multi-objective FLPs, it can be further implemented to solve multi-objective, dynamic and unequal area FLPs.

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## Appendix A

## A. 1 Demonstration steps of BBO algorithm with an illustrative example of $\mathbf{n}=5$ department problem

Step 1: Randomly generate islands $(k=5)$. Each layout is considered as one island. Since there are 5 departments, string length of each island consists of 5 random numbers.

| $\{0.8147$ | 0.0975 | 0.1576 | 0.1419 | $0.6557\}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\{0.9058$ | 0.2785 | 0.9706 | 0.4218 | $0.0357\}$ |
| $\{0.1270$ | 0.5469 | 0.9572 | 0.9157 | $0.8491\}$ |
| $\{0.9134$ | 0.9575 | 0.4854 | 0.7922 | $0.9340\}$ |
| $\{0.6324$ | 0.9649 | 0.8003 | 0.9595 | $0.6787\}$ |

Step 2: Obtain the integer numbers for departments by sorting and indexing each row of above matrix. Evaluate OFV of each layout.

$$
\begin{array}{ll}
\mathrm{L}_{1}: 2-4-3-5-1 ; & \text { OFV: } 186 \\
\mathrm{~L}_{2}: 5-2-4-1-3 ; & \text { OFV: } 168 \\
\mathrm{~L}_{3}: 1-2-5-4-3 ; & \text { OFV: } 179 \\
\mathrm{~L}_{4}: 3-4-1-5-2 ; & \text { OFV: } 151 \\
\mathrm{~L}_{5}: 1-5-3-4-2 ; & \text { OFV: } 163
\end{array}
$$

From initial layouts, the minimum OFV is 151 and the corresponding layout is the best layout.
Step 3: Start first iteration. The emigration rate $\mu_{s}$ and immigration rate $\lambda_{s}$ are calculated using Eqs. (11) and (12) respectively.

$$
\begin{aligned}
\mu_{s} & =\{0.2,0.4,0.6,0.8,1\} \\
\lambda_{s} & =\{0.8,0.6,0.4,0.2,0\}
\end{aligned}
$$

Probability of the species $P_{s}$ is calculated using Eq. (13)

$$
P_{s}=\{0.0313,0.1563,0.3125,0.3125,0.1563,0.0313\}
$$

Step 4: Start migration operation. Select two islands for emigration and immigration. Using $\mu_{s}$, fourth island is selected for emigration and using $\lambda_{s}$, first island is selected for immigration. Randomly select SIVs from fourth and first island to replace their positions. Suppose fifth and second SIVs are selected from fourth and first island respectively and their positions are replaced. The numbers replaced are shown in bold. The new islands will be as follows:

| $\{0.8147$ | $\mathbf{0 . 9 3 4 0}$ | 0.1576 | 0.1419 | $0.6557\}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\{0.9058$ | 0.2785 | 0.9706 | 0.4218 | $0.0357\}$ |
| $\{0.1270$ | 0.5469 | 0.9572 | 0.9157 | $0.8491\}$ |
| $\{0.9134$ | 0.9575 | 0.4854 | 0.7922 | $\mathbf{0 . 0 9 7 5}\}$ |
| $\{0.6324$ | 0.9649 | 0.8003 | 0.9595 | $0.6787\}$ |

Step 5: Start mutation operation. The mutation rate is calculated using Eq. (17).

$$
\mathrm{m}=\{0.8998,0.4998,0,0,0.4998,0.8998\}
$$

Using $P_{s}$, two islands are selected. In each island, one SIV is selected to replace with randomly generated number. Suppose first and fifth islands are selected for mutation. From first island, third SIV is selected and from fifth island, fourth SIV is selected to replace with random numbers $r_{1}=0.3781$ and $r_{2}=0.4424$, respectively. After mutation the islands obtained are

$$
\left\{\begin{array}{lllll}
0.8147 & 0.9340 & 0.3781 & 0.1419 & 0.6557
\end{array}\right\}
$$

| $\{0.9058$ | 0.2785 | 0.9706 | 0.4218 | $0.0357\}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\{0.1270$ | 0.5469 | 0.9572 | 0.9157 | $0.8491\}$ |
| $\{0.9134$ | 0.9575 | 0.4854 | 0.7922 | $0.0975\}$ |
| $\{0.6324$ | 0.9649 | 0.8003 | 0.4424 | $0.6787\}$ |

