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Multi-objective multi-item solid transportation problem with fuzzy inequality constraints

Dipankar Chakraborty¹, Dipak Kumar Jana^{2*} and Tapan Kumar Roy³

*Correspondence:

dipakjana@gmail.com

²Department of Applied Science,
Haldia Institute of Technology,
Purba Midnapur, Haldia, West
Bengal 721657, India

Full list of author information is
available at the end of the article

Abstract

Zimmermann (Int. J. Gen. Syst. 2:209-215, 1976) first introduced the concept of fuzzy inequality in the field of linear programming problem (LPP). But this concept is hardly used in any real life applications of LPP. So, in this paper, a multi-objective multi-item solid transportation problem (MMSTP) with fuzzy inequality constraints is modeled. Representing different preferences of the decision maker for transportation, three different types of models are formulated and analyzed. Fuzzy inequality solid transportation problem is converted to parameter solid transportation problem by an appropriate choice of flexible index, and then the crisp solid transportation problem is solved by the algorithm (Cao in *Optimal Models and Methods with Fuzzy Quantities*, 2010) for decision values. Fuzzy interactive satisfied method (FISM), global criterion method (GCM) and convex combination method (CCM) are applied to derive optimal compromise solutions for MMSTP by using MatLab and Lingo-11.0. The models are illustrated with numerical examples and some sensitivity analysis is also presented.

Keywords: solid transportation problem; fuzzy inequality; global criterion method; fuzzy interactive satisfied method; convex combination method

1 Introduction

The solid transportation problem (STP) is a generalization of the traditional transportation problem in which three-dimensional properties (supply, demand, convenience) are taken into account in the objective and constraint set instead of source and destination. The necessity of considering this special type of transportation problem arises when heterogeneous conveyances are available for shipment of products. The STP is used in public distribution systems. In many industrial problems, a homogeneous product is delivered from its origin to a destination by means of different modes of transport called conveyances, such as trucks, cargo flights, goods trains, ships, *etc.* These conveyances are taken as the third dimension. A solid transportation problem can be converted to a classical transportation problem by considering only a single type of conveyance.

The STP was first introduced by Haley [1] in 1962. In recent years, there have been numerous papers in this area. Some papers only minimize the total transportation cost. For example, Ojha *et al.* [2], Pramanik *et al.* [3] considered an STP for an item with fixed charge, vehicle cost and price discounted varying charge. However, in practical programming problems, the decision maker (DM) usually needs to optimize several objectives.

Thus, the DM cannot obtain the optimal values of all the objectives simultaneously. The growing literature of STP focuses on multiple objective problems, that is, multiple objective solid transportation problems (MOSTPs). For example, Bit *et al.* [4] used a fuzzy programming approach to solve a MOSTP; Ida *et al.* [5] presented a neural network method to solve a MOSTP; Gao and Liu [6] developed two-phase fuzzy algorithms to solve multi-objective STP; Tao and Xu [7] developed a class of rough multiple objective programming and its application to a solid transportation problem.

If more than one objective is to be optimized in an STP, then the problem is called multi-objective solid transportation problem (MOSTP). If we consider more than one item, then it is called multi-item solid transportation problem. If we consider more than one item and more than one objective at a time in an STP, then it is called a multi-objective multi-item solid transportation problem (MMSTP). The MMSTP model was given by Kundu *et al.* [8]. Recently, Pramanik *et al.* [9] have developed a multi-objective STP in a fuzzy random environment.

Nowadays, in a very often changing market, the business of a single item does not pay much profit to a retailer. For this reason, almost all businessmen in the fields of transportation (Sancak and Salman [10]) do the business of several items. Generally, in all the cases of STP (multi-objective, multi-item and multi-objective multi-item ones), the inequality has been considered as a general inequality. But we can consider this inequality in the fuzzy environment named fuzzy inequality [11, 12]. Fuzzy inequality means it will essentially satisfy that inequality condition. Flexible index is used (Cao [13]) to convert it into the general inequality, so that it will give you a chance to choose the appropriate decision value. Two algorithms were given by Cao [13] to find the decision value. We have taken one of them to find the decision value. That decision value will give us a more general optimal solution and an optimal value to minimize the objectives.

The following developments are made in the formulation and solution of MMSTP models:

- Various types of examples have been used to illustrate the single-objective fuzzy inequality constraints.
- MMSTP has been solved in a fuzzy inequality constraint environment.
- Three different soft computing techniques FISM, GCM and CCM have been used to make the comparison between optimal solutions in multi-objective problems.
- Two different soft-computing tools (MATLAB and LINGO-11.0) have been used to solve the examples.

The rest of this paper is organized as follows. In Section 2, we recall some preliminary knowledge about fuzzy inequality constraint linear programming. Section 3 provides the notation and assumption used throughout this paper. In Section 4, we formulate multi-objective multi-item LPP with fuzzy inequality constraint and also provides general information about the fuzzy interactive satisfied method, global criterion method and convex combination method. MMSTP with fuzzy inequality constraints and its two special cases are developed in Section 5. In Section 6 we discuss the solution procedure to those models using the fuzzy interactive satisfied method. A numerical example is solved, and results obtained using the mentioned techniques are compared in Section 7. Section 8 summarizes the paper and also discusses the scope of future work.

2 Preliminaries about fuzzy inequality constraint linear programming

Let us consider the fuzzy inequality constraint linear programming (FICP)

$$\begin{aligned} \max z &= cx \\ \widetilde{LP} \quad \text{s.t. } Ax &\lesssim b, \\ x &\geq 0, \end{aligned} \tag{1}$$

its corresponding parameter linear programming is given by (Cao [13])

$$\begin{aligned} \max z &= cx \\ LP_\alpha \quad \text{s.t. } Ax &\leq b + (1 - \alpha)d, \\ x &\geq 0, \end{aligned} \tag{2}$$

where $\alpha \in [0, 1]$ and $d \geq 0$. In the given discussion, we will use x_α as an optimal solution, B_α denotes an optimal basis and z_α denotes an optimal value of linear programming (LP_α).

Definition 2.1 [13] Let B be one of the optimal basis matrices of (LP_α). If an interval $[\alpha_1, \alpha_2]$ exists, satisfying that B is an optimal basis matrix of (LP_α) ($\forall \alpha \in [\alpha_1, \alpha_2]$) while B is not an optimal basis matrix for each $\alpha \notin [\alpha_1, \alpha_2]$, we call α_1 and α_2 critical values of (LP_α) and $[\alpha_1, \alpha_2]$ a characteristic interval.

Theorem 1 (LP_α) has a finite characteristic interval on the interval $[0, 1]$.

Theorem 2 Let B be an optimal basis matrix of (LP_α) on a characteristic interval $[\alpha_1, \alpha_2]$. If $(B^{-1}b)_i \neq 0$ ($1 \leq i \leq m$), then

$$\begin{aligned} \alpha_1 &= \max \left[\frac{[B^{-1}(b+d)]_i}{[B^{-1}b]_i}, 0 \mid (B^{-1}d)_i < 0 \ (1 \leq i \leq m) \right], \\ \alpha_2 &= \min \left[\frac{[B^{-1}(b+d)]_i}{[B^{-1}b]_i}, 0 \mid (B^{-1}d)_i > 0 \ (1 \leq i \leq m) \right] \end{aligned}$$

are derived, where $(B^{-1}(b+d))_i$ and $(B^{-1}d)_i$ are the i th components of $B^{-1}(b+d)$ and $B^{-1}d$, respectively.

Property 2.1 Let B be an optimal matrix of (LP_α) on the characteristic interval $[\alpha_i, \alpha_j]$. Then $x_\alpha = B^{-1}(b + (1 - \alpha)d)$ ($\alpha_i \leq \alpha \leq \alpha_j$) is a linear vector function about variable α . The optimal value function $z_\alpha = C_B B^{-1}(b + (1 - \alpha)d)$ is a linear function about variable α and decreases with the increase of variable α .

Property 2.2 The optimal value of function z_α of (LP_α) continues on the interval $[0, 1]$.

2.1 Algorithm for fuzzy inequality constraint linear programming

Let z_1 be an optimal value of (LP_1) and z_0 be an optimal value of (LP_0), $d_0 = z_0 - z_1 > 0$. Based on the above conclusion, the algorithm (Cao [13]) of fuzzy linear programming is given as follows.

Step 1: Let the optimal solutions of (LP_0) and (LP_1) be x_0 and x_1 , the optimal values of (LP_0) and (LP_1) be z_0 and z_1 , and the optimal basis matrix of (LP_0) be B_0 .

Step 2: Solve $[B_0^{-1}(b + (1 - \alpha)d)]_i = 0$.

Assume the solution as

$$\alpha_1, \alpha_2, \dots, \alpha_{n-1} \quad (0 < \alpha_1 < \alpha_2 < \dots < \alpha_{n-1} < 1).$$

Let $\alpha_0 = 1, \alpha_n = 1, \alpha = \alpha_1, k = 1$.

Step 3: Solve (LP_α) .

Let the optimal value be Z_α . If $Z_\alpha \leq Z_1 + d_0\alpha$, turn to Step 4, otherwise let $k = k + 1, \alpha = \alpha_k$, turn to Step 3.

