

MULTI-PRODUCT FIRMS AND TRADE LIBERALIZATION*

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December 2006

Abstract

This paper develops a general equilibrium model of multi-product firms and analyzes their behavior during trade liberalization. Firm productivity in a given product is modeled as a combination of firm-level “ability” and firm-product-level “expertise”, both of which are stochastic and unknown prior to the firm’s payment of a sunk cost of entry. Higher firm-level ability raises a firm’s productivity across all products, which induces a positive correlation between a firm’s intensive (output per product) and extensive (number of products) margins. Trade liberalization fosters productivity growth within and across firms and in aggregate by inducing firms to shed marginally productive products and forcing the lowest-productivity firms to exit. Though exporters produce a smaller range of products after liberalization, they increase the share of products sold abroad as well as exports per product. All of these adjustments are shown to be relatively more pronounced in countries’ comparative advantage industries.

Keywords: heterogeneous firms, endogenous product scope, love of variety, core competency

JEL classification: F12, F13, L11

*Bernard and Schott thank the National Science Foundation (SES-0241474 and SES-0550190) and Redding thanks Princeton University and the Centre for Economic Performance (CEP) for research support. We thank Jonathan Eaton, Gordon Hanson, Marc Melitz, Guy Michaels, Peter Neary, Henry Overman, Esteban Rossi-Hansberg and seminar participants at the NBER, LSE, Penn State, University of Pennsylvania, University of British Columbia, UC Davis and UC San Diego for helpful comments and discussion. The research in this paper was conducted while Bernard and Schott were Special Sworn Status researchers of the U.S. Census Bureau at the Boston and New York Research Data Centers. Research results and conclusions expressed are those of the authors and do not necessarily reflect the views of the Census Bureau, the NBER, or any other institution to which the authors are affiliated.

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1. Introduction

Firms producing multiple products dominate domestic production and international trade. In the United States, firms manufacturing more than one product account for more than 90 percent of total manufacturing shipments, while firms that export multiple products represent more than 95 percent of total exports.¹ Despite the overwhelming empirical importance of multi-product firms, comparatively little theoretical research examines how firms' product scope (i.e., the number of goods produced or exported) is determined or how it is influenced by globalization.

This paper develops a general equilibrium model of international trade in which firms that are heterogeneous in both firm-specific ability and firm-product-specific expertise choose endogenous ranges of products to manufacture and sell abroad. The model is motivated by a series of stylized facts about multi-product firms that have emerged recently, most notably a positive correlation between the number of products firms produce and their output per product. The framework captures these stylized facts and returns a number of novel theoretical implications that can be taken to the data.

A key prediction of the model is that trade liberalization induces adjustments along firms' extensive (number of goods) and intensive (sales per product) margins as well as entry and exit. As trade costs fall, reallocations of resources occur both within firms, as surviving firms drop their marginally productive products, and across firms, as firms with the lowest overall productivity exit. Though exporters are among the firms dropping their least-productive products, the share of products they export as well as their sales per exported product rise. Significantly, productivity increases both within firms and within industries as trade costs fall. As a result, the model provides an intuitive micro-foundation for the much-discussed idea that international trade spurs firms to rationalize production, but one that does not imply money being left on the table prior to liberalization.²

Our framework represents a significant departure from existing models of heterogeneous firms (e.g., Jovanovic 1982, Hopenhayn 1992, Ericson and Pakes 1995, Melitz 2003 and Bernard, Redding and Schott 2006a) which restrict firms to production of a single product. The introduction of endogenous product scope creates a new margin of firm adjustment, which generates novel firm and industry dynamics in response to trade liberalization. As in existing heterogeneous firm models, the firms considered in this paper vary in terms of productivity. Here, however, firm "productivity" in a given product is a combination of two capabilities: firm-level "ability" and firm-product-level "expertise". We assume these two components of overall firm productivity to be stochastic, indepen-

¹These figures come from Bernard, Redding and Schott (2006b) and Bernard, Jensen and Schott (2005) and are discussed further below. They define products as five-digit SIC and ten-digit Harmonized System categories, respectively.

²The hypothesis that trade liberalization raises firm productivity by reducing X-inefficiency has a long tradition, but theories of this form face the challenge of explaining X-inefficiency as an equilibrium phenomenon (see for example the discussion in Lawrence 2000).

dent, and unknown prior to firms paying a sunk cost of entry. These assumptions are meant to capture the idea that even though some firms are successful in a broad range of activities (e.g., General Electric), even the most successful firm may not be the most efficient producer of any given product. Though we assume that firm ability and product expertise are uncorrelated, and that a firm's expertise across products is uncorrelated, a firm's productivity across products is nevertheless positively correlated because higher firm ability raises productivity in all products.³

The model works as follows. After a firm incurs the sunk entry cost, and after firm ability as well as product expertise in *all* products are revealed, the firm decides which (if any) of a continuum of products to manufacture. Firms that decide to enter at least one product market must pay a fixed headquarters cost which is independent of the range of products manufactured, as well as fixed production costs for each product market they enter. Marginal costs of production vary across products, declining with firm ability and product expertise. Ability varies across firms and expertise varies across products for a firm with a particular ability. Firms with higher abilities have a lower zero-profit cutoff for product expertise, i.e., the level of product expertise at which variable profit exceeds the fixed production costs. As a result, firms with greater ability can profitably manufacture a larger range of products, extending their product scope. Though firms with high ability enjoy higher average productivity than firms with lower ability, the percentage difference in average productivity is smaller than the percentage difference in ability as a result of their endogenous expansion into lower-expertise products.

The equilibrium of the model exhibits conventional self-selection of the most productive firms into production. Here, self selection across firms is driven by the interaction of stochastic firm ability and fixed headquarters costs. Firms drawing low ability are unable to generate enough variable profit to cover fixed headquarters costs from the small number of products in which they have high expertise. They have low average productivity and exit immediately.

With endogenous product scope, the model also features self-selection *within* firms. Firms' product selection is explained by the interaction of stochastic product expertise and fixed production costs for each product. Firms that become active in a product have systematically higher productivity in that product than in the products they choose not to produce. They also have systematically higher productivity in that product than firms that decide not to produce it.

Stochastic variation in firm ability yields a positive correlation between firms' intensive and extensive margins: more productive firms are on average larger not only because they sell more of a given product but because they manufacture more products. This positive correlation between intensive and extensive margins magnifies inequality in the firm-size distribution and has general equilibrium implications for firms' entry and exit

³An example of a firm ability that influences productivity across all products is proficiency in the organization of production, including the use flexible manufacturing methods and other productivity-enhancing techniques: see for example Milgrom and Roberts (1990).

decisions. As higher-ability firms expand along the extensive margin into lower-expertise products, demand for labor rises, wages increase and the profitability of lower-ability firms declines. Relative to a setting with exogenous product scope, this mechanism drives up the zero-profit threshold for ability above which firm entry is profitable.

One manifestation of the influence of endogenous product scope is the role it plays in dampening firm entry and exit responses to exogenous shocks to market structure. In conventional heterogeneous-firm models, an increase in sunk costs of entry, for example, reduces the zero-profit productivity cutoff necessary for survival, allowing firms with previously unprofitable levels of productivity to enter the market. As the sunk entry cost rises above the expected value of entry, entry from the fringe declines, competition is diminished and the *ex post* profitability of production increases. As a result the zero-profit productivity cutoff falls until the expected value of entry is equal to the new higher sunk entry cost. When product scope is endogenous, the reduction in competition induced by the higher sunk entry cost allows firms to expand along their extensive margins into lower-expertise products. As a result, some of the increase in the *ex post* profitability required for the expected value of entry to equal the new higher sunk cost of entry is achieved through an expansion of product scope, implying a dampened decline in the zero-profit productivity cutoff. While there is less reallocation through firm entry and exit than in a model with exogenous product scope, firm entry and exit is only part of overall reallocation which now also includes reallocation across products within firms.⁴

The open economy equilibrium permits examination of a wide range of firm responses to globalization. When international trade with fixed and variable trade costs is possible, the model reproduces the conventional result that only the highest-productivity firms are profitable enough to overcome trade costs and become exporters. With endogenous product scope, however, exporters ship only a relatively small fraction of their overall output abroad. While exporters' high-expertise products are profitable enough to overcome trade costs and are sent abroad, their lower-expertise goods are confined to the domestic market. In equilibrium the same product might be both exported and sold domestically by some firms and only sold locally by others.

A key result of the model is that trade liberalization pressures firms to focus on their "core competencies".⁵ When product scope is endogenous, symmetric reductions in countries' fixed or variable trade costs cause all firms to drop their lowest-expertise products. This result is driven by the general equilibrium influence of growing export opportunities on labor markets. As trade costs fall, exporters' foreign markets expand,

⁴For empirical evidence on firm entry and exit as a source of reallocation and productivity growth, see Dunne et al. (1989), Baily et al. (1992) and Foster et al. (2001) among many others. For evidence on the role of reallocation across products within firms, see Bernard, Redding and Schott (2006b).

⁵The concept of core competencies appears in both the theory of the firm and the business strategy literature (see for example Aghion and Tirole 1995, Ossa 2006, Porter 1998 and Sutton 2005). As formalized here, core competencies refer to the products where firms have the greatest expertise.

driving up their demand for labor and therefore wages. As wages rise, the profitability of firms' lowest-expertise products falls below the point at which they can cover fixed costs. However, even though product scope contracts for all firms, declining trade costs result in an increase in both the share of products exporters sell abroad as well as their level of exports in each product. Because these reactions are driven by greater export opportunities, they are all relatively more pronounced in countries' comparative-advantage industries.

Firms' intensive- and extensive-margin responses to trade liberalization drive productivity growth both within firms and across firms within industries. As firms drop their lowest-expertise products, they raise the average productivity of products that survive and therefore raise overall firm productivity. Therefore, although firm ability and product expertise are themselves fixed parameters, the model generates firm-level productivity growth following trade liberalization due to reallocations of resources across products within firms. This contrasts with existing models of heterogeneous firms and trade, where firms manufacture a single product whose productivity is unaffected by trade liberalization. Along with the conventional result that symmetric reductions in trade costs cause the failure of low-productivity firms, endogenous product scope contributes to aggregate productivity growth that is strongest in countries' comparative-advantage industries. Trade liberalization gives rise to endogenous Ricardian productivity differences at both the firm and industry level that magnify Heckscher-Ohlin-based comparative advantage and provide an additional source of welfare gains from trade.

The positive correlation between firms' intensive and extensive margins influences adjustment in the domestic and export markets. As trade costs fall, there is a contraction in production for the domestic market, through both a reduction in the amount of each product supplied and the range of products offered. At the same time, there is an expansion in production for the export market, through both the amount of each product exported and the range of products shipped abroad. Therefore, trade liberalization has a magnified impact on the firm-size distribution relative to a model with exogenous product scope. High-ability exporting firms are larger than low-ability non-exporting firms, not only because they sell more of a given product in the domestic market, but also because they manufacture a larger range of products and export a range of products. Again these implications of trade liberalization vary with comparative advantage, so that firms in comparative-advantage industries export a greater share of their output of a given product and a greater fraction of their products than those in comparative disadvantage industries.

The research in this paper complements existing models of product scope in industrial organization and international trade. Early research on multi-product firms in the industrial organization literature by Baumol (1977), Panzar and Willig (1977) and others (see the survey in Bailey and Friedlaender 1982) emphasized supply-side economies of scope. More recent contributions by Brander and Eaton (1984), Shaked and Sut-

ton (1990), Eaton and Schmidt (1994) and Johnson and Myatt (2003) have introduced demand-side considerations and strategic interaction between firms. In contrast to these studies, we examine multi-product firms within a general equilibrium model of industry dynamics where firms are heterogeneous and entry and exit is endogenous. Klette and Kortum (2004) develop a model of industry dynamics with these features, in which products are symmetric within firms, and research and development (R&D) discoveries expand product scope and drive firm growth. In contrast, our approach emphasizes the interaction of general competencies (“ability”) and product-specific competencies (“expertise”), which together explain within-firm heterogeneity, and generate the positive correlation between firms’ intensive and extensive margins. While the industrial organization literature concentrates on a closed economy, a key concern of our analysis is the role of international trade in shaping both firm and industry productivity.

In the international trade literature, Baldwin and Ottaviano (2001) use a model of multi-product multinationals to explain two-way Foreign Direct Investment (FDI) within industries. In the presence of trade costs, multi-product firms are able to reduce inter-variety competition by locating production of some varieties abroad through FDI. Eckel and Neary (2006) develop a theory of multi-product firms, in which each firm has an ideal product but can produce additional products by incurring an adaptation cost that rises with the number of products. Firms compete under conditions of Cournot oligopoly subject to quasi-linear preferences. This framework captures strategic interaction between firms but, in contrast to the analysis below, assumes that the number of multi-product firms is exogenous and that multi-product firms are homogeneous within industries.⁶ Nocke and Yeaple (2006) develop a theory of multi-product firms in which firms vary in terms of a single dimension, “organizational capability”, and in which a firm’s productivity for all of its products declines with the number of products manufactured. In equilibrium, firms with higher organizational capability produce a greater number of products but have lower overall productivity than firms with low organizational capability. In contrast with this paper, and the stylized facts discussed below, their model predicts an equal distribution of output across products within multi-product firms, and a negative correlation between firms’ intensive and extensive margins.

The assumptions upon which our model is based are motivated by the large body of empirical evidence that firm entry and exit and within-industry reallocation across heterogeneous firms are key features of trade liberalizations (see for example Pavcnik 2002, Treffer 2004 and the survey by Tybout 2003). Our model highlights the way in which trade liberalization raises the expected value of industry entry and induces endogenous changes in firm entry and exit that cause multi-product firms to focus on their core competencies. The correlation between firms’ intensive and extensive margins,

⁶A large empirical literature documents substantial heterogeneity across firms within industries and between exporters and non-exporters. See, for example, the survey by Bartelsman and Doms (2000) and Bernard and Jensen (1995). In a partial equilibrium version of their analysis, Eckel and Neary (2006) consider firm heterogeneity.

which is a key prediction of our framework and which receives empirical support, is driven by the within-industry heterogeneity in firm ability across multi-product firms.

The remainder of the paper is structured as follows. Section 2 summarizes the empirical motivation for the model. Section 3 introduces the structure of the model and Section 4 solves for the closed economy equilibrium. In Section 5 we characterize a number of properties of the closed economy equilibrium. Section 6 opens the economy to international trade. Section 7 solves for the open economy equilibrium and Section 8 develops a number of predictions for the effects of trade liberalization. Section 9 generalizes the model to two industries and factors of production to consider the influence of comparative advantage. Section 10 concludes. Appendices A and B contain data and theory appendices respectively.

2. Empirical Motivation

Our model and its assumptions are motivated by a series of stylized facts about multi-product firms reported by Bernard, Redding and Schott (2006b) and Bernard, Jensen and Schott (2005), henceforth BRS and BJS, respectively. These facts document the importance of multi-product firms for aggregate production and trade as well as the substantial heterogeneity of products within firms. In this section we discuss these stylized facts and offer others that are inspired by our main theoretical results.

Multi-product firms dominate U.S. manufacturing and trade. Analyzing quinquennial manufacturing censuses from 1972 to 1997, BRS find that an average 41 percent of U.S. manufacturing firms produce more than a single product, but that these firms account for an average of 91 percent of output. Moreover, net product adding and dropping by incumbent firms accounts for roughly one third of aggregate U.S. manufacturing growth between 1972 and 1997, a contribution that dwarfs that of new firm entry and exit. Similarly, BJS (2005), analyzing U.S. trade data for 2000, show that firms that export more than ten products make up 17 percent of exporting firms but account for 94 percent of all exports.⁷

BRS demonstrate that the goods produced by multi-product firms exhibit substantial heterogeneity. The same firm, for example, often produces products in more than a single four- or even two-digit SIC industry. In addition, firms' product-size distribution is highly skewed, with a firm's largest product accounting for an average of forty percent of overall firm output. BRS also show that product characteristics such as relative size and tenure with the firm are associated with firms' decisions to drop products from their

⁷In the U.S. production data, BRS define a product as a five-digit Standard Industrial Classification (SIC) category. This level of aggregation breaks manufacturing into roughly 1800 products. In the U.S. trade data, BRS define a product as a ten-digit Harmonized System (HS) category. This level of aggregation breaks all U.S. exports into roughly 8,500 products, with manufacturing products making up approximately two thirds of this total. We discuss the relationship between the levels of aggregation in the production and trade data below.

portfolios. After controlling for firm and product fixed effects, a firm is more likely to exit a product where its output and tenure are small relative to those of its rivals.

