Multi-objective input signal design for plant friendly identification of process systems

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***Abstract*— System Identification is the process of constructing an accurate and reliable dynamic mathematical model of the system from observed data and available knowledge. The choice of inputs used for perturbing the system is critical in the identification and model building exercise. One of the major objectives of system identification is accurate estimation of the system parameters. Identification of chemical process plants is carried out on running plants in real time. The practitioner would thus prefer a 'plant friendly' input signal. We propose unified multi-objective formulations and solution methods for the input design for two particular cases. The input can be evaluated as a solution to a multi-objective optimization problem.**

I. INTRODUCTION

System Identification is the process of constructing an accurate and reliable dynamic mathematical model of the process or system from observed data and available knowledge. A number of excellent reviews on system identification and input signal design are available [1]–[3]. Such models are commonly used for estimation, prediction and control, fault diagnosis, simulation, operator training etc. It is common practice to perturb the system with specially tailored inputs and the consequential input/output data are used to build the system model. The quality of the model depends strongly on the experiment design and identification and hence the input used for perturbing the system should be carefully selected [1].

A considerable amount of literature exists on statistical experiment design. The input design problem, as applied to a dynamic system for system identification, has received much less attention [4]–[7]. The issue of experiment design with respect to the intended model application, which is often control, has received considerable attention recently [8]–[10] .

System identification, in practice is carried out by perturbing processes or plants in operation. The concept of plant friendly identification has received attention amongst members of the process control and identification community recently [11]–[14]. Techniques for synthesizing multi-harmonic signals with low crest factors which are attractive from a plant friendly perspective have been reported [1], [15].

There has been some recent application of multi-objective optimization based methods to identification and control. [16]–[18]. However, such an approach to input design has not been attempted. We present two illustrative formulations for evaluating the optimal input as a solution to a multi-objective optimization problem.

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II. OBJECTIVES

A. Experiment design

Consider a system, described in a general manner as

$$y(t) = f(u, t, \theta, v) \tag{1}$$

where, $y(t)$, $u(t)$, $v(t)$, θ are the output, input, noise and parameter respectively. It is required to select a design for the input $u(t) \in \Omega_u$ such that a suitable cost function related to the end use of the identified model is minimized. The accuracy of parameter estimates is most conveniently expressed in terms of its bias and the covariance. For input design purposes, it is convenient to assume an unbiased efficient estimator, so that the covariance of the parameter estimates is given by the Cramer-Rao bound, viz., the inverse of the Fisher information matrix, and can be computed irrespective of the choice of the estimator. The Fisher information matrix is defined as

$$M = E\left[\frac{\partial}{\partial\theta}\ln p(y|\theta)\left(\frac{\partial}{\partial\theta}\ln p(y|\theta)\right)^t\right] \tag{2}$$

Some of the commonly used measures of performance are

- A-optimality $\min_U \mathtt{tr}(M^{-1})$, i.e, minimize the average variance of the parameters
- E-optimality $\min_U \lambda_{\max}(M^{-1})$, i.e, minimize the maximum eigenvalue
- D-optimality $\det(M^{-1})$, i.e, minimize the volume of the uncertainty ellipsoid
- G optimality The min-max design problem is to choose the optimal input such that it minimizes the maximum of $\mathtt{cov}y(t, \hat{\theta})$ for all possible input signals.
- Maximize the trace of the information matrix. This is commonly done as direct minimization of the trace of M^{-1} is a nonlinear optimization problem [6].

Though different experiment design criteria can be used for optimal experiment design, it has been observed that a 'good' or optimal design corresponding to one criterion will, in general be deemed 'good' or optimal with respect to other criteria too. However, the choice of the experiment criterion is important as it is possible that inputs designed based on some criteria may not be persistently exciting [4]. Techniques for designing optimal input signals in the time and frequency domain have been published [3], [4], [6].

B. Plant friendly identification

Multi-variable model based control strategies are commonly used in chemical process industries. Plant friendly identification has received the attention of researchers in recent times [11]–[14]. Identification experiments in process industries are carried out on running plants under real time operating conditions. While a persistently rich excitation with high signal to noise ratio is theoretically preferred, operational, safety, environmental and economic considerations have to be taken into account during identification.