Step 4: Solve the optimal decision

$$\alpha^* = \frac{Z_1\alpha_k - Z_1\alpha_{k-1} - Z_{\alpha_{k-1}}\alpha_k + Z_{\alpha_k}\alpha_{k-1}}{Z_{\alpha_k} - Z_{\alpha_{k-1}} - \alpha_k d_0 + \alpha_{k-1} d_0}.$$

Step 5: Solve linear programming (LP_{α^*}) , and we can obtain an optimal solution x_{α^*} and an optimal value z_{α^*} .

Example 2.1 Calculate

$$\max 4x_1 + 3x_2 + 3x_3$$

subject to

$$2x_1 + 3x_2 + 2x_3 \lesssim 65,$$

$$3x_1 + x_2 + x_3 \lesssim 61,$$

$$\widetilde{LP} \quad x_2 + x_3 \lesssim 24, \tag{3}$$

$$x_2 \lesssim 14,$$

$$x_1 + x_3 \lesssim 10,$$

$$x_1, x_2, x_3 \geq 0,$$

where $d = (0, 0, 0, 0, 7)$.

The corresponding parametric linear programming problem of the above \widetilde{LP} is presented as follows:

$$\max 4x_1 + 3x_2 + 3x_3$$

subject to

$$2x_1 + 3x_2 + 2x_3 \leq 65,$$

$$3x_1 + x_2 + x_3 \leq 61,$$

$$LP_\alpha \quad x_2 + x_3 \leq 24, \tag{4}$$

$$x_2 \leq 14,$$

$$x_1 + x_3 \leq 10 + 7(1 - \alpha),$$

$$0 \leq \alpha \leq 1, \quad x_1, x_2, x_3 \geq 0.$$

Now, using algorithm given in Section 2.1, we obtain $Z_0 = 98.833$, $Z_1 = 82$ and $d_0 = 16.833$ by calculating (LP_0) and (LP_1) . The inverse of the optimal matrix in (LP_0) is

$$B_0^{-1} = \begin{pmatrix} 1.667 & -0.5 & 0 & 0 & 1.1667 \\ 0.333 & 0 & 0 & 0 & -0.6667 \\ -0.5 & 0.5 & 1 & 0 & -0.5 \\ -0.333 & 0 & 0 & 1 & 0.6667 \\ -0.1667 & 0.5 & 0 & 0 & -0.1667 \end{pmatrix}.$$

Now, calculating the equations $[B_0^{-1}(65, 61, 24, 14, 10 - 7\alpha)]_i = 0$ ($i = 1, 2, \dots, 5$), respectively, we obtain $\alpha_1 = 0.0204$, $\alpha_2 = 0.7857$, assume $\alpha_0 = 0$ and $\alpha_3 = 1$.

Now solving (LP_{α_1}) we get the optimal solution as $x_1 = 16.8571$, $x_2 = 10.4286$ and $x_3 = 0$ and the optimal value as $Z_{\alpha_1} = Z_{0.0204} = 98.714$.

Since $Z_{0.0204} = 98.714 > Z_1 + 0.0204 * 16.833 = 82.3434$, we must continue to solve the linear programming (LP_{α_2}) . By solving (LP_{α_2}) , we obtain the optimal solution as $x_1 = 11.5$, $x_2 = 14$ and $x_3 = 0$ and the optimal value as $Z_{\alpha_2} = Z_{0.7857} = 88$.

Now $Z_{0.7857} = 88 < Z_1 + 0.7857 * 16.833 = 95.2257$, so we stop here and calculate optimal decision α^* .

Now

$$\begin{aligned} \alpha^* &= \frac{Z_1\alpha_2 - Z_1\alpha_1 - Z_{\alpha_1}\alpha_2 + Z_{\alpha_2}\alpha_1}{Z_{\alpha_2} - Z_{\alpha_1} - \alpha_2d_0 + \alpha_1d_0} \\ &= 0.5514. \end{aligned}$$

Now calculating $(LP_{0.5514})$, we obtain the optimal solution as $x_1 = 13.1405$, $x_2 = 12.9063$ and $x_3 = 0$ and the optimal value as $Z_{0.5514}^* = 91.2811$.

3 Notations and assumptions

3.1 Notations

In this solid transportation problem, the following notations are used:

- (i) M = number of sources of the transportation problem.
- (ii) N = number of destinations of the transportation problem.
- (iii) K = number of conveyances, *i.e.*, different modes of transportation.
- (iv) a_i^p = amount of product available at i th origin for p th item.
- (v) b_j^p = demand at j th destination of p th item.
- (vi) e_k = conveyances of the transportation problem.
- (vii) T = number of items.
- (viii) x_{ijk}^p = the amount to be transported from i th origin to j th destination by means of k th conveyance of p th item (decision variables).
- (ix) C_{ijk}^{tp} = per unit transportation cost from i th origin to j th destination by k th conveyance of p th item and t th objective.

3.2 Assumptions

In this solid transportation problem, the following assumptions are made.

- (i) Homogeneous product should be transported from sources to destinations.
- (ii) During transportation no items are damaged, *i.e.*, the amount of received items in destination is the same as the one sent from sources.

4 Multi-objective multi-item LPP with fuzzy inequality constraint

Consider the following multi-objective linear programming problems with fuzzy constraint:

$$\begin{cases} \min_x C_l^{(p)} x^{(p)}, & l = 1, 2, \dots, k \text{ and } p = 1, 2, \dots, T \\ \text{s.t. } \begin{cases} A_i^{(p)}(x^{(p)}) \lesssim B_i^{(p)}, & i = 1, 2, \dots, r \text{ and } \forall p, \\ x^{(p)} \geq 0 & \forall p, \end{cases} \end{cases} \quad (5)$$

where x is an n -dimensional decision variable column vector.

Its corresponding parametric linear programming is given by

$$\begin{cases} \min_x C_l^{(p)} x^{(p)}, & l = 1, 2, \dots, k \text{ and } p = 1, 2, \dots, T \\ \text{s.t. } \begin{cases} A_i^{(p)}(x^{(p)}) \leq B_i^{(p)} + V_i, & i = 1, 2, \dots, r \text{ and } \forall p, \\ x^{(p)} \geq 0 & \forall p, \end{cases} \end{cases} \quad (6)$$

where V_i ($1 \leq i \leq r$) is a flexible index by an appropriate choice.

Now the solution methodology of a multi-objective decision making problem by converting into a single-objective problem is discussed as follows.

4.1 Fuzzy interactive satisfied method

We introduce the interactive fuzzy satisfied method (FISM) proposed by Sakawa [14], Xu and Zhou [15]. We consider the following multi-objective decision-making model:

$$\begin{cases} \max [G_i(x), i = 1, 2, \dots, M] \\ \text{s.t. } x \in X. \end{cases} \quad (7)$$

The objective function of equation (7) is to maximize $G_i(x)$, so for each objective we introduce the fuzzy objective 'G_i(x) approximately more than some value', and the membership function is

$$\mu_i(G_i(x)) = \begin{cases} 1 & \text{for } G_i(x) > G_i^1, \\ 1 - \frac{G_i(x) - G_i^0}{G_i^1 - G_i^0} & \text{for } G_i^0 \leq G_i(x) \leq G_i^1, \\ 0 & \text{for } G_i(x) < G_i^0. \end{cases} \quad (8)$$

In equation (8), the membership is 1 and 0 when the values of $G_i(x)$ are G_i^1 and G_i^0 , respectively,

$$G_i^1 = \max_{x \in X} G_i(x), \quad G_i^0 = \min_{x \in X} G_i(x), \quad i = 1, 2, \dots, M. \quad (9)$$

For model $\min_{x \in X} G_i(x)$, its optimal solution should be gotten at the boundary of the convex set X . If there exists no solution of $\max_{x \in X} G_i(x)$ or $\min_{x \in X} G_i(x)$, or $G_i^1 = \infty$, $G_i^0 = -\infty$, the decision maker may set the value of G_i^1, G_i^0 subjectively. Hence, equation (9) could be transformed into the following form:

$$\begin{cases} \min [\mu_1(G_1(x)), \mu_2(G_2(x)), \dots, \mu_m(G_m(x))] \\ \text{s.t. } x \in X. \end{cases} \quad (10)$$

For each objective function $\mu(G_i(x))$, let the decision maker give the reference value of membership function $\bar{\mu}_i$ to reflect the ideal value of membership function. Through solving the minmax problem (10), we obtain an efficient solution of equation (7) as follows:

$$\begin{cases} \min \max_{i=1,2,\dots,M} [\bar{\mu}_i - \mu_i(G_i(x))] \\ \text{s.t. } x \in X. \end{cases} \quad (11)$$

Equation (11) is equivalent to

$$\begin{cases} \min \lambda \\ \text{s.t. } \begin{cases} \bar{\mu}_i - \mu_i(G_i(x)) \leq \lambda, & j = 1, 2, \dots, p, \\ 0 \leq \lambda \leq 1, & x \in X. \end{cases} \end{cases} \quad (12)$$

4.2 Global criteria method

The global criteria method gives a compromise solution for a multi-objective problem. Actually this method is a way of achieving compromise in minimizing the sum in derivations of the ideal solutions from the respective objective functions. The solution procedure is as follows.