Our model highlights the contribution of firms' extensive margins to the firm-size distribution. Here, we augment the existing stylized facts on multi-product firms by examining this contribution empirically across U.S. firms. Total firm output (Y_f) is the sum of output of individual products (y_{pf}), which equals the number of products (n_f) times average output per product (\bar{y}_f),

$$Y_f = n_f \bar{y}_f, \quad \bar{y}_f \equiv \frac{1}{n_f} \sum_p y_{pf} \quad (1)$$

where f and p index firms and products respectively. This relationship provides the basis for a regression decomposition of variation in firm size into the contributions of the extensive and intensive margins. The log number of products and log average output per product are each separately regressed on log total firm output,

$$\begin{aligned} \ln n_f &= \alpha_1 + \beta_1 \ln Y_f + u_f \\ \ln \bar{y}_f &= \alpha_2 + \beta_2 \ln Y_f + \varepsilon_f \end{aligned} \quad (2)$$

where u_f and ε_f are stochastic errors and, by the properties of OLS, $\beta_1 + \beta_2 = 1$. The coefficients β_1 and β_2 capture the partial correlations between total firm output and the extensive and intensive margins respectively. When total firm output increases, β_1 captures the percentage contribution from the number of products (the extensive margin), holding constant average output per product (the intensive margin). Similarly, β_2 captures the percentage contribution of the intensive margin, holding constant the extensive margin. A similar decomposition may be undertaken for total firm exports into the contributions of the number of products exported and average exports per product.

Regression decompositions for both production and exporting are contained in Table 1. For brevity we report results for β_1 given that $\beta_2 = 1 - \beta_1$. The first two columns of the table summarize the results of regressing the log number of five-digit SIC manufacturing products produced by U.S. multi-product firms in 1997 on their log total output in that year. The dataset is identical to that examined by BRS and is drawn from the Longitudinal Research Database of the United States Bureau of the Census (see Appendix A). Column one displays the decomposition from an OLS regression, where we find that firms' extensive margins account for 13 percent of the variation in total firm output.

Column two of Table 1 reports an analogous decomposition of U.S. firm-product exports in 2000 using trade data from BJS.⁸ Here, we find that variation in the number

⁸These data differ from the manufacturing data in two important ways. First, the product classification is much more disaggregated. Second, the products span all sectors, i.e., they are not confined to manufacturing. Undertaking the decomposition at a common level of aggregation (SIC4) for manufacturing alone, we find that the contributions of the intensive and extensive margins are of a similar magnitude for export and production data.

Table 1: Regression Decompositions of U.S. Firm Shipments and Exports

	Production (1997)	Exporting (2000)
In Firm Shipments	0.1321 0.0002	
In Firm Exports		0.1382 0.0002
Observations	109,267	63,322
R-Squared	0.85	0.83

Notes: Table reports firm-level OLS regressions of the number of products produced and exported by U.S.-based firms on the total shipments and exports of the firm, respectively. Data are for noted years, and sample includes all multi-product firms in the noted year. Both coefficient estimates are significant at the 1 percent level.

of products exported by firms is responsible for 14 percent of the variation in their total exports.

One of the distinguishing features of the model that we develop below is a positive correlation between firms' extensive and intensive margins. To examine this correlation in the data, we regress firms' average output or exports per product on the number of products the firm produces or exports,

$$\ln \bar{y}_f = \eta + \gamma \ln n_f + \chi_f, \quad (3)$$

where χ_f is a stochastic error.⁹ Results are reported in Table 2. As indicated in the table, firms' intensive and extensive margins are correlated positively and significantly in both production and exporting. These results demonstrate that larger firms produce and export a larger number of products and also exhibit larger output or exports per product than smaller firms.¹⁰

A central feature of our model is that firm productivity consists of a component that is common across all products (ability) and a component that is specific to individual products (expertise). Since log firm sales in the model are proportional to log productivity, variation in firm-product sales can be similarly decomposed. Here, we regress the log of firm-product shipments for the year 1997 on a full set of product fixed effects and a full set of firm fixed effects. We examine the R^2 of these regressions to gauge their

⁹We note that because \bar{y}_f includes $1/n$, measurement error in the number of products n induces a negative correlation between firms' intensive and extensive margins. As a result, the results reported in Table 2 likely provide a lower bound to the strength of the positive correlation between firms' intensive and extensive margins.

¹⁰Our findings on the role of the extensive margin in accounting for variation in total firm size or exports are robust across a number of specifications. We re-estimated the specifications as five-year differences, which eliminates the variation in the number of products and average firm output per product due to time-invariant factors. We also re-estimated the specification separately for each two-digit industry, where we allocate firms to two-digit industries based on the product that accounts for the majority of their shipments. In each case, we find a substantial role for the extensive margin and a positive correlation between firm's intensive and extensive margins.

Table 2: Correlation of U.S. Firms' Extensive and Intensive Margins

	Production (1997)	Exporting (2000)
In Number of Products	0.491 0.011	0.643 0.006
Observations	109,267	63,322
R-Squared	0.02	0.16

Notes: Table reports firm-level OLS regressions of the average log shipments or exports per product of U.S. firms on the log number of products produced and exported, respectively, in the noted years. Sample includes all multi-product firms in the noted year. All estimates are significant at the 1 percent level.

relative importance.¹¹

The first column of Table 3 reports the OLS adjusted- R^2 for the specification with product fixed effects, which control for heterogeneity in average shipment values across products. We find that product fixed effects account for 30 percent of the variation in the firm-product shipments of multi-product firms and 28 percent of the variation for all firms. The second column of Table 3 contains the adjusted- R^2 for the regression with both product and firm fixed effects, where the firm fixed effects capture the common component of firm shipments across all products (i.e., ability). We find that the adjusted- R^2 of the regression rises to 49 percent for multi-product firms and 54 percent for all firms. These adjusted- R^2 imply that there is a common component of firm shipments that is not only statistically significant but also explains a substantial proportion of the differences in shipments across firms. The unexplained variation in the data is 51 percent for multi-product firms and 46 percent for all firms, confirming the importance of considerations that are idiosyncratic to individual pairings of firms and products (i.e., expertise).

Table 3: Common and Idiosyncratic Components of Firm-Product Shipments

	Product Fixed Effects	Product and Firm Fixed Effects	Observations
Multiple-Product Firms	0.30	0.49	212,691
All Firms	0.28	0.54	305,623

Notes: Table reports adjusted R^2 's of OLS regressions of 1997 firm-product shipments in 1997 on product as well as firm and product fixed effects. First row reports results for multiple-product firms. Results in the second row are for all firms.

¹¹Shipments vary across individual firm-product pairs and so the data have a panel structure, with dimensions of firms and products, permitting the inclusion of the both firm and product fixed effects. We report adjusted- R^2 to control for the number of fixed effects being estimated. We report results for multi-product firms as well as for all firms, since firm fixed effects perfectly explain variation in the shipments of single-product firms. As indicated in the table, including single product firms in the sample results in a higher contribution of the firm fixed effects.

3. The Model

We consider a framework where firms can choose whether to participate in multiple product markets and compete with one another in supplying horizontally differentiated varieties within those product markets. Products are imperfect substitutes in demand and require specific production expertise. The model can be interpreted as capturing an economy consisting of many products or as capturing an industry (e.g. consumer electronics) with many products (e.g. DVD-players, Televisions and MP3-players) of which firms supply distinct varieties (e.g. Sony, JVC and Panasonic). In section 9, we consider a multi-industry version of the model.

3.1. Preferences

The representative consumer derives utility from the consumption of a continuum of products which we normalize to the interval $[0, 1]$. There is a constant elasticity of substitution across products so that the utility function takes the following form:

$$U = \left[\int_0^1 C_i^\nu di \right]^{\frac{1}{\nu}}, \quad 0 < \nu < 1. \quad (4)$$

where i indexes products. Within each product market, a continuum of firms produce differentiated varieties of the product, so that C_i is a consumption index which also takes the constant elasticity of substitution form:

$$C_i = \left[\int_{\omega \in \Omega} c_i(\omega)^\rho d\omega \right]^{\frac{1}{\rho}}, \quad 0 < \rho < 1. \quad (5)$$

where ω indexes varieties.¹² We make the natural assumption that the elasticity of substitution across varieties within products is greater than the elasticity of substitution across products, $\sigma \equiv \frac{1}{1-\rho} > \kappa \equiv \frac{1}{1-\nu} > 1$, and the price index dual to (5) is:

$$P_i = \left[\int_{\omega \in \Omega} p_i(\omega)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (6)$$

3.2. Production Technology

The specification of entry and production follows Melitz (2003). However, we augment that model to allow firms to manufacture multiple products and to allow for uncertainty about productivity that is specific to individual product markets. There is a competitive fringe of potential firms who are identical prior to entry. In order to enter, firms must incur a sunk entry cost of $f_e > 0$ units of labor. After the sunk cost is paid,

¹²Our model focuses on firms that supply multiple products for final consumption. While vertical integration provides another reason why firms may produce multiple products (intermediate and final), it is clear that many manufacturing firms supply multiple products for final consumption.

firms draw a productivity in each product. Motivated by the empirical findings above, we model a firm's productivity in a product as depending upon two components: "ability" $\varphi \in (0, \infty)$ that is common to all products and drawn from a distribution $g(\varphi)$, and "expertise" $\lambda_i \in (0, \infty)$ that is specific to each product and drawn from a distribution $z(\lambda)$. Therefore, while φ varies across firms, λ_i is specific to individual pairings of firms and products. This conceptualization of firm productivity captures the idea that some components of firm productivity enhance efficiency for the firm as a whole while other components (e.g. specialized technical knowledge) only apply within particular product markets.¹³ The two distributions $g(\varphi)$ and $z(\lambda)$ are independent of one another and common to all firms, and the product expertise distribution is independently and identically distributed across products. The continuous cumulative distribution functions for firm ability and product expertise are denoted by $G(\varphi)$ and $Z(\lambda)$. We focus on an interior equilibrium where no firm manufactures all products and where each firm derives positive profits from a range of products. Therefore we assume the following regularity conditions for the distributions of ability and expertise: $\lim_{\varphi \rightarrow 0} g(\varphi) > 0$, $\lim_{\varphi \rightarrow \infty} G(\varphi) \rightarrow 1$, $\lim_{\lambda \rightarrow 0} z(\lambda) > 0$, and $\lim_{\lambda \rightarrow \infty} Z(\lambda) \rightarrow 1$.

Incurring the sunk entry cost creates the blueprint for one horizontally differentiated variety of each product.¹⁴ Once the sunk cost has been paid, and firm ability and product expertise are observed, firms decide whether to enter and in which product markets to participate. For now, labor is the sole factor of production and is in inelastic supply L (which also indexes the size of the economy).¹⁵ There is a fixed corporate headquarters cost of $f_h > 0$ units of labor which the firm must incur irrespective of the number of product markets in which it is active and a fixed production cost of $f_p > 0$ units of labor for each product. There is a constant marginal cost of production for each product which depends on the interaction of firm ability and product expertise. Total labor employed by a firm is therefore:

$$l(\varphi) = f_h + \int_0^1 I_i \left[f_p + \frac{q(\varphi, \lambda_i)}{\varphi \lambda_i} \right] di, \quad (7)$$

where I_i is an indicator variable which equals one if a firm produces product i and zero otherwise, and $q(\varphi, \lambda_i)$ denotes output of product i by a firm with firm ability φ and product expertise λ_i .

Even though the distributions $g(\varphi)$ and $z(\lambda)$ have been assumed to be independent of one another, a firm's productivity in a product market i , $\theta_i \equiv \varphi \lambda_i$, depends on both

¹³The two features that are important for our analysis are the variation in productivity across products within firms and the variation in average productivity across firms. While our formulation captures these features in an intuitive and tractable way, one could also generate idiosyncratic variation in firm-product productivity from a pairwise interaction between firm and product characteristics.

¹⁴Our framework can be extended to allow for endogenous measures of varieties within each product, introducing the potential for strategic interaction since firms are no longer of measure zero relative to each product.

¹⁵We analyze a setting with two factors of production, skilled and unskilled labor, in Section 9.

firm ability and product expertise. In particular, a higher level of firm ability φ raises productivity θ_i across all products i . Therefore, productivity (the combination of firm ability and product expertise) is positively correlated across products, but the correlation is imperfect due to the stochastic variation in product expertise.¹⁶

3.3. Firm-Product Profitability

Demand for a product variety depends upon the own variety price, the price index for the product and the price indices for all other products. If a firm is active in a product market, it manufactures one of a continuum of varieties and so is unable to influence the price index for the product. At the same time, the price of firm's variety in one product market only influences the demand for its varieties in other product markets through the price indices. Therefore, the firm's inability to influence the price indices implies that its profit maximization problem reduces to choosing the price of each product variety separately to maximize the profits derived from that product variety.¹⁷ This optimization problem yields the standard result that the equilibrium price of a product variety is a constant mark-up over marginal cost:

$$p_i(\varphi, \lambda_i) = \frac{1}{\rho} \frac{w}{\varphi \lambda_i}, \quad (8)$$

where we choose the wage for the numeraire and so $w = 1$.

Substituting for the pricing rule, and using the choice of numeraire, equilibrium revenue and profits from a product variety are:

$$r_i(\varphi, \lambda_i) = R_i (\rho P_i \varphi \lambda_i)^{\sigma-1}, \quad \pi_i(\varphi, \lambda_i) = \frac{r_i(\varphi, \lambda_i)}{\sigma} - f_p, \quad (9)$$

where R_i denotes aggregate expenditure on product i . From equation (9), the ratio of revenue for two varieties of the same product depends solely on their relative productivities:

$$r_i(\varphi'', \lambda_i'') = (\varphi''/\varphi')^{\sigma-1} (\lambda_i''/\lambda_i')^{\sigma-1} r_i(\varphi', \lambda_i'). \quad (10)$$

A firm with a particular ability draw φ and expertise draw λ_i decides whether or not to produce a product based on a comparison of revenue and fixed production costs for the product. For each firm ability φ , there is a **zero-profit cutoff for product expertise** $\lambda_i^*(\varphi)$, such that the firm will enter the product if it draws a value of λ_i equal

¹⁶Note this property does not depend on firm ability and product expertise combining multiplicatively ($\theta_i \equiv \varphi \lambda_i$). All we require is that, for two firms with the same expertise (λ_i), the firm with a higher ability (φ) has the higher productivity (θ_i) in the product. This would be satisfied by for example a linear specification ($\theta_i \equiv \varphi + \lambda_i$).

¹⁷The structure of our model eliminates strategic interaction within or between firms. This choice of model structure enables us to focus purely on the implications of introducing multi-product firms into a standard model of industry dynamics with ongoing entry, exit and firm heterogeneity.

to or greater than $\lambda_i^*(\varphi)$. For each firm ability φ , the value of $\lambda_i^*(\varphi)$ is defined by the following zero-profit condition:

$$r_i(\varphi, \lambda_i^*(\varphi)) = \sigma f_p. \quad (11)$$

We refer to the lowest level of ability where a firm finds it profitable to incur the fixed headquarters cost and enter as the **zero-profit cutoff for firm ability** φ^* . The zero-profit cutoff for product expertise for a firm with ability φ^* is defined by:

$$r_i(\varphi^*, \lambda_i^*(\varphi^*)) = \sigma f_p. \quad (12)$$

Since there is a continuum of identical products, the zero-profit cutoff for product expertise is the same across all products: $\lambda_i^*(\varphi) = \lambda^*(\varphi)$ for all i . Furthermore, since product expertise is independently and identically distributed, the law of large numbers implies that the fraction of products manufactured by a firm with a particular level of ability φ equals the probability of that firm drawing a product expertise above $\lambda^*(\varphi)$, that is $[1 - Z(\lambda^*(\varphi))]$.