Again find the integer numbers by sorting and indexing each row of above matrix to get the layouts and their OFVs after mutation.

$$
\begin{array}{ll}
\mathrm{L}_{1}: 4-3-5-1-2 ; & \text { OFV: } 166 \\
\mathrm{~L}_{2}: 5-2-4-1-3 ; & \text { OFV: } 168 \\
\mathrm{~L}_{3}: 1-2-5-4-3 ; & \text { OFV: } 179 \\
\mathrm{~L}_{4}: 5-3-4-1-2 ; & \text { OFV: } 172 \\
\mathrm{~L}_{5}: 4-1-5-3-2 ; & \text { OFV: } 148
\end{array}
$$

Step 7: After first iteration, the best layout and its OFV obtained is:

$$
4-1-5-3-2 ; \quad \text { OFV }=148
$$

Step 8: Repeat steps 3 to 7 until maximum number of iterations is reached.

## A. 2 Demonstration steps of NSBBO with an illustrative example of 8 departments.

Step 1. Generate initial islands/population $(\mathrm{P}=10)$ of random numbers. Set the values of iterations/generations (G), mutation rate, sharing fitness ( $\sigma_{\text {share }}$ ) and dummy fitness. The random numbers are indexed and sorted to obtain layouts. Calculate both the objectives of material handling cost (MHC) and closeness rating (CR) for each layout.

| Sr . <br> No | Randomly generated islands |  |  |  |  |  |  |  | Layout |  |  |  |  |  |  | MHC |  | CR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.7772 | 0.6679 | 0.0615 | 0.7989 | 0.1103 | 0.9705 | 0.0866 | 0.1006 | 3 | 7 | 8 | 5 | 2 | 1 | 4 | 6 | 244 | 212 |
| 2 | 0.9051 | 0.6034 | 0.7801 | 0.7343 | 0.1174 | 0.8669 | 0.7719 | 0.2940 | 5 | 8 | 2 | 4 | 7 | 3 | 6 | 1 | 247 | 227 |
| 3 | 0.5337 | 0.5261 | 0.3375 | 0.0513 | 0.6407 | 0.0862 | 0.2056 | 0.2373 | 4 | 6 | 7 | 8 | 3 | 2 | 1 | 5 | 226 | 198 |
| 4 | 0.1091 | 0.7297 | 0.6078 | 0.0728 | 0.3288 | 0.3664 | 0.3882 | 0.5308 | 4 | 1 | 5 | 6 | 7 | 8 | 3 | 2 | 269 | 237 |
| 5 | 0.8258 | 0.7072 | 0.7412 | 0.0885 | 0.6538 | 0.3691 | 0.5517 | 0.0914 | 4 | 8 | 6 | 7 | 5 | 2 | 3 | 1 | 261 | 234 |
| 6 | 0.3380 | 0.7813 | 0.1048 | 0.7983 | 0.7491 | 0.6850 | 0.2289 | 0.4053 | 3 | 7 | 1 | 8 | 6 | 5 | 2 | 4 | 264 | 226 |
| 7 | 0.2939 | 0.2879 | 0.1278 | 0.9430 | 0.5831 | 0.5979 | 0.6419 | 0.1048 | 8 | 3 | 2 | 1 | 5 | 6 | 7 | 4 | 232 | 210 |
| 8 | 0.7463 | 0.6925 | 0.5495 | 0.6837 | 0.7400 | 0.7893 | 0.4844 | 0.1122 | 8 | 7 | 3 | 4 | 2 | 5 | 1 | 6 | 243 | 216 |
| 9 | 0.0103 | 0.5566 | 0.4852 | 0.1320 | 0.2348 | 0.3676 | 0.1518 | 0.7844 | 1 | 4 | 7 | 5 | 6 | 3 | 2 | 8 | 252 | 218 |
| 10 | 0.0484 | 0.3965 | 0.8904 | 0.7227 | 0.7349 | 0.2060 | 0.7819 | 0.2915 | 1 | 6 | 8 | 2 | 4 | 5 | 7 | 3 | 241 | 221 |