Step-I: Solve the multi-objective problem using each time only one objective f_t ($t = 1, 2, \dots, R$) ignoring all the other objectives.

Step-II: From the results of Step-I, determine the ideal objective vector, say $(f_1^{\min}, f_2^{\min}, \dots, f_R^{\min})$ and the corresponding values of $(f_1^{\max}, f_2^{\max}, \dots, f_R^{\max})$.

Step-III: Formulate the following auxiliary problem:

$$\begin{aligned} &\min G(x) \\ &\text{subject to } g_j(x) \leq 0, \quad j = 1, 2, \dots, M, \\ &x \geq 0, \end{aligned}$$

where $G(x) = \min\{\sum_{t=1}^R (\frac{f_t(x)-f_t^{\min}}{f_t^{\max}-f_t^{\min}})^q\}^{\frac{1}{q}}$ or $G(x) = \min\{\sum_{t=1}^R (\frac{f_t(x)-f_t^{\min}}{f_t^{\max}-f_t^{\min}})^q\}^{\frac{1}{q}}$, where $1 \leq q < \infty$. A usual value of q is 2. This method is then called global criteria method in L_2 norms.

4.3 Convex combination method

We consider the following multi-objective model:

$$\begin{cases} \min [f_i(x), i = 1, 2, \dots, M] \\ \text{s.t. } g_j \geq 0, \quad j = 1, 2, \dots, N, \\ x \in X. \end{cases} \quad (13)$$

Then, by the convex combination method, we transfer the above problem into the following form:

$$\begin{cases} \min \sum_{i=1}^M w_i f_i(x), \quad \text{where } \sum_{i=1}^M w_i = 1, 0 < w_i < 1 \\ \text{s.t. } g_j \geq 0, \quad j = 1, 2, \dots, N, \\ x \in X. \end{cases} \quad (14)$$

Corresponding x and $f_i(x)$ is solution.

5 Formulation of different models of STP

5.1 Model-1: multi-objective multi-item STP

Let p ($p = 1, 2, \dots, T$) items be transported from M origins (or sources) a_i ($i = 1, 2, \dots, M$), N destinations (*i.e.*, demands) b_j ($j = 1, 2, \dots, N$) and K conveyances e_k ($k = 1, 2, \dots, K$). K conveyances, *i.e.*, different modes of transport, may be trucks, cargo flights, goods trains, ships, *etc.* Let $a_i^{(p)}$ be the product available at i th origin for items p ($p = 1, 2, \dots, T$), $b_j^{(p)}$ be the demand at j th destination for items p ($p = 1, 2, \dots, T$), and let e_k represent the amount of product which can be carried by k th conveyance. The variable $x_{ijk}^{(p)}$ represents the unknown quantity to be transported from origin $a_i^{(p)}$ to destination $b_j^{(p)}$ by means of k th conveyance for item $p = 1, 2, \dots, T$. Then we propose the mathematical model for the fuzzy inequality constraint. Single-objective and p ($p = 1, 2, \dots, T$)-item problem is to minimize the total transportation cost as follows:

$$f_t^{(p)} = \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K c_{ijk}^{(tp)} x_{ijk}^{(p)} \quad \forall p \text{ and } \forall t. \quad (15)$$

From the discussion above, we develop mathematical formulations of the objectives as follows:

$$\min f_t = \sum_{p=1}^T \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K c_{ijk}^{(tp)} x_{ijk}^{(p)} \quad \forall t. \quad (16)$$

As mentioned by Haley [1], the constraints are divided into three types: source constraint, destination constraint and conveyance capacity constraint. In the fuzzy environment, the quantity from a source is essentially less than equal to the supply capacity of products for different items, that is,

$$\sum_{j=1}^N \sum_{k=1}^K x_{ijk}^{(p)} \lesssim a_i^{(p)}, \quad i = 1, 2, 3, \dots, M \quad \forall p. \quad (17)$$

In the fuzzy environment, the quantity of product transported to a destination is essentially greater than equal to its demand for different items, that is,

$$\sum_{i=1}^M \sum_{k=1}^K x_{ijk}^{(p)} \gtrsim b_j^{(p)}, \quad j = 1, 2, 3, \dots, N \quad \forall p. \quad (18)$$

In the fuzzy environment, the transportation quantity of conveyance is essentially less than equal to its capacity, that is,

$$\sum_{p=1}^T \sum_{i=1}^M \sum_{j=1}^N x_{ijk}^{(p)} \lesssim e_k, \quad k = 1, 2, 3, \dots, K. \quad (19)$$

It is natural to require the nonnegativity of decision variable $x_{ijk}^{(p)}$, that is,

$$x_{ijk}^{(p)} \geq 0 \quad \forall i, j, k \text{ and } \forall p. \quad (20)$$

It is noted that the decision maker (DM) and the modeling analyst are often different individuals. In the transportation problem, the DM is the manager of transport enterprise,

while the modeling analyst may be an expert in transportation problems or a researcher in the enterprise. With the complexity of feasible region, the DM may give an appropriately large region so that all the feasible solutions are included in it. Hence, the above MMSTP with fuzzy inequality constraint can be written as

$$\begin{aligned}
 \min f_t &= \min \sum_{p=1}^T \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K c_{ijk}^{(tp)} x_{ijk}^{(p)}, \quad t = 1, 2, \dots, R \\
 \text{s.t.} \quad &\sum_{j=1}^N \sum_{k=1}^K x_{ijk}^{(p)} \lesssim a_i^{(p)}, \quad i = 1, 2, 3, \dots, M \quad \forall p, \\
 &\sum_{i=1}^M \sum_{k=1}^K x_{ijk}^{(p)} \gtrsim b_j^{(p)}, \quad j = 1, 2, 3, \dots, N \quad \forall p, \\
 &\sum_{p=1}^T \sum_{i=1}^M \sum_{j=1}^N x_{ijk}^{(p)} \lesssim e_k, \quad k = 1, 2, 3, \dots, K, \\
 &x_{ijk}^{(p)} \geq 0 \quad \forall i, j, k,
 \end{aligned} \tag{21}$$

where \lesssim means ‘essentially smaller than equal to’ and \gtrsim means ‘essentially greater than equal to’.

5.2 Model-2: multi-objective single-item STP

We consider a multi-objective single-item solid transportation problem with fuzzy inequality constraint. Then the model may be written as

$$\begin{aligned}
 \min f_t &= \min \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K c_{ijk}^{tp} x_{ijk} \\
 \text{s.t.} \quad &\sum_{j=1}^N \sum_{k=1}^K x_{ijk} \lesssim a_i, \quad i = 1, 2, 3, \dots, M, \\
 &\sum_{i=1}^M \sum_{k=1}^K x_{ijk} \gtrsim b_j, \quad j = 1, 2, 3, \dots, N, \\
 &\sum_{i=1}^M \sum_{j=1}^N x_{ijk} \lesssim e_k, \quad k = 1, 2, 3, \dots, K, \\
 &x_{ijk} \geq 0 \quad \forall i, j, k.
 \end{aligned} \tag{22}$$

5.3 Model-3: single-objective multi-item STP

We consider a single-objective multi-item solid transportation problem with fuzzy inequality constraint. Then the model may be written as

$$\begin{aligned}
 \min f &= \min \sum_{p=1}^T \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K c_{ijk}^{(p)} x_{ijk}^{(p)} \\
 \text{s.t.} \quad &\sum_{j=1}^N \sum_{k=1}^K x_{ijk}^{(p)} \lesssim a_i^{(p)}, \quad i = 1, 2, 3, \dots, M \quad \forall p,
 \end{aligned}$$

$$\sum_{i=1}^M \sum_{k=1}^K x_{ijk}^{(p)} \gtrsim b_j^{(p)}, \quad j = 1, 2, 3, \dots, N \quad \forall p, \tag{23}$$

$$\sum_{p=1}^T \sum_{i=1}^M \sum_{j=1}^N x_{ijk}^{(p)} \lesssim e_k, \quad k = 1, 2, 3, \dots, K,$$

$$x_{ijk}^{(p)} \geq 0 \quad \forall i, j, k.$$

6 Solution of proposed models

6.1 Model-1

Let us consider that p different items are to be transported from i th origin to j th destination by means of k th conveyance. Here we have considered a two-objective function. Let $\max_{x \in X} f_1 = f_1^U$, $\min_{x \in X} f_1 = f_1^L$, $\max_{x \in X} f_2 = f_2^U$ and $\min_{x \in X} f_2 = f_2^L$. Then the membership functions of f_1 and f_2 are given by

$$\mu_1(f_1(x)) = \begin{cases} 0 & \text{for } f_1(x) > f_1^U, \\ \frac{f_1^U - f_1(x)}{f_1^U - f_1^L} & \text{for } f_1^L < f_1(x) < f_1^U, \\ 1 & \text{for } f_1(x) < f_1^L, \end{cases}$$

$$\mu_2(f_2(x)) = \begin{cases} 0 & \text{for } f_2(x) > f_2^U, \\ \frac{f_2^U - f_2(x)}{f_2^U - f_2^L} & \text{for } f_2^L < f_2(x) < f_2^U, \\ 1 & \text{for } f_2(x) < f_2^L. \end{cases}$$

