

Multi-Photon Quantum Interference

Zhe-Yu Jeff Ou

Multi-Photon Quantum Interference

 Springer

Zhe-Yu Jeff Ou
Department of Physics
Indiana University-Purdue University Indianapolis
Indianapolis, IN 46202

Library of Congress Control Number: 2007922125

ISBN 978-0-387-25532-3

e-ISBN 978-0-387-25554-5

© 2007 Springer Science+Business Media, LLC

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC, 233 Spring Street, New York, NY 10013, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

9 8 7 6 5 4 3 2 1

springer.com

For my parents,
who did so much for me.

Preface

Quantum interference, as a fundamental phenomenon of quantum mechanics, is what makes quantum physics different from classical Newtonian physics. Optical interference has played some essential roles in the understanding of light. It has fascinated Dirac, the pioneer of the quantum theory of light, since the very beginning, as seen in his famous statement on photon interference: “*Each photon ... interferes only with itself. Interference between different photons never occurs.*” Feynman, who was one of the founders of quantum electrodynamics, wrote, in his well-known lecture series on physics, that the interference phenomenon “*has in it the heart of quantum mechanics..., it contains the only mystery.*” As we explore into the quantum regime in the 21st century, we will find even more presence of quantum interference in our life.

Most commonly-occurring interference phenomena are optical interference, in the form of some beautiful interference fringe patterns. These phenomena have been well-studied by the classical theory of coherence, and documented in Born and Wolf’s classic book *Principle of Optics*. In terms of the language of photon, these phenomena can be categorized as the single-photon interference effect and described by the first part of Dirac’s statement given above. On the other hand, in the situation when more than one photon is involved, the second part of Dirac’s statement is not correct. In this book, we try to understand the phenomena of quantum interference through the multi-photon effects of photon correlation. Our major concern is the temporal correlation among photons and how it influences the interference effect. Because of this, we resort to the multi-frequency description of an optical field.

The multi-photon interference effects discussed in this book will find their applications in many of the quantum information protocols, such as, quantum cryptography and quantum state teleportation. However, the emphasis of this book is on the fundamental physical principle in those protocols. Therefore, we will not cover the broad topics of quantum information processing. Nevertheless, readers may find the multi-frequency description of optical fields to be a good complement to the single-mode treatment found in most discussions on quantum information, and closer to a real experimental environment.

This book is organized into two parts. The first part deals mainly with the two-photon interference effect. The second part studies the effects of more than two photons. In addition to the interference effects, Chapter 2 is devoted to the generation and the spectral properties of a two-photon state in the process of parametric down-conversion, which is the main photon source for the effects studied in this book. We also investigate the coherence of the multi-photon source in Chapter 7, which is the preparation for Chapters 8-10. The complementary principle of quantum mechanics is demonstrated in a quantitative fashion in Chapter 9, when we discuss the relation between photon distinguishability and multi-photon interference effects.

This book is based on a tutorial lecture series held during the Yellow Mountain Workshop on Quantum Information in 2001. I would like to thank Professor Guang-can Guo of the University of Science and Technology of China for inviting me to the workshop and for his generous support.

Indianapolis, September, 2006

Zhe-Yu Jeff Ou

Contents

Part I Two-Photon Interference

1	Historical Background and General Remarks	3
1.1	Interference between Independent Lasers: Magyar-Mandel and Pfleegor-Mandel Experiments	4
1.2	Two-Photon Interpretation of Pfleegor-Mandel Experiment ...	6
1.3	Two-Photon Interference with Quantum Sources	8
	References	15
2	Quantum State from Parametric Down-Conversion	17
2.1	Introduction	17
2.2	Spontaneous Parametric Down-Conversion Process	21
2.3	Phase Matching Condition and Spectral Bandwidth.....	27
2.3.1	Type-I Phase Matching	27
2.3.2	Type-II Phase Matching.....	30
2.4	Quantum State with a Narrow Band Pump Field	34
2.5	Quantum State with a Wide Band Pump Field	39
	References	41
3	Hong-Ou-Mandel Interferometer	43
3.1	Single-Mode Consideration	43
3.2	Multi-Mode Treatment and Hong-Ou-Mandel Dip	45
3.2.1	Narrow Band Pumping.....	49
3.2.2	Wide Band Pumping	52
3.2.3	Dispersion Cancellation	55
3.3	A Nonlocal Two-Photon Interference Effect	56
3.4	Photon Bunching in Hong-Ou-Mandel Interferometer	59
	References	61

4	Phase-Independent Two-Photon Interference	63
4.1	Two-Photon Polarization Entanglement	63
4.1.1	Polarization Hong-Ou-Mandel Interferometer and Violation of Bell's Inequalities.....	63
4.1.2	Photon Anti-Bunching Effect and Bell State Measurements	67
4.2	Two-Photon Frequency Entanglement and Spatial Beating....	70
4.3	Two-Photon Interference Fringes and Ghost Fringes.....	73
4.3.1	Two-Photon Interference Fringes	74
4.3.2	Spatial Correlation and Ghost Fringes	78
	References	82
5	Phase-Dependent Two-Photon Interference: Two-Photon Interferometry	83
5.1	Two-Photon Interferometer with Two Down-Converters	83
5.1.1	Phase Memory by Entanglement with Vacuum	83
5.1.2	Multi-Mode Theory.....	85
5.2	Franson Interferometer	89
5.2.1	Entanglement in Time	89
5.2.2	Two-Photon Coherence versus One-Photon Coherence: Multi-Mode Analysis	92
5.3	Two-Photon De Broglie Interferometer	95
5.3.1	Maximally Entangled Photon State – the NOON State .	95
5.3.2	Detailed Analysis of Two-Photon De Broglie Interferometer	96
	References	99
6	Interference between a Two-Photon State and a Coherent State	101
6.1	Anti-Bunching by Two-Photon Interference	101
6.2	Multi-Mode Analysis I: CW Case	104
6.3	Multi-Mode Analysis II: Pulsed Case.....	106
	References	108