- An input requiring aggressive and frequent movement of valves and actuators is not desirable as this can lead to equipment wear and tear.
- Identification experimentation time has to be kept to a minimum so as to minimize off-spec products and consumption of utilities. Tests using the popular Pseudo-Random Binary Signals (PRBS) usually require days to conduct [19].
- Output deviations should be reduced to ensure that the product quality differs as little as possible from the set point and remains within specified permissible limits.

It must be noted that some of these practical considerations may be in conflict with the theoretical requirements of identification [14].

Input friendliness factor Φ_i

For discrete input signals, which are commonly used in identification practice, a quantitative plant friendliness index in terms of the number of changes from one level to another has been proposed [13]. For a deterministic sequence, the input plant friendliness Φ_i can be defined as

$$\Phi_i = 100\left(1 - \frac{n_t}{N-1}\right) \tag{3}$$

where N is the total length and n_t is the total number of switches. For a discrete level stochastic signal, the definition needs to be changed and the friendliness factor is simply defined as the probability of non-transition from one state to another, i.e, the probability that the signal continues to be in the same state.

Although the above definition is intuitively appealing, a drawback is that it might be difficult to represent this quantity in a closed form in an optimization formulation. A closed form definition which retains the same intuitive appeal could be [12]

$$\begin{aligned}\Phi_i &= 1 - \frac{\sum(u_k - u_{k-1})^2}{(N-1)\max(u_k - u_{k-1})^2} \\ &= 1 \quad if\ u_k = u_{k-1} \quad \forall k \le N \end{aligned} \tag{4}$$

For a continuous system, the definition for input friendliness can be modified as follows.

$$\Phi_i = 1 - \frac{\int_0^T \dot{u}^t \dot{u} dt}{T\mathtt{max}(\dot{u}^t \dot{u})} \tag{5}$$

A multi-sine signal is a signal obtained by the addition of a finite number of harmonically related sinusoids and can be represented as

$$x(t) = \sum_{u=1}^{N_u} a_u \cos(2\pi k_u t/T + \alpha_u) \tag{6}$$

where k_u are monotonically increasing harmonic numbers $k_u \in \mathbb{N},\ u = 1, 2, \ldots N_u$. The input friendliness factor is often expressed in terms of the peak factor (Pf) or crest factor (Cf).

$$Pf = \frac{x_{max} - x_{min}}{2\sqrt{2}x_{rms}} \tag{7}$$

$$Cf = \frac{\max|x|}{x_{rms}} \tag{8}$$

where $x_{min}, x_{max}, x_{rms}$ denote the minimum, maximum and rms values of the signal respectively. For a specified amplitude spectrum $\{a_u\}$, it is possible to synthesize an input signal with a low crest factor by using a generalization of Polya's algorithm [15]. The crest factor is thus the ratio of the L_∞ norm and the L_2 norm where the general L_p norm of the function $x(t)$ over the interval $[0, T]$ is defined as

$$L_p(x) = \left[\frac{1}{T}\int_0^T |x(t)|^p dt\right]^{(1/p)} \tag{9}$$

and the L_∞ norm is $\max|x(t)|$.

Output friendliness factor

One possible definition could be the crest factor which has been defined in (8) [14], [15]. However, a major drawback of the reported procedure is that the output is assumed to be given by

$$y(t) = \sum_{u=1}^{N_u} a_u G_u \cos(2\pi k_u t/T + \alpha_u + \phi_u) \tag{10}$$

where, G_u and ϕ_u are the *a priori* amplitude ratio and phase shift evaluated at the frequency $2\pi k_u/T$. While, the above expression correctly describes the steady state output of a Linear Time Invariant (LTI) system subjected to the multi-sine input, it does not capture the system transients. A suitable output friendliness factor Φ_o that takes into account the output variability, the time spent in out of control region, the spectral energies or crest factor needs to be investigated. It must be noted that a minimum crest factor input does not imply a minimum or low crest factor output.