Now, using FISM in Section 4.1, we present the equivalent crisp linear programming of (21) as follows:

$$\left\{ \begin{array}{l} \min_x \lambda \\ \left\{ \begin{array}{l} f_1 \leq f_1^U - (\mu_1 - \lambda)(f_1^U - f_1^L), \\ f_2 \leq f_2^U - (\mu_2 - \lambda)(f_2^U - f_2^L), \\ \sum_{j=1}^N \sum_{k=1}^K x_{ijk}^{(p)} \leq a_i^{(p)} + (1 - \alpha)d_i^{(p)}, \quad i = 1, 2, 3, \dots, M, \\ \sum_{i=1}^M \sum_{k=1}^K x_{ijk}^{(p)} \geq b_j^{(p)} + (1 - \alpha)v_j^{(p)}, \quad j = 1, 2, 3, \dots, N, \\ \sum_{p=1}^T \sum_{i=1}^M \sum_{j=1}^N x_{ijk}^{(p)} \leq e_k + (1 - \alpha)u_k, \quad k = 1, 2, 3, \dots, K, \\ 0 \leq \alpha \leq 1, \\ x_{ijk}^{(p)} \geq 0 \quad \forall i, j, k \text{ and } \forall p. \end{array} \right. \end{array} \right.$$

6.2 Model-2

Let $\max_{x \in X} f_1 = f_1^U$, $\min_{x \in X} f_1 = f_1^L$, $\max_{x \in X} f_2 = f_2^U$ and $\min_{x \in X} f_2 = f_2^L$. Then the membership functions of f_1 and f_2 are given by

$$\mu_1(f_1(x)) = \begin{cases} 0 & \text{for } f_1(x) > f_1^U, \\ \frac{f_1^U - f_1(x)}{f_1^U - f_1^L} & \text{for } f_1^L < f_1(x) < f_1^U, \\ 1 & \text{for } f_1(x) < f_1^L, \end{cases}$$

$$\mu_2(f_2(x)) = \begin{cases} 0 & \text{for } f_2(x) > f_2^U, \\ \frac{f_2^U - f_2(x)}{f_2^U - f_2^L} & \text{for } f_2^L < f_2(x) < f_2^U, \\ 1 & \text{for } f_2(x) < f_2^L. \end{cases}$$

Now, using FISM in Section 4.1, we present the equivalent crisp linear programming of (22) as follows:

$$\left\{ \begin{array}{l} \min_x \lambda \\ \text{s.t.} \left\{ \begin{array}{l} f_1 \leq f_1^U - (\mu_1 - \lambda)(f_1^U - f_1^L), \\ f_2 \leq f_2^U - (\mu_2 - \lambda)(f_2^U - f_2^L), \\ \sum_{j=1}^N \sum_{k=1}^K x_{ijk} \leq a_i + (1 - \alpha)d_i, \quad i = 1, 2, 3, \dots, M, \\ \sum_{i=1}^M \sum_{k=1}^K x_{ijk} \geq b_j + (1 - \alpha)v_j, \quad j = 1, 2, 3, \dots, N \\ \sum_{i=1}^M \sum_{j=1}^N x_{ijk}^{(p)} \leq e_k + (1 - \alpha)u_k, \quad k = 1, 2, 3, \dots, K, \\ 0 \leq \alpha \leq 1, \\ x_{ijk}^{(p)} \geq 0 \quad \forall i, j, k. \end{array} \right. \end{array} \right.$$

6.3 Model-3

Now corresponding parametric linear programming of equation (23) is presented as follows:

$$\begin{aligned} \min \quad & \sum_{p=1}^T \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^K c_{ijk}^{(p)} x_{ijk}^{(p)} \\ \text{s.t.} \quad & \sum_{j=1}^N \sum_{k=1}^K x_{ijk}^{(p)} \leq a_i^{(p)} + (1 - \alpha)d_i^{(p)}, \quad i = 1, 2, 3, \dots, M \text{ and } \forall p, \\ & \sum_{i=1}^M \sum_{k=1}^K x_{ijk}^{(p)} \geq b_j^{(p)} + (1 - \alpha)v_j^{(p)}, \quad j = 1, 2, 3, \dots, N \text{ and } \forall p, \\ & \sum_{p=1}^T \sum_{i=1}^M \sum_{j=1}^N x_{ijk}^{(p)} \leq e_k + (1 - \alpha)u_k, \quad k = 1, 2, 3, \dots, K, \\ & 0 \leq \alpha \leq 1, \quad x_{ijk} \geq 0 \quad \forall i, j, k, \end{aligned} \tag{24}$$

where $d_i^{(p)}$ for $i = 1, 2, 3, \dots, M$, $v_j^{(p)}$ for $j = 1, 2, 3, \dots, N$ and u_k for $k = 1, 2, 3, \dots, K$ are flexible index values $\forall p$.

7 Numerical experiment

7.1 Input data for Model-1 and Model-3

Let us consider a multi-objective multi-item solid transportation problem with two types of items (*i.e.*, $T = 2$), three origins (*i.e.*, $M = 3$), two destinations (*i.e.*, $N = 2$) and two types of conveyances (*i.e.*, $K = 2$). The parameters are given as follows.

Transportation cost for 1st objective and 1st item [$c_{ijk}^{(11)}$]

$$\begin{bmatrix} c_{111}^{(11)} & c_{112}^{(11)} & c_{121}^{(11)} \\ c_{122}^{(11)} & c_{131}^{(11)} & c_{132}^{(11)} \\ c_{211}^{(11)} & c_{212}^{(11)} & c_{221}^{(11)} \\ c_{222}^{(11)} & c_{231}^{(11)} & c_{232}^{(11)} \end{bmatrix} = \begin{bmatrix} 11 & 10 & 11 \\ 13 & 11 & 10 \\ 10 & 11 & 13 \\ 14 & 10 & 12 \end{bmatrix}.$$

Transportation cost for 1st objective and 2nd item [$c_{ijk}^{(12)}$]

$$\begin{bmatrix} c_{111}^{(12)} & c_{112}^{(12)} & c_{121}^{(12)} \\ c_{122}^{(12)} & c_{131}^{(12)} & c_{132}^{(12)} \\ c_{211}^{(12)} & c_{212}^{(12)} & c_{221}^{(12)} \\ c_{222}^{(12)} & c_{231}^{(12)} & c_{232}^{(12)} \end{bmatrix} = \begin{bmatrix} 18 & 11 & 12 \\ 12 & 13 & 15 \\ 12 & 11 & 13 \\ 14 & 15 & 12 \end{bmatrix}.$$

Transportation cost for 2nd objective and 1st item [$c_{ijk}^{(11)}$]

$$\begin{bmatrix} c_{111}^{(21)} & c_{112}^{(21)} & c_{121}^{(21)} \\ c_{122}^{(21)} & c_{131}^{(21)} & c_{132}^{(21)} \\ c_{211}^{(21)} & c_{212}^{(21)} & c_{221}^{(21)} \\ c_{222}^{(21)} & c_{231}^{(21)} & c_{232}^{(21)} \end{bmatrix} = \begin{bmatrix} 16 & 52 & 32 \\ 21 & 14 & 26 \\ 25 & 35 & 25 \\ 11 & 24 & 13 \end{bmatrix}.$$

Transportation cost for 2nd objective and 2nd item [$c_{ijk}^{(12)}$]

$$\begin{bmatrix} c_{111}^{(22)} & c_{112}^{(22)} & c_{121}^{(22)} \\ c_{122}^{(22)} & c_{131}^{(22)} & c_{132}^{(22)} \\ c_{211}^{(22)} & c_{212}^{(22)} & c_{221}^{(22)} \\ c_{222}^{(22)} & c_{231}^{(22)} & c_{232}^{(22)} \end{bmatrix} = \begin{bmatrix} 22 & 19 & 15 \\ 18 & 19 & 13 \\ 22 & 14 & 12 \\ 11 & 8 & 15 \end{bmatrix}.$$

Amount of items available at origin [a_i^p]

$$\begin{bmatrix} a_1^{(1)} & a_2^{(1)} \\ a_1^{(2)} & a_2^{(2)} \end{bmatrix} = \begin{bmatrix} 35 & 30 \\ 28 & 30 \end{bmatrix}.$$

The demand amount of items at destination [$b_j^{(p)}$]

$$\begin{bmatrix} b_1^{(1)} & b_2^{(1)} & b_3^{(1)} \\ b_1^{(2)} & b_2^{(2)} & b_3^{(1)} \end{bmatrix} = \begin{bmatrix} 9 & 7 & 8 \\ 7 & 7 & 6 \end{bmatrix}.$$

