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MULTI-OBJECTIVE OPTIMISATION MODEL OF SHUTTLE-BASED STORAGE AND RETRIEVAL SYSTEM

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Abstract. This paper presents a multi-objective optimisation solution procedure for the design of the Shuttle-Based Storage and Retrieval System (SBS/RS). An efficient SBS/RS design should take into account multi-objectives for optimization. In this study, we considered three objective functions in the design concept which are the minimization of average cycle time of transactions (average throughput time), amount of energy (electricity) consumption and total investment cost. By also considering the amount of energy consumption as an objective function for minimization, we aimed to contribute to an environmentally friendly design concept. During the optimization procedure, we considered seven design variables as number of aisles, number of tiers, number of columns, velocities of shuttle carriers, acceleration/deceleration of shuttle carriers, velocity of the elevators lifting tables and acceleration/deceleration of the elevators lifting tables. Due to the non-linear property of the objective function, we utilized the Non-Dominated Sorting Genetic Algorithm II (NSGA II) genetic algorithm for facilitating the solution. Lastly, we searched Pareto optimal solutions to find out the optimum results. We believe that this study provides a useful and a flexible tool for warehouse planners and designers, while choosing a particular type of SBS/RS at the early stage of the warehouse design.

Keywords: warehouses; shuttle-based storage and retrieval systems; multi-criteria optimization problem; performance analysis.

Abbreviations

AS/RS – automated storage and retrieval systems;
AVS/RS – autonomous vehicle storage and retrieval systems;
AVs – autonomous vehicles;
CO_2 – carbon dioxide;
CBAS/RS – crane-based automated storage and retrieval systems;
DCC – double command cycle;
GA – genetic algorithm;
I/O – input and output location;
NSGA II – non-dominated sorting genetic algorithm II;
ES – one way travel time component;
SBS/RS – shuttle-based storage and retrieval systems;
SCC – single command cycle;
SR – storage rack;
SIT – square-in-time;
TB – travel between time component.

Objective Functions

$f_T(x_i)$ – objective function throughput time;
$f_{EC}(x_i)$ – objective function energy consumption;
$f_{TC}(x_i)$ – objective function total cost.

Design Variables

A – number of aisles in the SBS/RS;
M – number of tiers in the SBS/RS;
C – number of columns in the SBS/RS;
v_x – velocity of the shuttle carrier;
a_x – acceleration/deceleration of the shuttle carrier;
v_y – velocity of the elevator's lifting table;
a_y – acceleration/deceleration of the elevator's lifting table.

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Operational Parameters

a_y^+ – acceleration of the elevator's lifting table;

a_x^+ – acceleration of the shuttle carrier;

g – acceleration of gravity;

F_B – braking force;

a_y^- – deceleration of the elevator's lifting table;

a_x^- – deceleration of the shuttle carrier;

d_i – distance;

l_{cvel} – distance of the shuttle carrier when travelling with constant velocity;

l_{acc} – distance of the shuttle carrier during acceleration;

l_{dec} – distance of the shuttle carrier during deceleration;

h_{acc} – distance of the elevator's lifting table during acceleration;

h_{cvel} – distance of the elevator's lifting table when hoisting with constant velocity;

h_{dec} – distance of the elevator's lifting table during deceleration;

η – efficiency;

T_η – efficiency of the SBS/RS;

EC – energy consumption;

$T_{project}$ – expected lifetime of the SBS/RS;

$E(ES)_{SCAR}$ – expected one way travel time of the shuttle carrier;

$E(TB)_{SCAR}$ – expected travel-between time of the shuttle carrier;

$E(SCC)_{SCAR}$ – expected single command cycle time of the shuttle carrier;

$E(DCC)_{SCAR}$ – expected dual command cycle time of the shuttle carrier;

$E(ES)_{LIFT}$ – expected one way travel time of the elevator's lifting table;

$E(TB)_{LIFT}$ – expected travel-between time of the elevator's lifting table;

$E(SCC)_{LIFT}$ – expected single command cycle time of the elevator's lifting table;

$E(DCC)_{LIFT}$ – expected dual command cycle time of the elevator's lifting table;

$E(DCC)_{AISLE}$ – expected dual command cycle time of a single SBS/RS;

k_{ir} – factor which takes into account the impact resistance of the rotating masses with variable vehicle velocity;

G – force of gravity;

F_R – force of rolling friction;

F_{iT} – force of inertia;

H_{SR} – height of the storage rack of the SBS/RS;

h_{cell} – height of the storage location;

L_{SR} – length of the storage rack of the SBS/RS;

l_{cell} – length of the storage location;

x_l – lower bound of the design space;

m_{tote} – mass of the tote;

m_{table} – mass of the lifting table;

m_{scar} – mass of the shuttle carrier;

v_{max} – maximum velocity;

n – number of elevator's lifting tables/shuttle carriers;

n_{weeks} – number of weeks;

n_{wd} – number of working days in a week;

T_{shift} – number of working hours in a shift;

$t_{P/S}$ – pick-up and set-down times of the elevator's lifting table/shuttle carrier;

P – power;

$F_i(z)$ – probability distribution function;

$f_i(z)$ – probability density function;

k_r – rolling resistance coefficient;

P_{SCAR} – root mean square power of the shuttle carrier;

P_{LIFT} – root mean square power of the elevator's lifting table;

λ – throughput capacity;

$\lambda_{SBS/RS}$ – throughput performance of the SBS/RS;

$\lambda(DCC)_{SCAR}$ – throughput capacity of the shuttle carrier in the case of DCC;

$\lambda(DCC)_{LIFT}$ – throughput capacity of the elevator's lifting table in the case of DCC;

$T_{SBS/RS}$ – throughput time;

t_{acc} – time for accelerating the elevator's lifting table/shuttle carrier to reach the maximum velocity;

t_{cvel} – time for travelling of the elevator's lifting table/shuttle carrier with constant velocity;

t_{dec} – time for decelerating the elevator's lifting table/shuttle carrier until it stops;

F_T – traction force;

$t(d_i)$ – travel time of the elevator's lifting table to the most distant tier in the SBS/RS/ travel time of the shuttle carrier to the most distant storage location (cell) in the SBS/RS;

x_l – upper bound of the design space;

Q – warehouse volume;

w_{cell} – width of the storage location;

$W_{SBS/RS}$ – width of the SBS/RS;

k – this symbol stands for the number of transaction, which is two (2) in case of DCC (one tote is going inside of the SR and one tote is leaving the SR).

Operational Cost Parameters

C_{LIFT} – cost of the selected elevator with lifting tables;

C_{SCAR} – cost of the selected shuttle carrier;

C_{SP} – cost of the storage location;

C_{SA} – cost of storage area per square meter per year;

$C_{EC\,LIFT}$ – cost of the energy consumption of the elevator's lifting tables;

$C_{EC\,SCAR}$ – cost of the energy consumption of shuttle carriers;

c_{EC} – cost for 1 kWh of electricity;

I_{LIFT} – investment in elevators with lifting tables;

I_{SCAR} – investment in shuttle carriers;

I_{SP} – investment in storage locations;

I_{SA} – investment in the storage area of one aisle of the SBS/RS;

TC – total cost.

Introduction

Warehouses are critical for the supply chain.

Basically, there are three types of warehouses:

- distribution warehouses;
- production warehouses;
- contract warehouses.

In distribution warehouses, products from different suppliers are received and stored (sometimes they are assembled) for delivery to several customers. Production warehouses are usually used for storing raw materials, semi-finished products and finished products in a production facility. In a contract warehouse, warehousing operations are performed on behalf of one or more customers. Although many companies tend to follow make-to-order policies, due to the demand variabilities they may cause, companies may need warehouses. To be able to increase space utilization in a warehouse, managers prefer a high-rise storage area with a relatively small foot-print where storage locations on racks are accessible via narrow aisles. Having high-rise storage design creates the necessity for a fast moving automated material handling system for storing/retrieving loads to/from storage locations to increase the throughput capacity. Advances in technology could provide such a technology ensuring greater responsiveness and additional flexibility in fulfilling orders. Therefore, warehouses in a supply chain are moving beyond traditional CBAS/RS technologies towards AVS/RS technologies offering additional flexibilities in warehouse operations (Malmborg 2002). In this technology, throughput capacity can be varied by changing the number of AVs in the system. The main components of an AVS/RS are lifts, AVs and a system of rails in the storage rack area. Lifts provide vertical movement for transactions to travel among tiers and AVs provide horizontal movement for transactions within the tier. AVS/RS technology is first introduced for heavy unit-load transactions (Malmborg 2002). Recently, increasing trends towards more product variety with small size and short response times have created a new AVS/RS called Shuttle-Based Storage and Retrieval System (SBS/RS) as an alternative system to mini-load CBAS/RS (Carlo, Vis 2012; Marchet *et al.* 2013, 2012; Lerher *et al.* 2015a, 2015b, 2014, 2013). A typical SBS/RS is a tier-captive AVS/RS design where AVs – i.e. shuttles – can travel within a tier and each aisle has a lift mechanism. The main advantages of this system are that it is light-weight and has a high transaction throughput rate. Shuttles carry the loads in totes hence this system is also known as AVS/RS with product totes (Marchet *et al.* 2013). In literature reviews, previous researchers have mostly focused on AVS/RS (Malmborg 2003, 2002; Kuo *et al.* 2008, 2007; Fukunari, Malmborg 2009, 2008; Ekren, Heragu 2011; Ekren *et al.* 2014, 2013, 2010; Ekren 2011; Roy *et al.* 2012; Zhang *et al.* 2009) configurations, whereas 'tier captive' AVS/RS and SBS/RS seem to have been disregarded, notwithstanding its higher adoption by a number of industrial applications for tote handling. This study not only fills this gap in the literature but also provides a significant contribution to the energy efficient design concept of AVS/RS. It is important to design SBS/RS right the first time due to the relative inflexibility of the physical layout and the equipment. In this study, we provide a multi-objective optimization model for designing SBS/RS by optimizing the average cycle (throughput) times of transactions, amount of energy consumption and total investment costs. In other words, our aim was to find out the best SBS/RS design for minimizing the average cycle times of transactions, total amounts of energy consumption and maximising the throughput capacity.