Step 2: Start first iteration. Rank the objectives according to non-dominated solutions as shown below:


Step 3. Calculate shared fitness, niche count and dummy fitness using Eqs.8-10 respectively. Calculate probability and cumulative probability for selection of islands.

| Sr. No. | Niche count | Dummy fitness | Shared fitness | Probability | Cumulative probability |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.2 | 50.0000 | 41.6666 | 0.3949 | 0.3949 |
| 2 | 1.2 | 41.5631 | 34.6359 | 0.3282 | 0.7232 |
| 3 | 4.2 | 34.1715 | 8.1360 | 0.0771 | 0.8003 |
| 4 | 4.2 | 34.1715 | 8.1360 | 0.0771 | 0.8774 |
| 5 | 4.2 | 34.1715 | 8.1360 | 0.0771 | 0.9545 |
| 6 | 4.2 | 8.0685 | 1.9210 | 0.0182 | 0.9910 |
| 7 | 4.2 | 8.0685 | 1.9210 | 0.0182 | 0.9948 |
| 8 | 4.2 | 1.7263 | 0.4110 | 0.0038 | 0.9987 |
| 9 | 4.2 | 1.7263 | 0.4110 | 0.0038 | 1.0000 |
| 10 | 1.2 | 0.1529 | 0.1274 | 0.0012 |  |

Step 4: Generate random number for each member and sort the islands with respect to probability and cumulative probability.

| Rand no. | Sorted | Islands sorted with respect to cumulative probability and random no. |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.6554 | 2 | 0.2939 | 0.2879 | 0.1278 | 0.9430 | 0.5831 | 0.5979 | 0.6419 | 0.1048 |
| 0.1097 | 4 | 0.7463 | 0.6925 | 0.5495 | 0.6837 | 0.7400 | 0.7893 | 0.4844 | 0.1122 |
| 0.9337 | 5 | 0.0484 | 0.3965 | 0.8904 | 0.7227 | 0.7349 | 0.2060 | 0.7819 | 0.2915 |
| 0.1874 | 10 | 0.1091 | 0.7297 | 0.6078 | 0.0728 | 0.3288 | 0.3664 | 0.3882 | 0.5308 |
| 0.2661 | 9 | 0.3380 | 0.7813 | 0.1048 | 0.7983 | 0.7491 | 0.6850 | 0.2289 | 0.4053 |
| 0.7978 | 7 | 0.0103 | 0.5566 | 0.4852 | 0.1320 | 0.2348 | 0.3676 | 0.1518 | 0.7844 |
| 0.4876 | 1 | 0.5337 | 0.5261 | 0.3375 | 0.0513 | 0.6407 | 0.0862 | 0.2056 | 0.2373 |
| 0.7689 | 8 | 0.8258 | 0.7072 | 0.7412 | 0.0885 | 0.6538 | 0.3691 | 0.5517 | 0.0914 |
| 0.3960 | 6 | 0.9051 | 0.6034 | 0.7801 | 0.7343 | 0.1174 | 0.8669 | 0.7719 | 0.2940 |
| 0.2729 | 3 | 0.7772 | 0.6679 | 0.0615 | 0.7989 | 0.1103 | 0.9705 | 0.0866 | 0.1006 |

Step 5: Apply migration operator to the islands by using migration probability. Mutation is done by using mutation probability. The updated islands are given below:

| Sr. No. | Islands after migration |  |  |  |  |  |  |  | Islands after mutation |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.2939 | 0.2879 | 0.1278 | 0.9430 | 0.5831 | 0.5979 | 0.6419 | 0.1048 | 0.2939 | 0.2879 | 0.1278 | 0.9430 | 0.5831 | 0.5979 | 0.6419 | 0.1048 |
| 2 | 0.7463 | 0.6925 | 0.5495 | 0.6837 | 0.7400 | 0.7893 | 0.4844 | 0.1122 | 0.7463 | 0.6925 | 0.5495 | 0.6837 | 0.7400 | 0.7893 | 0.4844 | 0.1122 |
| 3 | 0.0484 | 0.3965 | 0.8904 | 0.0728 | 0.5831 | 0.2060 | 0.7819 | 0.2915 | 0.0484 | 0.3965 | 0.8904 | 0.0728 | 0.5831 | 0.2060 | 0.7819 | 0.2915 |
| 4 | 0.1091 | 0.7297 | 0.6078 | 0.7983 | 0.2348 | 0.6850 | 0.3882 | 0.5308 | 0.1091 | 0.7297 | 0.6078 | 0.7983 | 0.2348 | 0.6850 | 0.3882 | 0.5308 |
| 5 | 0.0484 | 0.7813 | 0.1048 | 0.7983 | 0.2348 | 0.6850 | 0.2289 | 0.4053 | 0.0484 | 0.7813 | 0.1048 | 0.7983 | 0.2348 | 0.6850 | 0.2289 | 0.4053 |
| 6 | 0.0103 | 0.6679 | 0.7412 | 0.1320 | 0.1174 | 0.3676 | 0.7719 | 0.0914 | 0.0103 | 0.6679 | 0.7412 | 0.1320 | 0.1174 | 0.3676 | 0.7719 | 0.0914 |
| 7 | 0.0103 | 0.5261 | 0.7412 | 0.0513 | 0.6407 | 0.7893 | 0.7719 | 0.1006 | 0.0103 | 0.5261 | 0.7412 | 0.0513 | 0.6407 | 0.7893 | 0.7719 | 0.1006 |
| 8 | 0.9051 | 0.6034 | 0.8904 | 0.0513 | 0.5831 | 0.3691 | 0.3882 | 0.1006 | 0.9051 | 1.5346 | 0.8904 | 0.0513 | 0.5831 | 0.3691 | 0.3882 | 0.1006 |
| 9 | 0.9051 | 0.6034 | 0.7801 | 0.7343 | 0.1174 | 0.8669 | 0.7719 | 0.2940 | 0.9051 | 0.6034 | 0.7801 | 0.7343 | 0.1174 | 0.8669 | 0.7719 | 0.2940 |
| 10 | 0.7772 | 0.6679 | 0.0615 | 0.7989 | 0.1103 | 0.9705 | 0.0866 | 0.1006 | 0.8667 | 0.6679 | 0.0615 | 0.7989 | 0.1103 | 0.9705 | 0.0866 | 0.1006 |

Step 6: Evaluate the objective function values after first iteration as follows:

| Sr. No. | Layouts |  |  |  |  |  |  |  | MHC | CR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 3 | 2 | 1 | 5 | 6 | 7 | 4 | 232 | 210 |
| 2 | 8 | 7 | 3 | 4 | 2 | 5 | 1 | 6 | 243 | 216 |
| 3 | 1 | 4 | 6 | 8 | 2 | 5 | 7 | 3 | 220 | 216 |
| 4 | 1 | 5 | 7 | 8 | 3 | 6 | 2 | 4 | 250 | 215 |
| 5 | 1 | 3 | 7 | 5 | 8 | 6 | 2 | 4 | 254 | 231 |
| 6 | 1 | 8 | 5 | 4 | 6 | 2 | 3 | 7 | 256 | 233 |
| 7 | 1 | 4 | 8 | 2 | 5 | 3 | 7 | 6 | 252 | 222 |
| 8 | 4 | 8 | 6 | 7 | 5 | 3 | 1 | 2 | 234 | 219 |
| 9 | 5 | 8 | 2 | 4 | 7 | 3 | 6 | 1 | 247 | 227 |
| 10 | 3 | 7 | 8 | 5 | 2 | 4 | 1 | 6 | 244 | 207 |

Step 7: The non-dominated solutions after first iteration are given below:

| Sr. No. | Layouts |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 3 | 7 | 5 | 8 | 6 | 2 | 4 | 254 |
| 2 | 1 | 8 | 5 | 4 | 6 | 2 | 3 | 7 | 256 |

Appendix B: Tables B. 1 - B. 8 shows the material flow and closeness rating for multi-objective FLPs.
Table B. 1
Flow-matrix of $\mathrm{n}=6$

| $\boldsymbol{f}_{i j}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 0 | 4 | 6 | 2 | 4 | 4 |
| $\mathbf{2}$ | 4 | 0 | 4 | 2 | 8 |  |
| $\mathbf{3}$ | 6 | 2 | 2 | 2 | 6 |  |
| $\mathbf{4}$ | 2 | 2 | 2 | 6 | 6 | 2 |
| $\mathbf{5}$ | 4 | 8 | 6 | 2 | 0 | 10 |
| $\mathbf{6}$ | 4 |  |  | 10 | 0 |  |