Constraints

The experiment design has to take into account certain constraints that may be imposed on the conditions. Some of these constraints are

- Amplitude constraints on inputs, $a_t \le u(t) \le b_t$, outputs $c_t \le y(t) \le d_t$ or state variables.
- Energy constraints on inputs,

$$\begin{aligned} &\int u^t u\ dt \le b \quad ;\mathtt{u\ continuous} \\ &U^t U \le b \ \ ;\ U^t = [u_1, u_2, \ldots, u_N], \end{aligned} \tag{11}$$

- Total time available for the experiment
- Number of samples
- Physical constraints on actuators, valves
- Process, safety and environmental constraints.

III. Multi-objective optimization

In traditional single objective optimization problems, the aim is to find a globally optimal solution, if it exists. Unlike single objective optimization problems, in optimization with possibly conflicting objectives, there is no unique optimal solution. System and real world design usually involves tradeoffs between different objectives and more than one decision maker. There has been considerable activity in recent times in the field of multi-objective or multi criteria decision making [20]. A fair amount of subjectivity and user influenced decision making is a characteristic feature of multi-objective problems. There are several possible approaches for solving a multi-objective optimization problem.

A. Single weighted cost function

One simple and common approach to multi-objective optimization is to formulate a single weighted objective function from the individual costs J_i.

$$J = \alpha_1 J_1 + \ldots + \alpha_n J_n \tag{12}$$

such that the weights $\alpha_i \geq 0$. This results in the following single optimization problem that can be solved by standard methods of optimization.

$$\text{Minimize}\{\text{J}\} \quad \texttt{s.t} \begin{cases} L_i \leq 0 & \forall i \\ E_j = 0 & \forall j \end{cases} \tag{13}$$

where E_j and L_i are constraints that we wish to impose.

B. Pareto optimal sets

In multi-objective optimization problems, the interaction among different objectives gives rise to a set of solutions, called the Pareto optimal solutions (see Fig. 1). Solutions A, D, B form a Pareto optimal front and no one solution in this set can be said to be better than another in pure quantitative terms. However, solution C is dominated by solution D as solution D is better than C in both objectives. A set is called a global Pareto-optimal set, if no solution in the search space dominates any member in it. The optimization algorithm should attain two goals :- search for the global Pareto-optimal front and maintain population diversity in the optimal front so that no bias towards any particular objective function exists [21].

Formally, in a minimization problem, an objective vector, $z^* = [z_1^*, z_2^*, \ldots, z_k^*]^t$ is Pareto optimal if, for any other vector, z, $z_i \leq z_i^* \; \forall i \leq k$ implies that $z = z^*$. A related concept is that of a weak Pareto optimal solution. In a minimization problem, an objective vector $z^* = [z_1^*, z_2^*, \ldots, z_k^*]^t$ is weakly Pareto optimal, if there does not exist another objective vector $z = [z_1, z_2, \ldots, z_k]^t$ such that $z_i < z_i^* \; \forall i \leq k$ [20]. The set of Pareto optimal solutions is a subset of the set of weakly Pareto optimal solutions.

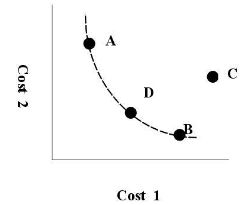

Fig. 1. Pareto optimal sets

The final choice or result of the design problem is based on subjective or higher level knowledge of the user. Thus, it is not necessary to explicitly specify the weighting coefficients *a priori*, unlike the single cost function approach. Under certain conditions, it is possible to describe the Pareto front by solving the weighted objective problem for different choices of the weights [20]. However, the procedure for varying the weights to generate the complete Pareto front is usually not clear.

In the succeeding sections, we present 2 problem formulations:- the first is a single weighted cost function approach for solution of the optimal input in the time domain for a system described by the state-space equations and the second involves description of a Pareto front for the optimal input in the frequency domain.

IV. Single weighted cost function method for time domain input signal

In this section, we consider the problem of synthesizing an optimal input in the time domain for a system described by the following state space model

$$\dot{x}(t) = Fx(t) + Gu(t) \tag{14}$$

$$y(t) = Hx(t) + v(t) \tag{15}$$

where $x(t)$ is an $n \times 1$ state vector, $u(t)$ is a $q \times 1$ input vector, $y(t)$ is a $p \times 1$ output vector, $v(t)$ is a $p \times 1$ measurement noise vector, $F(n{\times}n)$,$G(n{\times}q)$, $H(p{\times}n)$, are state space matrices of the system. $R(p{\times}p)$ is the covariance matrix of the measurement noise.