1. Literature Review of AS/RS and SBS/RS

The AS/RS is a major category of material handling equipment. There are primarily two types of AS/RS, unit-load AS/RS and the mini-load AS/RS. AS/RS usually consists of conveyors, SR and an automated S/R machine that can travel along narrow aisles between the SR to store and retrieve loads. The S/R machine can manipulate either pallets (unit-load system) or totes (mini-load system). Over the past 50 years, many studies of AS/RS have been performed within the material handling research community. The intensive development of AS/RS began with the development of informational and computer science. Hausman *et al.* (1976), Graves *et al.* (1977) presented travel time models for AS/RS assuming that the SR was square-in-time, which meant that times to the most distant column $t_x = L_{SR} / v_x$ and tier $t_y = H_{SR} / v_y$ were both equal ($t_x = t_y$). They analysed different storage strategies, e.g. randomised, turnover-based and class-based storage assignment rules. Gudehus (1973) presented principles for calculations of the cycle times for the SCC and DCC. In the case of the SCC the S/R machine could perform one storage or retrieval request, only. More advanced is the DCC where the storage and the retrieval request are done simultaneously by the S/R machine. With regard to other cycle time expressions, he considered the impact of the acceleration and deceleration on travel times. Bozer and White (1984) presented an analytical travel time model for calculating the SCC and DCC for non-SIT racks. Their models were based on randomized storage and retrieval with different I/O configurations of the input queue. Their analytical travel time model was based on the assumption that the S/R machine travels all the time at constant velocity. Hwang and Lee (1990) presented travel time models by considering the operating characteristics of the S/R machines for AS/RS and non-SIT racks. Sari *et al.* (2005) presented closed-form travel-time expressions for flow-rack AS/RS. Lerher *et al.* (2006) developed analytical travel time models for multi-aisle AS/RS by considering the operating characteristics of the S/R machine. Using the proposed analytical travel time models, average travel time can be evaluated. Gu *et al.* (2007) presented a comprehensive review of research on warehouse operation. Roodbergen and Vis (2009) presented a comprehensive explanation of the current state-of-the-art in AS/RS. Rouwenhorst *et al.* (2000) presented a comprehensive review of warehouse design and control. Lerher *et al.* (2011) presented simulation analysis of a mini-load multi-shuttle AS/RS. Recently Bortolini *et al.* (2015a) proposed an extension for analytical models when computing the expected travel time for the SCCs and DCCs of AS/RS in three-class-based storage systems. Later, Bortolini *et al.* (2015b) proposed non-conventional easy-applicable configuration for unit load warehouses with diagonal cross-aisles. Accorsi *et al.* (2015) presented time and energy based assignment strategy for unit-load AS/RS warehouses. Janilionis *et al.* (2016) presented a comparison between routing algorithms for storage and retrieval mechanisms in cylindrical AS/RS.

The SBS/RS is composed of shuttle carriers, SR and the elevator with lifting tables that are attached on a mast. The shuttle carrier function is a S/R machine in this system. Each tier of the SR has usually one shuttle carrier (tier-captive SBS/RS). There can be some other designs, where shuttle carriers serve for multi-tiers. Totes (loads) are carried to the destination tiers via elevator's lifting tables. Each aisle has one elevator mounted along the SR. There are typically two buffer positions in each tier where totes are dropped off from the elevator's lifting tables or shuttle carriers. In the storage process, the totes are picked up from these buffer positions by the shuttle carriers to be stored at storage locations in the SR. Based on the literature review, there are very few studies related to SBS/RS. Carlo and Vis, 2012 studied an SBS/RS developed by the Vanderlande Industries (*http://www.vanderlande.com*) where two non-passing lifting systems are mounted along the SR. In this paper, they focused on the scheduling problem of lifts where two (piece-wise linear) functions are introduced to evaluate candidate solutions. They developed an integrated look-ahead heuristic for the solution procedure to improve the total handling time (in terms of throughput). Marchet *et al.* (2012) modelled an SBS/RS via open queuing network to estimate the performance of the system in terms of utilisations of lifts and shuttles as well as waiting times for lifts and queues. They compared the analytical results with the simulation models in order to validate them. The proposed analytical models demonstrated good estimates for the performance measures. Later, Marchet *et al.* (2013) presented main design trade-offs for SBS/RS using simulation. They completed their study for several warehouse design scenarios for tier-captive shuttle vehicles. They presented several performance measures from the system – utilizations of lifts and shuttles, average flow time, waiting times, as well as the costs for the pre-defined SR designs.

Concerning multi-objective studies, Diao *et al.* (2011) studied a trade-off problem on the time-cost-quality performance of a project. A computer-based Pareto multi-objective optimisation approach applying NSGA II GA algorithm was utilized for solving trade-off problems. Based on their proposed approach, decision-making can become easy according to the sorted non-dominated solutions and project preferences. Lerher *et al.* (2013) studied multi-objective optimization for a CBS/RS, where the objective functions were defined as: minimisation of cost, average travel times of transactions and maximization of quality. The NSGA II GA algorithm was used for the solution procedure. The primary reason for using the evolutionary algorithm is its ability to find out the Pareto optimal solution. Recently, Lerher (2013), Lerher *et al.* (2015a, 2015b, 2014) studied SBS/RS by considering the energy efficient concept within the system's design. The proposed models provided several warehouse designs and their performances. Designs were considered in terms of velocity profiles of material handling devices while performances were considered as the amounts of energy (electricity) consumption, amount of CO_2 oscillation and throughput capacity. These studies provide a significant contribution towards automated warehouse planning by taking into consideration the environmentally friendly design concept. Smew *et al.* (2013) presented a simulation study of trade-offs between the conflicting objectives of maximising customer service levels and minimising work-in-process. Bekker (2013) presented a computationally economic approach for optimizing the throughput rate and allocated buffer space, which are the two conflicting objectives of the buffer allocation problem.

According to the above mentioned, SBS/RS compared to mini-load AS/RS offers customers the following benefits: maximum flexibility, optimum utilisation of space and volume, minimum energy consumption due to low moving masses, energy recovery, decentralised communication between shuttles and a warehouse management system, modular and scalable design, etc. Of course, the final decision always lies with the customer if the customer is willing to invest in new technology such SBS/RS instead of mini-load AS/RS.

According to the literature review, the majority of the reviewed papers used a different approach for analysing the system performance of SBS/RS. More importantly, in this paper by considering the multi-objective function and the total energy consumption of the system as one of these multi-objectives, we believe that we have contributed to literature and the environment. In this study, we considered three objective functions as minimisation of:

- average cycle (throughput) time of transactions (maximisation of throughput capacity);

- total amount of energy consumption;
- total investment cost.

Due to the non-linearity of the multi-objectives, we utilised GA to solve the problem (Holland 1992). Specifically, we utilised Pareto optimal solutions and the NSGA II GA to obtain optimal solutions (Deb *et al.* 2002).

2. Model for Designing Shuttle-Based Storage and Retrieval Systems

2.1. Shuttle-Based Storage and Retrieval Systems

Shuttle-based storage and retrieval systems are composed of the elevator with lifting tables that are attached on a mast, shuttle carriers, buffer positions and the SR (Fig. 1). The elevator through the lifting table moves the totes up and down to the prescribed tier in the SR. To be able to process more work, two independent lifting tables may be attached to the elevator, one of which is on the right side and the other one on the left side of the mast. In this case, the performance of the elevator can be doubled. An elevator can move lifting tables with velocities up to v_y = 1.5 m/s theoretically. An energy regeneration system module can be applied during lowering (braking) the lifting table. Elevators are often bottlenecks in this system so they usually determine the performance of the whole system (Lerher 2013).

A shuttle carrier is a small AV with four wheels that transports totes from the buffer position to the storage locations on the SR (Fig. 1). A shuttle carrier is equipped with a telescopic attachment for manipulating totes. The maximal weight of a tote should not exceed 50 kg/shuttle carrier. A shuttle carrier can travel with velocities up to v_x = 4 m/s theoretically. An energy regeneration system module can also be applied during its braking. Usually there is a single shuttle carrier in each level of a storage rack. This assumption can be released if we use a special shuttle elevator at the back of the storage rack, for moving shuttle carriers up and down to the prescribed level in the SR. The buffer position is placed at each level of the SR and is used for buffering totes from the elevator and shuttle carriers. The SR is composed of storage compartments that can receive *n* loads. By multiplying storage compartments in the horizontal and vertical directions, the capacity of the storage rack is calculated. The SR can be implemented as a single or a double deep system (Lerher 2013).

2.2. Optimisation Model of the SBS/RS

This section presents the optimisation model, which aims to show the trade-off between the average throughput time, total energy consumption and total cost.

When building the proposed model, the following assumptions and notations were considered (Lerher 2013):

- the SBS/RS is divided into two sides within an aisle; therefore totes can be stored on either side in a tier;
- the input/output (I/O_{aisle}) location of the single SBS/RS is located at the first tier, next to the lift location (Fig. 1);
- the SR is divided by columns and tiers; At each tier, there are two buffer positions (left and right, Fig. 1) and a single shuttle carrier (aisle-captive system);
- the elevator manipulates two lifting tables independently, one of which is located at the left side and the other one on at the right side of the elevator; each lifting table can serve one tote at a time;
- the elevator and the shuttle carrier complete SCCs and DCCs;
- velocity and acceleration/deceleration (v_y, a_y) of the elevators lifting tables as well as the height H_{SR} of the SR are known;
- velocity and acceleration/deceleration of shuttle carriers (v_x, a_x) as well as the length L_{SR} of the SR are known;
- the height H_{SR} and length L_{SR} of the SR are large enough for the elevators lifting tables and shuttle carriers to reach their maximum velocity v_{max} in the vertical and horizontal directions;
- the elevator's lifting table and the shuttle carrier travel on basis of the velocity-time dependence, which is associated with acceleration, constant velocity and deceleration;

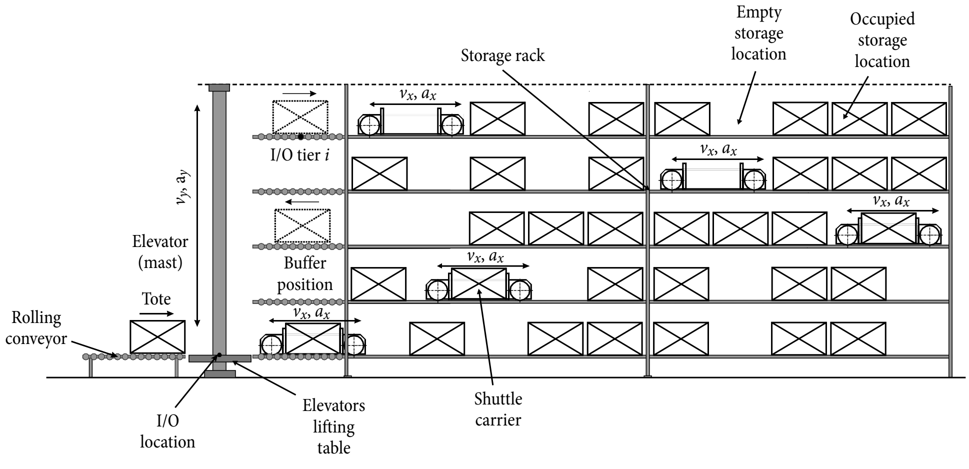

Fig. 1. SBS/RS (Lerher *et al.* 2015a)

- randomised assignment policy is considered which means that any buffer position in the vertical direction and any storage position in the horizontal direction are equally likely to be selected for storage or retrieval assignments to be processed.

The model minimises the average throughput time $T_{SBS/RS}$, total cost TC and total energy consumption EC of a SBS/RS according to project restraints and conditions using the following seven design variables x:

$$\text{integer: } A,\ M,\ C;$$
$$\text{real: } v_x,\ a_x,\ v_y,\ a_y, \tag{1}$$

where: A refers to the number of aisles; M refers to the number of tiers; C refers to the number of columns; v_x refers to the velocities of shuttle carriers for traveling in the horizontal direction; a_x refers to the acceleration/deceleration of the shuttle carriers for travelling in the horizontal direction; v_y refers to the velocities of the elevator's lifting tables for moving in the vertical direction; a_y refers to the acceleration/deceleration of the elevator's lifting tables for moving in the vertical direction. The proposed model is represented by a mathematical model, which includes design variables, all relevant operational, physical parameters, and investment/operational costs, and is detailed in the following section.