Amount of items transported by conveyances [c_k]

$$[e_1 \quad e_2] = [25 \quad 26].$$

Optimum result for Model-1

With the above input data, f_1 and f_2 are calculated using GRG, and we get

$$f_1^U = 691.5, \quad f_1^L = 602.5, \quad f_2^U = 1,106, \quad f_2^L = 670.$$

So, we can get the membership functions of f_1 and f_2 (Figure 1) as follows:

$$\mu_1(f_1(x)) = \begin{cases} 0 & \text{for } f_1(x) > 691.5, \\ \frac{691.5 - f_1(x)}{691.5 - 602.5} & \text{for } 602.5 < f_1(x) < 691.5, \\ 1 & \text{for } f_1(x) < 602.5, \end{cases}$$

$$\mu_2(f_2(x)) = \begin{cases} 0 & \text{for } f_2(x) > 1,106, \\ \frac{1,106 - f_2(x)}{1,106 - 670} & \text{for } 1,106 < f_2(x) < 670, \\ 1 & \text{for } f_2(x) < 670. \end{cases}$$

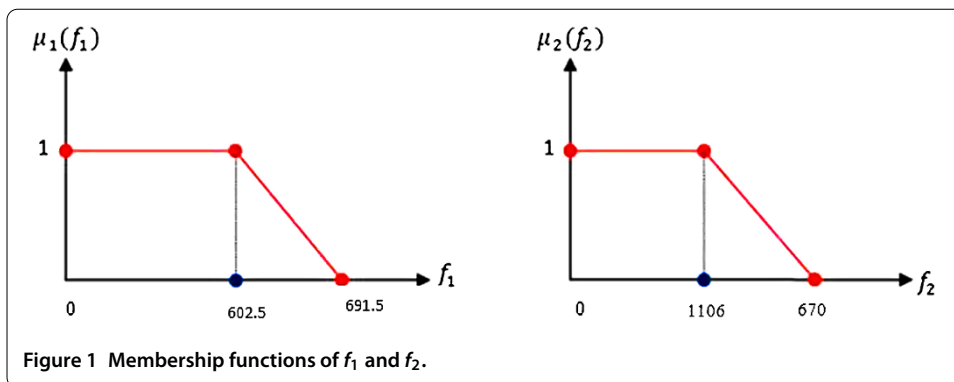


Table 1 Employ the interactive fuzzy satisfied method based on the fuzzy inequality constraint of Model-1

μ_1	μ_2	f_1	f_2	$\mu_1(f_1)$	$\mu_2(f_2)$	Values of decision variables	λ
1	1	634.3	826.09	0.641	0.641	$x_{111}^1 = 9, x_{121}^1 = 2.04, x_{131}^1 = 8, x_{222}^1 = 4.95,$ $x_{121}^2 = 9.45, x_{122}^2 = 1.04, x_{212}^2 = 10.5, x_{232}^2 = 9.50$	0.35
1	0.9	630.98	853.13	0.679	0.579	$x_{111}^1 = 9, x_{121}^1 = 3.17, x_{131}^1 = 8, x_{222}^1 = 3.82,$ $x_{121}^2 = 8.32, x_{122}^2 = 2.17, x_{212}^2 = 10.5, x_{232}^2 = 9.5$	0.32
0.9	1	637.76	799.15	0.603	0.703	$x_{111}^1 = 9, x_{121}^1 = 0.91, x_{131}^1 = 8, x_{222}^1 = 6.08,$ $x_{121}^2 = 10.5, x_{122}^2 = 10.5, x_{232}^2 = 9.5$	0.296

Then we compute the following model to get the interactive satisfied solution:

$$\left\{ \begin{array}{l} \min_x \lambda \\ \text{s.t.} \left\{ \begin{array}{l} (691.5 - f_1(x)) \geq (\mu_1 - \lambda)(691.5 - 602.5), \\ (1,106 - f_2(x)) \geq (\mu_2 - \lambda)(1,106 - 670), \\ x_{111}^1 + x_{112}^1 + x_{121}^1 + x_{122}^1 + x_{131}^1 + x_{132}^1 \leq 35, \\ x_{211}^1 + x_{212}^1 + x_{221}^1 + x_{222}^1 + x_{231}^1 + x_{232}^1 \leq 30, \\ x_{111}^1 + x_{112}^1 + x_{211}^1 + x_{212}^1 \geq 9, \\ x_{121}^1 + x_{122}^1 + x_{221}^1 + x_{222}^1 \geq 7, \\ x_{131}^1 + x_{132}^1 + x_{231}^1 + x_{232}^1 \geq 8, \\ x_{111}^2 + x_{112}^2 + x_{121}^2 + x_{122}^2 + x_{131}^2 + x_{132}^2 \leq 28, \\ x_{211}^2 + x_{212}^2 + x_{221}^2 + x_{222}^2 + x_{231}^2 + x_{232}^2 \leq 30 + 7(1 - \alpha), \\ x_{111}^2 + x_{112}^2 + x_{211}^2 + x_{212}^2 \geq 7 + 7(1 - \alpha), \\ x_{121}^2 + x_{122}^2 + x_{221}^2 + x_{222}^2 \geq 7 + 7(1 - \alpha), \\ x_{131}^2 + x_{132}^2 + x_{231}^2 + x_{232}^2 \geq 6 + 7(1 - \alpha), \\ x_{111}^1 + x_{121}^1 + x_{131}^1 + x_{211}^1 + x_{221}^1 + x_{231}^1 + x_{111}^2 \\ \quad + x_{121}^2 + x_{131}^2 + x_{211}^2 + x_{221}^2 + x_{231}^2 \leq 25 + 7(1 - \alpha), \\ x_{112}^1 + x_{122}^1 + x_{132}^1 + x_{212}^1 + x_{222}^1 + x_{232}^1 + x_{112}^2 \\ \quad + x_{122}^2 + x_{132}^2 + x_{212}^2 + x_{222}^2 + x_{232}^2 \leq 26 + 7(1 - \alpha), \\ 0 \leq \alpha \leq 1, \quad x_{ijk} \geq 0 \quad \forall i, j, k, \end{array} \right. \end{array} \right.$$

where $\alpha = 0.5$ has been calculated in Example 9.1. Here we solve Model-1, to get the satisfied solutions, which are listed in Table 1.

The first line of Table 1 lists each reference value of membership function $\mu_1(f_1)$, when the initialized membership function is 1, the value of objective function $f_1(x)$, and its corresponding solution x . If the decision maker hopes to improve $f_2(x)$ on the basis of sacrifice

Table 2 Optimum results by using the convex combination method of Model-1

w_1	w_2	f_1	f_2	Values of decision variables
0.5	0.5	691.5	670	$x_{111}^1 = 9, x_{131}^1 = 6.5, x_{222}^1 = 7, x_{232}^1 = 1.5, x_{212}^2 = 10.5, x_{222}^2 = 10.5, x_{231}^2 = 9.5$
0.6	0.4	688	673.5	$x_{111}^1 = 9, x_{131}^1 = 8, x_{222}^1 = 7, x_{212}^2 = 10.5, x_{221}^2 = 2, x_{222}^2 = 8.5, x_{231}^2 = 9.5$
0.7	0.3	651	748.5	$x_{111}^1 = 9, x_{131}^1 = 8, x_{222}^1 = 7, x_{212}^2 = 10.5, x_{221}^2 = 10.5, x_{232}^2 = 9.5$

Table 3 Comparison of optimum results of Model-1

Method	f_1	f_2	Values of decision variables
FISM	630.98	799.15	$x_{111}^1 = 9, x_{121}^1 = 0.91, x_{131}^1 = 8, x_{222}^1 = 6.08, x_{212}^2 = 10.5, x_{212}^2 = 10.5, x_{232}^2 = 9.5$
Convex combination	651	748.5	$x_{111}^1 = 9, x_{131}^1 = 8, x_{222}^1 = 7, x_{212}^2 = 10.5, x_{221}^2 = 10.5, x_{232}^2 = 9.5$
Global criteria	669	711.2	$x_{111}^1 = 9, x_{131}^1 = 6.75, x_{222}^1 = 7, x_{232}^1 = 1.24, x_{221}^2 = 7.95, x_{212}^2 = 10, x_{231}^2 = 4.78, x_{222}^2 = 2.54, x_{232}^2 = 4.71, GC = 0.1265$

$f_1(x)$, we may consider resetting the reference value of membership function (μ_1, μ_2) , e.g., we set $(\mu_1, \mu_2) = (0.9, 1)$ or $(\mu_1, \mu_2) = (1, 0.9)$. The corresponding results are listed in the second and third lines. Suppose that when the reference value of membership function is $(\mu_1, \mu_2) = (0.9, 1)$, the decision maker is satisfied, then the interactive process is stopped, so we obtain that the satisfied solutions for different items are

$$\begin{bmatrix} x_{111}^{(1)} & x_{112}^{(1)} & x_{121}^{(1)} \\ x_{122}^{(1)} & x_{131}^{(1)} & x_{132}^{(1)} \\ x_{211}^{(1)} & x_{212}^{(1)} & x_{221}^{(1)} \\ x_{222}^{(1)} & x_{231}^{(1)} & x_{232}^{(1)} \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0.91 \\ 0 & 8 & 0 \\ 0 & 0 & 0 \\ 6.08 & 0 & 0 \end{bmatrix},$$

$$\begin{bmatrix} x_{111}^{(2)} & x_{112}^{(2)} & x_{121}^{(2)} \\ x_{122}^{(2)} & x_{131}^{(2)} & x_{132}^{(2)} \\ x_{211}^{(2)} & x_{212}^{(2)} & x_{221}^{(2)} \\ x_{222}^{(2)} & x_{231}^{(2)} & x_{232}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 10.5 \\ 0 & 0 & 0 \\ 0 & 10.5 & 0 \\ 0 & 0 & 9.5 \end{bmatrix}$$

and the corresponding optimal values for different items are

$$\begin{bmatrix} f_1^{(1)} & f_1^{(2)} \\ f_2^{(1)} & f_2^{(2)} \end{bmatrix} = \begin{bmatrix} 282.26 & 355.5 \\ 352.15 & 447 \end{bmatrix}.$$

Applying the convex combination method stated in Section 4.3, we get Table 2 for different weights on f_1 and f_2 . The comparison between the optimum results calculated by different methods for Model-1 is given in Table 3.