3.4. Firm Profitability

A firm with a particular ability draw φ decides whether or not to enter based on a comparison of the fixed headquarters cost and total profits across those products where its product expertise draw λ_i is greater than the zero-profit cutoff $\lambda^*(\varphi)$.

With a continuum of identical products and independent draws for product expertise, the law of large numbers implies that a firm's expected revenue across the continuum of products equals its expected revenue for an individual product. Expected revenue for an individual product is a function of firm ability φ and equals the probability of drawing a product expertise above the zero-profit cutoff $\lambda^*(\varphi)$ times expected revenue conditional on profitable production. Therefore total firm revenue across the unit interval of products is:

$$\begin{aligned} r(\varphi) &= \int_0^1 \left[\int_0^\infty r(\varphi, \lambda_i) \mu_z(\lambda_i) d\lambda_i \right] di \\ &= [1 - Z(\lambda^*(\varphi))] \int_{\lambda^*(\varphi)}^\infty r(\varphi, \lambda) \left(\frac{z(\lambda)}{1 - Z(\lambda^*(\varphi))} \right) d\lambda \\ &= \int_{\lambda^*(\varphi)}^\infty r(\varphi, \lambda) z(\lambda) d\lambda, \end{aligned} \quad (13)$$

where the *ex post* distribution for product expertise $\mu_z(\lambda_i)$ is a truncation of the *ex ante* distribution $z(\lambda)$ above the zero-profit cutoff for product expertise $\lambda^*(\varphi)$.

Total profits across products equal expected profits for an individual product minus fixed headquarters costs. Firm profits are therefore equal to the probability of drawing a product expertise above the zero-profit cutoff $\lambda^*(\varphi)$ times expected profits conditional on profitable production minus fixed headquarters costs:

$$\begin{aligned}\pi(\varphi) &= [1 - Z(\lambda^*(\varphi))] \left[\int_{\lambda^*(\varphi)}^{\infty} \left(\frac{r(\varphi, \lambda)}{\sigma} - f_p \right) \left(\frac{z(\lambda)}{1 - Z(\lambda^*(\varphi))} \right) d\lambda \right] - f_h \quad (14) \\ &= \int_{\lambda^*(\varphi)}^{\infty} \left(\frac{r(\varphi, \lambda)}{\sigma} - f_p \right) z(\lambda) d\lambda - f_h.\end{aligned}$$

The lower the ability of a firm φ , the higher the zero-profit cutoff for product expertise $\lambda^*(\varphi)$, and so the lower the probability of drawing a product expertise sufficiently high to profitably manufacture a product. Firms with lower abilities have lower expected profits in an individual product market and manufacture a smaller fraction of products $[1 - Z(\lambda^*(\varphi))]$. For sufficiently low draws of firm ability, the excess of revenue over fixed production costs in the small range of profitable products falls short of the fixed headquarters cost. Therefore, the requirement that firm profits net of fixed headquarters costs are greater than or equal to zero defines a zero-profit cutoff for firm ability φ^* , such that only firms who draw an ability equal to or greater than φ^* enter:

$$\pi(\varphi^*) = 0. \quad (15)$$

3.5. Free Entry

Firms from the competitive fringe decide whether or not to enter based on a comparison of the expected value of entry and the sunk entry cost. We assume that there is a constant exogenous probability of firm death (δ), as a result of *force majeure* events beyond the manager's control, which generates ongoing firm entry and exit. Therefore, the expected value of entry (v_e) is the *ex ante* probability of successful entry, multiplied by average firm profits conditional on entry ($\bar{\pi}$), and discounted by the probability of firm death. Free entry requires:

$$v_e = \frac{[1 - G(\varphi^*)] \bar{\pi}}{\delta} = f_e, \quad (16)$$

where the *ex ante* probability of successful entry is $[1 - G(\varphi^*)]$.

Firm revenue for a particular value of ability φ is given in equation (14). Taking expectations over values for ability yields average firm revenue conditional on entry:

$$\begin{aligned}\bar{r} &= \int_0^{\infty} r(\varphi) \mu_g(\varphi) d\varphi, \quad (17) \\ &= \int_{\varphi^*}^{\infty} r(\varphi) \left(\frac{g(\varphi)}{1 - G(\varphi^*)} \right) d\varphi,\end{aligned}$$

where the *ex post* distribution for ability $\mu_g(\varphi)$ is a truncation of the *ex ante* distribution $g(\varphi)$ above the zero-profit cutoff for ability φ^* where entry occurs. The expression for average firm profits conditional on entry $\bar{\pi}$ is analogous and takes expectations over values for ability in equation (14).

3.6. Goods and Labor Markets

The steady-state equilibrium is characterized by a constant mass of firms entering each period, M_e , and a constant mass of firms producing, M . Each of the firms producing is active in a subset of product markets. The steady-state mass of firms manufacturing an individual product, M_p , is a constant fraction of the mass of firms producing, and this fraction is the same across products. The steady-state equilibrium is also characterized by stationary distributions of firm ability and product expertise that are determined by the zero-profit cutoffs φ^* and $\lambda^*(\varphi)$ and that are again the same across products.

From the equilibrium pricing rule (8), the aggregate price index for each product can be written as a function of the mass of firms manufacturing the product and the price of a variety with a weighted average of firm ability and product expertise:

$$P = M_p^{\frac{1}{1-\sigma}} \frac{1}{\rho \tilde{\varphi}}, \quad (18)$$

where the weighted average $\tilde{\varphi}$ depends on the *ex post* distributions of firm ability and product expertise and is defined in the appendix.

Of the mass of firms with a particular ability φ , a fraction $[1 - Z(\lambda^*(\varphi))]$ are active in an individual product market. Therefore, the mass of firms manufacturing an individual product is the following fraction of the mass of firms:

$$M_p = \left[\int_{\varphi^*}^{\infty} [1 - Z(\lambda^*(\varphi))] \left(\frac{g(\varphi)}{1 - G(\varphi^*)} \right) d\varphi \right] M. \quad (19)$$

With a constant mass of firms producing in steady-state, the mass of firms that draw an ability sufficiently high to enter must equal the mass of firms that die, implying the following steady-state stability condition:

$$[1 - G(\varphi^*)] M_e = \delta M. \quad (20)$$

Finally, labor market clearing implies that the demand for labor in production and entry equals the economy's aggregate supply of labor:

$$L_q + L_e = \bar{L}, \quad (21)$$

where the subscripts q and e respectively denote labor used in production and entry.

4. Closed Economy Equilibrium

The closed economy equilibrium is completely referenced by the quadruple $\{\varphi^*, \lambda^*(\varphi^*), P, R\}$. All other endogenous variables can be written as functions of these four elements of the equilibrium vector.

4.1. Multi-Product Firms

We begin by determining the zero-profit cutoff for expertise for every firm as a function of its ability and the zero-profit cutoff for ability φ^* . Combining the zero-profit cutoff for expertise for a firm with ability φ in equation (11), the zero-profit cutoff for expertise for a firm with ability φ^* in equation (12), and the relationship between the revenues of varieties from equation (10), we obtain:

$$\lambda^*(\varphi) = \left(\frac{\varphi^*}{\varphi}\right) \lambda^*(\varphi^*). \quad (22)$$

Intuitively, an increase in ability φ reduces a firm's zero-profit cutoff for expertise $\lambda^*(\varphi)$ because higher ability raises productivity in each product, and so sufficient revenue to cover fixed production costs is generated at a lower value of expertise. In contrast, an increase in the zero-profit cutoff for ability φ^* raises a firm's zero-profit cutoff for expertise $\lambda^*(\varphi)$ because it raises the average productivity of rival firms' products, and so intensifies product market competition, and hence increases the value for expertise at which sufficient revenue is generated to cover fixed production costs.

To determine $\lambda^*(\varphi^*)$, we combine the requirement that zero profits are made by a firm with ability φ^* in equation (15) with the expression for firm profits in equation (14). We substitute for product revenue using the relationship between relative variety revenues in equation (10) and the zero-profit condition for product expertise in equation (12). Together these equations yield the following expression that implicitly defines a unique value of $\lambda_i^*(\varphi^*)$ as a function of model parameters alone:

$$\int_{\lambda^*(\varphi^*)}^{\infty} \left[\left(\frac{\lambda}{\lambda^*(\varphi^*)} \right)^{\sigma-1} - 1 \right] f_p z(\lambda) d\lambda = f_h, \quad \Rightarrow \quad \lambda^*(\varphi^*) = \bar{\lambda}. \quad (23)$$

Equations (22) and (23) jointly determine the zero-profit cutoffs for expertise for a firm as a function of its ability and the zero-profit-cutoff ability. Three parameters influence a firm's zero-profit cutoff for expertise for given values of its ability (φ) and the zero-profit-cutoff ability (φ^*): the fixed production cost (f_p), the elasticity of substitution (σ) and the fixed headquarters cost (f_h).

From equation (23) it can be seen that an increase in the fixed production cost or the elasticity of substitution raises $\lambda^*(\varphi^*)$, which in turn raises the zero-profit cutoff for expertise for firms of all abilities, $\lambda^*(\varphi)$. Intuitively, a higher fixed production cost raises the equilibrium level of product revenue needed for zero profits to be achieved. Since product revenue is increasing in expertise, the higher level of product revenue needed for zero profits implies a higher value of the zero-profit cutoff for product expertise $\lambda^*(\varphi)$. Similarly, increases in the elasticity of substitution reduce the revenue of low-expertise products relative to that of higher expertise products. Once again, this implies that a higher level of product expertise is required to achieve zero profits and so $\lambda^*(\varphi)$ rises. Note higher elasticities of substitution correspond to less love of variety and so diminish firms' incentives to expand along the extensive margin into lower-expertise products.

An increase in the fixed headquarters cost has the opposite effect of an increase in fixed production costs. Higher fixed headquarter costs reduce $\lambda^*(\varphi^*)$ and thereby reduce the zero-profit cutoff for expertise for firms of all abilities, $\lambda^*(\varphi)$. This third and somewhat surprising comparative static is a property of general equilibrium. As the fixed headquarters cost rises, the *ex post* profits of firms across all products decline, depressing entry and diminishing product-market competition. As competition declines, firms with lower product-expertise draws can generate sufficient profits to cover fixed production costs, and so the zero-profit cutoff for product expertise falls.

The zero-profit cutoff for expertise determines the fraction of product markets where a firm produces, $[1 - Z(\lambda^*(\varphi))]$. Therefore, equations (22) and (23) together determine endogenous product scope as a function of a firm's ability φ , the zero-profit cutoff for ability φ^* and model parameters. The relationship between a firm's product scope and its ability is illustrated graphically in Figure 1. For a firm that draws an ability less than φ^* , average profits across products are insufficient to cover fixed headquarters costs, and the firm exits immediately. A firm that draws an ability φ^* manufactures varieties in a range of products $[1 - Z(\bar{\lambda})]$. The range of products manufactured by a firm is monotonically increasing in ability and converges asymptotically towards one. Our analysis is consistent with a wide range of distributions for product expertise, and the exact shape of the relationship shown in Figure 1 will vary with different functional forms for the product expertise distribution.

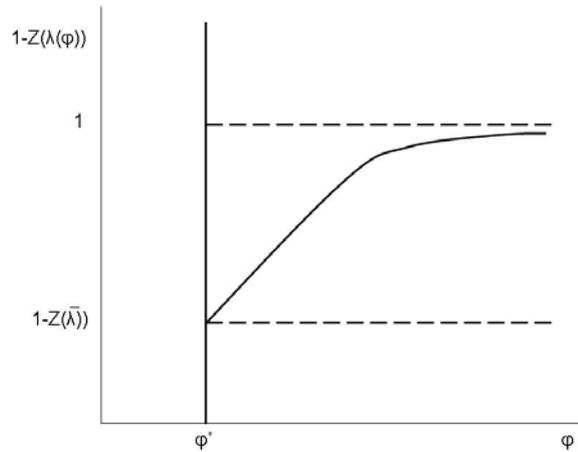


Figure 1: Product Scope and Firm Ability

4.2. Free Entry

As shown in the appendix, average firm revenue and average firm profits can be re-written so that they are functions of only the zero-profit-cutoff ability (φ^*) and the

zero-profit-cutoff expertise for a firm with this ability ($\lambda^*(\varphi^*) = \bar{\lambda}$). Once rewritten in this way, it is clear that average firm revenue, $\bar{r} = \bar{r}(\varphi^*, \bar{\lambda})$, and average firm profits, $\bar{\pi} = \bar{\pi}(\varphi^*, \bar{\lambda})$, are monotonically decreasing in both φ^* and $\bar{\lambda}$. Intuitively, a higher value of φ^* implies a higher average ability of a firm's competitors and so lower average prices of competing varieties, which reduces average firm revenue and average firm profits. Similarly, a higher value of $\bar{\lambda}$ implies a higher average product expertise of the varieties manufactured by competitors and hence lower average prices of competing varieties.

Substituting for average profits in the free entry condition, the requirement that the expected value of entry equals the sunk entry cost becomes:

$$v_e = \frac{1}{\delta} \int_{\varphi^*}^{\infty} \left[\int_{\lambda^*(\varphi)}^{\infty} \left[\left(\frac{\lambda}{\lambda^*(\varphi)} \right)^{\sigma-1} - 1 \right] f_p z(\lambda) d\lambda - f_h \right] g(\varphi) d\varphi = f_e, \quad (24)$$

which is monotonically decreasing in φ^* , since $\lambda^*(\varphi) = (\varphi^*/\varphi) \bar{\lambda}$. The free entry condition in equation (24) implicitly defines a unique zero-profit cutoff for ability as a function of model parameters alone.

The comparative statics of the free entry condition are intuitive. As the product fixed production cost (f_p) rises, so does the zero-profit cutoff for ability (φ^*), since a higher productivity is required in each product to generate sufficient revenue to cover the fixed production cost, and greater ability raises productivity in all products. Increases in either the fixed headquarters cost (f_h) or the sunk entry cost (f_e) reduce the zero-profit cutoff for ability via the same logic discussed above: as the fixed headquarters cost rises, the *ex post* profits of firms across all products decline, thereby depressing entry and diminishing product market competition. As product market competition declines, firms with lower ability are capable of generating sufficient profits across all products to cover fixed headquarters cost and so the zero-profit cutoff for ability falls. A rise in the sunk cost of entry also reduces entry, and so has a similar effect on the zero-profit cutoff for ability as a rise in the fixed headquarters cost.

4.3. Goods and Labor Markets

So far we have solved for the zero-profit-cutoff ability φ^* and the zero-profit-cutoff expertise for a firm with this level of ability $\lambda^*(\varphi^*) = \bar{\lambda}$, which implies from equation (22) that we have pinned down the zero-profit cutoff for expertise $\lambda^*(\varphi)$ and hence equilibrium product scope for firms of all abilities φ . Together φ^* and $\bar{\lambda}$ determine the *ex post* distributions of firm ability and product expertise and hence pin down average variety prices as well as average firm revenue ($\bar{r} = \bar{r}(\varphi^*, \bar{\lambda})$) and average firm profits ($\bar{\pi} = \bar{\pi}(\varphi^*, \bar{\lambda})$).

Total payments to labor used in production equal aggregate revenue minus aggregate profits, $L_q = R - \Pi$. Combining the steady-state stability and free entry conditions, equations (20) and (16), it can be shown that total payments to labor used in entry

equal aggregate profits, $L_e = \Pi$. Therefore, aggregate revenue equals the economy's aggregate supply of labor, $R = L$, and the labor market clears.

We are now in a position to determine the price index for each product, which depends on average variety prices and the mass of firms producing each product in equation (18). Average variety prices have already been determined above. The mass of firms producing each product, M_p , is a constant fraction of the mass of firms producing any product, M , in equation (19). The mass of firms producing any product equals aggregate revenue divided by average firm revenue, $M = R/\bar{r}$, where we have solved for both aggregate and average firm revenue. The constant fraction of firms that produce each product depends solely on the zero-profit cutoff for firm ability φ^* and the zero-profit cutoffs for expertise $\lambda^*(\varphi)$ for which we have already solved. Therefore, we have determined the price index and completed the characterization of the equilibrium quadruple $\{\varphi^*, \lambda^*(\varphi^*), P, R\}$.