2.2.1. Minimising the Average Throughput Time

Throughput time $T_{SBS/RS}$ in most material handling facilities (in our case SBS/RS) have related to the movement of material handling devices like elevator's lifting tables and shuttle carriers. Throughput time $T_{SBS/RS}$ could be minimised by using efficient drives for faster movement of shuttle carriers in the horizontal direction and hoisting of the elevator's lifting tables in the vertical direction. Beside the efficient drives, the number of columns C and the number of tiers M of the SBS/RS should be in an appropriate relationship. The average throughput time per item $T_{SBS/RS}$ is inversely dependent on the throughput capacity λ, measured in terms of the number of item movements that can be completed in a given time. According to the values of the throughput time $T_{SBS/RS}$ for the elevator's lifting tables and shuttle carriers, the throughput capacity $\lambda_{SBS/RS}$ of the SBS/RS has been defined. The objective was to minimise the throughput time $T_{SBS/RS}$ of the elevator's lifting tables and shuttle carriers, which is described as follows:

$$\min f_T\left(x_i\right);$$
$$f_T\left(x_i\right)=T_{SBS/RS}\left(v_x,a_x,v_y,a_y,A,C,M\right) \tag{2}$$

and is discussed in detail in the Chapter 3.1.

2.2.2. Minimising Total Energy Consumption

Total EC is comparatively relative to the throughput time $T_{\text{SBS/RS}}$ and consequently the total cost TC. Application of SBS/RS material handling devices with efficient drives (faster movement of shuttle carriers and hoisting of the elevators lifting tables) will no doubt increase the energy consumption of the material handling devices and consequently the total cost of the SBS/RS. The objective is to minimise the total EC, which is described as follows:

$$\min f_{EC}\left(x_i\right);$$
$$f_{EC}\left(x_i\right)=EC\left(v_x,a_x,v_y,a_y,A,C,M\right) \tag{3}$$

and is discussed in detail in the Chapter 3.2.

2.2.3. Minimising Total Cost

Total cost TC is comparatively relative to the throughput time $T_{SBS/RS}$. Application of SBS/RS material handling devices with efficient drives (faster movement of shuttle carriers and hoisting of the elevators lifting tables) will no doubt increase the cost of the SBS/RS and the maintenance costs of material handling devices. For the relationship between the TC and the $T_{SBS/RS}$, one can use a discrete function or continuous (linear/quadratic) function. The objective is to minimise the total cost TC, which is described as follows:

$$\min f_{TC}\left(x_i\right);$$
$$f_{TC}\left(x_i\right)=TC\left(v_x,a_x,v_y,a_y,A,C,M\right) \tag{4}$$

and is discussed in detail in the Chapter 3.3.

3. Definition of the Optimisation Model of the SBS/RS

3.1. Travel Time and Throughput Performance Definition

3.1.1. Travel Time Definition of the Elevators Lifting Table

The elevator's lifting table can work on a SCC, which means that only one tote can be handled at a time. More advanced is DCC, by which two totes are handled in a cycle. By assuming the distances between tiers to be large enough for the elevator's lifting table to achieve its maximum velocity v_{max} and a continuous sequence of loaded and unloaded moves between destination points (buffer positions), the expected cycle time using the probability theory has been developed.

In the case of SCC, the elevator's lifting table lifts a tote to tier i, unloads the tote and moves back to the I/O location (Fig. 2). The same sequence can also be performed in the reverse order. One way travel time $(ES)_{LIFT}$ corresponds to the variable travel time z for lifting of the elevators lifting table with a tote from I/O location to the selected i-th tier of the SR. By assuming a condition of uniform distribution in assigning tiers in SBS/RS, the probability distribution function $F_y(z)$ can be calculated. According to the lifting of the elevator's lifting table in the vertical direction, the expected SCC time $E(SCC)_{LIFT}$ for the SBS/RS is represented by the following expression:

$$E\left(SCC\right)_{LIFT}=2\cdot t_{P/S}+2\cdot E\left(ES\right)_{LIFT};$$
$$E\left(SCC\right)_{LIFT}=2\cdot t_{P/S}+2\cdot\frac{v_y}{a_y}+\frac{H_{SR}}{v_y}. \tag{5}$$

Note: for the verification of expression (5) see Appendix A.

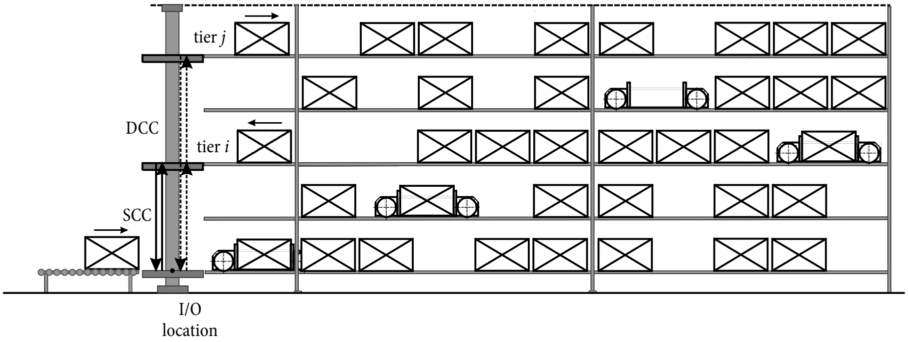

Fig. 2. SCC and DCC of the elevator's lifting table (Lerher *et al.* 2015a)

In the case of DCC, the elevators lifting table moves a tote to tier *i*, unloads the tote and moves further to the tier *j*, where a tote is retrieved (Fig. 2). After loading a tote at tier *j*, the elevators lifting table moves back to the I/O location. Travel time for DCC corresponds to the travel time for SCC to the randomly selected tier *i* and travel–between $(TB)_{LIFT}$ time component for DCC, where the retrieval request occurs in the *i*-th or *j*-th tiers (Fig. 2). According to lifting of the elevator's lifting table in the vertical direction, the expected DCC time $E(DCC)_{LIFT}$ for the SBS/RS is represented by the following expression:

$$E\left(DCC\right)_{LIFT} = 4 \cdot t_{P/S} + 2 \cdot E\left(ES\right)_{LIFT} + E\left(TB\right)_{LIFT};$$

$$E\left(DCC\right)_{LIFT} = 4 \cdot t_{P/S} + 3 \cdot \frac{v_y}{a_y} + \frac{4}{3} \cdot \frac{H_{SR}}{v_y}. \qquad (6)$$

Note: for the verification of expression (6) see Appendix A.

By considering expression (6) the throughput performance $\lambda(DCC)_{LIFT}$ of the DCC per hour is calculated by expression:

$$\lambda\left(DCC\right)_{LIFT} = \frac{3600}{E\left(DCC\right)_{LIFT}} \cdot k, \; k = 2. \qquad (7)$$

3.1.2. Travel Time Definition of the Shuttle Carrier

The shuttle carrier can work on a SCC, which means that only one tote can be handled at a time. More advanced is the DCC, by which two totes are handled in a cycle. By assuming the distances in the tier to be large enough for the shuttle carrier to achieve its maximum velocity v_{max} and a continuous sequence of loaded and unloaded moves between destination points (storage/retrieval), the expected cycle time using the probability theory was developed (Lerher 2013).

Under travelling of the shuttle carrier in the *i*-th tier, the shuttle carrier is capable of visiting a single storage or retrieval location (Fig. 3). The travel time depends on the kinematics properties of the shuttle carrier, the length of the storage rack L_{SR} and the selected storage assignment policy. One way travel time $(ES)_{SCAR}$ corresponds to the variable travel time *z* for travelling from the I/O$_{tier(i)}$ location to any randomly selected location in the *i*-th tier (Fig. 3). By assuming the condition of uniform distribution of the storage locations in the SBS/RS, the probability distribution function $F_x(z)$ can be calculated.

According to travelling of the shuttle carrier in the horizontal direction, the expected SCC time $E(SCC)_{SCAR}$ for the SBS/RS is represented by the following expression:

$$E\left(SCC\right)_{SCAR} = 2 \cdot t_{P/S} + 2 \cdot E\left(ES\right)_{SCAR};$$

$$E\left(SCC\right)_{SCAR} = 2 \cdot t_{P/S} + 2 \cdot \frac{v_x}{a_x} + \frac{L_{SR}}{v_x}. \qquad (8)$$

Note: For the verification of expression (8) see Appendix B.

In the case of the shuttle carrier, the operation of DCC considers storage and retrieval processes at a time (Fig. 3). According to the travelling of the shuttle carrier in the horizontal direction, the expected DCC time

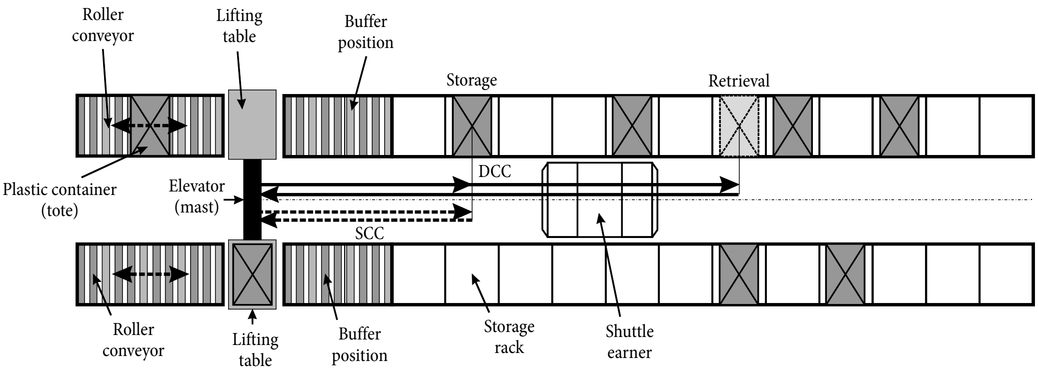

Fig. 3. SCC and DCC of the shuttle carrier (Lerher *et al.* 2015a)

$E(DCC)_{SCAR}$ for the SBS/RS is represented by the following expression:

$$E\left(DCC\right)_{SCAR}=4\cdot t_{P/S}+2\cdot E\left(ES\right)_{SCAR}+E\left(TB\right)_{SCAR};$$

$$E\left(DCC\right)_{SCAR}=4\cdot t_{P/S}+3\cdot\frac{v_x}{a_x}+\frac{4}{3}\cdot\frac{L_{SR}}{v_x}. \tag{9}$$

Note: for the verification of expression (9) see Appendix B.

