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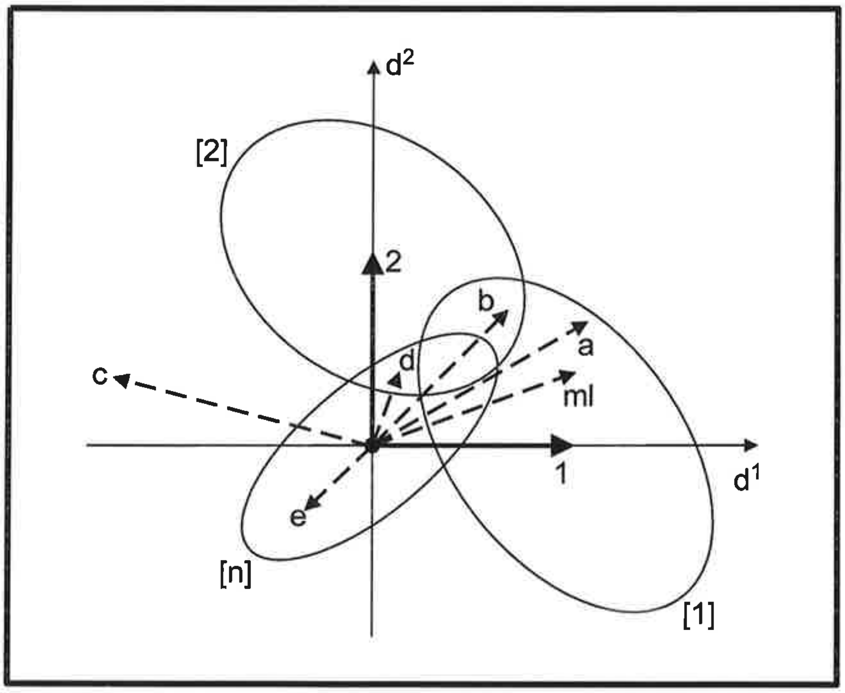

MULTI - PATTERN FINGERPRINT METHOD FOR DETECTION AND ATTRIBUTION OF CLIMATE CHANGE

by

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ABSTRACT
The multi-variate optimal fingerprint method for the detection of an externally forced climate change signal in the presence of natural internal variability is extended to the attribution problem. To determine whether a climate change signal which has been detected in observed climate data can be attributed to a particular climate forcing mechanism, or combination of mechanisms, the predicted space-time dependent climate change signal patterns for the candidate climate forcings must be specified. In addition to the signal patterns, the method requires input information on the space-time dependent covariance matrices of the natural climate variability and the predicted signal pattern errors. The detection and attribution problem is treated as a sequence of individual consistency tests applied to all candidate forcing mechanisms, as well as to the null hypothesis that no climate change has taken place, within the phase space spanned by the predicted climate change patterns. As output the method yields a significance level for the detection of a climate change signal in the observed data and individual confidence levels for the consistency of the retrieved climate change signal with each of the forcing mechanisms. A statistically significant climate change signal is regarded as consistent with a given forcing mechanism if the statistical confidence level exceeds a given critical value, but is attributed to that forcing only if all other climate change mechanisms are rejected at that confidence level. The analysis is carried out using tensor notation, with a metric given by the natural-variability covariance matrix. This clarifies the relation between the covariant signal patterns and their contravariant fingerprint counterparts. The signal patterns define the vector space in which the climate trajectories are analyzed, while the fingerprints are needed to project the climate trajectories onto this space.

1 Introduction

There is mounting evidence that the global warming due to increasing atmospheric greenhouse gas concentrations predicted by state-of-the-art coupled ocean-atmosphere global circulation models (CGCMs) is beginning to emerge from the background noise of natural climate variability (cf. summary in IPCC Second Assessment Report, Santer *et al*, 1995b). However, much of the evidence is still qualitative or circumstantial. There have been relatively few attempts to assign a quantitative measure to the probability that a climate change signal distinct from natural climate variability can be detected in observed climate data.