The objective of experiment design is to minimize the variance of the parameters and maximize the input friendliness and minimize identification time. In this case, we choose to maximize the trace of the information matrix as it leads to a convenient quadratic optimization function.

The information matrix can be expressed as [6]

$$M = \int_0^T (\nabla_\theta x)^t H^t R^{-1} H (\nabla_\theta x) dt \tag{16}$$

where $\nabla_\theta = \frac{\partial}{\partial \theta}$. Thus, in the case when the parameter to be estimated is a scalar, the trace is just the same as the information matrix. In the multi-dimensional case, the problem can be suitably modified by defining the augmented state vector as [5]

$$x_a = [x, \nabla_{\theta_1} x, \ldots, \nabla_{\theta_l} x]^t \tag{17}$$

The state equation can be written as

$$\dot{x_a} = F_a x_a + G_a u \tag{18}$$

where

$$F_a = \left[\begin{array}{c|c|c|c} F & 0 & \dots & 0 \\ \hline \nabla_{\theta_1} F & F & & \\ \hline \vdots & & \ddots & \\ \hline \nabla_{\theta_l} F & & & F \end{array}\right] \tag{19}$$

$$G_a = [G, \nabla_{\theta_1} G, \dots, \nabla_{\theta_l} G]^t \tag{20}$$

In addition, define, H_a, R_a as follows

$$H_a = \begin{bmatrix} 0 & H & & 0 \\ & & \ddots & \vdots \\ & & & 0 \\ 0 & 0 & 0 & H \end{bmatrix} \tag{21}$$

$$R_a^{-1} = \mathtt{diag}(R^{-1}) \tag{22}$$

Then,

$$tr(M) = \int_0^T x_a^t H_a^t R_a^{-1} H_a x_a dt \tag{23}$$

The objectives are

- Maximize $\int_0^T x_a^t H_a^t R_a^{-1} H_a x_a dt$
- Maximize Φ_i as defined in (5)
- Minimize time T

The multi-objective optimization problem is reduced to a single-objective optimization one by minimizing the modified cost function

$$J = -1/2[\int_0^T x_a^t H_a^t R_a^{-1} H_a x_a dt + q'\Phi_i] \tag{24}$$

It must be noted that time is not explicitly incorporated in the above cost function. Instead, the optimization problem is solved for different values of the terminal time. To ensure that the problem is well posed, the input energy is constrained.

$$\int_0^T u^t u dt = E \tag{25}$$

The cost function is modified to account for the input energy constraint as

$$J = -1/2\left[\int_0^T x_a^t H_a^t R_a^{-1} H_a x_a dt + q'\Phi_i - \mu \int_0^T (u^t u - \frac{E}{T}) dt\right] \tag{26}$$

where μ is a constant chosen so that the integral equation is satisfied. The case $q' = 0$ has been solved in literature [5], [6]. The multi-objective optimization problem (corresponding to $q' \neq 0$) can be solved by the Euler Lagrange method of the calculus of variations which results in the following boundary value problem.

$$\frac{d}{dt}\begin{bmatrix} x_a \\ \lambda \\ u \\ \dot{u} \end{bmatrix} = \begin{bmatrix} F_a & 0 & G_a & 0 \\ H_a^t R_a^{-1} H_a & -F_a^t & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & \frac{G_a^t}{q''} & \frac{\mu I}{q''} & 0 \end{bmatrix}\begin{bmatrix} x_a \\ \lambda \\ u \\ \dot{u} \end{bmatrix} \tag{27}$$

where $q'' = q'/(T\mathtt{max}(\dot{u}^t\dot{u}))$ and the boundary conditions are $x_a(0) = x_{a0}$, $\lambda(T) = 0$, $u(0) = u_0$, $\dot{u}(T) = 0$. The above differential equations are integrated to give $u(t)$ using the method of complementary functions. However, calculation of the optimal input requires knowledge of the system parameters F, G, H and noise characteristics R. This apparent paradox in determining optimal inputs has been recognized, but is not well known. A common technique is to design the optimal inputs based on an initial estimate of the model parameters and iteratively carry out design and identification [8]. The focus of the current work is to demonstrate and formulate the multi-objective optimization problem in input design and the optimal input is evaluated using the assumed model parameters.