5. Properties of the Closed Economy Equilibrium

The equilibrium range of products manufactured by a firm is shaped by two sets of considerations. On the one hand, the firm charges a lower price for products where it has higher productivity and as a result enjoys higher sales. This force for specialization causes the firm to expand along the intensive margin in those products where it has higher productivity. On the other hand, consumer preferences display a love of variety and there is diminishing marginal utility to consuming more of any one variety. Diminishing marginal utility implies that the firm faces a downward-sloping demand curve for each product, which constrains sales and limits expansion along the intensive margin. This force for diversification induces the firm to expand along the extensive margin into products where it has a lower productivity. Nonetheless, the firm must derive sufficient revenue from each product to cover fixed production costs, which constrains the firm's incentive to expand along the extensive margin.

Proposition 1 *There is a positive correlation between firms' intensive margins (how much of a product is produced) and their extensive margins (how many products are produced).*

Proof. See appendix. ■

Proposition 1 summarizes a key prediction of our framework, namely the positive correlation between firms' intensive and extensive margins which matches the empirical findings in Table 2. More productive firms are larger on average, not only because they sell more of a given product, but because they manufacture more products. The explanation for this positive correlation lies in firm ability's role as a component of productivity that is common across products. On one hand, higher ability raises a firm's productivity in a given product, thereby lowering the firm's price for that product, and increasing the quantity sold. On the other hand, higher ability also lowers the zero-profit

cutoff for expertise above which the firm finds it profitable to enter a product market, thereby expanding the range of products manufactured by the firm.¹⁸ Our framework endogenizes a new margin of firm decisions, the range of products to produce, which is typically left unmodelled through the explicit or implicit assumption of single-product firms. The distribution of products across firms is non-random, with higher ability firms characterized by systematically larger equilibrium ranges of products. Note that the correlation between firms' intensive and extensive margins exists even though expertise draws are uncorrelated across products by assumption. Assuming that expertise draws were positively correlated across products would only strengthen the correlation.

Corollary 1 *Multi-product firms' endogenous decisions about product scope magnify inequality in the firm-size distribution: the percentage difference in revenue between two firms with abilities φ'' and φ' (where $\varphi'' > \varphi'$) is strictly greater than when all firms have the same exogenous product scope ($\lambda^*(\varphi) = \lambda^*(\varphi^*) = \bar{\lambda}$ for all φ).*

Proof. See appendix. ■

The positive correlation between the intensive and extensive margin magnifies inequality in the firm-size production. Firms with higher abilities are larger than those with lower abilities for two reasons: they sell more of any given product and they manufacture more products. This prediction of the model has implications for the large theoretical and empirical literature concerned with the firm-size distribution (see for example Axtell 2001, Rossi-Hansberg and Wright 2005, and the survey by Sutton 1997). The positive correlation between firms' intensive and extensive margins in our model implies that a smaller variance in firm ability is required to explain the observed dispersion in firm sizes than in a model where product scope is exogenous. A given increase in firm ability leads to a larger increase in firm size than it would with exogenous product scope because of firms' endogenous expansion along their extensive margins.

Proposition 2 *(a) Firm weighted-average productivity $\tilde{\lambda}(\varphi)$ is monotonically increasing in firm ability φ , (b) The percentage difference in firm weighted average productivity $\tilde{\lambda}(\varphi)$ is smaller than the percentage difference in firm ability φ .*

Proof. See appendix. ■

While inequality in firm sizes is magnified by endogenous product scope, the dispersion in average productivity across firms in an industry is compressed. Note, a firm's

¹⁸We note that firms' intensive and extensive margins do not have to be positively correlated. Limited managerial time and other diseconomies of scope provide potential reasons why more productive firms may have a greater incentive to specialize in those products where they are most productive, inducing a negative correlation between the two margins. However, the data reveal a positive correlation between the intensive and extensive margins, as predicted by our framework where firms can "leverage" their ability φ across multiple product markets.

weighted-average productivity ($\tilde{\lambda}(\varphi)$) depends upon the productivities of all the products it manufactures, as defined formally in the appendix.¹⁹ As ability increases, a firm's weighted-average productivity rises because it becomes more productive in all of the products that it currently manufactures. At the same time, an increase in ability induces the firm to expand along the extensive margin into new products, where it has less expertise and so is less productive. The addition of these lower-expertise products reduces the firm's weighted-average productivity, which therefore rises less than proportionately with ability.²⁰ An implication of this finding is that measures of the productivity of multi-product firms based on total firm output and total firm inputs will be systematically biased. If product range were held constant, the difference in productivity between a high and low-ability firm would be greater than when product scope is endogenous, because the high-ability firm expands along the extensive margin into products where it has less expertise and so is less productive.²¹

The expansion of higher-ability firms along the extensive margin into lower-expertise products has a variety of general equilibrium implications. The most basic of these implications arises from the expansion of labor demand at higher-ability firms, which bids up factor prices and so reduces the profitability of lower-ability firms. As a result, the zero-profit cutoff for ability below which firms exit is higher when product scope is endogenous than when all firms manufacture the same exogenous range of products. A more subtle implication is for the economy's response to changes in model parameters such as the sunk cost of entry. The potential to respond along a new margin – the number of goods produced – dampens responses along existing adjustment margins such as the range of productivities where firms produce.

Proposition 3 *Endogenous adjustments along firms' extensive margins dampen the response of the zero-profit-cutoff ability φ^* to changes in the sunk cost of entry f_e .*

Proof. See appendix. ■

Following a rise in the sunk entry cost, the expected value of entry must rise in order for the free entry condition to continue to be satisfied in equilibrium. When product scope is exogenous, all of the adjustment in the expected value of entry occurs through a fall in the zero-profit cutoff below which firms exit, which expands the range of viable

¹⁹Although the proposition focuses on the weighted-average productivity of firms, a directly analogous result holds for the unweighted average productivity of firms.

²⁰As long as the cumulative distribution function for product expertise is continuous, the measure of products added as ability rises is small relative to the measure of products already manufactured. Therefore, as shown formally in the proof of Proposition 2, the reduction in firm weighted-average productivity from expanding into lower expertise products cannot outweigh the increase caused by enhanced productivity in existing products, which implies that firm weighted-average productivity is monotonically increasing in ability.

²¹See De Loecker (2006) for an empirical analysis of productivity measurement with multi-product firms.

firm abilities. In contrast, when product scope is endogenous, some of the adjustment in the expected value of entry occurs through an expansion in the range of products manufactured by firms, thereby dampening the necessary adjustment in the zero-profit cutoff ability.

Intuitively, as the sunk entry cost rises, entry declines and product market competition is diminished. With less competition, firms are able to derive positive profits from products which were not previously viable, and so expand their product scope, which increases the expected value of entry. Less competition also enables lower-ability firms who were not previously viable to make positive profits, and so the zero-profit-cutoff ability falls, which also increases the expected value of entry. But the zero-profit-cutoff ability falls by a less than it would if product scope were exogenous, because of the increase in labor demand created by firms' extensive margin expansion. This increase in labor demand bids up wages, and so reduces the profitability of low-ability firms, which leads to a smaller reduction in the zero-profit-cutoff ability compared to when product scope is exogenous.

6. Open Economy

In this section, we examine the implications of opening to trade within a framework of two symmetric countries, deferring asymmetry and the influence of comparative advantage to Section 9. We denote domestic market variables with a superscript d and export market variables with a superscript x . In line with a large body of empirical evidence, we assume that trade is costly. There are fixed costs of operating across national borders, such as the costs of mastering customs procedures and building distribution networks. Therefore we assume that, to begin exporting, a firm must incur additional fixed headquarters costs of $f_{xh} > 0$ which raise total headquarters costs to $f_h + f_{xh}$. There are also fixed costs of entering export markets that are specific to individual products, such as costs of market research, advertising and conforming to foreign regulatory standards. Hence we assume that, to export a particular product, a firm must incur an additional product-specific fixed cost of $f_x > 0$. As more products are exported, total fixed exporting costs rise, but average fixed exporting costs fall because the headquarters costs of becoming an exporter are spread over a larger number of products. Finally we allow for variable costs of trade, such as transportation costs, which take the standard "iceberg" form where a fraction $\tau > 1$ of a variety must be shipped in order for one unit to arrive.²²

6.1. Consumption and Production

Profit maximization implies that the equilibrium price of a product variety is again a constant mark-up over marginal cost, with export prices a constant multiple of domestic

²²For evidence on the magnitude of overall trade costs, see Anderson and van Wincoop (2004) and Hummels (2001). For evidence on the fixed costs of exporting, see Bernard and Jensen (2004), Eaton, Kortum and Kramatz (2005) and Roberts and Tybout (1997).

prices due to the variable costs of trade:

$$p_{ix}(\varphi, \lambda_i) = \tau p_{id}(\varphi, \lambda_i) = \tau \frac{1}{\rho} \frac{w}{\varphi \lambda_i}. \quad (25)$$

Given firms' pricing rule and symmetric countries, equilibrium product revenue in the export market is proportional to revenue in the domestic market: $r_{ix}(\varphi, \lambda_i) = \tau^{1-\sigma} r_{id}(\varphi, \lambda_i)$. The combined revenue from a product depends upon whether or not it is exported:

$$r(\varphi, \lambda_i) = \begin{cases} r_{id}(\varphi, \lambda_i) & \text{if the product is not exported} \\ (1 + \tau^{1-\sigma}) r_{id}(\varphi, \lambda_i) & \text{if the product is exported} \end{cases}. \quad (26)$$

Consumer love of variety and fixed production costs imply that no firm ever exports a product without also selling it in the domestic market. Therefore, we may separate a firm's profit from a product into components earned from domestic sales, $\pi_{id}(\varphi, \lambda_i)$, and export sales, $\pi_{ix}(\varphi, \lambda_i)$, where we apportion the entire fixed production cost to domestic profits and the fixed exporting cost to export profits.²³

$$\pi_{id}(\varphi, \lambda_i) = \frac{r_{id}(\varphi, \lambda_i)}{\sigma} - f_p, \quad \pi_{ix}(\varphi, \lambda_i) = \frac{r_{ix}(\varphi, \lambda_i)}{\sigma} - f_x. \quad (27)$$

The combined profit from a product is therefore:

$$\pi_i(\varphi, \lambda_i) = \pi_{id}(\varphi, \lambda_i) + \max\{0, \pi_{ix}(\varphi, \lambda_i)\}. \quad (28)$$

6.2. Firm-Product Profitability

Given a firm's ability (φ), the zero-profit condition in equation (11) determines a cutoff for product expertise, such that the firm sells a product domestically if it draws an expertise equal to or greater than $\lambda^*(\varphi)$. Manipulating the zero-profit condition for expertise for firms with different abilities φ yields the same expressions for the equilibrium range of products manufactured as in equations (22) and (23). However, the zero-profit cutoffs for firm ability (φ^*) and product expertise ($\lambda^*(\varphi)$) will differ between the closed and open economies and thus induce differences in equilibrium product scope.

Entry into export markets within products is determined in an analogous fashion. For each firm ability φ , there is an **exporting cutoff for product expertise**, i.e. the firm exports the product if it draws an expertise equal to or greater than $\lambda_{ix}^*(\varphi)$:

$$r_{ix}(\varphi, \lambda_{ix}^*(\varphi)) = \sigma f_x. \quad (29)$$

Since products are identical, the exporting cutoff for expertise is the same across products: $\lambda_{ix}^*(\varphi) = \lambda_x^*(\varphi)$ for all i . As expertise is independently and identically distributed across products, the fraction of products exported by a firm with ability φ is $[1 - Z(\lambda_x^*(\varphi))]$.

²³This is a convenient accounting device which simplifies the exposition. Rather than comparing revenue from exporting to the fixed cost of exporting, we could equivalently compare the sum of domestic and export revenue to the sum of fixed production and exporting costs.

Exporters vary in terms of their ability φ , and we refer to the lowest level of ability among exporters as the **exporting cutoff firm ability** φ_x^* . Using equation (29) for two exporting firms with abilities, φ and φ_x^* , we obtain the following expression for the firms' relative exporting cutoffs for product expertise:

$$\lambda_x^*(\varphi) = \left(\frac{\varphi_x^*}{\varphi} \right) \lambda_x^*(\varphi_x^*). \quad (30)$$

The intuition underlying this relationship in the export market is analogous to the intuition underlying the similar relationship in the domestic market (equation (22)). As ability φ increases, firms become more productive in each product, and so have a lower exporting cutoff for product expertise. As the exporting cutoff for ability φ_x^* increases, the average productivity of the product varieties supplied to the export market by rival firms increases, which intensifies export market competition, and so raises the exporting cutoff for product expertise.

Since revenue in the export market is proportional to revenue in the domestic market, there is an equilibrium relationship between the exporting and zero-profit cutoffs for expertise. Combining the expression for the relative revenue of a variety in the export and domestic markets, $r_{ix}(\varphi, \lambda_i) = \tau^{1-\sigma} r_{id}(\varphi, \lambda_i)$, the zero-profit cutoff condition for expertise (11), and the exporting cutoff condition for expertise (29) yields the following relationship between the exporting and zero-profit cutoffs for expertise for those firms that export:

$$\lambda_x^*(\varphi) = \tau \left(\frac{f_x}{f_p} \right)^{\frac{1}{\sigma-1}} \lambda^*(\varphi). \quad (31)$$

There is selection into export markets within products if the exporting cutoff for expertise lies above the zero-profit cutoff: $\lambda_x^*(\varphi) > \lambda^*(\varphi)$. In this case, for a given level of ability φ , a firm with a sufficiently high draw for expertise finds it profitable to export the product, while another firm with a lower draw for expertise only sells the product in the domestic market. Given the strong empirical evidence supporting the partitioning of firms by export status, we focus on parameter values where self-selection into export markets within products occurs: $\tau^{\sigma-1} f_x > f_p$ in equation (31) above.²⁴

6.3. Firm Profitability

We noted earlier that no firm ever exports a product without also selling it in the domestic market. For each product, we are therefore able to apportion fixed production costs (f_p) to profits from domestic sales and fixed exporting costs (f_x) to profits from export sales. Similarly, across the continuum of products, we are able to apportion fixed

²⁴For empirical evidence on selection into export markets, see Bernard and Jensen (1995, 1999, 2004), Clerides, Lach and Tybout (1998), and Roberts and Tybout (1997).

headquarters costs to firm profits from domestic sales and exporting headquarters costs (f_{xh}) to firm profits from export sales.²⁵

As in the closed economy, with a unit continuum of identical products, firm profits from domestic sales equal expected profits from domestic sales in an individual product market minus fixed headquarters costs. The requirement that firm profits net of fixed headquarters costs are greater than or equal to zero defines a zero-profit cutoff for firm ability φ^* , as in equations (14) and (15). Only firms who draw an ability equal to or greater than φ^* enter, and the zero-profit cutoff for ability varies between the closed and open economy, as established below. Following a similar line of reasoning, firm profits from export sales equal expected profits from export sales in an individual product market minus exporting headquarters costs:

$$\pi_x(\varphi) = \int_{\lambda_x^*(\varphi)}^{\infty} \left[\frac{r_x(\varphi, \lambda)}{\sigma} - f_x \right] z(\lambda) d\lambda - f_{xh}. \quad (32)$$

The lower the ability of a firm, the higher $\lambda_x^*(\varphi)$, and the lower the probability of drawing a product expertise sufficiently high to profitably export a product. Firms with lower abilities have lower expected profits from an individual export market and export a smaller fraction of products [$1 - Z(\lambda_x^*(\varphi))$]. For sufficiently low draws of ability, the excess of revenue over fixed exporting costs in the small range of products that the firm could profitably sell abroad falls short of the exporting headquarters cost. Therefore, the following zero-profit condition defines an exporting cutoff for ability φ_x^* , such that only firms who draw an ability equal to or greater than φ_x^* export:

$$\pi_x(\varphi_x^*) = 0. \quad (33)$$

When the exporting cutoff lies above the zero-profit cutoff for both firm-product expertise and firm ability, namely $\lambda_x^*(\varphi) > \lambda^*(\varphi)$ and $\varphi_x^* > \varphi^*$, there is selection into export markets within and across firms. A firm which draws an ability between φ^* and φ_x^* serves only the domestic market for all the products that it manufactures. Even though such a firm draws an expertise above the exporting cutoff in a fraction [$1 - Z(\lambda_x^*(\varphi))$] of products that it manufactures, the total export profits received across this fraction of products are insufficient to cover the exporting headquarters costs. A firm which draws an ability above φ_x^* does export, but manufactures a larger fraction of products than it exports, so that [$1 - Z(\lambda_x^*(\varphi))$] < [$1 - Z(\lambda^*(\varphi))$]. For some of the firm's products, the expertise draw is high enough to cover fixed *production* costs for the product, but too low given variable trade costs to cover fixed *exporting* costs for the product. Such products are supplied to the domestic market but not exported. The parameter values under which $\varphi_x^* > \varphi^*$ are determined below and are closely related to those discussed above under which $\lambda_x^*(\varphi) > \lambda^*(\varphi)$.