A basic obstacle for quantitative signal-to-noise analyses is that they require information on the space-time structure of both the predicted climate signal and the climate variability. While the predicted signal properties can be inferred from model computations, the estimation of the required space-time covariance structure of natural climate variability from model simulations and observations is more difficult. Thus although general multi-variate theories for the optimal detection of a space-time dependent climate change signal in the presence of natural climate variability noise have now been developed (Hasselmann, 1979, Bell, 1982, 1986, Hasselmann, 1993, referred to in the following as H, North *et al*, 1995, North and Kim, 1995), gives a summary of the results and presents some conclusions.

2 The detection problem

We review in this section briefly the multi-fingerprint method of multi-variate climate change detection, following the approach of H for the general space-time dependent problem, but returning – for better illustration of the interrelationship between fingerprint and signal patterns – to the co- and contra-variant tensor notation of Hasselmann's (1979) earlier analysis of the spatial signal-to-noise problem (see also Thacker, 1995).

Terminology

We shall use the term climate change in the following to denote the response of the climate system to external forcing, as opposed to natural internal climate variability generated by interactions within the climate system. According to this terminology, climate variations due to volcanic activity or variations in the solar constant are classed as (natural) climate change, rather than as climate variability. An alternative terminology refers to these variations also as natural variability, climate variability being regarded as a superposition of externally forced and internally generated components, the term climate change being reserved for anthropogenic climate modifications only. However, for the detection and attribution problem our definitions will be found to be more convenient. Thus climate change in our terminology can be of either natural or of anthropogenic orgin, while climate variability is always natural. The definition loses precision if interactions between climate change and internal natural climate variability are considered, but in our applications we shall regard the climate state to first order simply as a linear superposition of climate change and climate variability.

The present definitions are more consistent than alternative earlier attempts to distinguish between climate change and climate variability on the basis of time scales, or in terms of climate change 'events' as opposed to 'continuous' climate fluctuations. In practice, the time scales of internal climate variability and externally forced climate change overlap, so that for a given finite time scale it is not possible to distinguish between 'events' and 'continuous fluctuations'. Indeed, the impossibility of distinguishing between externally generated climate change and internal climate variability on the basis of time scale considerations alone is the essence of the detection and attribution problem.

We consider a vector time series $\phi_a(t)$ of climate data, which we assume can be represented as a superposition

$$\phi_a = \phi_a^s + \tilde{\phi}_a \tag{1}$$

of a climate change signal ϕ_a^s and a natural-variability component $\tilde{\phi}_a$. The index a refers to different types of climate data, e.g. temperature or precipitation, and to the location or averaging region of the data. The data set can represent either observed data or synthetic data from a model simulation. The climate vector ϕ_a need not represent a dynamically complete description of the climate state. In fact, the covariance matrix C_{ij}. Thus the operations of index raising and lowering are defined by

$$\begin{aligned} X_{\cdots}^{\cdots i \cdots} &= C^{ij} X_{\cdots j \cdots}^{\cdots} \\ X_{\cdots i \cdots}^{\cdots} &= C_{ij} X_{\cdots}^{\cdots j \cdots} \end{aligned} \tag{6}$$

The definition of the climate trajectory vector as a covariant vector is arbitrary in the present context. The role of co- and contravariant variables can be interchanged. We adopt here the original assignments of Hasselmann (1979).

For each trajectory ψ there exists a constant probability surface $\rho^2(\tilde{\psi}) = C^{ij}\tilde{\psi}_i\tilde{\psi}_j = \text{const} = C^{ij}\psi_i\psi_j$ which contains the vector ψ. We consider then the integral

$$\bar{P}_\rho = \int_{\tilde{\rho}^2 > \rho^2} p(\tilde{\psi}) d\tilde{\psi}_1 \cdots d\tilde{\psi}_n \tag{7}$$

of the n-dimensional probability density over the region $\tilde{\rho}^2(\tilde{\psi}) > \rho^2(\psi)$ outside the surface $\rho^2(\tilde{\psi}) = \rho^2(\psi) = \text{const}$. If $\bar{P}_\rho$ is small, 5%, say, the null hypothesis that ψ represents a realization of the natural variability ensemble is said to be rejected with a risk of $\bar{P}_\rho$. Conversely, a climate change signal is said to have been detected in the data at a significance level of $P_\rho = (1 - \bar{P}_\rho)$ (95%).