Part II Quantum Interference of More Than Two Photons

7	Coherence and Multiple Pair Production in Parametric Down-Conversion	113
7.1	Coherence Properties of Spontaneous Parametric Down-Conversion.....	113
7.1.1	Field Correlation Functions and Coherence of Parametric Down-Converted Fields	114
7.1.2	Generation of Transform-Limited Single-Photon Wave Packet by Gated Photon Detection	118

7.2	Multi-Pair Production and Stimulated Pair Emission	121
7.2.1	Pair Statistics and Photon Bunching	122
7.2.2	Stimulated Pair Emission	125
7.2.3	Induced Coherence without Induced Emission	130
7.3	Distinguishable or Indistinguishable Pairs of Photons	132
	References	136
8	Quantum Interference with Two Pairs of Down-Converted Photons	137
8.1	Hong-Ou-Mandel Interferometer for Independent Photons	137
8.1.1	Two-Photon Interference without Gating	138
8.1.2	Two-Photon Interference with Gating: Hong-Ou-Mandel Interferometer for Two Independent Photons	141
8.2	Quantum State Teleportation and Swapping	142
8.2.1	Quantum State Teleportation: Single-Mode Case	143
8.2.2	Quantum State Teleportation: Multi-Mode Case	145
8.2.3	Entanglement Swapping	147
8.3	Distinguishing a Genuine Polarization Entangled Four-Photon State from Two Independent EPR Pairs	149
8.3.1	Single-Mode Analysis	149
8.3.2	Multi-Mode Analysis	151
8.4	Hong-Ou-Mandel Interferometer for Two Pairs of Photons	155
8.4.1	Symmetric Beam Splitter	155
8.4.2	Asymmetric Beam Splitter	159
8.5	Generation of a NOON State by Superposition	163
8.5.1	Interference between a Coherent State and Parametric Down-Conversion	163
8.5.2	A Special N-Photon Interference Scheme	166
8.6	Multi-Photon De Broglie Wavelength by Projection Measurement	169
8.6.1	Projection by Asymmetric Beam Splitters	169
8.6.2	NOON State Projection Measurement	172
8.7	Stimulated Emission and Multi-Photon Interference	177
8.8	Remarks on \mathcal{E} and \mathcal{A} and General Discussion	181
	References	182
9	Temporal Distinguishability of a Multi-Photon State	185
9.1	Hong-Ou-Mandel Interferometer for Characterizing Two-Photon Temporal Distinguishability	186
9.2	Characterizing an N-Photon State in Time	187
9.2.1	General Description of a Multi-Mode N-Photon State	188
9.2.2	Direct Photon Detection from Glauber's Coherence Theory	190

9.2.3	A NOON-State Projection Measurement and Generalized Hong-Ou-Mandel Interferometer	195
9.3	The First Example of $ 1_H, 2_V\rangle$	197
9.4	The General Case of $ 1_H, N_V\rangle$	200
9.5	The General Case of $ k_H, N_V\rangle$	205
9.5.1	General Formula for the Visibility	206
9.5.2	The Special Cases of $ 2_H, 2_V\rangle$, $ 2_H, 3_V\rangle$, and $ 2_H, 4_V\rangle$	206
9.5.3	The Special Case of $ 3_H, 3_V\rangle$	208
9.6	The Scheme for Characterizing the Temporal Distinguishability by an Asymmetric Beam Splitter	208
9.6.1	The Temporal Distinguishability of $ 1_H, N_V\rangle$	210
9.6.2	The Case of $ 2_H, N_V\rangle$	212
9.7	Experimental Realization of the Cases of $ 2_H, 1_V\rangle$, $ 2_H, 2_V\rangle$ with Two Pairs of Down-Converted Photons	214
9.7.1	Generation of the State of $ 2_H, 1_V\rangle$ with Tunable Temporal Distinguishability	214
9.7.2	Distinguishing 4×1 Case from 2×2 Case for the State of $ 2_H, 2_V\rangle$	218
	References	224
10	Homodyne of a Single-Photon State: A Special Multi-Photon Interference	225
10.1	Interference with a Single-Photon State and an N-Photon State at a Symmetric Beam Splitter	225
10.2	Interference of a Single-Photon State and an Arbitrary State	228
10.3	Multi-Mode Consideration	232
	References	236
A	Lossless Beam Splitter	237
	References	243
B	Derivation of the Visibility for $k_H, N_V\rangle$	245
B.1	The Case of $ 2_H, N_V\rangle$	245
B.1.1	The Scenario of $2HmV + (N - m)V$	245
B.1.2	The Scenario of $1HmV + 1HnV + (N - n - m)V$	249
B.2	The Case of $ 3_H, N_V\rangle$	251
B.2.1	The Scenario of $3HmV + (N - m)V$	251
B.2.2	The Scenario of $2HmV + 1HnV + (N - m - n)V$	255
B.2.3	The Scenario of $1HmV + 1HnV + 1HpV + (N - m - n - p)V$	258
B.3	The General Case of $ k_H, N_V\rangle$	261
B.3.1	The Scenario of $k_1VmH + k_2VnH$	262
B.3.2	The Most General Scenario	265
	Index	267