According to expression (9) the throughput performance $\lambda(\mathrm{DCC})_{\mathrm{SCAR}}$ of the DCC is calculated by expression (10):

$$\lambda\left(DCC\right)_{SCAR}=\frac{3600}{E\left(DCC\right)_{SCAR}}\cdot k,\ \ k=2. \tag{10}$$

3.1.3. System Performance of the SBS/RS as a Whole

Three different objective functions were selected for the multi-objective optimisation of SBS/RS: average throughput time, total energy consumption, and total cost. Better solutions in terms of cost and energy consumption are those solutions with lower value of these objective functions (minimisation) whereas better solutions in terms of throughput capacity are the ones with higher values (maximisation). In this study, the throughput performance $\lambda_{SBS/RS}$ of the SBS/RS was placed with the average throughput time $T_{SBS/RS}$ of the SBS/RS, to simplify the model and is calculated by expression (11):

$$T_{SBS/RS}=\frac{3600}{\lambda_{SBS/RS}}=E\left(DCC\right)_{AISLE}\frac{1}{k\cdot A}, \tag{11}$$

where: $E(DCC)_{AISLE}$ represents the expected DCC time of the single SBS/RS; k is 2 due to the DCC and A represents the number of aisles (number of SBS/RS). The expression $E(DCC)_{AISLE}$ considers that there is a bottleneck in SBS/RS and equals $E(DCC)_{AISLE}=\max(T_{A\text{–}LIFT}, T_{A\text{–}SCAR})$, where $T_{A\text{–}LIFT}$ and $T_{A\text{–}SCAR}$ represent the expected DCC times of the elevator's lifting table and the shuttle carrier in one single SBS/RS. The expression $T_{A\text{–}LIFT}=E(DCC)_{LIFT}/n$, where $n=2$ because there are two lifting tables per lift. The expression $T_{A\text{–}SCAR}=E(DCC)_{SCAR}/M$, where M stands for number of tiers in the SBS/RS.

3.2. Energy Consumption Definition

This section provides calculations for the required engine power P_{SCAR} of the shuttle carrier and the required engine power P_{LIFT} of the elevators lifting table.

3.2.1. Required Engine Power for the Travelling of the Shuttle Carrier

Mean power of the shuttle carrier P_{SCAR} is calculated according to the expression for the root mean square power of the shuttle carrier for travelling in the horizontal direction in the case where one way travel time and equals the next expression:

$$P_{SCAR}=\sqrt{\frac{P_{Ta}^2\cdot t_{acc}+P_{Tv}^2\cdot t_{cvel}+P_B^2\cdot t_{dec}}{t_{acc}+t_{cvel}+t_{dec}}}. \tag{12}$$

Note: for the verification of expression (12) see Appendix C.

3.2.2. Hoisting of the Elevator's Lifting Table with Constant Velocity

The mean necessary power of the elevator's lifting table P_{LIFT} is calculated according to the expression for the root mean square power of the elevator's lifting table for hoisting in the vertical direction in the case where one way travel time equals the next expression:

$$P_{LIFT}=\sqrt{\frac{P_{Ta}^2\cdot t_{acc}+P_{Tv}^2\cdot t_{cvel}+P_B^2\cdot t_{dec}}{t_{acc}+t_{cvel}+t_{dec}}}. \tag{13}$$

Note: for the verification of expression (13) see Appendix C.

3.2.3. Calculation of Amount of Energy Consumption

Amount of energy consumption EC counted on a yearly basis depends on the root mean square power P of the elevators lifting table/shuttle carrier (P_{LIFT}/P_{SCAR}), efficiency of the elevators lifting table/shuttle carrier η, number of working hours in a shift T_{shift}, number of working days in a week n_{wd}, number of weeks n_{weeks}, number of elevators lifting tables/shuttle carriers n, number of aisles A and is calculated by:

$$EC=P\cdot\eta\cdot T_{shift}\cdot n_{wd}\cdot n_{weeks}\cdot n\cdot A. \tag{14}$$

Note: calculation of the energy consumption is based on the expression (14). In practice, the energy consumption is not completely equal to the product of root mean square power multiplied with the efficiency, number of working hours in a shift, number of working days in a week, number of weeks, etc., but is equal to the integral of the power consumption over time. We assumed that this simplified expression was good enough for our study, which is to analyse the influences of the multi-objective optimisation method in the design of the SBS/RS.

3.3. Cost Definition

In continuation, the cost definition will be presented.

The cost definition of the SBS/RS consist of:

- the investment for elevators with lifting tables;
- the investment for shuttle carriers;
- the investment for storage locations;
- the investment for the storage area of one aisle of SBS/RS;
- cost of energy consumption by the elevator's lifting tables;
- cost of energy consumption by the shuttle carriers.

Total cost TC of the SBS/RS is calculated by:

$$TC=I_{LIFT}+I_{SCAR}+I_{SP}+T_{project}\times\left(I_{SA}\cdot A+C_{EC\ LIFT}+C_{EC\ SCAR}\right), \tag{15}$$

where: I_{LIFT} [EUR] indicates the investment for the elevators with lifting tables; I_{SCAR} [EUR] indicates the investment for shuttle carriers; I_{SP} [EUR] indicates the investment for storage locations; I_{SA} [EUR/m^2·year] indicates the investment for storage area for one aisle of SBS/RS; $C_{EC\ LIFT}$ [EUR/year] indicates the cost for energy consumption of the elevator's lifting tables; $C_{EC\ SCAR}$ [EUR/year] indicates the cost for energy consumption of shuttle carriers; $T_{project}$ indicates the expected life time of the SBS/RS (15 years).

Note: for the verification of expression (15) see Appendix D.

3.4. Pareto Optimisation Design

While single objective optimisation problems may have a unique optimal solution, multi-objective problems can have an uncountable number of solutions (Pareto solutions) when the objective functions have an inverse relationship. This means that decreasing one objective function increases at least one of the other objective functions. At the end of the optimisation the user then chooses an acceptable solution from all Pareto solutions (Lerher *et al.* 2013). The aim of our study was to optimise the objective functions: travel time, energy consumption and total cost for a warehouse that has a defined minimum volume ($Q_{\min}$). Thus, this aim was formulated as a constrained multi-objective problem (16–19) where the optimisation procedure searches for the best solution in terms of design variable vector $\{x\}$ taking into account all prescribed constraints and design space bounds (x_l and x_u represent the lower and upper bounds of the design space). The components of the design variable vector are described in detail in expression (1).

$$\min\left\{f_T(x), f_{EC}(x), f_{TC}(x)\right\}; \tag{16}$$

$$Q_{\min} - Q(x) \leq 0; \tag{17}$$

$$x_l \leq x \leq x_u; \tag{18}$$

$$x = \left\{A, M, C, v_x, a_x, v_y, a_y\right\}. \tag{19}$$

To solve the proposed problem the non-dominated sorting genetic algorithm NSGA II was applied (Deb *et al.* 2002), which is designed to solve constrained multi-objective problems. The output of the algorithm is a set of solutions lying on or near the Pareto frontier.

Note: for a brief description of the NSGA II algorithm see Appendix E.

4. Illustrative Example and Optimisation Results

In this section, an illustrative example and optimisation results are presented. With the optimisation of design variables A, M, C, v_x, a_x, v_y and a_y in the proposed model, the optimal design of SBS/RS is defined in terms of three objective functions and one constraint. The objective function to be minimised is throughput time, total cost of SBS/RS and total energy consumption of the elevator's lifting tables and shuttle carriers, while the constraint was the volume of the SBS/RS in terms of $Q_{\min}$ = 10000 totes. The minimal and maximal values of the design variables A, M, C, v_x, a_x, v_y and a_y are presented in Table 1. The size of the SBS/RS is limited with respect to the maximum number of aisles A, number of tiers M, and number of columns C in such a way that the optimisation constraint in terms of Q, can be met.

Table 1. Design variables parameters used during the multi-criterion optimisation

Variable	Units of measure	Min value	Max value
A	–	1	100
M	–	1	20
C	–	1	180
v_x	m/s	1.5	4.0
a_x	m/s^2	1.5	3.0
v_y	m/s	1.5	2.0
a_y	m/s^2	1.5	2.5

Note: SBS/RS variable parameters are selected according to the references of material handling equipment producers and practical experiences of the authors.

In order to create a SBS/RS model, additional constant parameters are needed, which are as follows:

- ***constant parameters for the computation of SBS/RS:*** warehouse volume Q = 10000, length of the storage location l_{cell} = 0.5 m, width of the storage location w_{cell} = 0.35 m, height of the storage location h_{cell} = 0.6 m;
- ***constant parameters for the computation of the travel time:*** pick-up and set down time of the shuttle carrier $t_{P/S\ SCAR}$ = 3.0 sec, pick-up and set down time of the elevators lifting table $t_{P/S\ LIFT}$ = 1.5 sec;
- ***constant parameters for the computation of SCS/RS investment:*** cost of the shuttle carrier C_{SCAR} = 10000 EUR/unit, cost of the elevator with lifting tables C_{LIFT} = 50000 EUR/unit, cost of the storage location C_{SP} = 30 EUR/unit, cost of storage area per square meter year C_{SA} = 50 EUR/m^2·year, cost for 1 kWh of electricity c_{EC} = 0.2 EUR/kW·h;
- ***constant parameters for the computation of the energy consumption needed to operate the SBS/RS:*** mass of the tote m_{tote} = 20 kg, mass of the elevator's lifting table m_{table} = 60 kg, mass of the shuttle carrier m_{scar} = 40 kg, expected lifetime of the SBS/RS $T_{project}$ = 15 years, number of weeks n_{weeks} = 50 weeks, number of working days per week n_{wd} = 5 days, number of working hours per shift T_{shift} = 16 hours, efficiency of the SBS/RS T_η = 0.8, acceleration of gravity g = 9.81 m/s^2, rolling resistance coefficient k_r = 0.01, factor which take into account the impact resistance of rotating masses with variable vehicle velocity k_{ir} = 1.15, efficiency η = 0.9.

5. Analyses and Evaluation of Results

This section presents the analyses and the evaluation of the results of SBS/RS. For the optimisation of design variables, the NSGA II algorithm was used (Deb *et al.* 2002). The primary reason for using this evolutionary algorithm is its ability to find Pareto optimal solution front within a single simulation run. The optimisation of design variables x_i, was carried out according to the following evolutionary and genetics operators: the degree of crossover was set at 0.9; the degree of mutation was set at 0.1; the size of population was set to 100; the number of generations was set to 200. Values of crossover and mutation degrees were chosen in accordance with our previous analyses and the experiences of researchers who have been engaged in the development and application of the GA method.

The analysis was performed on a personal computer with an *Intel Core i7 920 CPU* and it took about 2 minutes to complete the optimisation run with 20000 samples.