Reduction of the detection space

In practice, this straightforward statistical detection test can be applied successfully only if the vector dimension n of the climate state trajectory is small. Unfortunately, the situation is normally just the reverse: the discretization of a set of time series of gridded climate data will normally yield a vector ψ of very high dimension. The problem or many dimensions is that even a relatively large climate change signal ψ^s relative to the noise component in some given but unknown direction in phase space cannot be detected in the presence of noise distributed over a large number of other components. For successful detection and attribution, the dimension of the detection space must be strongly reduced – ideally to a single dimension by specifying the direction of the anticipated climate change signal, or to a small number of climate change patterns if more than one candidate forcing mechanism is considered.

The impact of the number of dimensions on the detection power can best be demonstrated by transforming to ortho-normal variables

$$\psi'_i = T_i^{\ j}\psi_j \tag{8}$$
$$\psi'^i = \hat{T}^i_j \psi^j, \tag{9}$$

where $\hat{T}^i_j$ denotes the transposed inverse of the transformation matrix $T_i^{\ j}$,

$$T_i^{\ j}\hat{T}^i_k = \delta^j_k. \tag{10}$$

In the ortho-normal system, the covariance matrix and its inverse are transformed to the unit co- and contra-variant matrices I_{ij} and I^{ij}, respectively,

$$C'_{ij} = <\tilde{\psi}'_i\tilde{\psi}'_j> = T_i^{\ k}T_j^{\ l}C_{kl} = I_{ij} \tag{11}$$
$$C'^{ij} = <\tilde{\psi}'^i\tilde{\psi}'^j> = \hat{T}^i_k\hat{T}^j_l C^{kl} = I^{ij}. \tag{12}$$

The optimal fingerprint

In the ortho-normal coordinate system, it is self-evident from the isotropic symmetry of the problem that if the signal lies in the direction of the first coordinate, the univariate detection test should also be carried out with respect to the first coordinate. How does this result transform to a signal ψ_i^s oriented in some given guess-pattern direction g_i in an arbitrary coordinate system? To estimate the amplitude of the signal from the observed data ψ in the general case we write

$$\psi_i = d\, g_i + \psi_i^r \tag{18}$$

where the coefficient d (the *detection variable*) is determined by the scalar multiplication of the observed data with a suitably defined fingerprint f^i,

$$d = f^i \psi_i, \tag{19}$$

and ψ_i^r is a residual which we wish to minimize.

It is common practice in many applications to determine the coefficient d by minimizing the mean square error $\sum_i < (\psi_i^r)^2 >$. However, in the present case this is not appropriate. Firstly, the mean square error is not invariant with respect to linear transformations to other variables. Secondly, our goal for the purpose of detection is to not to maximize the explained variance in a particular reference system, but rather to maximize the squared signal-to-noise ratio $d^2 / < \tilde{d}^2 >$ for an arbitrary reference system, where $\tilde{d} = f^i \tilde{\psi}_i$ is the detection variable determined by the natural climate variability in the absence of a climate change signal. Since the signal-to-noise ratio is independent of the scaling of d, for detection applications we need to determine only the direction of the fingerprint. It was shown in H, and is shown again trivially below (see also Hasselmann, 1979), that the maximization of the signal-to-noise ratio yields the fingerprint

$$f^i = C^{ij} g_j \equiv g^i. \tag{20}$$

where the signal pattern and fingerprint can be normalized, without loss of generality, such that

$$\begin{aligned} C^{ij} g_i g_j &= 1, \quad \text{or} & (21)\\ C_{ij} f^i f^j &= f^i g_i = 1. & (22) \end{aligned}$$

Thus the optimal fingerprint represents the contravariant counterpart of the covariant guess pattern. (We nevertheless use different symbols for the fingerprint and signal rather than distinguishing the two only by the position of the index to emphasize the basic difference in the role of the two patterns. In the detection literature this distinction is sometimes overseen.)