7.2 Input data for Model-2

Let us consider a multi-objective single-item solid transportation problem with three origins (i.e., $M = 3$), two destinations (i.e., $N = 2$) and two types of conveyances (i.e., $K = 2$). The parameters are given as follows.

Transportation cost for 1st objective $[c_{ijk}^{(1)}]$

$$\begin{bmatrix} c_{111}^{(1)} & c_{112}^{(1)} & c_{121}^{(1)} \\ c_{122}^{(1)} & c_{131}^{(1)} & c_{132}^{(1)} \\ c_{211}^{(1)} & c_{212}^{(1)} & c_{221}^{(1)} \\ c_{222}^{(1)} & c_{231}^{(1)} & c_{232}^{(1)} \end{bmatrix} = \begin{bmatrix} 10 & 12 & 11 \\ 13 & 9 & 19 \\ 12 & 11 & 13 \\ 14 & 13 & 19 \end{bmatrix}.$$

Transportation cost for 2nd objective $[c_{ijk}^{(2)}]$

$$\begin{bmatrix} c_{111}^{(2)} & c_{112}^{(2)} & c_{121}^{(2)} \\ c_{122}^{(2)} & c_{131}^{(2)} & c_{132}^{(2)} \\ c_{211}^{(2)} & c_{212}^{(2)} & c_{221}^{(2)} \\ c_{222}^{(2)} & c_{231}^{(2)} & c_{232}^{(2)} \end{bmatrix} = \begin{bmatrix} 16 & 15 & 12 \\ 11 & 14 & 16 \\ 15 & 15 & 13 \\ 11 & 14 & 13 \end{bmatrix}.$$

Amount of items available at origin $[a_i]$

$$[a_1 \quad a_2] = [21 \quad 20].$$

The demand amount of items at destination $[b_j]$

$$[b_1 \quad b_2 \quad b_3] = [7 \quad 5 \quad 2].$$

Amount of items transported by conveyances $[c_k]$

$$[e_1 \quad e_2] = [20 \quad 22].$$

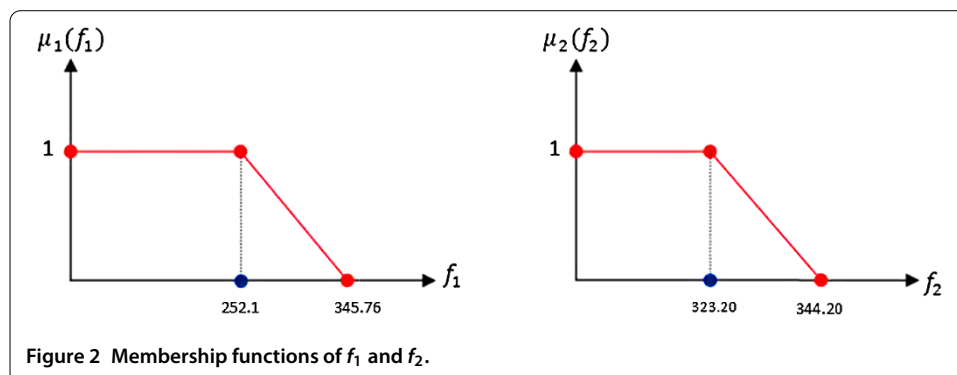
Optimum result for Model-2

With the above input data, f_1 and f_2 are calculated using the GRG technique, and we get

$$f_1^U = 345.76, \quad f_1^L = 252.1, \quad f_2^U = 344.20, \quad f_2^L = 323.20.$$

So we can get the membership functions of f_1 and f_2 (Figure 2) as follows:

$$\mu_1(f_1(x)) = \begin{cases} 0 & \text{for } f_1(x) > 345.76, \\ \frac{345.76 - f_1(x)}{345.76 - 252.1} & \text{for } 252.1 < f_1(x) < 345.76, \\ 1 & \text{for } f_1(x) < 252.1, \end{cases}$$



$$\mu_2(f_2(x)) = \begin{cases} 0 & \text{for } f_2(x) > 344.20, \\ \frac{344.20 - f_2(x)}{344.20 - 323.20} & \text{for } 323.20 < f_2(x) < 344.20, \\ 1 & \text{for } f_2(x) < 323.20. \end{cases}$$

Then we compute the following model to get the interactive satisfied solution:

$$\left\{ \begin{array}{l} \min_x \lambda \\ \text{s.t.} \end{array} \right. \begin{cases} (345.76 - f_1(x)) \geq (\mu_1 - \lambda)(345.76 - 252.1), \\ (344.20 - f_2(x)) \geq (\mu_2 - \lambda)(344.20 - 323.20), \\ x_{111} + x_{112} + x_{121} + x_{122} + x_{131} + x_{132} \leq 21, \\ x_{211} + x_{212} + x_{221} + x_{222} + x_{231} + x_{232} \leq 20, \\ x_{111} + x_{112} + x_{211} + x_{212} \geq 7 + 7(1 - \alpha), \\ x_{121} + x_{122} + x_{221} + x_{222} \geq 5 + 7(1 - \alpha), \\ x_{131} + x_{132} + x_{231} + x_{232} \geq 2 + 7(1 - \alpha), \\ x_{111} + x_{121} + x_{131} + x_{211} + x_{221} + x_{231} \leq 20 + 7(1 - \alpha), \\ x_{112} + x_{122} + x_{132} + x_{212} + x_{222} + x_{232} \leq 22 + 7(1 - \alpha), \\ x_{ijk} \geq 0 \quad \forall i, j, k, \quad 0 \leq \alpha \leq 1, \end{cases}$$

where $\alpha = 0.4974$ has been already calculated in Example 9.2. Here we solve Model-2 to get the satisfied solutions, which are listed in Table 4.

The first line of Table 4 lists each reference value of membership function $\mu_1(f_1)$, when the initialized membership function is 1, the value of objective function $f_1(x)$, and its corresponding solution x . If the decision maker hopes to improve $f_2(x)$ on the basis of sacrifice $f_1(x)$, we may consider resetting the reference value of membership function (μ_1, μ_2) , e.g., we set $(\mu_1, \mu_2) = (0.9, 1)$ or $(\mu_1, \mu_2) = (1, 0.9)$. The corresponding results are listed in the second and third lines. Suppose that when the reference value of membership function is $(\mu_1, \mu_2) = (1, 0.90)$, the decision maker is satisfied, then the interactive process is stopped, so we obtain that the satisfied solutions are $x_{121} = 1.34, x_{122} = 7.16, x_{131} = 5.51, x_{212} = 10.51$. The corresponding optimal values are

$$[f_1 \quad f_2] = [273.40 \quad 330.67].$$

Applying the convex combination method stated in Section 4.3, we get Table 5 for different weights on f_1 and f_2 . The comparison between optimum results calculated by different methods for Model-2 is given in Table 6.

Table 4 Employ the interactive fuzzy satisfied method based on fuzzy inequality constraint for Model-2

μ_1	μ_2	f_1	f_2	$\mu_1(f_1)$	$\mu_2(f_2)$	Values of decision variables	λ
1	1	276.55	328.68	0.73	0.73	$x_{122} = 8.51, x_{131} = 5.47, x_{212} = 10.51, x_{232} = 0.045$	0.261
0.9	1	283.03	328.03	0.66	0.76	$x_{122} = 8.51, x_{131} = 4.82, x_{212} = 10.51, x_{232} = 0.69$	0.2302
1	0.9	273.4	330.07	0.67	0.77	$x_{121} = 1.34, x_{122} = 7.16, x_{131} = 5.51, x_{212} = 10.51$	0.227

Table 5 Optimum results by using the convex combination method for Model-2

w_1	w_2	f_1	f_2	Values of decision variables
0.5	0.5	259.06	337.24	$x_{121} = 8.51, x_{131} = 5.51, x_{212}^1 = 10.51, x_{121}^1 = 8.51$
0.3	0.7	276.1	328.72	$x_{122} = 8.51, x_{131} = 5.51, x_{212}^1 = 10.51$
0.6	0.4	252.1	344.20	$x_{111} = 6.96, x_{121} = 8.51, x_{131}^1 = 5.51, x_{212}^1 = 3.55$