²⁵Following the earlier discussion, we could equivalently compare the sum of firm profits from domestic sales and export sales together to the sum of fixed headquarters costs and exporting headquarters costs.

6.4. Free Entry

The expected value of entry is now equal to the sum of two terms: the *ex ante* probability of successful entry times the average firm profits from the domestic market conditional on entry ($\bar{\pi}_d$), plus the *ex ante* probability of successful entry times the probability of exporting times the average firm profits from the export market conditional on exporting ($\bar{\pi}_x$), all discounted by the probability of firm death. The free entry condition requires that the expected value of entry equals the sunk entry cost:

$$v_e = \frac{[1 - G(\varphi^*)][\bar{\pi}_d + \chi\bar{\pi}_x]}{\delta} = f_e, \quad (34)$$

where the *ex ante* probability of successful entry is $[1 - G(\varphi^*)]$ and $\chi = [1 - G(\varphi_x^*)] / [1 - G(\varphi^*)]$ denotes the probability of exporting. The expression for average firm profits in the domestic market takes expectations over possible values for firm ability in equation (14); similarly, the expression for average firm profits in the export market takes expectations over possible values for firm ability in equation (32).

6.5. Goods and Labor Markets

The steady-state equilibrium is characterized by a constant mass of firms entering each period, M_e , a constant mass of firms producing, M , a constant mass of firms producing in each product market, M_p , as well as a constant mass of firms exporting in each product market, M_{px} . The steady-state equilibrium is also characterized by stationary distributions of firm ability and product expertise in domestic and export markets, which are determined by the zero-profit cutoffs φ^* and $\lambda^*(\varphi)$ and the exporting cutoffs φ_x^* and $\lambda_x^*(\varphi)$, and are the same across products.

Using the equilibrium pricing rule (25) and the symmetry of the two countries, the aggregate price index for each product may be written as a function of the mass of firms manufacturing the product for domestic and export markets, as well as the prices of products with weighted averages of firm ability and product expertise in the domestic and export markets:

$$P = \left[M_p \left(\frac{1}{\rho\tilde{\varphi}} \right)^{1-\sigma} + M_{px} \left(\frac{\tau}{\rho\tilde{\varphi}_x} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (35)$$

where the weighted averages $\tilde{\varphi}$ and $\tilde{\varphi}_x$ depend on the *ex post* distributions of firm ability and product expertise and are defined in the appendix.

Of the mass of firms with a particular ability φ , a fraction $[1 - Z(\lambda^*(\varphi))]$ serve the domestic market in a product, while a fraction $[1 - Z(\lambda_x^*(\varphi))]$ serve the export market in a product. Therefore, the expression for the mass of firms supplying the domestic market in a product remains as in equation (19) for the closed economy, while the expression for the mass of firms supplying the export market in a product is analogous:

$$M_x = \left[\int_{\varphi_x^*}^{\infty} [1 - Z(\lambda_x^*(\varphi))] \left(\frac{g(\varphi)}{1 - G(\varphi_x^*)} \right) d\varphi \right] M. \quad (36)$$

With a constant mass of firms producing in steady-state, the steady-state stability condition remains as before in equation (20). Finally, labor market clearing requires that the demand for labor in production for the domestic market, in production for the export market and in entry equals the economy's aggregate supply of labor.

7. Open Economy Equilibrium

The open economy equilibrium is completely referenced by the sextuple $\{\varphi^*, \varphi_x^*, \lambda^*(\varphi^*), \lambda_x^*(\varphi_x^*), P, R\}$. All other endogenous variables can be written as functions of these six elements of the equilibrium vector.

7.1. Multi-Product Exporters

The range of products manufactured by a firm in the open economy is governed by equations (22) and (23) as in the closed economy. We determine the range of products exported in a similar way. To determine the exporting cutoff for expertise for the lowest ability exporter, $\lambda_{ix}^*(\varphi_x^*)$, we combine the requirement that zero profits are made from export sales by a firm with ability φ_x^* in equation (33) with the expression for firm profits from export sales in equation (32). We substitute for product revenue from export sales using the relationship between relative variety revenues in equation (10) and the exporting cutoff condition for expertise in equation (29). Together these equations yield the following expression that implicitly defines a unique value of $\lambda_{ix}^*(\varphi_x^*)$ as a function of model parameters alone:

$$\int_{\lambda_x^*(\varphi_x^*)}^{\infty} \left[\left(\frac{\lambda}{\lambda_x^*(\varphi_x^*)} \right)^{\sigma-1} - 1 \right] f_x z(\lambda) d\lambda = f_{hx}, \quad \Rightarrow \quad \lambda_x^*(\varphi_x^*) = \bar{\lambda}_x. \quad (37)$$

Once $\lambda_x^*(\varphi_x^*)$ is known, the exporting cutoff for expertise for an exporter with an ability greater than φ_x^* can be determined from equation (30) as a function of the firm's ability φ and the exporting cutoff ability φ_x^* : $\lambda_x^*(\varphi) = (\varphi_x^*/\varphi) \bar{\lambda}_x$. Since the exporting cutoff for expertise determines the fraction of products that a firm exports (which equals $[1 - Z(\lambda_x^*(\varphi))]$), equations (30) and (37) together determine the equilibrium range of products exported for each value of firm ability.

The comparative statics of the range of products exported parallel those of the range of products manufactured. An increase in the fixed exporting cost for a product (f_x) or the elasticity of substitution (σ) raises the exporting cutoff for expertise ($\lambda_x^*(\varphi)$) for firms of all abilities, since a higher value for expertise is required to generate sufficient revenue to cover the fixed exporting cost. An increase in the exporting headquarters cost (f_{hx}) has the opposite effect, reducing the exporting cutoff for expertise ($\lambda_x^*(\varphi)$) for firms of all abilities. The third comparative static is again a property of general equilibrium. As the exporting headquarters cost rises, the total *ex post* profits of exporting firms decline, thereby reducing the expected value of entry, depressing entry, and diminishing

product market competition. As product market competition declines, firms with lower expertise draws can generate sufficient profits to cover fixed exporting costs, and so the exporting cutoff for expertise falls.

The ranges of products exported and manufactured are linked in general equilibrium. From the exporting cutoff condition for expertise (29) for a firm with ability φ_x^* , the zero-profit cutoff condition for expertise (11) for a firm with ability φ^* , the expression for relative revenue in the domestic and export markets, and the equation for relative variety revenues in (10), the exporting cutoff ability φ_x^* and the zero-profit-cutoff ability φ^* are related as follows:

$$\varphi_x^* = \Gamma \varphi^*, \quad \Gamma \equiv \tau \left(\frac{f_x}{f_p} \right)^{\frac{1}{\sigma-1}} \left(\frac{\bar{\lambda}}{\bar{\lambda}_x} \right), \quad (38)$$

where we have used $\lambda_x^*(\varphi_x^*) = \bar{\lambda}_x$ and $\lambda^*(\varphi^*) = \bar{\lambda}$.

The parameter values where $\varphi_x^* > \varphi^*$ (some firms export while others only serve the domestic market) are closely related to those where $\lambda_x^*(\varphi) > \lambda^*(\varphi)$ (exporters ship some products abroad while others are only supplied to the domestic market). From the discussion above, $\lambda_x^*(\varphi) > \lambda^*(\varphi)$ whenever $\tau^{\sigma-1} f_x > f_p$. From equation (38), $\varphi_x^* > \varphi^*$ whenever $\tau^{\sigma-1} f_x \bar{\lambda}^{\sigma-1} > f_p \bar{\lambda}_x^{\sigma-1}$, where $\bar{\lambda}$ is determined by equation (23) and depends on fixed production costs and fixed headquarters costs, while $\bar{\lambda}_x$ is determined by equation (37) and depends on fixed exporting costs and exporting headquarters costs. In the light of the overwhelming evidence of selection into export markets, and since the interior equilibrium is the most interesting one, we focus on parameter values where both sets of inequalities are satisfied, so that $\varphi_x^* > \varphi^*$ and $\lambda_x^*(\varphi) > \lambda^*(\varphi)$.

7.2. Free Entry

Average firm revenue and average firm profits in the domestic market can be re-written as decreasing functions of the zero-profit-cutoff ability (φ^*) and the zero-profit cutoff expertise for a firm with this ability ($\lambda^*(\varphi^*) = \bar{\lambda}$), as in the closed economy: $\bar{r}_d = \bar{r}_d(\varphi^*, \bar{\lambda})$ and $\bar{\pi}_d = \bar{\pi}_d(\varphi^*, \bar{\lambda})$. Similarly, average firm revenue and average firm profits in the export market can be re-written as decreasing functions of the exporting cutoff ability (φ_x^*) and the exporting cutoff expertise for a firm with this ability ($\lambda_x^*(\varphi_x^*) = \bar{\lambda}_x$): thus $\bar{r}_x = \bar{r}_x(\varphi_x^*, \bar{\lambda}_x)$ and $\bar{\pi}_x = \bar{\pi}_x(\varphi_x^*, \bar{\lambda}_x)$.

Substituting for average firm profits in the domestic and export markets in the free entry condition, the requirement that the expected value of entry equals the sunk entry cost becomes:

$$\begin{aligned} v_e = \frac{1}{\delta} \int_{\varphi^*}^{\infty} \left[\int_{\lambda^*(\varphi)}^{\infty} \left[\left(\frac{\lambda}{\lambda^*(\varphi)} \right)^{\sigma-1} - 1 \right] f_p z(\lambda) d\lambda - f_h \right] g(\varphi) d\varphi \\ + \frac{1}{\delta} \int_{\varphi_x^*}^{\infty} \left[\int_{\lambda_x^*(\varphi)}^{\infty} \left[\left(\frac{\lambda}{\lambda_x^*(\varphi)} \right)^{\sigma-1} - 1 \right] f_x z(\lambda) d\lambda - f_{xh} \right] g(\varphi) d\varphi = f_e, \end{aligned} \quad (39)$$

which is again monotonically decreasing in φ^* , since $\lambda^*(\varphi) = (\varphi^*/\varphi)\bar{\lambda}$, $\lambda_x^*(\varphi) = (\varphi_x^*/\varphi)\bar{\lambda}$ and $\varphi_x^* = \Gamma\varphi^*$. This equation implicitly defines a unique zero-profit cutoff for firm ability φ^* as a function of model parameters alone.

As in the closed economy, a rise in the fixed production cost (f_p) leads to a rise in the zero-profit cutoff for ability (φ^*), and an increase in either the fixed headquarters cost (f_h) or the sunk entry cost (f_e) leads to a decrease in the zero-profit cutoff for ability. A rise in the fixed exporting cost (f_x) has similar effects to a rise in the fixed production cost and an increase in the exporting headquarters cost (f_{xh}) has analogous effects to an increase in the fixed headquarters cost (f_h). The economic intuitions for these comparative statics are analogous to those discussed with respect to the closed economy equilibrium above.

7.3. Goods and Labor Markets

We have now solved for the zero-profit cutoff for expertise for a firm with ability φ^* ($\lambda^*(\varphi^*) = \bar{\lambda}$), the exporting cutoff for expertise for a firm with ability φ_x^* ($\lambda_x^*(\varphi_x^*) = \bar{\lambda}_x$), and the zero-profit cutoff ability φ^* as a function of model parameters. These three variables in turn determine the exporting cutoff ability (φ_x^*) in equation (38). The quadruple $\{\varphi^*, \varphi_x^*, \bar{\lambda}, \bar{\lambda}_x\}$ pins down the zero-profit cutoff for expertise ($\lambda^*(\varphi)$) and hence product scope, as well as the exporting cutoff for expertise ($\lambda_x^*(\varphi)$) and hence export scope, for firms of all abilities. Together these variables determine the *ex post* distributions of firm ability and product expertise in both domestic and export markets. We can therefore solve for average revenue and profits in the domestic market ($\bar{r}_d = \bar{r}_d(\varphi^*, \bar{\lambda})$ and $\bar{\pi}_d = \bar{\pi}_d(\varphi^*, \bar{\lambda})$), average revenue and profit in the export market ($\bar{r}_x = \bar{r}_x(\varphi_x^*, \bar{\lambda}_x)$ and $\bar{\pi}_x = \bar{\pi}_x(\varphi_x^*, \bar{\lambda}_x)$), as well as average variety prices in the domestic and export markets.

Combining the steady-state stability, free entry and labor market clearing conditions, it can again be shown that aggregate revenue equals the economy's aggregate supply of labor ($R = L$). The mass of firms producing any product equals aggregate revenue divided by average revenue, $M = R/\bar{r}$, where we have determined both aggregate and average revenue above. The mass of firms producing a product, M_p , and the mass of firms exporting a product, M_{px} , are constant fractions of the mass of firms producing any product, M . These fractions depend solely on the zero-profit-cutoff ability (φ^*), the zero-profit cutoffs for expertise ($\lambda^*(\varphi)$), the exporting cutoff ability (φ_x^*) and the exporting cutoffs for expertise ($\lambda_x^*(\varphi)$) for which we have solved. The price index for each product in equation (35) follows immediately from the mass of firms producing each product, M_p , and the mass of firms exporting each product, M_{px} , as well as average variety prices in the domestic and export market for which we have already solved. This completes the characterization of the equilibrium sextuple $\{\varphi^*, \varphi_x^*, \lambda^*(\varphi^*), \lambda_x^*(\varphi_x^*), P, R\}$.

8. Properties of the Open Economy Equilibrium

Multi-product firms' decisions about endogenous product scope introduce an additional adjustment margin along which economies can respond to trade liberalization. Examination of this new adjustment margin yields a number of novel implications. Most significantly, trade liberalization causes firms to drop marginal lower-expertise products to “focus on their core competencies” in higher-expertise products. Since low-expertise products are low-productivity products, this reallocation of resources within firms induces firm-level productivity growth, offering an intuitive micro-foundation for the idea that international trade rationalizes firm production.²⁶ This response contrasts with existing models of heterogeneous firms and trade where firm productivity is fixed by assumption and aggregate productivity growth is achieved entirely through resource reallocation *across* firms. As a result, our model provides a platform for analyzing firm as well as industry responses to globalization.

Proposition 4 *The opening of trade increases steady-state aggregate productivity through both: (a) **within-firm resource reallocation:** multi-product firms drop low-productivity products (focus on their “core competencies”), and therefore firm and aggregate weighted-average productivity rise, (b) **across-firm resource reallocation:** low-productivity firms exit and therefore aggregate weighted-average productivity rises.*

Proof. See appendix. ■

The key to understanding these results is recognizing that opening an economy to trade has heterogeneous effects across firms depending on whether they are exporters as well as heterogeneous effects across products within firms depending on whether a product is exported. As output of higher-expertise exported products rises, the resulting increase in labor demand bids up wages and reduces the profitability of lower-expertise products that are only supplied to the domestic market. As profitability declines in these lower-expertise products, previously viable products become unprofitable and are dropped. The zero-profit cutoff for expertise $\lambda^*(\varphi)$ rises for firms of all abilities, so that in equilibrium a firm with ability φ manufactures a smaller range of products $[1 - Z(\lambda^*(\varphi))]$. Thus, the opening of trade leads to a contraction in equilibrium product scope, as firms focus on their “core competencies” in a narrower range of higher-expertise products.