In the present co- and contravariant notation the result (20)-(22) follows immediately from the argument indicated above that in the special case of an ortho-normal reference system, $C'^{ij} = I^{ij}$ = unit matrix, the fingerprint and signal pattern must have the same directions for reasons of isotropic symmetry:

$$f'^i = I^{ij} g'_j = C'^{ij} g'_j, \tag{23}$$

of the guess patterns (applying the summation convention also to the indices ν of the p guess patterns), the condition that the quadratic form $\rho^{r2} = \rho^2(\psi^r)$, cf. eq.(4), for the residual is minimized (maximizing also the multi-variate signal-to-noise ratio for the coefficient vector $\mathbf{d} = (d^\nu)$) yields as determining equations for the coefficients d^ν of the retrieved climate change signal the set of p linear equations

$$D_{\nu\mu} d^\mu = f_\nu^i \psi_i \quad (\nu = 1, \ldots, p), \tag{29}$$

where

$$f_\nu^i = C^{ij} g_{\nu j} \; (= g_\nu^i) \tag{30}$$

denotes the fingerprint of the ν'th guess pattern, in analogy with the definition (20) in the single pattern case, and

$$D_{\nu\mu} = f_\nu^i g_{\mu i} = C^{ij} g_{\nu i} g_{\mu j}. \tag{31}$$

The solution can be expressed in a concise form by introducing the operations of index raising and lowering also for Greek guess-pattern indices, using as metric the matrix $D_{\nu\mu}$ defined by the scalar products of the signal patterns. Introducing the covariant multi-pattern detection coefficients, given, in analogy with the definition for the scalar single-pattern detection coefficient d, eq.(19), by

$$d_\nu = f_\nu^i \psi_i, \tag{32}$$

the contravariant detection coefficients may be expressed as

$$d^\nu = D^{\nu\mu} f_\mu^i \psi_i = f^{\nu i} \psi_i, \tag{33}$$

where $D^{\nu\mu}$ denotes the inverse of $D_{\nu\mu}$,

$$D^{\nu\mu} D_{\mu\lambda} = \delta_\lambda^\nu. \tag{34}$$

It follows from eq.(33) that $D^{\nu\mu}$ represents the covariance matrix of the natural variability components $\tilde{d}^\nu$ of the contravariant detection coefficients,

$$D^{\nu\mu} = <\tilde{d}^\nu \tilde{d}^\mu> = f^{\nu i} f^{\mu j} <\psi_i \psi_j> = f^{\nu i} g_i^\mu, \tag{35}$$

while

$$D_{\nu\mu} = <\tilde{d}_\nu \tilde{d}_\mu> = f_\nu^i f_\mu^j <\psi_i \psi_j> = f_\nu^i g_{\mu i} \tag{36}$$

represents the corresponding covariance matrix of the natural variability contribution of the covariant detection coefficients.

Depending on the context, the multi-pattern detection problem is seen to lead to a detection vector which can appear either in a co- or a contravariant form with respect to the metric $D_{\nu\mu}$. We shall refer to the contravariant detection coefficients d^ν, which appear in the original representation (28) of the climate trajectory in terms of the signal patterns, as *pattern amplitudes.* The covariant detection coefficients d_ν, defined by the straightforward generalization, eq.(32), of the expression (19) for the scalar detection variable, will be termed simply the *detection variables.* The detection variables are the variables which arise naturally in the multivariate attributed to internal natural climate variability. For the attribution problem we need to consider now further hypotheses regarding the cause of a detected climate change. We assume there exist generally several candidate mechanisms $\nu = 1, \dots, p$, each of which is characterized by a predicted climate change signal. In contrast to the detection problem, where we needed to know only the normalized directions $\mathbf{g}_\nu$ of the signal patterns, we specify now also the predicted amplitudes a^ν of the signals.