The size of population depends greatly on the number of design variables, which indirectly influences the necessary number of generations. Due to the proposed design variables x_i, the first optimization runs indicated that in most cases the GA finds Pareto optimal front within 200 generations. The results of the optimisation are presented in Fig. 4 and in Table 2. Comparison between the first and 200th generations is presented only in Fig. 4a–c while in Fig. 4d the final optimised Pareto front is represented. Fig. 4a–c represent the dependencies between pairs of objective functions. The development of the individual solutions can be noticed from Fig. 4a–c. At the beginning of the optimisation process, the single solutions are widely spread throughout the design space. Then SBS/RS that do not follow the required constraints, defined at the optimisation of the design variables are deleted and unconsidered in the next generations. This means that with the increase in the number of the generation, only the solutions with valid constraints remain. At the same time, the selection ensures that the good solutions are continually replaced by better solutions. In this way, the Pareto optimal solutions can be found. It can be seen from Fig. 4a–c that after 200 generations the Pareto optimal front was found. In order to depict the individual solutions of SBS/RS from the first generation, which do not meet the constraint criterion of the warehouse volume, the first generation of solutions is shown in two groups. The first group contains the SBS/RS whose volume Q is smaller than the given constraint and the second group contains the SBS/RS with an appropriate volume Q. It can be seen that all solutions from the second group of the first generation lie in the feasible solution space, while there are some solutions from the first group of the first generation, which have better objective function values than the Pareto optimal solutions but lie in the infeasible part of the design space. The diagram in Fig. 4d represents the dependencies between all three objective functions (handling time – energy consumption – total cost) in one diagram for the 200th generation of the optimisation. For better visualisation of the Pareto optimal front in space, all objective functions were normalised with the highest value of the objective function in the generation and a projection of the Pareto front is also depicted on the horizontal plane.

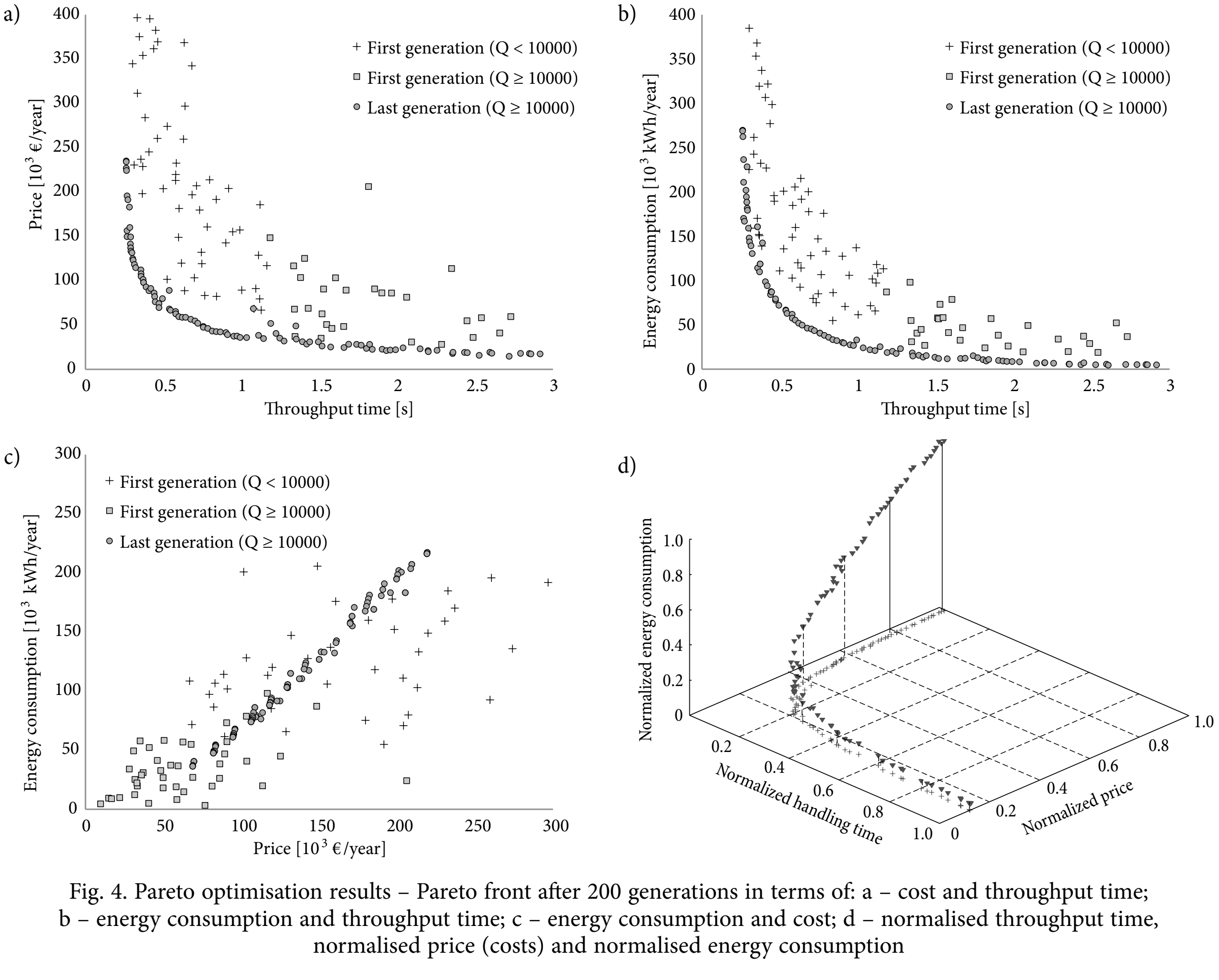

Fig. 4. Pareto optimisation results – Pareto front after 200 generations in terms of: a – cost and throughput time; b – energy consumption and throughput time; c – energy consumption and cost; d – normalised throughput time, normalised price (costs) and normalised energy consumption

Table 2. Pareto optimal solutions

ID	A [–]	M [–]	C [–]	v_x [m/s]	v_y [m/s]	a_x [m/s^2]	a_y [m/s^2]	$T_{SBS/RS}$ [s]	TC [10^3 EUR/year]	EC [10^3 kW·h/year]	Q
(*i*) Pareto solutions sorted by throughput time											
1	16	6	53	2.25	1.77	1.69	1.50	0.26	222.8	234.9	10176
2	16	6	53	2.25	1.77	1.69	1.50	0.26	222.8	234.9	10176
3	16	6	53	2.25	1.77	1.67	1.50	0.26	222.7	234.3	10176
4	16	6	53	2.23	1.77	1.61	1.50	0.26	221.7	229.1	10176
5	16	7	66	2.03	1.95	1.53	1.50	0.27	246.6	207.6	14784
6	16	7	53	1.81	1.99	1.71	1.50	0.27	226.9	185.0	11872
...	...	...	...	...	...	...	...	...	...	...	...
95	14	2	180	1.50	1.50	1.50	1.50	2.61	201.8	14.8	10080
96	2	17	151	1.50	1.55	1.50	1.50	2.72	67.1	18.1	10268
97	2	18	152	1.50	1.52	1.52	1.50	2.79	69.8	17.9	10944
98	2	19	152	1.50	1.52	1.52	1.50	2.85	72.3	17.8	11552
99	2	19	148	1.50	1.50	1.50	1.50	2.86	71.3	17.6	11248
100	2	20	125	1.50	1.50	1.50	1.50	2.92	68.1	17.4	10000
(*ii*) Pareto solutions sorted by total cost											
94	2	14	179	1.50	2.00	1.50	1.50	2.60	65.1	18.2	10024
92	2	14	179	1.71	1.97	1.50	1.50	2.45	65.9	22.0	10024
96	2	17	151	1.50	1.55	1.50	1.50	2.72	67.1	18.1	10268
100	2	20	125	1.50	1.50	1.50	1.50	2.92	68.1	17.4	10000
97	2	18	152	1.50	1.52	1.52	1.50	2.79	69.8	17.9	10944
99	2	19	148	1.50	1.50	1.50	1.50	2.86	71.3	17.6	11248
...	...	...	...	...	...	...	...	...	...	...	...
48	15	6	173	1.50	1.77	1.52	1.50	0.79	299.7	53.3	31140
52	16	5	178	1.50	1.77	1.52	1.50	0.90	300.9	45.8	28480
44	16	6	164	1.51	1.51	1.50	1.50	0.70	310.3	59.5	31488
45	16	6	169	1.50	1.83	1.50	1.50	0.72	315.5	57.9	32448
46	16	6	179	1.50	2.00	1.54	1.50	0.76	326.1	55.6	34368
47	16	6	179	1.50	2.00	1.54	1.50	0.76	326.1	55.6	34368
(*iii*) Pareto solutions sorted by energy consumption											
95	14	2	180	1.50	1.50	1.50	1.50	2.61	201.8	14.8	10080
93	14	2	180	1.56	1.50	1.50	1.50	2.53	202.1	15.9	10080
91	15	2	180	1.50	1.50	1.58	1.50	2.43	216.3	15.9	10800
90	15	2	180	1.56	1.50	1.50	1.50	2.36	216.5	17.0	10800
100	2	20	125	1.50	1.50	1.50	1.50	2.92	68.1	17.4	10000
89	6	5	173	1.50	1.77	1.52	1.50	2.36	111.0	17.4	10380
...	...	...	...	...	...	...	...	...	...	...	...
12	15	6	56	2.20	1.50	1.59	1.50	0.29	209.1	205.6	10080
5	16	7	66	2.03	1.95	1.53	1.50	0.27	246.6	207.6	14784
4	16	6	53	2.23	1.77	1.61	1.50	0.26	221.7	229.1	10176
3	16	6	53	2.25	1.77	1.67	1.50	0.26	222.7	234.3	10176
1	16	6	53	2.25	1.77	1.69	1.50	0.26	222.8	234.9	10176
2	16	6	53	2.25	1.77	1.69	1.50	0.26	222.8	234.9	10176

All SBS/RS configurations from the 200th generation which are shown in Fig. 4 represent Pareto optimal SBS/RS configurations and are at the same time different with respect to each other. In order to show the differences between the optimal solutions, the solutions were first given a unique solution ID as an identification number. Solutions were then sorted according to their objective function value. As the optimisation was performed using three different objective functions (handling time – energy consumption – total cost), this resulted in three differently sorted lists of SBS/RS configurations. The lists are shown in Table 2. To decrease the length of the Table 2, the solutions in the middle of each list were left out of Table 2.

Table 2 (*i* part) shows SBS/RS configurations sorted by handling time (throughput time) in the increasing direction. The best solution has a throughput time of 0.26 sec (throughput equals 13800 totes/h in the DCC) and is the one with the maximum number of aisles A. The worst solution is the one with the lowest number of aisles A and has a throughput time of approximately 3.0 sec (throughput equals 1200 totes/h in DCC), which is 11 times slower than the best solution. Table 2 (*ii* part) shows SBS/RS configurations sorted by total cost of the SBS/RS per year in the increasing manner. The solution with the smallest total cost (price) has the narrowest configuration with only two aisles $A = 2$ but is on the other hand quite high with $M = 14$ tiers and very long with $C = 179$ columns. The smallest total cost of all solutions equals 65100 EUR/year while the most expensive solution of SBS/RS equals 326100 EUR/year. The ratio between the most expensive solution and between the solution with the smallest price equaled 5. This ratio is smaller than the ratio in the throughput time, and can also be observed in Fig. 4a. If the throughput time is reduced, the total cost (price) of the SBS/RS starts to rise slowly at first and then the total cost (price) rises faster and faster which indicates the inverse relationship between the throughput times of the SBS/RS and its total cost (price). Table 2 (*iii* part) shows SBS/RS configurations sorted by the energy consumption needed to operate the SBS/RS over a one year time in an increasing manner. The solution of SBS/RS, which needs the smallest amount of energy consumption for its operation, was also one of the slowest solutions in the generation, which was somehow expected. This solution of SBS/RS needs 14.8 MW·h of energy for the operation in one year. The fastest SBS/RS in comparison needs 234.9 MW·h of energy for the operation in one year and is also one of the most expensive solutions. The fastest SBS/RS solution is 11 times faster, 3.3 times more expensive and consumes 13 times the amount of energy as the slowest SBS/RS solution.