The following Single Input, Single Output system is considered in order to demonstrate the multi-objective nature of the input design problem.

$$\dot{x}(t) = -ax(t) + u(t) \tag{28}$$

$$y(t) = x(t) + v(t) \tag{29}$$

where a ($a = 0.1$) is the parameter to be estimated and v is a zero mean, normally distributed white noise process with variance $\sigma^2 = 1$. The system is assumed to be at an initial state $x(0) = 0.1$. Optimal input signals are computed for evaluation of the parameter a and are compared in Tables I, II for different values of the terminal time T. The three inputs for $T = 1$ are plotted in Fig. 2. Input I is evaluated by maximizing (M) alone subject to the energy constraint . Input II maximizes the weighted sum of the input friendliness Φ_i and M with the initial value of the input assumed to be the same as Input I. Input III is evaluated by a procedure similar to that of II, except that the initial value of the input $u(0)$ is chosen such that weighted sum of the input friendliness Φ_i and M is maximum over different values of $u(0)$.

Thus, it is clear that it is possible to improve upon the input friendliness by accepting a lower value of the infor-

TABLE I

COMPARISON OF INPUT SIGNALS FOR T=1

	Input I	Input II	Input III
q'	0	1.6	1.6
μ	0.075	0.0843	0.064
$u(o)$	8.2	8.2	6.5
M	1.36	1.34	1.29
Φ_i	0.807	0.836	0.920
$M + q'\Phi_i$	1.36	2.67	2.76
Input Energy	17.2	17.2	17.2
Output Energy	6.43	6.59	6.80

TABLE II
COMPARISON OF INPUT SIGNALS FOR T=1.25

	Input I	Input II	Input III
q'	0	2	2
μ	0.176	0.179	0.166
$u(o)$	7.3	7.3	6.4
M	3.18	3.17	3.16
Φ_i	0.902	0.908	0.93
$M + q'\Phi_i$	3.18	4.99	5.02
Input Energy	17.2	17.2	17.2
Output Energy	9.78	9.90	10.09

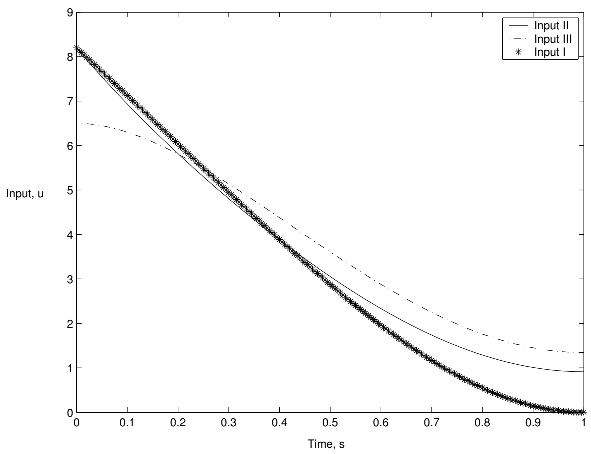

Fig. 2. Comparison of different inputs for T=1 s

mation or equivalently, a higher variance in the parameter estimates. Input I gives the best performance with respect to parameter accuracy amongst all inputs with the same energy. It is also possible to improve the plant friendliness by increasing the experiment time. Thus, there exists trade-offs between the different objectives- input friendliness, parameter variance and time. This is clearly brought out by solving the above multi-objective optimization problem. The standard machinery of optimal input generation as described in [6] does not allow the user this freedom. The use of the weighted cost function approach to generate the Pareto front needs to be investigated.