Additional intuition for these findings can be gained from comparing the free entry conditions in the closed and open economies (equations (24) and (39)). The opening

²⁶Firm-level productivity corresponds to a weighted average of firm ability and firm-product expertise ($\tilde{\lambda}(\varphi)$), as defined in the appendix, and rises following trade liberalization. We note that the equilibrium pricing rule implies proportionately lower prices for higher-expertise products, so that deflating revenue by firm-product prices eliminates differences in measured productivity in variable production costs across products. However, because there are fixed production costs for each product that are independent of expertise, real revenue per unit of factor input is higher for the higher-expertise products of a firm.

of trade increases the expected value of entry for a firm because of the positive *ex ante* probability of drawing an ability high enough to export. As a result, there is increased entry, which enhances product-market competition and raises the zero-profit-cutoff ability φ^* below which firms exit. This rise in φ^* elevates the average productivity of the varieties manufactured by competing firms, strengthens product market competition, and so induces surviving firms to drop lower-expertise products. The product expertise cutoff $\lambda^*(\varphi)$ of a firm with a particular ability φ in equations (22) and (23) increases as φ^* rises, which implies that the firm manufactures a smaller fraction of products $[1 - Z(\lambda^*(\varphi))]$. Figure 2 illustrates the impact of the opening of trade as a rightward shift in the schedule $[1 - Z(\lambda^*(\varphi))]$, where φ^{*A} and φ^{*CT} in the figure denote the zero-profit cutoffs for ability in the autarkic and costly trade economies, respectively. Although the product expertise cutoff for a firm with ability φ^* remains unchanged at $\lambda^*(\varphi^*) = \bar{\lambda}$, the value of φ^* for which the product expertise cutoff is equal to $\bar{\lambda}$ has increased.

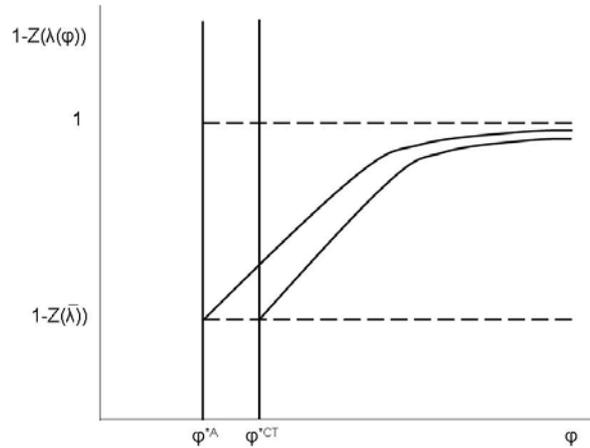


Figure 2: Trade Liberalization and Product Scope

The focusing on core competencies that follows the opening of trade leads to a change in the composition of firm output that generates firm-level productivity growth. Marginally viable low-expertise products are dropped; output of all surviving products for the domestic market contracts; and entry into exporting generates new output for the export market of higher-expertise products. Each of these responses shifts the composition of firm output towards higher-expertise products. Therefore, even though firm ability and product expertise are themselves parameters that are fixed by assumption, firm productivity rises with the within-firm reallocation of resources across products. This response contrasts with existing models of heterogeneous firms and trade in which firms are implicitly restricted to production of just a single product.

Our results also contrast with the predictions of an alternative framework where products are randomly allocated to firms. In our model, each firm optimizes on the decision of the number of products to produce, and so all firms focus on their core competencies in higher-expertise products following trade liberalization. Whereas in the alternative framework where firms are random groupings of products, only those firms who happen to have products with a productivity above the old and below the new zero-profit cutoff would drop products following trade liberalization. The systematic focusing on core competencies that occurs in our model following trade liberalization leads to pervasive firm-level productivity growth, while the correlation between firms' intensive and extensive margins has additional general equilibrium implications considered further below.

Although we have developed these results in the context of opening a closed economy to trade, similar results hold for reductions in trade costs once international trade occurs, due, for example, to improvements in transportation technology or multilateral trade liberalization. The proof of that result, not provided to conserve space, is directly analogous to the proof of Proposition 4 and involves differentiating the free entry condition with respect to trade costs. Trade liberalization again generates firm-level productivity growth and induces between-firm reallocations of resources from lower to higher-ability firms.²⁷

The prediction that trade liberalization leads to firm-level productivity growth is supported by an extensive body of empirical research.²⁸ In an influential study, Pavcnik (2002) finds that within-plant productivity growth accounts for one third (6.6 percentage points) of the 19 percent increase in aggregate productivity in the Chilean manufacturing sector following the trade reforms of the late 1970s and early 1980s.

The mechanism that generates firm-level productivity growth in our model also receives empirical support. Using census data on Canadian plants, Baldwin and Gu (2006) develop a measure of product diversification that captures the extent to which plants' output is concentrated in their largest products. Analyzing the Canada-US Free Trade Agreement (CUSFTA) and the North American Free Trade Agreement (NAFTA), the authors find a sharp decline in product diversification following these liberalizations. Bowen and Wiersema (2005) and Liu (2006) find a similar correlation between increased import competition and reduced product diversification using U.S. firm-level data.

Further implications of extensive-margin adjustments emerge from the distinction between domestic and export markets. While firms contract the range of products that they manufacture and supply to the domestic market as trade costs fall, exporting firms expand the range of products that they supply to the export market, so that

²⁷Throughout the following, we focus on reductions in variable trade costs, but reductions in the fixed costs of exporting have similar effects as long as there remains selection into export markets: $\lambda_x^*(\varphi) > \lambda^*(\varphi)$ and $\varphi_x^* > \varphi^*$.

²⁸See among others Tybout and Westbrook (1995), Pavcnik (2002), Trefler (2004), Bernard, Jensen and Schott (2006), and the survey by Tybout (2003).

exports account for an increasing fraction of firm sales. Thus, consistent with the stylized facts presented in Section 2, there is a positive correlation between firms' intensive and extensive margins in both the domestic and export market.

Proposition 5 *Reductions in variable trade costs (τ) lead to:*

- (a) *exit from the domestic market by low-ability firms*
- (b) *contraction in the domestic market along both the extensive margin (the number of products manufactured for the domestic market) and the intensive margin (domestic sales of each product)*
- (c) *entry into the export market by firms who previously only served the domestic market*
- (d) *expansion in the export market along both the extensive margin (the number of products exported) and the intensive margin (export sales of each product).*

Proof. See appendix. ■

The positive correlation between firms' intensive and extensive margins magnifies the size inequality between exporting and non-exporting firms relative to a framework where product scope is exogenous. High-ability exporting firms are larger than low-ability non-exporting firms, not only because they sell more of a given range of products in the domestic market, but also because they manufacture a larger range of products, and because they export a range of products that increases with their ability.

Proposition 6 *Multi-product firms decisions about endogenous product scope magnify the inequality in firm size between high-ability exporters and low-ability domestic producers relative to when firms manufacture and export exogenous ranges of products ($\lambda^*(\varphi) = \lambda^*(\varphi^*) = \bar{\lambda}$ and $\lambda_x^*(\varphi) = \lambda_x^*(\varphi^*) = \bar{\lambda}_x$ for all φ).*

Proof. See appendix. ■

The predictions of our model are consistent with a broad set of stylized facts about exporting, firm size and productivity. As in the existing theoretical literature, steady-state firm productivity is higher at exporters than non-exporters, and there are no steady-state differences in productivity growth across firms because productivity is drawn from a fixed distribution. Following reductions in trade costs, all firms experience increases in average productivity, but firms that switch from non-exporting to exporting experience an additional source of productivity growth. New exporters not only drop lower-expertise products, but also expand output of newly exported products, which shifts the composition of firm output towards higher-expertise products. This prediction is consistent with a number of empirical studies that find significant productivity increases in the year of transition to exporting (for example Bernard and Jensen 1999).

For continuing exporters, the implications of reductions in trade costs for productivity are more subtle. Continuing exporters experience productivity growth from the

expansion of output at previously exported products, which shifts the composition of output towards higher-expertise products. But continuing exporters also begin to export lower-expertise products, which reduces average productivity in the export market. Combining these two effects, it is ambiguous whether reductions in trade costs lead to higher or lower productivity growth at continuing exporters than at non-exporting firms, whose sole source of productivity growth is the reallocation of output across products in the domestic market.

In contrast, the effect of reductions in trade costs on the relative output growth of continuing exporters and non-exporters is unambiguous. From Proposition 6, the output of continuing exporters rises relative to non-exporters as a result of expansion along both the intensive and extensive margin in export markets. This combination of predictions for output and productivity is consistent with empirical findings for the United States that, over a period since the early 1970s during which trade costs have fallen, there is evidence of significant differences in output and employment growth between continuing exporters and non-exporters in the same industry, but no evidence of significant differences in productivity growth (see Bernard and Jensen 1999).

More generally, our framework features a richer and more realistic range of export behavior than in existing models of heterogeneous firms and trade with single-product firms. There is selection into export markets across firms (exporters and non-exporters), within products (some firms export a product while other firms only supply the product to the domestic market) and also within firms (some products are exported by a firm while other products are only sold domestically). This richer range of export behavior enables our model to better account for a number of features of the micro data on the trade exposure of firms and plants. In particular, the model is able to explain the positive correlation between exports per product and the number of products exported in Table 2 above. We are also able to account for empirical findings that exports comprise a small share of revenue for many exporters, while for other firms exports constitute a large share of revenue.²⁹ In our framework, the share of exports in firm revenue varies endogenously across exporters according to their ability. Higher-ability firms export a higher fraction of products and thus have higher export shares.³⁰

The expansion of multi-product firms along the extensive margin of the number of products exported helps to explain the extreme inequality that is observed in the distribution of export shipments across firms. The top 1 percent of firms are found by Bernard, Jensen and Schott (2005) to account for over 80 percent of the value of U.S.

²⁹For empirical evidence on the distribution of firm export shares, see Bernard and Jensen (1995), Bernard *et al.* (2003) and Brooks (2006).

³⁰Another complementary explanation for variation in export shares involves single-product firms and fixed exporting costs that are specific to individual export markets, so that only higher-productivity firms find it profitable to serve destinations with higher fixed exporting costs (see for example Eaton, Kortum and Kramarz 2005). While this alternative explanation accounts for variation in the share of exports in firm revenue, it does not explain the positive correlation between exports per product and the number of products exported by a firm.

exports in the year 2000. To explain this inequality in export shipments, models of heterogeneous firms and trade with single products require either an extremely unequal productivity distribution and/or a high elasticity of substitution. In our framework, higher ability increases firm export shipments through both greater exports per product and a larger range of products exported. Therefore, the observed inequality in export shipments can be explained with less inequality in firm ability than when product scope is exogenous, mirroring our earlier result for the domestic market in the closed economy.

9. Multi-Product Firms and Comparative Advantage

In this section we examine the interaction of endogenous product choice and comparative advantage by embedding our model within the two-factor, two-country and two-industry heterogeneous firm framework of Bernard, Redding and Schott (2006a).

The model described thus far can be viewed as capturing a single industry containing many products, with firms supplying differentiated varieties of these products. We now generalize the framework by analyzing two industries, $j = 1, 2$, each of which has this structure. The two industries enter an upper tier of the representative consumer's utility that takes the Cobb-Douglas form:

$$\Upsilon = U_1^{\alpha_1} U_2^{\alpha_2}, \quad \alpha_1 + \alpha_2 = 1, \quad \alpha_1 = \alpha, \quad (40)$$

where U_j is an index for industry j that is defined over the consumption C_i of a continuum of products i within the industry (as in equation (4)), and C_i is itself an index that is defined over the consumption $c_i(\omega)$ of a continuum of varieties ω within each product (as in equation (5)). "Industries" now constitute an upper tier of utility, "products" form an intermediate tier and "varieties" occupy a lower tier. We assume for simplicity that the two industries have the same elasticity of substitution across products within industries (κ) and across varieties within products (σ).

The industries differ in the intensity with which they use two factors of production: skilled and unskilled labor. To enter an industry, a firm from the competitive fringe must incur the sunk entry cost for that industry, which equals $f_{ej}(w_S)^{\beta_j}(w_L)^{1-\beta_j}$, where w_S denotes the skilled wage, w_L corresponds to the unskilled wage and β_j parameterizes industry factor intensity. After the sunk entry cost is paid, the firm draws its ability and values of product expertise for the industry that it enters as above. The distributions of firm ability and product expertise are identically and independently distributed across industries, so that information about productivity within an industry can only be obtained by incurring the sunk entry cost for that industry. The distributions of firm ability and product expertise are also identically and independently distributed across countries, which ensures consistency with the Heckscher-Ohlin model's assumption of common technologies across countries.

The technology for production has the same factor intensity as for entry.³¹ While the factor intensity of production varies across industries, all products within an industry are modelled symmetrically and therefore have the same factor intensity.³² To manufacture a variety of a product, a firm must incur a fixed and variable cost of production as above. The variable cost depends upon the firm's productivity in manufacturing the product, which is again determined by both the firm's ability and its expertise in that product. Fixed and variable costs use the two factors of production with the same proportions, so that the total cost of production for a firm in industry j is as follows:

$$\Psi_j = \left(f_{hj} + \int_0^1 I_{ji} \left[f_{pj} + \frac{q_j(\varphi_j, \lambda_{ji})}{\varphi_j \lambda_{ji}} \right] di \right) (w_S)^{\beta_j} (w_L)^{1-\beta_j}, \quad 1 > \beta_1 > \beta_2 > 0, (41)$$

where I_{ji} is an indicator variable which equals one if a firm in industry j manufactures product i and zero otherwise. The fixed costs of becoming an exporter and of exporting a particular product are modelled analogously, and use skilled and unskilled labor with the same factor intensity as production and entry.

We assume that industry 1 is skill-intensive relative to industry 2 and that the home country is skill-abundant relative to the foreign country. The combination of industry-level variation in factor intensity and country-level variation in factor abundance gives rise to endowment-driven comparative advantage. The skilled wage in the home country is chosen as the numeraire.

We determine general equilibrium using an approach very similar to that of the previous sections and therefore omit the reporting of similar equations here to conserve space. The range of products manufactured and exported by a firm with a particular ability can be determined using expressions analogous to those above. Similarly, the zero-profit cutoff and exporting cutoff for ability can be determined following the same line of reasoning as previously. Entry, production and exporting costs all have the same factor intensity, and therefore terms in factor prices cancel from the relevant expressions. There is however an important general equilibrium interaction between comparative advantage and firms' product scope and entry decisions. Because countries are no longer symmetric, comparative advantage and trade costs generate cross-country differences in industry price indices. As in Bernard, Redding and Schott (2006a), these differences, in turn, generate greater export opportunities in comparative-advantage industries than comparative disadvantage industries.

The relative price indices for the two industries vary across countries because of the combination of comparative-advantage-based specialization and trade costs. Specialization leads to a larger mass of domestic firms relative to foreign firms in a country's

³¹Allowing factor intensity differences between entry and production introduces additional interactions with comparative advantage as discussed in Bernard, Redding and Schott (2006a).

³²The symmetry of products within industries is clearly a simplification, but is useful for the law of large numbers results that determine the fraction of products manufactured by a firm with a particular ability.

comparative-advantage industry than in its comparative disadvantage industry. Variable trade costs introduce a wedge between domestic and export prices for a variety. Additionally, the fixed costs of becoming an exporter imply that not all firms export, and the fixed costs for exporting individual products imply that only a subset of the products manufactured by an exporter are shipped abroad. Combining specialization and trade costs, the comparative-advantage industry has a greater mass of lower-priced domestic varieties relative to the mass of higher-price foreign varieties than the comparative disadvantage industry. As a result, the price index in the domestic market is lower relative to the price index in the export market in the comparative-advantage industry. Hence, the degree of competition in the domestic market is higher relative to the degree of competition in the export market in the comparative-advantage industry. These differences in the degree of competition in turn imply that variable profits in the export market are greater relative to variable profits in the domestic market in the comparative-advantage industry than in the comparative disadvantage industry.