Conclusions

This paper presented a multi-objective optimisation for the design and optimization of SBS/RS. The analysis deals with the selection of SBS/RS dimensions (*A*, *M* and *C*), the velocity profile of the elevators lifting tables and tier-captive shuttle carriers for the operation of a DCC. When designing SBS/RS, there is generally one objective function to be optimised subject to some constraints. Objective functions are usually throughput capacity, cycle time, travel distance and other financially-oriented performance metrics. This kind of optimisation is called a single objective optimisation problem. As the SBS/RS are relatively complex material handling systems, the application of the multi-objective optimisation problem proves to be most efficient comparing to single objective optimisation problem. Therefore our optimisation model consists of three independent objective functions named throughput time – total cost – energy consumption. All three objective functions are represented by an analytical model, which includes design variables, operational parameters and costs. Due to the non-linearity, discrete shape of objective functions and proposed design variables, the method of NSGA II GA has been applied in order to optimise design variables in objective functions. Based on the results of varying different design variables in objective functions, all relevant operational and physical parameters, investment and operating costs, the following main conclusions of the analysis can be given. According to the results presented in the diagrams in Fig. 4 a single solution from the Pareto optimal frontier could be selected as the representative solution of the SBS/RS. For example, if we deal with the fastest material handling equipment installed in the SBS/RS this will have an impact on high-energy consumption and consequently on a high investment cost of the SBS/RS. When comparing different combination of factors in Table 2, it was found out, that high velocity profiles demand high engine power, which has a consequence further on the energy consumption and CO_2 emissions. In recent years, the producers have introduced to the market some equipment that is able to reach extremely high velocities. The question is do we really need such extremely fast material handling equipment, which further has an increasingly high impact on the energy consumption and CO_2 emissions. Opposite if, the throughput time is relatively high, this will have impact on relatively low energy consumption and relatively low investment cost of the SBS/RS. In this case, the investment in the warehouse is moderate in comparison with the selection of highly efficient material handling equipment. According to the proposed model for design and optimisation of SBS/RS, we can conclude, that the presented results are useful for engineering practice. Based on the results shown in Fig. 4 and in Table 2, one can relatively quickly select the most efficient type of SBS/RS. For the future work the velocity-time relationship, which is associated with the acceleration and deceleration only next to the current velocity-time relationship (acceleration, constant velocity and deceleration) will be included in the proposed travel-time model.

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References

Accorsi, R.; Bortolini, M.; Faccio, M.; Gamberi, M.; Manzini, R.; Pilati, F. 2015. Time and energy based assignment strategy for unit-load AS/RS warehouses, in *Proceedings of the 23rd International Conference on Production Research* (ICPR23), 2–5 August 2015, Manila, Philippines, 1–15.

Bekker, J. 2013. Multi-objective buffer space allocation with the cross-entropy method, *International Journal of Simulation Modelling* 12(1): 50–61. http://doi.org/10.2507/IJSIMM12(1)5.228

Bortolini, M.; Accorsi, R.; Gamberi, M.; Manzini, R.; Regattieri, A. 2015a. Optimal design of AS/RS storage systems with three-class-based assignment strategy under single and dual command operations, *The International Journal of Advanced Manufacturing Technology* 79(9): 1747–1759. http://doi.org/10.1007/s00170-015-6872-1

Bortolini, M.; Faccio, M.; Gamberi, M.; Manzini, R. 2015b. Diagonal cross-aisles in unit load warehouses to increase handling performance, *International Journal of Production Economics* 170(C): 838–849. http://doi.org/10.1016/j.ijpe.2015.07.009

Bozer, Y. A.; White, J. A. 1984. Travel-time models for automated storage/retrieval systems, *IIE Transactions* 16(4): 329–338. http://doi.org/10.1080/07408178408975252

Carlo, H. J.; Vis, I. F. A. 2012. Sequencing dynamic storage systems with multiple lifts and shuttles, *International Journal of Production Economics* 140(2): 844–853. http://doi.org/10.1016/j.ijpe.2012.06.035

Deb, K.; Pratap, A.; Agarwal, S.; Meyarivan, T. 2002. A fast and elitist multiobjective genetic algorithm: NSGA-II, *IEEE Transactions on Evolutionary Computation* 6(2): 182–197. http://doi.org/10.1109/4235.996017

Diao, X.; Li, H.; Zeng, S.; Tam, V. W. Y.; Guo, H. 2011. A Pareto multi-objective optimization approach for solving time-cost-quality tradeoff problems, *Technological and Economic Development of Economy* 17(1): 22–41. http://doi.org/10.3846/13928619.2011.553988

Ekren, B. Y. 2011. Performance evaluation of AVS/RS under various design scenarios: a case study, *The International Journal of Advanced Manufacturing Technology* 55(9): 1253–1261. http://doi.org/10.1007/s00170-010-3137-x

Ekren, B. Y.; Heragu, S. S. 2011. Simulation based performance analysis of an autonomous vehicle storage and retrieval system, *Simulation Modelling Practice and Theory* 19(7): 1640–1650. http://doi.org/10.1016/j.simpat.2011.02.008

Ekren, B. Y.; Heragu, S. S.; Krishnamurthy, A.; Malmborg, C. J. 2014. Matrix-geometric solution for semi-open queuing network model of autonomous vehicle storage and retrieval system, *Computers & Industrial Engineering* 68: 78–86. http://doi.org/10.1016/j.cie.2013.12.002

Ekren, B. Y.; Heragu, S. S.; Krishnamurthy, A.; Malmborg, C. J. 2013. An approximate solution for semi-open queueing network model of an autonomous vehicle storage and retrieval system, *IEEE Transactions on Automation Science and Engineering* 10(1): 205–215. http://doi.org/10.1109/TASE.2012.2200676

Ekren, B. Y.; Heragu, S. S.; Krishnamurthy, A.; Malmborg, C. J. 2010. Simulation based experimental design to identify factors affecting performance of AVS/RS, *Computers & Industrial Engineering* 58(1): 175–185. http://doi.org/10.1016/j.cie.2009.10.004

Fukunari, M.; Malmborg, C. J. 2009. A network queuing approach for evaluation of performance measures in autonomous vehicle storage and retrieval systems, *European Journal of Operational Research* 193(1): 152–167. http://doi.org/10.1016/j.ejor.2007.10.049

Fukunari, M.; Malmborg, C. J. 2008. An efficient cycle time model for autonomous vehicle storage and retrieval systems, *International Journal of Production Research* 46(12): 3167–3184. http://doi.org/10.1080/00207540601118454

Gudehus, T. 1973. *Grundlagen der Kommissioniertechnik. Dynamik der Warenverteil- und Lagersysteme*. Cornelsen Verlag GmbH + C. 214 S. (in German).

Graves, S. C.; Hausman, W. H.; Schwarz, L. B. 1977. Storage-retrieval interleaving in automatic warehousing systems, *Management Science* 23(9): 935–945. http://doi.org/10.1287/mnsc.23.9.935

Gu, J.; Goetschalckx, M.; McGinnis, L. F. 2007 Research on warehouse operation: a comprehensive review, *European Journal of Operational Research* 177(1): 1–21. http://doi.org/10.1016/j.ejor.2006.02.025

Hausman, W. H.; Schwarz, L. B.; Graves, S. C. 1976. Optimal storage assignment in automatic warehousing systems, *Management Science* 22(6): 629–638. http://doi.org/10.1287/mnsc.22.6.629

Holland, J. H. 1992. *Adaptation in Natural and Artificial Systems: an Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence*. MIT Press. 211 p.

Hwang, H.; Lee, S. B. 1990. Travel-time models considering the operating characteristics of the storage and retrieval machine, *International Journal of Production Research* 28(10): 1779–1789. http://doi.org/10.1080/00207549008942833

Janilionis, V. V.; Bazaras, Ž.; Janilionis, V. 2016. Comparison of routing algorithms for storage and retrieval mechanism in cylindrical AS/RS, *Transport* 31(1): 11–21. http://doi.org/10.3846/16484142.2014.995130

Kuo, P.-H.; Krishnamurthy, A.; Malmborg, C. J. 2008. Performance modelling of autonomous vehicle storage and retrieval systems using class-based storage policies, *International Journal of Computer Applications in Technology* 31(3–4): 238–248. http://doi.org/10.1504/IJCAT.2008.018160

Kuo, P.-H.; Krishnamurthy, A.; Malmborg, C. J. 2007. Design models for unit load storage and retrieval systems using autonomous vehicle technology and resource conserving storage and dwell point policies, *Applied Mathematical Modelling* 31(10): 2332–2346. http://doi.org/10.1016/j.apm.2006.09.011

Lerher, T. 2013. Modern automation in warehousing by using the shuttle-based technology, in D. Arent, M. Freebush (Eds.). *Automation Systems of the 21st Century: New Technologies, Applications and Impacts on the Environment & Industrial Processes*, 51–86.

Lerher, T.; Borovinšek, M.; Šraml, M. 2013. A multi objective model for optimization of automated warehouses, in J. Cheung, H. Song (Eds.). *Logistics: Perspectives, Approaches and Challenges*, 87–110.