Table 6 Comparison of optimum results of Model-2

Method	f_1	f_2	Values of decision variables
FISM	273.40	330.07	$x_{121} = 1.34, x_{122} = 7.16, x_{131} = 5.51, x_{212} = 10.51$
Convex combination	252.1	344.20	$x_{111} = 6.96, x_{121} = 8.51, x_{131} = 5.51, x_{212} = 3.55$
Global criteria	259.06	337.24	$x_{121} = 8.51, x_{131} = 5.51, x_{212} = 10.51, GC = 0.0514$

7.3 Optimum result for Model-3

To solve Model-3, we will solve $MSSTP_\alpha$ (Example 9.1) and the optimum solution has come for decision $\alpha^* = 0.5$, and the optimal solutions for different items are

$$\begin{bmatrix} x_{111}^{(1)} & x_{112}^{(1)} & x_{121}^{(1)} \\ x_{122}^{(1)} & x_{131}^{(1)} & x_{132}^{(1)} \\ x_{211}^{(1)} & x_{212}^{(1)} & x_{221}^{(1)} \\ x_{222}^{(1)} & x_{231}^{(1)} & x_{232}^{(1)} \end{bmatrix} = \begin{bmatrix} 0 & 2.70 & 7 \\ 0 & 0 & 2.47 \\ 6.29 & 0 & 0 \\ 0 & 5.52 & 0 \end{bmatrix},$$

$$\begin{bmatrix} x_{111}^{(2)} & x_{112}^{(2)} & x_{121}^{(2)} \\ x_{122}^{(2)} & x_{131}^{(2)} & x_{132}^{(2)} \\ x_{211}^{(2)} & x_{212}^{(2)} & x_{221}^{(2)} \\ x_{222}^{(2)} & x_{231}^{(2)} & x_{232}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 4.81 & 8.32 \\ 2.17 & 0 & 0 \\ 0 & 5.68 & 0 \\ 0 & 0 & 9.5 \end{bmatrix}$$

and the corresponding optimal values for different items are

$$\begin{bmatrix} f_1^{(1)} & f_1^{(2)} \\ f_2^{(1)} & f_2^{(2)} \end{bmatrix} = \begin{bmatrix} 247 & 355.5 \\ 719.10 & 477.59 \end{bmatrix}.$$

8 Conclusion

The multi-objective multi-item solid transportation problem in fuzzy inequality constraints has been explored in this paper. Three different models have been derived. First, a fuzzy inequality solid transportation problem has been converted to a parametric solid transportation problem using flexible index, and then the fuzzy inequality solid transportation problem has been solved by using the decision making technique. The fuzzy interactive satisfied method, global criterion method and convex combination method have been applied to calculate the optimal compromise solutions of multi-objective STP problem, and then it was solved by using MatLab and Lingo-11.0. The models are illustrated with numerical examples and corresponding results are compared. This paper only researches the problem under fuzzy inequality constraints, and the problem in other more complex environments or multi-objective uncertain transportation problem may become new topics in further research. The present formulation and solution procedures can be applied to other fuzzy transportation models with different fuzzy numbers.

Appendix

Example 9.1 Consider the following FICP:

$$\begin{aligned} \min & 11x_{111}^1 + 10x_{112}^1 + 11x_{121}^1 + 13x_{122}^1 + 11x_{131}^1 + 10x_{132}^1 + 10x_{211}^1 + 11x_{212}^1 \\ & + 13x_{221}^1 + 14x_{222}^1 + 10x_{231}^1 + 12x_{232}^1 + 18x_{111}^2 + 11x_{112}^2 + 12x_{121}^2 + 12x_{122}^2 \\ & + 13x_{131}^2 + 15x_{132}^2 + 12x_{211}^2 + 11x_{212}^2 + 13x_{221}^2 + 14x_{222}^2 + 15x_{231}^2 + 12x_{232}^2 \end{aligned}$$

$$\begin{aligned}
 \text{s.t. } & x_{111}^1 + x_{112}^1 + x_{121}^1 + x_{122}^1 + x_{131}^1 + x_{132}^1 \lesssim 35, \\
 & x_{211}^1 + x_{212}^1 + x_{221}^1 + x_{222}^1 + x_{231}^1 + x_{232}^1 \lesssim 30, \\
 & x_{111}^1 + x_{112}^1 + x_{211}^1 + x_{212}^1 \gtrsim 9, \\
 & x_{121}^1 + x_{122}^1 + x_{221}^1 + x_{222}^1 \gtrsim 7, \\
 & x_{131}^1 + x_{132}^1 + x_{231}^1 + x_{232}^1 \gtrsim 8, \\
 & x_{111}^2 + x_{112}^2 + x_{121}^2 + x_{122}^2 + x_{131}^2 + x_{132}^2 \lesssim 28, \\
 & x_{211}^2 + x_{212}^2 + x_{221}^2 + x_{222}^2 + x_{231}^2 + x_{232}^2 \lesssim 30, \\
 & x_{111}^2 + x_{112}^2 + x_{211}^2 + x_{212}^2 \gtrsim 7, \\
 \widetilde{MMSTP} \quad & x_{121}^2 + x_{122}^2 + x_{221}^2 + x_{222}^2 \gtrsim 7, \\
 & x_{131}^2 + x_{132}^2 + x_{231}^2 + x_{232}^2 \gtrsim 6, \\
 & x_{111}^1 + x_{121}^1 + x_{131}^1 + x_{211}^1 + x_{221}^1 + x_{231}^1 + x_{111}^2 \\
 & \quad + x_{121}^2 + x_{131}^2 + x_{211}^2 + x_{221}^2 + x_{231}^2 \lesssim 25, \\
 & x_{112}^1 + x_{122}^1 + x_{132}^1 + x_{212}^1 + x_{222}^1 + x_{232}^1 + x_{112}^2 \\
 & \quad + x_{122}^2 + x_{132}^2 + x_{212}^2 + x_{222}^2 + x_{232}^2 \lesssim 26, \\
 & x_{ijk}^p \geq 0 \quad \forall i, j, k \text{ and } p = 1, 2, \quad d = (0, 0, 0, 0, 0, 0, 7, 7, 7, 7, 7, 7).
 \end{aligned}$$

The corresponding parametric linear programming problem of \widetilde{MMSTP} is presented as follows:

$$\begin{aligned}
 \min & 11x_{111}^1 + 10x_{112}^1 + 11x_{121}^1 + 13x_{122}^1 + 11x_{131}^1 + 10x_{132}^1 \\
 & + 10x_{211}^1 + 11x_{212}^1 + 13x_{221}^1 + 14x_{222}^1 + 10x_{231}^1 + 12x_{232}^1 \\
 & + 18x_{111}^2 + 11x_{112}^2 + 12x_{121}^2 + 12x_{122}^2 + 13x_{131}^2 + 15x_{132}^2 \\
 & + 12x_{211}^2 + 11x_{212}^2 + 13x_{221}^2 + 14x_{222}^2 + 15x_{231}^2 + 12x_{232}^2 \\
 \text{s.t. } & x_{111}^1 + x_{112}^1 + x_{121}^1 + x_{122}^1 + x_{131}^1 + x_{132}^1 \leq 35, \\
 & x_{211}^1 + x_{212}^1 + x_{221}^1 + x_{222}^1 + x_{231}^1 + x_{232}^1 \leq 30, \\
 & x_{111}^1 + x_{112}^1 + x_{211}^1 + x_{212}^1 \geq 9, \\
 & x_{121}^1 + x_{122}^1 + x_{221}^1 + x_{222}^1 \geq 7, \\
 & x_{131}^1 + x_{132}^1 + x_{231}^1 + x_{232}^1 \geq 8, \\
 & x_{111}^2 + x_{112}^2 + x_{121}^2 + x_{122}^2 + x_{131}^2 + x_{132}^2 \leq 28, \\
 & x_{211}^2 + x_{212}^2 + x_{221}^2 + x_{222}^2 + x_{231}^2 + x_{232}^2 \leq 30 + 7(1 - a), \\
 & x_{111}^2 + x_{112}^2 + x_{211}^2 + x_{212}^2 \geq 7 + 7(1 - \alpha), \\
 \widetilde{MMSTP}_\alpha \quad & x_{121}^2 + x_{122}^2 + x_{221}^2 + x_{222}^2 \geq 7 + 7(1 - \alpha), \\
 & x_{131}^2 + x_{132}^2 + x_{231}^2 + x_{232}^2 \geq 6 + 7(1 - \alpha), \\
 & x_{111}^1 + x_{121}^1 + x_{131}^1 + x_{211}^1 + x_{221}^1 + x_{231}^1 + x_{111}^2
 \end{aligned} \tag{25}$$

$$\begin{aligned}
 &+ x_{121}^2 + x_{131}^2 + x_{211}^2 + x_{221}^2 + x_{231}^2 \leq 25 + 7(1 - \alpha), \\
 &x_{112}^1 + x_{122}^1 + x_{132}^1 + x_{212}^1 + x_{222}^1 + x_{232}^1 + x_{112}^2 \\
 &+ x_{122}^2 + x_{132}^2 + x_{212}^2 + x_{222}^2 + x_{232}^2 \leq 26 + 7(1 - \alpha), \\
 &x_{ijk}^p \geq 0 \quad \forall i, j, k, p = 1, 2 \quad \text{and} \quad 0 \leq \alpha \leq 1.
 \end{aligned}$$

Now, using the algorithm given in Section 2.1, we obtain $Z_0 = 725$, $Z_1 = 480$ and $d_0 = 245$ by calculating $(MMSTP_0)$ and $(MMSTP_1)$ corresponding to (25). Let the inverse of the optimal matrix in $(MMSTP_0)$ be

$$B_0^{-1} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Now, calculating the equations $[B_0^{-1}(35, 30, 9, 7, 8, 28, 30 + 7(1 - \alpha), 7 + 7(1 - \alpha), 7 + 7(1 - \alpha), 6 + 7(1 - \alpha), 25 + 7(1 - \alpha), 26 + 7(1 - \alpha))]_i = 0$ ($i = 1, 2, \dots, 12$), respectively, we obtain $\alpha_1 = 0.8571$ assume $\alpha_0 = 0$ and $\alpha_2 = 1$.