Proposition 7 *Other things equal, the opening of trade leads to:*

- (a) *greater focusing on core competencies and greater firm-level productivity growth in the comparative-advantage industry than the comparative disadvantage industry ($\Delta\lambda_1^{*H}(\varphi) > \Delta\lambda_2^{*H}(\varphi)$ and $\Delta\lambda_2^{*F}(\varphi) > \Delta\lambda_1^{*F}(\varphi)$),*
- (b) *a larger increase in the zero-profit cutoff for ability below which firms exit in the comparative-advantage industry than the comparative disadvantage industry ($\Delta\varphi_1^{*H} > \Delta\varphi_2^{*H}$ and $\Delta\varphi_2^{*F} > \Delta\varphi_1^{*F}$),*
- (c) *a larger increase in weighted-average productivity in the comparative-advantage industry than in the comparative disadvantage industry ($\Delta\tilde{\varphi}_1^{*H} > \Delta\tilde{\varphi}_2^{*H}$ and $\Delta\tilde{\varphi}_2^{*F} > \Delta\tilde{\varphi}_1^{*F}$).*

Proof. See Appendix. ■

Following the opening of trade, countries specialize according to comparative advantage, which leads to a rise in the mass of firms in the comparative-advantage industry relative to the comparative disadvantage industry. While the comparative-advantage industry expands and the comparative disadvantage industry contracts, the shedding of low productivity products (i.e., the focus on “core competencies”) is greater in the comparative-advantage industry. The opening of trade therefore leads to greater firm-level productivity growth in the comparative-advantage industry. In addition to the greater focusing on core competencies in the comparative-advantage industry, there is also a larger rise in the zero-profit cutoff for ability below which firms exit. Hence, the opening of trade causes a larger increase in average industry productivity in the comparative-advantage industry because of both stronger firm-level productivity growth and greater across-firm reallocations of resources.

Consideration of the greater export opportunities that exist in comparative-advantage industries is the key to understanding these results. Following the opening of trade, there

is a larger increase in labor demand at exported products in the comparative-advantage industry than in the comparative disadvantage industry. This larger increase in labor demand leads to a rise in the relative price of the abundant factor, because this factor is used intensively in the comparative-advantage industry. As the relative price of the abundant factor rises, the difference in factor intensity between the two sectors implies that production costs in the comparative-advantage industry increase relative to production costs in the comparative disadvantage industry. This change in relative production costs reduces the profits derived from products that are supplied only to the domestic market in the comparative-advantage industry relative to the comparative disadvantage industry. The larger reduction in domestic market profits induces greater shedding of low-productivity products and hence greater firm-level productivity growth in the comparative-advantage industry. Similarly, the larger reduction in domestic market profits leads to a greater rise in the zero-profit cutoff for ability and hence greater between-firm reallocations of resources in the comparative-advantage industry than in the comparative disadvantage industry.

As a result, the interaction of comparative advantage and endogenous product selection causes a number of distinctive firm-level responses to trade liberalization that vary with comparative advantage. The measured productivity of a firm with a particular ability depends on comparative advantage because it shapes the endogenous range of products that the firm chooses to manufacture. Significantly, multi-product firms' endogenous extensive-margin adjustments have general equilibrium consequences. The greater firm-level productivity growth and between-firm reallocations of resources in the comparative-advantage industry induce Ricardian differences in average productivity which are non-neutral across firms and industries and provide an additional source of welfare gains from trade.

10. Conclusions

Although multi-product firms dominate world production and trade they have received relatively little theoretical attention among international trade economists. This paper introduces a general equilibrium model of multi-product firms that features firm- as well as industry-level responses to trade liberalization. Though firm productivity in each product is fixed (once drawn) by assumption, both firm- and industry-level productivity are shown to increase with reductions in trade costs as a result of resource reallocation within and across firms. As trade costs fall, firms drop low-productivity products while the lowest-productivity firms exit. Our framework therefore provides a micro-foundation for the idea that international competition rationalizes firm production – in marked contrast to existing models of heterogeneous firms and trade where firms manufacture a single product. Our model and its implications are supported by a large body of stylized facts as well as more recent findings that trade integration causes firms to increasingly concentrate production in their most successful products.

The model provides intuition for a number of distinctive and often surprising predictions, including the result that firms intensive (the quantity of each product is produced) and extensive (the number of products) margins exhibit a positive correlation. During trade liberalization, exporting firms concomitantly expand the range of products supplied to the export market and contract the range of products they manufacture. Endogenous product scope magnifies inequality in the firm-size distribution but reduces dispersion in average firm productivity. The adjustments of firms along their extensive margins dampen the response of overall firm entry and exit decisions to changes in model parameters.

Although we model endogenous product scope in general equilibrium, the framework is highly tractable and amenable to numerous extensions. One which we include in the paper considers the influence of comparative advantage. Another might include allowing for several stages of production so that firms can supply multiple intermediate and final goods. A further extension might include introducing a continuum of export destinations to which each product can be exported. If there are fixed costs of exporting to individual destinations and these fixed costs vary across destinations, high-firm ability firms not only export a wider range of products, but also supply each product to a larger number of destinations. Such an extension would further enrich the dimensions of firm responses to trade liberalization. Other extensions might include adding stochastic shocks to firm ability and product expertise, modifying the model to introduce strategic interaction, and considering asymmetric trade liberalizations.

While our framework is necessarily an abstraction, the introduction of endogenous product scope to a heterogeneous firm model with entry and exit represents an important step towards greater understanding of multiple-product firms and their responses to trade liberalization in general equilibrium. Testing the implications of our framework will provide direction for further modeling of firms and trade.

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A Data Appendix³³

The empirical results employ two data sources. For results on firm output, output per product and product count, we use the data from Bernard, Redding and Schott (2006b). Those data are derived from the U.S. Census of Manufactures of the Longitudinal Research Database (LRD) maintained by the U.S. Census Bureau. Manufacturing Censuses are conducted every five years and we examine data from 1972 to 1997. The sampling unit for each Census is a manufacturing “establishment”, or plant, and the sampling frame in each Census year includes information on the mix of products produced by the plant. Very small manufacturing plants (referred to as Administrative Records) are excluded from the analysis unless otherwise noted because data on their mix of products are unavailable. Because product-mix decisions are made at the level of the firm, we aggregate the LRD to that level for our analysis.

³³The description here draws upon that in Bernard, Redding and Schott (2006b).

Our definition of “product” is based upon 1987 Standard Industrial Classification (SIC) categories, which segment manufacturing output generally according to its end use.³⁴ We refer to five-digit SIC categories as products or goods. In the LRD, aggregate manufacturing contains 1848 products. For each firm in each Census year, we record the set of products in which the firm produces. We also observe firms’ total and product-level output. There are an average of 141,561 surviving firms in each Census year for which such extensive-margin adjustments can be observed. For further details about the construction of the dataset, see Bernard, Redding and Schott (2006b).

For export value, exports per product and the count of exported products we use the Linked/Longitudinal Firm Trade Transaction Database (LFTTD) from Bernard, Jensen and Schott (2005) which links individual trade transactions to firms in the United States. This dataset has two components. The first, foreign trade data assembled by the U.S. Census Bureau and the U.S. Customs Bureau, captures all U.S. international trade transactions between 1993 and 2000 inclusive. For each flow of goods across a U.S. border, this dataset records the product classification and the value. Products in the LFTTD are tracked according to ten-digit Harmonized System (HS) categories, which break exported goods into 8572 products. The second component of the LFTTD is the Longitudinal Business Database (LBD) of the U.S. Census Bureau, which records annual employment and survival information for most U.S. establishments.³⁵ Employment information for each establishment is collected in March of every year and we aggregate the establishment data up to the level of the firm. Matching the annual information in the LBD to the transaction-level trade data yields the LFTTD. For further details about the construction of the dataset, see Bernard, Jensen and Schott (2005).

B Theory Appendix

B1. Weighted Average Productivity

Aggregate weighted-average productivity is:

$$\tilde{\varphi} \equiv \left[\frac{1}{1 - G(\varphi^*)} \int_{\varphi^*}^{\infty} \tilde{\lambda}(\varphi) g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}, \quad (42)$$

where $\tilde{\lambda}(\varphi)$ denotes weighted-average productivity for a firm of ability φ :

$$\tilde{\lambda}(\varphi) = \frac{1}{1 - Z(\lambda^*(\varphi))} \int_{\lambda^*(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda) d\lambda. \quad (43)$$

³⁴The SIC classification scheme was revised substantially in 1977 and again in 1987. Industry identifiers from Censuses prior to 1987 have been concorded to the 1987 scheme. Our results are not sensitive to this concordance: we get substantially similar results if we focus exclusively on the 1987 to 1997 portion of the LRD.

³⁵This dataset excludes the U.S. Postal Service and firms in agriculture, forestry and fishing, railroads, education, public administration and several smaller sectors. See Jarmin and Miranda (2002) for an extensive discussion of the LBD and its construction.

Similarly, aggregate weighted-average productivity in the export market is:

$$\tilde{\varphi}_x \equiv \left[\frac{1}{1 - G(\varphi_x^*)} \int_{\varphi_x^*}^{\infty} \tilde{\lambda}_x(\varphi) g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}, \quad (44)$$

where $\tilde{\lambda}_x(\varphi)$ denotes weighted-average productivity in the export market for a firm of ability φ :

$$\tilde{\lambda}_x(\varphi) = \frac{1}{1 - Z(\lambda_x^*(\varphi))} \int_{\lambda_x^*(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda). \quad (45)$$

B2. Closed Economy Average Firm Revenue and Profit

Combining equations (17), (10), (11), (22) and (23), average revenue and profits can be re-written as:

$$\begin{aligned} \bar{r} &= \int_{\varphi^*}^{\infty} \left[\int_{\lambda^*(\varphi)}^{\infty} \left(\frac{\lambda}{\lambda^*(\varphi)} \right)^{\sigma-1} \sigma f_p z(\lambda) d\lambda \right] \left(\frac{g(\varphi)}{1 - G(\varphi^*)} \right) d\varphi \\ \bar{\pi} &= \int_{\varphi^*}^{\infty} \left[\int_{\lambda^*(\varphi)}^{\infty} \left[\left(\frac{\lambda}{\lambda^*(\varphi)} \right)^{\sigma-1} - 1 \right] f_p z(\lambda) d\lambda - f_h \right] \left(\frac{g(\varphi)}{1 - G(\varphi^*)} \right) d\varphi. \end{aligned} \quad (46)$$

where $\lambda^*(\varphi) = (\varphi^*/\varphi) \bar{\lambda}$.

B3. Open Economy Average Firm Revenue and Profit

Combining equations (17), (10), (11), (22) and (23), the expression for average revenue and profits in the domestic market are the same as those in equation (46). Combining equations (32), (10), (29), (30) and (37), average revenue and profits in the export market can be re-written as:

$$\begin{aligned} \bar{r}_x &= \int_{\varphi_x^*}^{\infty} \left[\int_{\lambda_x^*(\varphi)}^{\infty} \left(\frac{\lambda}{\lambda_x^*(\varphi)} \right)^{\sigma-1} \sigma f_x z(\lambda) d\lambda \right] \left(\frac{g(\varphi)}{1 - G(\varphi_x^*)} \right) d\varphi \\ \bar{\pi}_x &= \int_{\varphi_x^*}^{\infty} \left[\int_{\lambda_x^*(\varphi)}^{\infty} \left[\left(\frac{\lambda}{\lambda_x^*(\varphi)} \right)^{\sigma-1} - 1 \right] f_x z(\lambda) d\lambda - f_{xh} \right] \left(\frac{g(\varphi)}{1 - G(\varphi_x^*)} \right) d\varphi. \end{aligned} \quad (47)$$

where $\lambda_x^*(\varphi) = (\varphi_x^*/\varphi) \bar{\lambda}_x$.

B4. Proof of Proposition 1

Proof. The relative revenue of two firms with abilities φ'' and φ' and expertises λ'' and λ' within a product market is: $IM \equiv r(\varphi'', \lambda'') / r(\varphi', \lambda') = (\varphi''/\varphi')^{\sigma-1} (\lambda''/\lambda')^{\sigma-1}$, where λ is independently distributed across firms and products. The relative fraction of products manufactured by the two firms is: $EM \equiv [1 - Z(\lambda^*(\varphi''))] / [1 - Z(\lambda^*(\varphi'))]$. Note that $\lambda^*(\varphi)$ in equations (22) and (23) is monotonically decreasing in φ , while $Z(\lambda^*(\varphi))$ is a cumulative distribution function which is monotonically increasing in its argument. It follows that IM and EM are both monotonically increasing in (φ''/φ') and are positively correlated. ■

B5. Proof of Corollary 1

Proof. Consider two firms with abilities φ'' and φ' , where $\varphi'' > \varphi'$. Combining the equation for firm revenue in (13), the relationship between variety revenues in (10), the zero-profit cutoff for product expertise in (11), and the relationship between product zero-profit cutoffs in (22) yields the following expression for the relative revenues of the two firms:

$$r(\varphi'') = \left(\frac{\varphi''}{\varphi'}\right)^{\sigma-1} r(\varphi') + \int_{\lambda^*(\varphi'')}^{\lambda^*(\varphi')} \left(\frac{\lambda}{\lambda^*(\varphi')}\right)^{\sigma-1} \sigma f_p z(\lambda) d\lambda, \quad (48)$$

which is strictly greater than the relative revenue of the two firms with a common exogenous product scope: $r(\varphi'') = (\varphi''/\varphi')^{\sigma-1} r(\varphi')$. ■

B6. Proof of Proposition 2

Proof. (a) Differentiating firm weighted average productivity $\tilde{\lambda}(\varphi)$ in equation (43), we obtain:

$$\begin{aligned} \frac{d\tilde{\lambda}(\varphi)}{d\varphi} &= \underbrace{\left(\frac{z(\lambda^*(\varphi))}{[1-Z(\lambda^*(\varphi))]} \right) \frac{d\lambda^*(\varphi)}{d\varphi} \left(\frac{1}{[1-Z(\lambda^*(\varphi))]} \right) \int_{\lambda^*(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda) d\lambda}_{\text{Term A} < 0} \\ &+ \underbrace{\left(\frac{1}{[1-Z(\lambda^*(\varphi))]} \right) \int_{\lambda^*(\varphi)}^{\infty} (\sigma-1)(\lambda)^{\sigma-1} (\varphi)^{\sigma-2} z(\lambda) d\lambda}_{\text{Term B} > 0} \\ &- \underbrace{\left(\frac{z(\lambda^*(\varphi))}{[1-Z(\lambda^*(\varphi))]} \right) \left(\frac{d\lambda^*(\varphi)}{d\varphi}\right) (\varphi\lambda^*(\varphi))^{\sigma-1}}_{\text{Term C} > 0} > 0, \end{aligned}$$

where $d\lambda^*(\varphi)/d\varphi < 0$. Since $z(\lambda)$ is a continuous distribution function, $z(\lambda^*(\varphi)) / [1-Z(\lambda^*(\varphi))]$ is of measure zero, and so the derivative $d\tilde{\lambda}(\varphi)/d\varphi > 0$.