Table B. 2
Closeness rating of $\mathrm{n}=6$

| $\boldsymbol{c}_{i j}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | 0 | 5 | 3 | 2 | 6 | 4 |
| $\mathbf{2}$ | 5 | 0 | 5 | 2 | 6 | 2 |
| $\mathbf{3}$ | 3 | 5 | 0 | 1 | 2 | 1 |
| $\mathbf{4}$ | 2 | 2 | 1 | 0 | 2 | 2 |
| $\mathbf{5}$ | 6 | 6 | 2 | 2 | 0 | 6 |
| $\mathbf{6}$ | 4 | 2 | 1 | 2 | 6 | 0 |

Table B. 3
Flow matrix n=8

| $\boldsymbol{f}_{i \boldsymbol{j}}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 0 | 6 | 1 | 1 | 8 | 2 | 4 | 4 |
| $\mathbf{2}$ | 6 | 0 | 1 | 2 | 3 | 3 | 6 | 2 |
| $\mathbf{3}$ | 1 | 1 | 0 | 5 | 2 | 3 | 1 | 10 |
| $\mathbf{4}$ | 1 | 2 | 5 | 0 | 2 | 8 | 3 | 3 |
| $\mathbf{5}$ | 8 | 3 | 2 | 2 | 0 | 4 | 10 | 10 |
| $\mathbf{6}$ | 2 | 3 | 3 | 8 | 4 | 0 | 8 | 8 |
| $\mathbf{7}$ | 4 | 6 | 1 | 3 | 10 | 8 | 0 | 2 |
| $\mathbf{8}$ | 4 | 2 | 10 | 3 | 10 | 8 | 2 | 0 |

Table B. 4
Closeness rating $\mathrm{n}=8$

| $\boldsymbol{c}_{i j}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | 0 | 6 | 5 | 5 | 6 | 4 | 5 | 2 |
| $\mathbf{2}$ | 6 | 0 | 3 | 5 | 3 | 2 | 6 | 2 |
| $\mathbf{3}$ | 5 | 3 | 0 | 6 | 3 | 1 | 2 | 2 |
| $\mathbf{4}$ | 5 | 5 | 6 | 0 | 2 | 2 | 3 | 1 |
| $\mathbf{5}$ | 6 | 3 | 3 | 2 | 0 | 5 | 6 | 6 |
| $\mathbf{6}$ | 4 | 2 | 1 | 2 | 5 | 0 | 6 | 6 |
| $\mathbf{7}$ | 5 | 6 | 2 | 3 | 6 | 6 | 0 | 4 |
| $\mathbf{8}$ | 2 | 2 | 2 | 1 | 6 | 6 | 4 | 0 |

Table B. 5
Flow matrix $\mathrm{n}=12$

| $\boldsymbol{f}_{i j}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ | $\mathbf{1 2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 0 | 2 | 4 | 6 | 3 | 5 | 7 | 8 | 6 | 5 | 7 | 1 |
| $\mathbf{2}$ | 2 | 0 | 4 | 3 | 2 | 5 | 1 | 4 | 3 | 7 | 2 | 6 |
| $\mathbf{3}$ | 4 | 4 | 0 | 1 | 8 | 8 | 6 | 9 | 1 | 1 | 10 | 9 |
| $\mathbf{4}$ | 6 | 3 | 1 | 0 | 2 | 10 | 11 | 7 | 10 | 11 | 11 | 12 |
| $\mathbf{5}$ | 3 | 2 | 8 | 2 | 0 | 10 | 20 | 20 | 9 | 19 | 19 | 20 |
| $\mathbf{6}$ | 5 | 5 | 8 | 10 | 10 | 0 | 20 | 18 | 13 | 15 | 14 | 12 |
| $\mathbf{7}$ | 7 | 1 | 6 | 11 | 20 | 20 | 0 | 11 | 13 | 15 | 12 | 16 |
| $\mathbf{8}$ | 8 | 4 | 9 | 7 | 20 | 18 | 11 | 0 | 3 | 14 | 17 | 13 |
| $\mathbf{9}$ | 6 | 3 | 1 | 10 | 9 | 13 | 13 | 3 | 0 | 19 | 21 | 15 |
| $\mathbf{1 0}$ | 5 | 7 | 1 | 11 | 19 | 15 | 15 | 14 | 19 | 0 | 8 | 22 |
| $\mathbf{1 1}$ | 7 | 2 | 10 | 11 | 19 | 14 | 12 | 17 | 21 | 8 | 0 | 13 |
| $\mathbf{1 2}$ | 1 | 6 | 9 | 12 | 20 | 12 | 16 | 13 | 15 | 22 | 13 | 0 |