Lerher, T.; Edl, M.; Rosi, B. 2014. Energy efficiency model for the mini-load automated storage and retrieval systems, *The International Journal of Advanced Manufacturing Technology* 70(1): 97–115. http://doi.org/10.1007/s00170-013-5253-x

Lerher, T.; Ekren, B. Y.; Dukic, G.; Rosi, B. 2015a. Travel time model for shuttle-based storage and retrieval systems, *The International Journal of Advanced Manufacturing Technology* 78(9): 1705–1725. http://doi.org/10.1007/s00170-014-6726-2

Lerher, T.; Ekren, Y. B.; Sari, Z.; Rosi, B. 2015b. Simulation analysis of shuttle based storage and retrieval systems, *In-*

ternational Journal of Simulation Modelling 14(1): 48–59. http://doi.org/10.2507/IJSIMM14(1)5.281

Lerher, T.; Sraml, M.; Kramberger, J.; Potrc, I.; Borovinsek, M.; Zmazek, B. 2006 Analytical travel time models for multi aisle automated storage and retrieval systems, *The International Journal of Advanced Manufacturing Technology* 30(3): 340–356. http://doi.org/10.1007/s00170-005-0061-6

Lerher, T.; Šraml, M.; Potrč, I. 2011. Simulation analysis of mini-load multi-shuttle automated storage and retrieval systems, *The International Journal of Advanced Manufacturing Technology* 54(1): 337–348. http://doi.org/10.1007/s00170-010-2916-8

Malmborg, C. J. 2003. Interleaving dynamics in autonomous vehicle storage and retrieval systems, *International Journal of Production Research* 41(5): 1057–1069. http://doi.org/10.1080/0020754021000033887

Malmborg, C. J. 2002. Conceptualizing tools for autonomous vehicle storage and retrieval systems, *International Journal of Production Research* 40(8): 1807–1822. http://doi.org/10.1080/00207540110118668

Marchet, G.; Melacini, M.; Perotti, S.; Tappia, E. 2013. Development of a framework for the design of autonomous vehicle storage and retrieval systems, *International Journal of Production Research* 51(14): 4365–4387. http://doi.org/10.1080/00207543.2013.778430

Marchet, G.; Melacini, M.; Perotti, S.; Tappia, E. 2012. Analytical model to estimate performances of autonomous vehicle storage and retrieval systems for product totes, *International Journal of Production Research* 50(24): 7134–7148. http://doi.org/10.1080/00207543.2011.639815

Roodbergen, K. J.; Vis, I. F. A. 2009. A survey of literature on automated storage and retrieval systems, *European Journal of Operational Research* 194(2): 343–362. http://doi.org/10.1016/j.ejor.2008.01.038

Rouwenhorst, B.; Reuter, B.; Stockrahm, V; Van Houtum, G. J.; Mantel, R. J.; Zijm, W. H. M. 2000. Warehouse design and control: framework and literature review, *European Journal of Operational Research* 122(3): 515–533. http://doi.org/10.1016/S0377-2217(99)00020-X

Roy, D.; Krishnamurthy, A.; Heragu, S. S.; Malmborg, C. J. 2012. Performance analysis and design trade-offs in warehouses with autonomous vehicle technology, *IIE Transactions* 44(12): 1045–1060. http://doi.org/10.1080/0740817X.2012.665201

Sari, Z.; Saygin, C.; Ghouali, N. 2005 Travel-time models for flow-rack automated storage and retrieval systems, *The International Journal of Advanced Manufacturing Technology* 25(9): 979–987. http://doi.org/10.1007/s00170-003-1932-3

Smew, W.; Young, P.; Geraghty, J. 2013. Supply chain analysis using simulation, gaussian process modelling and optimisation, *International Journal of Simulation Modelling* 12(3): 178–189. http://doi.org/10.2507/IJSIMM12(3)4.239

Zhang, L.; Krishnamurthy, A.; Malmborg, C. J.; Heragu, S. S. 2009. Variance-based approximations of transaction waiting times in autonomous vehicle storage and retrieval systems, *European Journal of Industrial Engineering* 3(2): 146–168. http://doi.org/10.1504/EJIE.2009.023603

APPENDIXES

Appendix A. Verification of Expressions (5) and (6)

A.1. Verification of the Expression (5)

Travel time of the elevator's lifting table to the most distant tier in the SBS/RS is calculated by:

$$t\left(d_y\right)=\frac{v_y}{a_y}+\frac{d_y}{v_y},\quad 0\le z\le d_y. \tag{20}$$

Under the randomized storage policy, the probability distribution function $F_y(z)$ and probability density function $f_y(z)$ of z_i ($i=1, 2, \ldots, n$) are as follows. Probability distribution function $F_y(z)$ from Bozer and White (1984) is calculated by:

$$F_y\left(z\right)=\begin{cases}\dfrac{z}{d_y}, & 0\le z\le d_y;\\ 1, & z\ge d_y.\end{cases} \tag{21}$$

Probability density function $f_y(z)$ is calculated by:

$$f_y\left(z\right)=\frac{dF_y\left(z\right)}{dz}=\begin{cases}\dfrac{1}{d_y}, & 0\le z\le d_y;\\ 0, & z\ge d_y.\end{cases} \tag{22}$$

The expected one way travel time $E(ES)_{LIFT}$ for the lifting of the elevator's lifting table in the SBS/RS is equal to the next expression:

$$E\left(ES\right)_{LIFT}=\frac{v_y}{a_y}+\frac{1}{v_y}\cdot\int_0^{d_y} z\cdot f_y\left(z\right)dz=\frac{v_y}{a_y}+\frac{d_y}{2\cdot v_y}. \tag{23}$$

The expected SCC time $E(SCC)_{LIFT}$ when distance d_y equals H_{SR}, is calculated by:

$$E\left(SCC\right)_{LIFT}=2\cdot t_{P/S}+2\cdot E(ES)_{LIFT};$$

$$E\left(SCC\right)_{LIFT}=2\cdot t_{P/S}+2\cdot\left(\frac{v_y}{a_y}+\frac{d_y}{2\cdot v_y}\right);$$

$$E\left(SCC\right)_{LIFT}=2\cdot t_{P/S}+2\cdot\frac{v_y}{a_y}+\frac{H_{SR}}{v_y}, \tag{24}$$

where: $t_{P/S}$ stands for the pick-up and set-down times for the elevator's lifting table.

A.2. Verification of the Expression (6)

Probability distribution function $F_y(z)$ from Bozer and White (1984) is calculated by:

$$F_y(z)=\begin{cases}\dfrac{2z}{d_y}-\dfrac{z^2}{d_y{}^2}, & 0\le z\le d_y;\\ 1, & z\ge d_y.\end{cases} \tag{25}$$

Probability density function $f_y(z)$ is calculated by:

$$f_y(z)=\frac{dF_y(z)}{dz}=\begin{cases}\frac{2}{d_y}-\frac{2z}{d_y^{2}}, & 0\le z\le d_y;\\ 0, & z\ge d_y.\end{cases} \tag{26}$$

The expected travel-between time $E(TB)_{LIFT}$ time for two randomly selected tiers $M_1(y_1)$ and $M_2(y_2)$ in the SBS/RS is equal to:

$$E(TB)_{LIFT}=\frac{v_y}{a_y}+\frac{1}{v_y}\cdot\int_0^{d_y} z\cdot\left(\frac{2}{d_y}-\frac{2z}{d_y^{2}}\right)dz=$$

$$\frac{v_y}{a_y}+\frac{d_y}{3\cdot v_y}. \tag{27}$$

The expected DCC time $E(DCC)_{LIFT}$ when distance d_y equals H_{SR}, is calculated by:

$$E(DCC)_{LIFT}=4\cdot t_{P/S}+2\cdot E(ES)_{LIFT}+E(TB)_{LIFT};$$

$$E(DCC)_{LIFT}=4\cdot t_{P/S}+2\cdot\left(\frac{v_y}{a_y}+\frac{d_y}{2\cdot v_y}\right)+\frac{v_y}{a_y}+\frac{d_y}{3\cdot v_y};$$

$$E(DCC)_{LIFT}=4\cdot t_{P/S}+3\cdot\frac{v_y}{a_y}+\frac{4}{3}\cdot\frac{H_{SR}}{v_y}, \tag{28}$$

where: $t_{P/S}$ stands for the pick-up and set-down times for the elevator's lifting table.

Appendix B. Verification of Expressions (8) and (9)

B.1. Verification of the Expression (8)

Travel time of the shuttle carrier to the most distant storage location (cell) in the SBS/RS is calculated by:

$$t(d_x)=\frac{v_x}{a_x}+\frac{d_x}{v_x}, \tag{29}$$

$$0\le z\le d_x.$$

Under the randomized storage policy, the probability distribution function $F_x(z)$ and probability density function $f_x(z)$ of z_i ($i=1, 2, \ldots, n$) are as follows. Probability distribution function $F_x(z)$ from Bozer and White (1984) is calculated by:

$$F_x(z)=\begin{cases}\frac{z}{d_x}, & 0\le z\le d_x;\\ 1, & z\ge d.\end{cases} \tag{30}$$

Probability density function $f_x(z)$ is calculated by:

$$f_x(z)=\frac{dF_x(z)}{dz}=\begin{cases}\frac{1}{d_x}, & 0\le z\le d_x;\\ 0, & z\ge d_x.\end{cases} \tag{31}$$

The expected one way travel time $E(ES)_{SCAR}$ for the travelling of the shuttle carrier in the SBS/RS is equal to the next expression:

$$E(ES)_{SCAR}=\frac{v_x}{a_x}+\frac{1}{v_x}\cdot\int_0^{d_x} z\cdot f_x(z)dz=$$

$$\frac{v_x}{a_x}+\frac{d_x}{2\cdot v_x}. \tag{32}$$

The expected SCC time $E(SCC)_{SCAR}$ when distance d_x equals L_{SR} is calculated by:

$$E(SCC)_{SCAR}=2\cdot t_{P/S}+2\cdot E(ES)_{SCAR};$$

$$E(SCC)_{SCAR}=2\cdot t_{P/S}+2\cdot\left(\frac{v_x}{a_x}+\frac{d_x}{2\cdot v_x}\right);$$

$$E(SCC)_{SCAR}=2\cdot t_{P/S}+2\cdot\frac{v_x}{a_x}+\frac{L_{SR}}{v_x}, \tag{33}$$

where: $t_{P/S}$ stands for the pick-up and set-down times for the shuttle carrier.

B.2. Verification of the Expression (9)

Probability distribution function $F_x(z)$ from Bozer and White (1984) is calculated by:

$$F_x(z)=\begin{cases}\frac{2z}{d_x}-\frac{z^2}{d_x^{2}}, & 0\le z\le d_x;\\ 1, & z\ge d_x.\end{cases} \tag{34}$$

Probability density function $f_x(z)$ is calculated by:

$$f_x(z)=\frac{dF_x(z)}{dz}=\begin{cases}\frac{2}{d_x}-\frac{2z}{d_x^{2}}, & 0\le z\le d_x;\\ 0, & z\ge d_x.\end{cases} \tag{35}$$

The expected travel-between time $E(TB)_{SCAR}$ for two randomly selected location (cells) $L_1(x_1)$ and $L_2(x_2)$ in the SBS/RS is equal to the following expression:

$$E(TB)_{SCAR}=\frac{v_x}{a_x}+\frac{1}{v_x}\cdot\int_0^{d_x} z\cdot\left(\frac{2}{d_x}-\frac{2z}{d_x^{2}}\right)dz=$$

$$\frac{v_x}{a_x}+\frac{d_x}{3\cdot v_x}. \tag{36}$$

The expected DCC time $E(DCC)_{SCAR}$ when distance d_x equals L_{SR}, is calculated by:

$$E(DCC)_{SCAR}=4\cdot t_{P/S}+2\cdot E(ES)_{SCAR}+E(TB)_{SCAR};$$

$$E(DCC)_{SCAR}=4\cdot t_{P/S}+2\cdot\left(\frac{v_x}{a_x}+\frac{d_x}{2\cdot v_x}\right)+\frac{v_x}{a_x}+\frac{d_x}{3\cdot v_x};$$

$$E(DCC)_{SCAR}=4\cdot t_{P/S}+3\cdot\frac{v_x}{a_x}+\frac{4}{3}\cdot\frac{L_{SR}}{v_x}, \tag{37}$$

where: $t_{P/S}$ stands for the pick-up and set-down times for the shuttle carrier.