Now, solving $(MMSTP_{\alpha_1})$, we get an optimal solution and an optimal value as $Z_{\alpha_1} = Z_{0.8571} = 515.01$.

Now, $Z_{0.8571} = 515.01 < Z_1 + 0.8571 \times 245 = 689.98$, so we stop here and calculate optimal decision α^* . Now

$$\begin{aligned}
 \alpha^* &= \frac{Z_1\alpha_1 - Z_1\alpha_0 - Z_{\alpha_0}\alpha_1 + Z_{\alpha_1}\alpha_0}{Z_{\alpha_1} - Z_{\alpha_0} - \alpha_1d_0 + \alpha_0d_0} \\
 &= 0.5.
 \end{aligned}$$

Now, solving $(MMSTP_{0.5})$, we obtain the optimal value $Z_{0.5}^* = 602.5$.

Example 9.2 Consider the following FICP:

$$\begin{aligned}
 &\min 10x_{111} + 12x_{112} + 11x_{121} + 13x_{122} \\
 &\quad + 9x_{131} + 19x_{132} + 12x_{211} + 11x_{212} \\
 &\quad + 13x_{221} + 14x_{222} + 13x_{231} + 19x_{232} \\
 &\text{s.t. } x_{111} + x_{112} + x_{121} + x_{122} + x_{131} + x_{132} \lesssim 21, \\
 &\quad x_{211} + x_{212} + x_{221} + x_{222} + x_{231} + x_{232} \lesssim 20,
 \end{aligned}$$

$$\begin{aligned}
 & x_{111} + x_{112} + x_{211} + x_{212} \gtrsim 7, \\
 & x_{121} + x_{122} + x_{221} + x_{222} \gtrsim 5, \\
 \widetilde{MSSTP} \quad & x_{131} + x_{132} + x_{231} + x_{232} \gtrsim 2, \\
 & x_{111} + x_{121} + x_{131} + x_{211} + x_{221} + x_{231} \lesssim 20, \\
 & x_{112} + x_{122} + x_{132} + x_{212} + x_{222} + x_{232} \lesssim 22, \\
 & x_{ijk} \geq 0 \quad \forall i, j, k, \quad d = (0, 0, 7, 7, 7, 7, 7).
 \end{aligned} \tag{26}$$

The corresponding parametric linear programming problem of \widetilde{MSSTP} is presented as follows:

$$\begin{aligned}
 & \min 10x_{111} + 12x_{112} + 11x_{121} + 13x_{122} \\
 & \quad + 9x_{131} + 19x_{132} + 12x_{211} + 11x_{212} \\
 & \quad + 13x_{221} + 14x_{222} + 13x_{231} + 19x_{232} \\
 \text{s.t.} \quad & x_{111} + x_{112} + x_{121} + x_{122} + x_{131} + x_{132} \leq 21, \\
 & x_{211} + x_{212} + x_{221} + x_{222} + x_{231} + x_{232} \leq 20, \\
 & x_{111} + x_{112} + x_{211} + x_{212} \geq 7 + 7(1 - \alpha), \\
 & x_{121} + x_{122} + x_{221} + x_{222} \geq 5 + 7(1 - \alpha), \\
 \widetilde{MSSTP}_\alpha \quad & x_{131} + x_{132} + x_{231} + x_{232} \geq 2 + 7(1 - \alpha), \\
 & x_{111} + x_{121} + x_{131} + x_{211} + x_{221} + x_{231} \leq 20 + 7(1 - \alpha), \\
 & x_{112} + x_{122} + x_{132} + x_{212} + x_{222} + x_{232} \leq 22 + 7(1 - \alpha), \\
 & x_{ijk} \geq 0 \quad \forall i, j, k, \quad 0 \leq \alpha \leq 1.
 \end{aligned} \tag{27}$$

Now, using the algorithm given in Section 2.1, we obtain $Z_0 = 367$, $Z_1 = 143$ and $d_0 = 224$ by calculating $(MSSTP_0)$ and $(MSSTP_1)$ corresponding to (27). The inverse of the optimal matrix in $(MSSTP_0)$ is

$$B_0^{-1} = \begin{pmatrix} -1 & 0 & -1 & -1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix}.$$

Now, calculating the equations $[B_0^{-1}(21, 20, 7 + 7(1 - \alpha), 5 + 7(1 - \alpha), 2 + 7(1 - \alpha), 20 + 7(1 - \alpha), 22 + 7(1 - \alpha))]_i = 0$ ($i = 1, 2, \dots, 7$), respectively, we obtain $\alpha_1 = 0.8571$ assume $\alpha_0 = 0$ and $\alpha_2 = 1$.

Now, solving $(MSSTP_{\alpha_1})$, we get an optimal solution as $x_{122} = 9$, $x_{311} = 9$ and $x_{321} = 2$ and an optimal value as $Z_{\alpha_1} = Z_{0.8571} = 173.009$.

Now $Z_{0.8571} = 173.009 < Z_1 + 0.8571 \times 224 = 334.99$, so we stop here and calculate optimal decision α^* . Now

$$\begin{aligned}\alpha^* &= \frac{Z_1\alpha_1 - Z_1\alpha_0 - Z_{\alpha_0}\alpha_1 + Z_{\alpha_1}\alpha_0}{Z_{\alpha_1} - Z_{\alpha_0} - \alpha_1d_0 + \alpha_0d_0} \\ &= 0.4974.\end{aligned}$$

Now, solving $(MSSTP_{0.4974})$, we obtain an optimal value as $Z_{0.4974}^* = 252.1$.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Author details

¹Department of Mathematics, Heritage Institute of Technology, Anandapur, Kolkata, West Bengal 700107, India.

²Department of Applied Science, Haldia Institute of Technology, Purba Midnapur, Haldia, West Bengal 721657, India.

³Department of Mathematics, Bengal Engineering and Science University, Shibpur, Howrah, West Bengal 711103, India.

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References

1. Haley, K: The solid transportation problem. *Oper. Res.* **10**, 448-463 (1962)
2. Ojha, A, Das, B, Mondal, S, Maiti, M: A solid transportation problem for an item with fixed charge, vehicle cost and price discounted varying charge using genetic algorithm. *Appl. Soft Comput.* **10**, 100-110 (2010)
3. Pramanik, S, Jana, DK, Maiti, K: A multi objective solid transportation problem in fuzzy, bi-fuzzy environment via genetic algorithm. *Int. J. Adv. Oper. Manag.* **6**(1), 4-26 (2014)
4. Bit, AK, Biswal, MP, Alam, SS: Fuzzy programming approach to multi-objective solid transportation problem. *Fuzzy Sets Syst.* **57**, 183-194 (1993)
5. Ida, K, Gen, M, Li, Y: Neural networks for solving multicriteria solid transportation problem. *Comput. Ind. Eng.* **31**, 873-877 (1996)
6. Gao, SP, Liu, SY: Two-phase fuzzy algorithms for multi-objective transportation problem. *J. Fuzzy Math.* **12**, 147-155 (2004)
7. Tao, Z, Xu, J: A class of rough multiple objective programming and its application to solid transportation problem. *Inf. Sci.* **188**, 215-235 (2012)
8. Kundu, P, Kar, S, Maiti, M: Multi-objective multi-item solid transportation problem in fuzzy environment. *Appl. Math. Model.* **37**, 2028-2038 (2013)
9. Pramanik, S, Jana, DK, Maiti, M: Multi-objective solid transportation problem in imprecise environment. *J. Transp. Secur.* **6**, 131-150 (2013)
10. Sancak, E, Salman, S: Multi-item dynamic lot-sizing with delayed transportation policy. *Int. J. Prod. Econ.* **131**, 595-603 (2011)
11. Zimmermann, HJ: Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets Syst.* **1**, 45-55 (1978)
12. Zimmermann, HJ: Description and optimization of fuzzy systems. *Int. J. Gen. Syst.* **2**, 209-215 (1976)
13. Cao, BY: *Optimal Models and Methods with Fuzzy Quantities*. Springer, Berlin (2010)
14. Sakawa, K: *Fuzzy Sets and Interactive Multiobjective Optimization*. Plenum, New York (1993)
15. Xu, J, Zhou, X: *Fuzzy Like Multiple Objective Decision Making*. Springer, Berlin (2011)

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