Table B. 6
Closeness rating $\mathrm{n}=12$

| $\boldsymbol{c}_{i j}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ | $\mathbf{1 2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | 0 | 5 | 4 | 1 | 1 | 1 | 1 | 1 | -1 | 1 | 2 | 2 |
| $\mathbf{2}$ | 5 | 0 | 3 | 1 | 4 | 4 | 1 | 5 | 1 | 3 | 2 | 2 |
| $\mathbf{3}$ | 4 | 3 | 0 | 3 | 1 | 1 | 1 | 1 | -1 | 1 | 2 | 2 |
| $\mathbf{4}$ | 1 | 1 | 3 | 0 | 4 | 3 | 3 | 3 | 1 | 1 | 2 | 2 |
| $\mathbf{5}$ | 1 | 4 | 1 | 4 | 0 | 5 | 4 | 1 | 1 | 1 | 2 | 2 |
| $\mathbf{6}$ | 1 | 4 | 1 | 3 | 5 | 0 | 5 | 1 | 1 | 1 | 2 | 2 |
| $\mathbf{7}$ | 1 | 1 | 1 | 3 | 4 | 5 | 0 | 1 | 1 | 1 | 2 | 2 |
| $\mathbf{8}$ | 1 | 5 | 1 | 3 | 1 | 1 | 1 | 0 | 1 | 4 | 2 | 2 |
| $\mathbf{9}$ | -1 | 1 | -1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 2 | 2 |
| $\mathbf{1 0}$ | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 4 | 1 | 0 | 2 | 2 |
| $\mathbf{1 1}$ | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 |
| $\mathbf{1 2}$ | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 0 |

Table B. 7
Flow matrix $n=15$

| $\boldsymbol{f}_{i j}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ | $\mathbf{1 2}$ | $\mathbf{1 3}$ | $\mathbf{1 4}$ | $\mathbf{1 5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 0 | 4 | 5 | 8 | 12 | 16 | 3 | 7 | 2 | 6 | 8 | 9 | 12 | 17 | 12 |
| $\mathbf{2}$ | 15 | 0 | 6 | 8 | 12 | 0 | 9 | 35 | 4 | 7 | 3 | 6 | 9 | 8 | 12 |
| $\mathbf{3}$ | 15 | 16 | 0 | 9 | 2 | 0 | 15 | 6 | 17 | 6 | 12 | 3 | 5 | 8 | 9 |
| $\mathbf{4}$ | 1 | 25 | 9 | 0 | 2 | 6 | 5 | 8 | 0 | 0 | 0 | 0 | 3 | 2 | 9 |
| $\mathbf{5}$ | 5 | 0 | 15 | 6 | 0 | 2 | 0 | 8 | 0 | 9 | 0 | 6 | 6 | 3 | 2 |
| $\mathbf{6}$ | 8 | 5 | 0 | 6 | 9 | 0 | 5 | 8 | 8 | 0 | 6 | 0 | 3 | 5 | 8 |
| $\mathbf{7}$ | 0 | 3 | 5 | 0 | 8 | 9 | 0 | 9 | 9 | 8 | 7 | 12 | 0 | 13 | 15 |
| $\mathbf{8}$ | 16 | 17 | 0 | 16 | 15 | 11 | 14 | 0 | 1 | 4 | 12 | 2 | 15 | 15 | 13 |
| $\mathbf{9}$ | 17 | 6 | 9 | 11 | 25 | 6 | 12 | 12 | 0 | 5 | 7 | 8 | 9 | 2 | 6 |
| $\mathbf{1 0}$ | 3 | 8 | 9 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 6 | 0 | 7 |
| $\mathbf{1 1}$ | 8 | 9 | 0 | 12 | 15 | 16 | 0 | 0 | 2 | 4 | 0 | 5 | 0 | 5 | 7 |
| $\mathbf{1 2}$ | 6 | 7 | 2 | 8 | 9 | 12 | 15 | 5 | 5 | 0 | 0 | 0 | 7 | 0 | 5 |
| $\mathbf{1 3}$ | 6 | 4 | 3 | 8 | 9 | 0 | 11 | 5 | 15 | 15 | 10 | 10 | 0 | 5 | 10 |
| $\mathbf{1 4}$ | 12 | 6 | 9 | 8 | 10 | 11 | 15 | 5 | 16 | 12 | 10 | 10 | 5 | 0 | 7 |
| $\mathbf{1 5}$ | 2 | 3 | 4 | 8 | 0 | 12 | 15 | 16 | 12 | 0 | 0 | 0 | 0 | 2 | 0 |