To decide whether the climate change signal $\psi^o_{(\nu)}$ inferred from observations is consistent with a given signal $\psi^m_{(\nu)}$ predicted from a model simulation, we must assign to each predicted climate change signal an error covariance matrix – in analogy with the natural variability covariance matrix required for the detection test. We assume again that the error distributions are Gaussian. The consistency of the retrieved climate change signal with the predicted signal is then tested by comparing the difference between the two signals with the differences which could be expected from the estimated signal errors. We shall be concerned only with the distinction between different signals in the space spanned by the p predicted signal patterns. Thus we need consider only the projection of the signal pattern errors in this signal pattern space.

We assume that the p predicted signal patterns are linearly independent and therefore do indeed span a p-dimensional space. However, we can allow also additional forcing mechanisms which generate climate change signals lying in this space (for example, by explicitly considering linear combinations of the p basic forcing mechanisms, such as a combined greenhouse gas and aerosol forcing, cf. Hegerl *et al*, 1996b). If the pattern amplitudes of such linearly combined climate change signals are prescribed, the attribution (or consistency) tests can be applied in the same way to these signals as to the p base signals. Formally, one needs only to replace one of the original base signals by the linear combination selected for the consistency test (note that the signal patterns $\mathbf{g}_\nu$ are assumed to be normalized by eq.(21), but are not necessarily orthogonal).

The consistency test described in the following is carried out for each forcing mechanism separately. The outcome can be that one, none, or some sub-set of the forcings is consistent with the inferred climate change. If the observations are found to be consistent with exactly one forcing mechanism, and the null hypothesis that the retrieved climate change signal is consistent with natural climate variability is rejected, the retrieved climate change is attributed to that mechanism.

3.1 Consistency and attribution tests

Having retrieved the observed climate change signal

$$\psi^o = d^\mu \mathbf{g}_\mu, \tag{43}$$

with pattern amplitudes d^μ given by the solutions of eqs. (29), we investigate now for each proposed forcing mechanism ν whether the retrieved signal is consistent with the predicted climate change signal

$$\psi^m = a^{(\nu)} \mathbf{g}_{(\nu)} \tag{44}$$

For the consistency test we apply the same approach as in the detection test. The null hypothesis is replaced now by the consistency hypothesis, and the retrieved pattern amplitude vector by the difference amplitude vector. Apart from this change in terminology, the concepts are identical to those introduced for the detection test. For any given amplitude difference vector $\epsilon_{(\nu)}$ there exists a surface $\rho_\epsilon^2 = \text{const}$ which contains the vector. We consider then the integral

$$\bar{P}_{\rho_\epsilon} = \int_{\tilde{\rho}_\epsilon^2 > \rho_\epsilon^2} p_\epsilon \left(\tilde{\epsilon}_{(\nu)}\right) d\tilde{\epsilon}_{(\nu)}^1 \cdots d\tilde{\epsilon}_{(\nu)}^p \tag{51}$$

of the p-dimensional probability density p_ϵ over the region $\tilde{\rho}_\epsilon^2(\tilde{\epsilon}_{(\nu)}) > \rho_\epsilon^2(\epsilon_{(\nu)})$ outside the surface $\tilde{\rho}_\epsilon^2(\tilde{\epsilon}_{(\nu)}) = \rho_\epsilon^2(\epsilon_{(\nu)}) = \text{const}$.

If $\bar{P}_{\rho_\epsilon}$ is small, 5%, say, the hypothesis that the retrieved climate change signal is consistent with the forcing mechanism ν is said to be rejected with a risk of $\bar{P}_{\rho_\epsilon}$, or at a significance level of $P_{\rho_\epsilon} = (1 - \bar{P}_{\rho_\epsilon})$ (95%).