(b) Consider two firms with abilities φ'' and φ' , where $\varphi'' > \varphi'$, and so $\lambda^*(\varphi'') < \lambda^*(\varphi')$. From the definition of firm weighted-average productivity in equation (43):

$$\tilde{\lambda}(\varphi'') = \left[\underbrace{\left(\frac{1-Z(\lambda^*(\varphi'))}{1-Z(\lambda^*(\varphi''))}\right)}_{\text{Term A}} + \underbrace{\left(\frac{Z(\lambda^*(\varphi'))-Z(\lambda^*(\varphi''))}{1-Z(\lambda^*(\varphi''))}\right)}_{\text{Term B}} \underbrace{\left(\frac{\tilde{\lambda}_0}{\tilde{\lambda}(\varphi')}\right)}_{\text{Term C}} \right] \left(\frac{\varphi''}{\varphi'}\right) \tilde{\lambda}(\varphi'), \quad (49)$$

$$\text{where } \tilde{\lambda}_0 \equiv \frac{1}{Z(\lambda^*(\varphi'))-Z(\lambda^*(\varphi''))} \int_{\lambda^*(\varphi'')}^{\lambda^*(\varphi')} (\varphi\lambda)^{\sigma-1} z(\lambda) d\lambda < \tilde{\lambda}(\varphi'),$$

and where the sum of the terms in square parenthesis in equation (49) is strictly less than one, since Terms A and B sum to one, and Term C is strictly less than one. ■

B7. Proof of Proposition 3

Proof. Consider an increase in the sunk entry cost: $\Delta f_e > 0$. Denote the value of variables before and after the change in the sunk entry cost by the subscripts 1 and 2. With endogenous product scope: $\varphi_1^* > \varphi_2^*$ and $\lambda_1^*(\varphi) > \lambda_2^*(\varphi)$. Taking differences in the free entry condition (24) before and after the parameter change, we obtain:

$$\begin{aligned} \Delta v_e = & \underbrace{\frac{1}{\delta} \int_{\varphi_1^*}^{\infty} \left[\int_{\lambda_1^*(\varphi)}^{\infty} \left(\left(\frac{\lambda}{\lambda_2^*(\varphi)} \right)^{\sigma-1} - \left(\frac{\lambda}{\lambda_1^*(\varphi)} \right)^{\sigma-1} \right) f_p z(\lambda) d\lambda - f_h \right] g(\varphi) d\varphi}_{\text{Term A} > 0} \\ & + \underbrace{\frac{1}{\delta} \int_{\varphi_1^*}^{\infty} \left[\int_{\lambda_2^*(\varphi)}^{\lambda_1^*(\varphi)} \left(\left(\frac{\lambda}{\lambda_2^*(\varphi)} \right)^{\sigma-1} - 1 \right) f_p z(\lambda) d\lambda - f_h \right] g(\varphi) d\varphi}_{\text{Term B} > 0} \\ & + \underbrace{\frac{1}{\delta} \int_{\varphi_2^*}^{\varphi_1^*} \left[\int_{\lambda_2^*(\varphi)}^{\infty} \left(\left(\frac{\lambda}{\lambda_2^*(\varphi)} \right)^{\sigma-1} - 1 \right) f_p z(\lambda) d\lambda - f_h \right] g(\varphi) d\varphi}_{\text{Term C} > 0} = \Delta f_e. \end{aligned}$$

In contrast, with exogenous product scope: $\varphi_1^* > \varphi_2^*$ and $\lambda^*(\varphi) = \lambda_1^*(\varphi)$. Taking differences in the free entry condition before and after the parameter change, we obtain:

$$\Delta v_e = \underbrace{\frac{1}{\delta} \int_{\varphi_2^*}^{\varphi_1^*} \left[\int_{\lambda_1^*(\varphi)}^{\infty} \left(\left(\frac{\lambda}{\lambda_1^*(\varphi)} \right)^{\sigma-1} - 1 \right) f_p z(\lambda) d\lambda - f_h \right] g(\varphi) d\varphi}_{\text{Term D} > 0} = \Delta f_e.$$

where Term D < Term C, since $\lambda_1^*(\varphi) > \lambda_2^*(\varphi)$.

For both endogenous and exogenous product scope, we require: $\Delta v_e = \Delta f_e$. For a given pair of values of φ_1^* and φ_2^* , note that Term A > 0, Term B > 0 and Term C > Term D > 0. Therefore, since Δv_e is monotonically increasing in $\Delta \varphi^* = \varphi_1^* - \varphi_2^* > 0$, the change in the zero-profit-cutoff ability $\Delta \varphi^*$ must be smaller when product scope is endogenous than when it is exogenous. ■

B8. Proof of Proposition 4

Proof. (a) The expected value of entry for the open economy in equation (39) equals that for the closed economy in equation (24) plus an additional positive term. Since the expected value of entry v_e is monotonically decreasing in φ^* , the equilibrium value of φ^* must be higher in the open economy than in the closed economy in order to equate v_e with the unchanged sunk entry cost. The product expertise cutoff $\lambda^*(\varphi) = (\varphi^*/\varphi) \bar{\lambda}$ is monotonically increasing in φ^* and therefore rises following the opening of trade. Firm weighted average productivity in equation (43) is monotonically increasing in $\lambda^*(\varphi)$ and so also rises, which leads to an increase in aggregate weighted average productivity $\tilde{\varphi}$ in equation (42).

(b) From the proof of Proposition 2, firms with higher abilities φ have higher weighted average productivities $\tilde{\lambda}(\varphi)$. Therefore, the increase in φ^* raises aggregate weighted average productivity $\tilde{\varphi}$ in equation (42), not only because each surviving firm has a higher weighted average productivity $\tilde{\lambda}(\varphi)$, but also because of the selection of firms with higher abilities. ■

B9. Proof of Proposition 5

Proof. (a) From the expression for the expected value of entry v_e in equation (39): $dv_e/d\tau < 0$. Therefore, since (39) is monotonically decreasing in φ^* , $d\varphi^*/d\tau < 0$.

(b) First, we consider the extensive margin in the domestic market. From equations (22) and (23), $d\lambda^*(\varphi)/d\varphi^* > 0$ and so $d\lambda^*(\varphi)/d\tau < 0$. Since the fraction of products manufactured $[1 - Z(\lambda^*(\varphi))]$ is decreasing in $\lambda^*(\varphi)$, the rise in $\lambda^*(\varphi)$ following a reduction in τ diminishes the range of products manufactured for the domestic market.

Second, we consider the intensive margin in the domestic market. The revenue received in the domestic market from a product with expertise λ at a firm with ability φ can be written as the following decreasing function of $\lambda^*(\varphi)$: $r_d(\varphi, \lambda) = (\lambda/\lambda^*(\varphi))^{\sigma-1} \sigma f_p$. Therefore, since $d\lambda^*(\varphi)/d\tau < 0$, $dr_d(\varphi, \lambda)/d\tau > 0$.

(c) Differentiating the expression for equilibrium φ_x^* in equation (38) with respect to τ :

$$\left(\frac{d\varphi_x^*}{d\tau} \frac{\tau}{\varphi_x^*} \right) = 1 + \left(\frac{d\varphi^*}{d\tau} \frac{\tau}{\varphi^*} \right), \quad (50)$$

where, from (a), $d\varphi^*/d\tau < 0$.

Define $\Omega = v_e - f_e$. By the implicit function theorem, $d\varphi^*/d\tau = -(d\Omega/d\tau) / (d\Omega/d\varphi^*)$. Applying the implicit function theorem and using $d\lambda_x^*(\varphi)/d\tau = \lambda_x^*(\varphi)/\tau$, $d\lambda_x^*(\varphi)/d\varphi^* = \lambda_x^*(\varphi)/\varphi^*$, $d\varphi_x^*/d\tau = \varphi_x^*/\tau$ and $d\varphi_x^*/d\varphi^* = \varphi_x^*/\varphi^*$, it can be shown that $(d\varphi^*/d\tau) / (\tau/\varphi^*) > -1$. Therefore, from equation (50), $d\varphi_x^*/d\tau > 0$.

(d) First, we consider the extensive margin in the export market. From equations (31), (22) and (23):

$$\left(\frac{d\lambda_x^*(\varphi)}{d\tau} \frac{\tau}{\lambda_x^*(\varphi)} \right) = 1 + \left(\frac{d\varphi^*}{d\tau} \frac{\tau}{\varphi^*} \right) > 0, \quad (51)$$

where we have used the result, from (c), that $(d\varphi^*/d\tau) / (\tau/\varphi^*) > -1$. Since the fraction of products exported $[1 - Z(\lambda_x^*(\varphi))]$ is decreasing in $\lambda_x^*(\varphi)$, the decline in $\lambda_x^*(\varphi)$ following a reduction in τ expands the range of products exported.

Second, we consider the intensive margin in the export market. The revenue received in the export market from a product with expertise λ at a firm with ability φ can be written as the following decreasing function of $\lambda_x^*(\varphi)$: $r_x(\varphi, \lambda) = (\lambda/\lambda_x^*(\varphi)) \sigma f_x$. Therefore, since $d\lambda_x^*(\varphi)/d\tau > 0$, $dr_x(\varphi, \lambda)/d\tau < 0$. ■

B10. *Proof of Proposition 6*

Proof. Consider an exporting firm of ability φ'' and a non-exporting firm of ability φ' , where $\varphi'' > \varphi'$. When product scope is exogenous, $\lambda^*(\varphi) = \lambda^*(\varphi^*) = \bar{\lambda}$ and $\lambda_x^*(\varphi) = \lambda_x^*(\varphi_x^*) = \bar{\lambda}_x$, and the relative revenue of the two firms is:

$$r(\varphi'') = \left(\frac{\varphi''}{\varphi'}\right)^{\sigma-1} r(\varphi') + \int_{\bar{\lambda}_x}^{\infty} \left(\frac{\varphi}{\varphi_x^*}\right)^{\sigma-1} \left(\frac{\lambda}{\bar{\lambda}_x}\right)^{\sigma-1} \sigma f_x z(\lambda) d\lambda.$$

When the ranges of products manufactured and exported are endogenous, the relative revenues of the two firms may be determined by combining the equation for firm revenue in (13), the relationship between variety revenues in (10), the expression for the zero-profit expertise cutoff in (11), the relationship between the zero-profit expertise cutoffs in (22), the equation for the exporting expertise cutoff in (29), and the relationship between exporting expertise cutoffs in (30):

$$\begin{aligned} r(\varphi'') &= \left(\frac{\varphi''}{\varphi'}\right)^{\sigma-1} r(\varphi') + \int_{\bar{\lambda}_x}^{\infty} \left(\frac{\varphi}{\varphi_x^*}\right)^{\sigma-1} \left(\frac{\lambda}{\bar{\lambda}_x}\right)^{\sigma-1} \sigma f_x z(\lambda) d\lambda \\ &\quad + \int_{\lambda^*(\varphi'')}^{\lambda^*(\varphi')} \left(\frac{\lambda}{\lambda^*(\varphi')}\right)^{\sigma-1} \sigma f_p z(\lambda) d\lambda + \int_{\lambda_x^*(\varphi'')}^{\bar{\lambda}_x} \left(\frac{\varphi}{\varphi_x^*}\right)^{\sigma-1} \left(\frac{\lambda}{\bar{\lambda}_x}\right)^{\sigma-1} \sigma f_x z(\lambda) d\lambda, \end{aligned}$$

which is strictly greater than the expression above. ■

B11. *Proof of Proposition 7*

Proof. The introduction of comparative advantage implies that countries are no longer symmetric. Therefore, the relative revenue from a product i in the domestic and export markets in industry j depends on price indices and aggregate revenue in the two countries: $r_{jix}(\varphi_j, \lambda_{ji}) = \tau_j^{1-\sigma} (P_{ji}^F/P_{ji}^H)^{\sigma-1} (R^F/R^H) r_{jid}(\varphi_j, \lambda_{ji})$. In equilibrium, the price indices are the same for all products within an industry and country due to symmetry: $P_{ji}^F = P_j^F$ and $P_{ji}^H = P_j^H$ for all i .

The equations determining the equilibrium range of products manufactured as a function of the zero-profit-cutoff firm ability φ^* (equations (22) and (23)) and the equations determining the equilibrium range of products exported as a function of the exporting cutoff firm ability φ_x^* (equations (30) and (37)) remain exactly as in previous sections. These relationships only compare the relative revenue of products within a particular market within an individual industry, and are therefore unchanged by the introduction of country asymmetries. The additional terms in factor prices due to the introduction of skilled and unskilled labor cancel from the left and right-hand sides of equations (23) and (37).

The free entry condition also takes the same form as previously (equation (39)), since average profits in the domestic and export market are evaluated separately relative to the lowest expertise product supplied by a firm in each market, and terms in factor

prices again cancel from the left and right-hand sides of the equation.

However, the introduction of country asymmetries changes the relationship between the exporting cutoff firm ability φ_{xj}^* and the zero-profit-cutoff firm ability φ_j^* , which instead of equation (38) is given by the following expression for the home country:

$$\varphi_{xj}^{*H} = \Gamma_j^H \varphi_j^{*H}, \quad \Gamma_j^H \equiv \tau_j \left(\frac{f_{xj} R^H}{f_{pj} R^F} \right)^{\frac{1}{\sigma-1}} \left(\frac{P_j^H}{P_j^F} \right) \left(\frac{\bar{\lambda}_j}{\bar{\lambda}_{xj}} \right), \quad (52)$$

where an analogous expression holds for the foreign country. Since $\lambda_{xj}^*(\varphi_j) = (\varphi_{xj}^*/\varphi_j) \bar{\lambda}_{xj}$, the change in the relationship between φ_{xj}^* and φ_j^* affects the exporting cutoffs for product expertise $\lambda_{xj}^*(\varphi_j)$ and hence average profits from the export market in the free entry condition (39).

Comparing the free entry conditions in the open and closed economies (equations (39) and (24)), the expected value of entry in the open economy is equal to the value for the closed economy plus an additional positive term which captures the expected profits from the export market. Since $\lambda_{xj}^*(\varphi_j) = (\varphi_{xj}^*/\varphi_j) \bar{\lambda}_{xj}$ and $\varphi_{xj}^* = \Gamma_j \varphi_j^*$, this additional positive term is larger, the smaller the value of Γ_j . Dividing equation (52) for the two industries:

$$\frac{\varphi_{x1}^{*H}/\varphi_1^{*H}}{\varphi_{x2}^{*H}/\varphi_2^{*H}} = \frac{\Gamma_1^H}{\Gamma_2^H} = \frac{\tau_1}{\tau_2} \left(\frac{f_{x1}/f_{p1}}{f_{x2}/f_{p2}} \right)^{\frac{1}{\sigma-1}} \left(\frac{\bar{\lambda}_1/\bar{\lambda}_{x1}}{\bar{\lambda}_2/\bar{\lambda}_{x2}} \right) \left(\frac{P_1^H/P_2^H}{P_1^F/P_2^F} \right)$$

The remainder of the proof follows the same structure as the proof of Proposition 4 in Bernard, Redding and Schott (2006b) and we present an abbreviated version here. The price index for the skill-intensive industry relative to the labor-intensive industry is lower in the skill-abundant country than the labor-abundant country: $P_1^H/P_2^H < P_1^F/P_2^F$. Therefore, in the absence of other differences in parameters across industries except factor intensity (common values of τ_i , f_h , f_p , f_{xh} , f_x and f_e across industries): $\Gamma_1^H < \Gamma_2^H$ and similarly $\Gamma_2^F < \Gamma_1^F$. Hence, the additional positive term in the free entry condition capturing expected profits in the export market is larger in the comparative-advantage industry than the comparative disadvantage industry.

Noting that $\lambda_j^*(\varphi) = (\varphi_j^*/\varphi_j) \bar{\lambda}_j$, $\lambda_{xj}^*(\varphi) = (\varphi_{xj}^*/\varphi_j) \bar{\lambda}_{xj}$, and $\varphi_{xj}^{*H} = \Gamma_j^H \varphi_j^{*H}$, the expected value of entry in the free entry condition (39) is monotonically decreasing in φ_j^* . Therefore, the comparative-advantage industry's larger increase in the expected value of entry following the opening of trade requires a larger rise in the zero-profit cutoff for firm ability φ_j^* in order to restore equality between the expected value of entry and the unchanged sunk entry cost: $\Delta\varphi_1^{*H} > \Delta\varphi_2^{*H}$ and $\Delta\varphi_2^{*F} > \Delta\varphi_1^{*F}$.

Since $\lambda_j^*(\varphi) = (\varphi_j^*/\varphi_j) \bar{\lambda}_j$, the comparative-advantage industry's larger rise in the zero-profit cutoff for firm ability φ_j^* implies a greater focusing on core competencies in the comparative-advantage industry than in the comparative disadvantage industry: $\Delta\lambda_1^{*H}(\varphi) > \Delta\lambda_2^{*H}(\varphi)$ and $\Delta\lambda_2^{*F}(\varphi) > \Delta\lambda_1^{*F}(\varphi)$.

From the definition of weighted average productivity in equations (42) and (43), the

comparative-advantage industry's larger rise in the zero-profit cutoff for firm ability and greater focusing on core competencies implies a larger increase in weighted average productivity in the comparative-advantage industry than in the comparative disadvantage industry: $\Delta\tilde{\varphi}_1^H > \Delta\tilde{\varphi}_2^H$ and $\Delta\tilde{\varphi}_2^F > \Delta\tilde{\varphi}_1^F$. ■