Appendix C. Verification of Expressions (12) and (13)

C.1. Verification of the Expression (12)

This section provides calculations for the required engine power P of the shuttle carrier for travelling in the horizontal direction.

Travelling of shuttle carrier with constant velocity

When the shuttle carrier travels with constant velocity only ($a_x^+ = 0$), the traction force F_{Tv} on the rear driving wheel equals:

$$F_{Tv} = G \cdot k_r. \tag{38}$$

It can be noticed that the traction force F_{Tv} on the rear driving wheel overcomes the rolling resistance ($F_R = G \cdot k_r$) only.

The size of the traction force F_{Tv} depends on the:
- combined mass of the shuttle carrier and the tote;
- rolling resistance coefficient k_r.

Required engine power P_{Tv} of the shuttle carrier in case of travelling with constant velocity ($a_x^+ = 0$) only, is calculated by:

$$P_{Tv} = \frac{F_{Tv} \cdot v_x}{1000 \cdot \eta}. \tag{39}$$

Travelling of the shuttle carrier with variable velocity

When shuttle carrier accelerates ($a_x^+ \neq 0$), the traction force F_{Ta} on the rear driving wheel equals:

$$F_{Ta} = G \cdot k_r + \frac{G}{g} \cdot a_x^+ \cdot k_{ir}. \tag{40}$$

It can be noticed that the traction force F_{Ta} on the rear driving wheel overcomes the rolling resistance ($F_R = G \cdot k_r$) and the inertial resistance F_{iT}.

The size of the traction force F_{Ta} depends on the:
- combined mass of the shuttle carrier and the tote;
- rolling resistance coefficient k_r;
- size of the acceleration a_x^+;
- factor k_{ir} that takes into account the resistance of rotating masses with variable vehicle speed.

Required engine power P_{Ta} of the shuttle carrier in case of acceleratin g ($a_x^- \neq 0$), is calculated by:

$$P_{Ta} = \frac{F_{Ta} \cdot v_x}{1000 \cdot \eta}. \tag{41}$$

When shuttle carrier decelerates ($a_x^- \neq 0$), the braking force F_B on the rear driving wheel equals (42):

$$F_B = \frac{G}{g} \cdot a_x^- \cdot k_{ir} - G \cdot k_r. \tag{42}$$

It can be noticed that the braking force F_B on the rear driving wheel overcomes the inertial resistance F_{iT}, which is reduced for the rolling resistance F_R. The size of the braking force F_B depends on:
- mass of the shuttle carrier with the container;
- size of deceleration a_x^-;
- factor k_{ir} that takes into account the resistance of rotating masses with variable vehicle speed;
- rolling resistance coefficient k_r.

Required engine power P_B of the shuttle carrier in case of deceleration ($a_x^- \neq 0$), is calculated by:

$$P_B = \frac{F_B \cdot v_x}{1000 \cdot \eta}. \tag{43}$$

Time for acceleration t_{acc} of the shuttle carrier to reach it maximum velocity v_x, time for travelling of the shuttle carrier with constant velocity t_{cvel} and time for deceleration of the shuttle carrier until it stops at the storage location t_{dec} are calculated according to velocity-time expressions of the shuttle carrier:

$$t_{acc} = \frac{v_x}{a_x^+},\ l_{acc} = \frac{v_x^2}{2 \cdot a_x^+};$$

$$t_{dec} = \frac{v_x}{a_x^-},\ l_{dec} = \frac{v_x^2}{2 \cdot a_x^-};$$

$$t_{cvel} = \frac{l_{cvel}}{v_x},\ l_{cvel} = \left(\frac{L_{SR}}{2}\right) - \left(l_{acc} + l_{dec}\right). \tag{44}$$

C.2. Verification of the Expression (13)

This section provides calculations for the required engine power P of the elevator's lifting table for moving in the vertical direction.

Hoisting of the elevator's lifting table with constant velocity

When the elevator's lifting table is hoisted with constant velocity only ($a_y^+ = 0$), the traction force F_{Tv} equals:

$$F_{Tv} = G. \tag{45}$$

It can be noticed that the traction force F_{Tv} overcomes the force of gravity G only. The size of the traction force F_{Tv} depends on the mass of the elevators lifting table with mass of the tote.

Required engine power P_{Tv} of the elevator's lifting table in case of hoisting with the constant velocity ($a_y^+ = 0$), is calculated by:

$$P_{Tv} = \frac{F_{Tv} \cdot v_y}{1000 \cdot \eta}. \tag{46}$$

Hoisting of the elevator's lifting table with non-constant velocity

When the elevator's lifting table accelerates ($a_y^+ \neq 0$), the traction force F_{Ta} is calculated by:

$$F_{Ta} = G + \frac{G}{g} \cdot a_y^+ \cdot k_{ir}. \tag{47}$$

It can be noticed that the traction force F_{Ta} overcomes the force of gravity G and the inertial resistance F_{iT}.

The size of the traction force F_{Ta} depends on the:
- mass of the elevators lifting table along with mass of the tote;
- size of acceleration a_y^+;
- factor k_{ir} that take into account the resistance of rotating masses with variable vehicle speed.

Required engine power P_{Ta} of the elevator's lifting table in the case of hoisting with non-constant velocity ($a_y^+ \neq 0$), is calculated by:

$$P_{Ta} = \frac{F_{Ta} \cdot v_y}{1000 \cdot \eta}. \tag{48}$$

When the elevator's lifting table decelerates ($a_y^- \neq 0$), the braking force F_B is calculated by:

$$F_B = G + \frac{G}{g} \cdot a_y^- \cdot k_{ir} . \tag{49}$$

It can be noticed that the braking force F_B overcomes the force of gravity G and the inertial resistance F_{iT}. The size of the braking force F_B depends on the:

- mass of the elevator's lifting table along with the container;
- size of deceleration a_y^-;
- factor k_{ir} that takes into account the resistance of rotating masses with variable speed.

Required engine power P_B of the elevator's lifting table in case of deceleration ($a_y^- \neq 0$), is calculated by:

$$P_B = \frac{F_B \cdot v_z}{1000 \cdot \eta} . \tag{50}$$

Time for acceleration t_{acc} of the elevator's lifting table to reach the maximum velocity v_y, time for hoisting of the elevator's lifting table with constant velocity t_{cvel} and time for deceleration of the elevator's lifting table until it stops at the storage location t_{dec} are calculated according to velocity-time expressions of the elevator's lifting table:

$$t_{acc} = \frac{v_y}{a_y^+} , \; h_{acc} = \frac{v_y^2}{2 \cdot a_y^+} ;$$

$$t_{dec} = \frac{v_y}{a_y^-} , \; h_{dec} = \frac{v_y^2}{2 \cdot a_y^-} ;$$

$$t_{cvel} = \frac{h_{cvel}}{v_y} , \; h_{cvel} = \left(\frac{H_{SR}}{2} \right) - \left(h_{acc} + h_{dec} \right) . \tag{51}$$

Appendix D. Verification of Expression (15)

The cost definition of the SBS/RS consist of the investment for the elevators with lifting tables, the investment for shuttle carriers, the investment for the storage location, the investment for the storage area, cost for the energy consumption of elevators lifting tables, cost for the energy consumption of shuttle carriers and finally the total cost.

The investment I_{LIFT} for elevator's with lifting tables is calculated by:

$$I_{LIFT} = C_{LIFT} \cdot A , \tag{52}$$

where: C_{LIFT} [EUR/unit] indicates the cost of the selected elevator with lifting tables; A indicates the number of aisles (number of SBS/RS).

The investment I_{SCAR} for shuttle carriers is calculated by:

$$I_{SCAR} = C_{SCAR} \cdot M \cdot A , \tag{53}$$

where: C_{SCAR} [EUR/unit] indicates the cost of the selected shuttle carrier; M indicates the number of tiers; A indicates the number of aisles (number of SBS/RS).

The investment I_{SP} for storage locations is calculated by:

$$I_{SP} = C_{SP} \cdot M \cdot C \cdot 2A , \tag{54}$$

where: C_{SP} [EUR/unit] indicates the cost of storage locations; M indicates the number of tiers; C indicates the number of columns; A indicates the number of aisles (number of SBS/RS).

The investment I_{SA} for storage area of one aisle of SBS/RS is calculated by:

$$I_{SA} = C_{SA} \cdot L_{SR} \cdot W_{SBS/RS} \cdot H_{SR} , \tag{55}$$

where: C_{SA} [EUR/m^2 · year] indicates the cost of storage area per square meter year; L_{SR} indicates the length of the SBS/RS; $W_{\text{SBS/RS}}$ indicates the width of the SBS/RS; H_{SR} indicates the height of the SBS/RS.

Cost for energy consumption of the elevator's lifting tables is calculated by:

$$C_{EC\ LIFT} = c_{EC} \cdot P_{LIFT} \cdot \eta_{LIFT} \cdot T_{shift} \cdot n_{wd} \cdot n_{weeks} \cdot n \cdot A . \tag{56}$$

Amount of cost for energy consumption $C_{EC\ LIFT}$ [EUR/year] counted on a yearly basis of the elevator's lifting tables depends on the cost for 1 kWh of electricity c_{EC}, engine power of the elevator's lifting table P_{LIFT}, efficiency of the elevator's lifting table η_{LIFT}, number of working hours in a shift T_{shift}, number of working days in a week n_{wd}, number of weeks n_{weeks}, number of lifting tables n and number of aisles A.

Cost for energy consumption of shuttle carriers is calculated by:

$$C_{EC\ SCAR} = c_{EC} \cdot P_{SCAR} \cdot \eta_{SCAR} \cdot T_{shift} \cdot n_{wd} \cdot n_{weeks} \cdot M \cdot A . \tag{57}$$

Amount of cost for energy consumption $C_{EC\ SCAR}$ [EUR/year] counted on a yearly basis of shuttle carriers depends on the cost for 1 kWh of electricity c_{EC}, engine power of the shuttle carrier P_{SCAR}, efficiency of the shuttle carrier η_{SCAR}, number of working hours in a shift T_{shift}, number of working days in a week n_{wd}, number of weeks n_{weeks}, number of tiers M and number of aisles A.

Appendix E. A Brief Description of the NSGA II Algorithm

The first population is initialised as by ordinary GA. Then the population is sorted based on non-domination into individual fronts. The first front being the completely non-dominant set in the current population and the second front being dominated only by the individuals in the first front and so on. Each individual in the front is assigned a rank (fitness) values based on the front it belong to. Individuals in the first front have a fitness value of 1, individuals in the second front have a fitness value as 2 and so on.

In addition to fitness value a new parameter called crowding distance is calculated for each individual. The crowding distance is a measurement of how close an individual is to its neighbours. Large average crowding distance of the population results in larger diversity of the population.

Then two parents are selected from the population by binary tournament selection based on the rank and crowding distance. From two random candidates a candidate with smaller rank or, when both ranks are equal, smaller crowding distance is selected as a first parent. The same procedure is then repeated for the second parent. From the parents offspring is generated based on crossover and mutation operators.

Current population and current offspring are added into combined population, which is again sorted based on non-domination. Then the best N individuals are then selected for the new generation, where *N* is the population size. The selection is again based on the rank and on the crowding distance.