We note that a positive outcome of the statistical detection test (i.e. the rejection of the null hypothesis) is formally analagous to a negative outcome of the consistency test (i.e. the rejection of the consistency hypothesis). A positive outcome of the consistency test should therefore be expressed formally in the double negative form that the retrieved climate change signal is not inconsistent with the proposed forcing mechanism at a given significance level P. However, if the chosen significance level P is high, 95%, say, this statement is rather weak (a high significance level is normally chosen to yield a strong statement for the converse case that the attribution test is rejected). To avoid the cumbersome double negative wording, while at the same time conveying more accurately the statistical significance of a positive outcome of a consistency test, we shall replace the statement that 'a retrieved climate change signal is *not inconsistent* with a given forcing mechanism at a *significance level* of P (95%)' by the simpler positive statement that 'a climate change signal is *consistent* with the forcing mechanism within the *P (95%)- confidence region*' (in analogy with the terminology of power spectral analysis) or 'at a *confidence level* of $\bar{P}$ (5%)'. Note that the stringency of the consistency test increases with decreasing P or increasing $\bar{P}$. For $P \to 0$, the confidence region contracts to zero, requiring zero error between the retrieved and predicted pattern amplitudes for a positive outcome of the consistency test, while the confidence level $\bar{P}$ for a consistent signal increases to 100%. For the acceptance of a consistency test as positive, it will generally be advisable to select a consistency confidence level somewhat higher than 5%, of the order of 10% - 20%. Still higher confidence levels, however, incur the risk of erroneously rejecting valid attributions.

As outcome of the combined multi-pattern detection/attribution exercise we can then assign a statistical *significance level*, defined by eq.(42), for the detection of a climate change signal within the space spanned by the p predicted signal patterns; and a *consistency confidence* level for each proposed climate change mechanism ν, defined, in analogy with the risk associated with the null hypothesis, by eq.(51).

The result of the test will consist generally of one of the following combinations (cf. Figure 1):

1. A statistically significant climate change signal a consisting of a superposition of predicted climate change signals is detected in the observed data at a given

5. The retrieved climate change signal e is not statistically significant and the retrieved climate change signal is not consistent with any of the predicted signals.

We note that the attribution of a detected climate change signal to a particular forcing mechanism is successful only in the first of these possible outcomes.

One can consider various modifications of the test procedure outlined above. Rather than determining the retrieved climate change signal in the p-dimensional space of all proposed signal patterns, the detection and attribution test can be carried out as a single-pattern analysis separately for each individual mechanism (yielding the same set of possible test outcomes). This has the advantage of enhancing the probability of detection of any given forcing signal. However, it provides less discrimination between competing mechanisms when the signal patterns are not orthogonal. The signal pattern a of Figure 1, for example, fails the consistency test for the forcing mechanism 2 in the full signal pattern space, but would pass an individual pattern consistency test for this process (as is apparent from a visual projection of the retrieved signal vector onto the direction of the signal pattern 2). Thus in contrast to the two-pattern analysis, a unique attribution is no longer achieved in this case using individual single pattern consistency tests (see also the similar example discussed in Hegerl *et al*, 1996b)).

Another modification is suggested if one of the predicted signals is consistent with a zero amplitude with acceptable probability, and the detection/attribution test also returns a small amplitude for that signal. One can then repeat the test leaving out that forcing mechanism, in the expectation that the significance and confidence levels for the detection and attribution of the other signals are thereby enhanced.

We note, however, that in our formulation of the attribution problem we have not considered the possibility that a proposed forcing mechanism, once introduced, simply does not exist. A proposed mechanism can only be rejected as not consistent statistically with the observations, or the retrieved signal, although consistent statistically with the predicted signal, can be so small that it is nevertheless not distinguishable statistically from zero.