Table B. 8
Closeness rating $\mathrm{n}=15$

| $\boldsymbol{c}_{i j}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ | $\mathbf{1 2}$ | $\mathbf{1 3}$ | $\mathbf{1 4}$ | $\mathbf{1 5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 0 | 2 | 0 | 3 | 0 | -1 | 2 | 0 | 3 | 1 | 4 | -1 | 2 | 2 | 0 |
| $\mathbf{2}$ | 2 | 0 | 3 | 1 | 2 | 3 | 2 | 4 | 0 | 3 | 0 | 1 | 1 | 0 | 1 |
| $\mathbf{3}$ | 0 | 3 | 0 | 0 | 3 | 1 | 0 | 2 | -1 | 0 | 4 | 0 | 2 | 1 | 0 |
| $\mathbf{4}$ | 3 | 1 | 0 | 0 | -1 | 0 | 1 | 0 | 4 | 4 | 0 | 1 | 2 | 0 | 2 |
| $\mathbf{5}$ | 0 | 2 | 3 | -1 | 0 | 0 | 1 | 3 | 0 | 2 | 1 | 0 | 1 | 0 | 0 |
| $\mathbf{6}$ | -1 | 3 | 1 | 0 | 0 | 0 | 2 | 0 | 0 | 3 | 0 | 1 | 3 | 3 | 0 |
| $\mathbf{7}$ | 2 | 2 | 0 | 1 | 1 | 2 | 0 | 0 | 0 | 0 | -1 | 2 | 1 | 0 | 1 |
| $\mathbf{8}$ | 0 | 4 | 2 | 0 | 3 | 0 | 0 | 0 | 1 | 4 | 3 | 2 | 3 | 3 | 2 |
| $\mathbf{9}$ | 3 | 0 | -1 | 4 | 0 | 0 | 0 | 1 | 0 | 3 | 0 | 2 | 1 | 0 | 3 |
| $\mathbf{1 0}$ | 1 | 3 | 0 | 4 | 2 | 3 | 0 | 4 | 3 | 0 | 1 | 0 | 1 | 0 | 1 |
| $\mathbf{1 1}$ | 4 | 0 | 4 | 0 | 1 | 0 | -1 | 3 | 0 | 1 | 0 | 3 | 2 | 1 | 1 |
| $\mathbf{1 2}$ | -1 | 1 | 0 | 1 | 0 | 1 | 2 | 2 | 2 | 0 | 3 | 0 | -1 | 1 | 1 |
| $\mathbf{1 3}$ | 2 | 1 | 2 | 2 | 1 | 3 | 1 | 3 | 1 | 1 | 2 | -1 | 0 | 0 | 1 |
| $\mathbf{1 4}$ | 2 | 0 | 1 | 0 | 0 | 3 | 0 | 3 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| $\mathbf{1 5}$ | 0 | 1 | 0 | 2 | 0 | 0 | 1 | 2 | 3 | 1 | 1 | 1 | 1 | 0 | 0 |

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