V. INPUT-OUTPUT CREST FACTOR MINIMIZATION-PARETO OPTIMAL FRONT

In this section, we propose to solve the multi-objective optimization problem by constructing a Pareto optimal front. The use of multi-sine signals as universal, flexible broad band excitation signals has been recommended especially for identification in the frequency domain [1], [15]. A multi-sine signal is a signal obtained by the addition of a finite number of harmonically related sinusoids as defined in (6).The design requirement is to phase the harmonics so that large peaks are avoided. This is achieved by choosing the phases so that the crest factor as defined in (8) is minimized. Another advantage of minimizing the crest factor is that a signal with a low crest factor can result in improved accuracy in identification [15]. Since the L_2 norm is invariant with respect to the phases α_u, minimizing the crest factor is equivalent to minimizing the L_∞ norm. Minimizing the crest factor is not straightforward due to the nonlinear appearance of the phases in the multi-sine and the nondifferentiable nature of the L_∞ norm. One popularly used technique is to sequentially minimize the differentiable L_p norm for increasing values of p, typically powers of 2 such as 4,8,16,32, 64 ... [15]. The above technique has been reported to give better results than other crest factor minimization methods.

The norms of a vector valued function $[x(t), y(t)]^t$ are defined as [15].

$$L_p(x,y) = \left[\frac{1}{2T}\int_0^T (|x(t)|^p + |y(t)|^p)dt\right]^{1/p} \quad (30)$$

$$\begin{aligned} \|(x,y)\|_\infty &= \max\max(|x(t)|, |y(t)|) \\ &= \max(L_\infty(x), L_\infty(y)) \end{aligned} \quad (31)$$

When the function $y(t)$ is the output of a LTI system as in (10), minimizing $\|x,y\|_\infty$ is equivalent to minimizing the maximum of the input and output peak values. As mentioned in Section II-B, this would not capture the dynamic evolution or transients in the output. Other solutions are also possible by using weighting factors, i.e minimizing $\|x/w_x, y/w_y\|_\infty$. By choosing the weights appropriately, we can minimize either the input or output crest factors. It must be noted that a minimum crest factor input signal does not necessarily imply a minimum crest factor output signal. Thus, we are faced with a multi-objective optimization problem of minimizing the input and output crest factors or equivalently $L_\infty(x)$ and $L_\infty(y)$. In fact, the solution obtained by minimizing $\|x/w_x, y/w_y\|_\infty$is weakly Pareto optimal. This can be demonstrated as follows. Assume that the vector $p^* = [\alpha_1^*, \alpha_2^*, \ldots, \alpha_{N_u}^*]^t$ minimizes $\|x/w_x, y/w_y\|_\infty$. If p^* were not weakly Pareto optimal, $\exists$ another vector of phases $p = [\alpha_1, \alpha_2, \ldots, \alpha_{N_u}]^t$ such that

$$\begin{aligned} L_\infty(x(p,t)) &< L_\infty(x(p^*,t)) \\ L_\infty(y(p,t)) &< L_\infty(y(p^*,t)) \end{aligned} \quad (32)$$

Therefore, we have

$$\|x(p)/w_x, y(p)/w_y\|_\infty < \|x(p^*)/w_x, y(p^*)/w_y\|_\infty \quad (33)$$

which contradicts the assumption that p^* is an optimal solution minimizing $\|x/w_x, y/w_y\|_\infty$. Thus, we must have that both inequalities in (32) cannot be true simultaneously. Hence, any p^* that minimizes $\|x/w_x, y/w_y\|_\infty$ is weakly Pareto optimal. It has been reported that the solution of L_p problem is Pareto optimal [20]. Also, since sequential minimization of L_p norms for increasing p converges to the solution obtained by minimization of L_∞ norm [15], [22], it is possible that the above solution is also Pareto optimal. This conjecture needs to be formally investigated. This approach can be extended to Multiple Input Multiple Output systems.

Thus, we have provided a constructive method for generating the weak Pareto optimal front (of which the Pareto front is a subset) when the objectives are to minimize the crest factor of the input and output. It is implicitly assumed that *a priori* knowledge of the dynamic characteristics of the system is available. One idea is to use the model generated by the previous identification cycle [14]. It has been suggested that minor inaccuracies in this knowledge do not affect the crest factor minimization procedure significantly [15]. This however, can be guaranteed only if the model is known within reasonable limits of accuracy. The role of the uncertainty of the model used for output predictions and accuracy of parameter estimates in the final choice of the input signal would be investigated.

VI. Conclusions and Future Work

A multi-objective optimization problem for input design for system identification was formulated and methods of solution outlined and results from a simple case study presented. Further generation of the Pareto front and guidelines for selection of the input from the Pareto front would be investigated.

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