To establish an optimal trade-off between a high detection significance level (requiring a small number of patterns) and the ability to discriminate between different competing climate forcing mechanisms (requiring a larger number of patterns), one can apply also a series of detection/attribution tests at different levels, each successive level involving an increase in the number of patterns. A similar optimal trade-off between statistical significance and the number of predictors has been applied in the construction of a hierarchy of statistical linear prediction models from a finite data set, cf. Barnett and Hasselmann (1979).

3.2 Maximum likelihood estimate of the climate change signal

If a detected climate change signal has been successfully attributed to a particular forcing mechanism ν, one may ask whether the climate change signal retrieved from the observations is necessarily the best estimate of the climate change signal. The retrieved signal is determined by projection of the observed climate trajectory onto climate change signal is always trivially consistent in this limit to the proposed forcing mechanism, since the retrieved climate change signal will always lie within the very large error bounds of the prediction.

The opposite limit of large $\hat{E}^{(\nu)}_{\mu\lambda}$ compared with $D_{\mu\lambda}$, i.e. very accurately determined differences between the predicted and retrieved pattern coefficients relative to the statistical errors in the retrieved optimal-detection pattern coefficients, formally yields the solution

$$d^{\mu}_{(\nu)} = \delta^{\mu}_{(\nu)} a^{(\nu)}, \tag{54}$$

i.e. the maximum likelihood signal is identical to the predicted signal. However, this limit is unaccessable, since the errors in the differences between the predicted and retrieved pattern amplitudes are always larger, according to eq.(47), than the statistical errors in the retrieved optimal-detection pattern coefficients. The largest values of $\hat{E}^{(\nu)}_{\mu\lambda}$ are obtained when the model errors $M^{\mu\lambda}_{(\nu)}$ vanish, so that eq.(47) yields $E^{\mu\lambda}_{(\nu)} = D^{\mu\lambda}$. In this case eq.(53) reduces to

$$2D_{\mu\lambda} d^{\lambda}_{(\nu)} = d_{\mu} + \hat{D}_{\mu(\nu)} a^{(\nu)}. \tag{55}$$

Multiplication from the left with $D^{\sigma\mu}$ yields the solution

$$d^{\sigma}_{(\nu)} = \frac{1}{2}\left(d^{\sigma} + \delta^{\sigma}_{(\nu)} a^{(\nu)}\right), \tag{56}$$

i.e. the maximum likelihood solution is given by the mean of the predicted and original retrieved solution.

In practice, neither limiting case will apply, and the maximum likelihood solution will lie somewhere between the original retrieved climate change signal and the limiting, maximally modified solution (56) (cf. Figure 1, signal vector ml).

4 Summary and conclusions

The general multi-pattern optimal fingerprint method for the detection of a space-time dependent climate change signal in the presence of natural climate variability can be readily extended to the problem of attribution. A co- and contra-variant tensor notation, based on a metric given by the space-time dependent covariance matrix C_{ij} of the natural climate variability, simplifies the analysis considerably. The optimal fingerprint patterns f^i_{ν} for detection are identified as the contravariant counterparts of the covariant signal patterns, $f^i_{\nu} = C^{ij} g_{\nu j} = g^i_{\nu}$. For the multi-pattern problem it is useful to introduce a second metric $D_{\nu\mu}$, defined by the scalar products $D_{\nu\mu} = g_{\nu i} g_{\mu j} C^{ij}$, in the p-dimensional space of signal patterns $g_{\nu i}$. The covariant detection variables $d_{\nu} = f^i_{\nu} \psi_i$ represent then the simplest set of coefficients for establishing the detection significance level, while the contravariant coefficients $d^{\nu} = D^{\nu\mu} d_{\mu}$, which require the inversion of the metric $D_{\nu\mu}$, define the amplitudes of the signal patterns g_{ν} estimated from the observed data. The matrices $D_{\nu\mu}$ and $D^{\nu\mu}$ represent also the covariance matrices of the natural variability of the co- and contravariant detection coefficients, respectively.

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