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To cite this version:

Gaëtan Le Goïc, Hugues Favrelière, Serge Samper, Fabien Formosa. Multi scale modal decomposition of primary form, waviness and roughness of surfaces. Scanning, Wiley-Blackwell: No OnlineOpen, 2011, 33 (5), pp.332-341. 10.1002/sca.20253 . hal-02136791

HAL Id: hal-02136791
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SCANNING VOL. 33, 332–341 (2011)
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Multi Scale Modal Decomposition of Primary Form, Waviness and Roughness of Surfaces

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Summary: This article introduces an innovative method for the multi-scale analysis of high value-added surfaces, which consists of applying a method based on a new parameterization. This kind of surface parameterization refers to natural modes of vibration, and is therefore named modal parameterization. It allows us to characterize the form, waviness and roughness defects of a surface. This parameterization opens up new fields of analysis, such as the appearance quality of surfaces. It is thereby possible to decompose a measured surface in a vector basis, of which vectors are represented by plane natural eigenmodes sorted by frequency and complexity. Different filtering operations can then be produced, such as extracting the primary form of the surface. To analyze the perceived quality of surfaces, these investigations focus on two approaches: that appearance defects have small periodicity, and that there is a link between curvatures and the visual impact of an anomaly. This methodology is applied to two prestige lighters, whose surfaces were measured by extended field confocal microscopy. Moreover, a prospect of this work is to develop an augmented-reality-type monitoring tool for sensory experts. SCANNING 33: 332–341, 2011. © 2011 Wiley Periodicals, Inc.

Key words: surface analysis, surface defect, appearance defect, metrology, 3-D reconstruction

Contract grant sponsor: European research program INTERREG IV, ARVE INDUSTRIE Mont Blanc.

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Received 28 December 2010; Accepted with revision 11 May 2011

DOI 10.1002/sca.20253
Published online 21 June 2011 in Wiley Online Library (wileyonlinelibrary.com)

Introduction

To deal with future economic challenges and to sharpen its global competitive edge, enterprises have to focus on cutting-edge economic sectors, such as high value-added products. In various fields, including micro-technologies, biomedicine or horology, the quality of high value-added products depends on its flawless geometry. The geometry is usually decomposed in six orders of defects defined by technological as well as mathematical criteria (ISO 8785, '98). The two first orders are size and position, and are covered by models from the theory of mechanisms (Favrelière, 2009). Therefore, this work focuses on the following orders that are primary form, waviness and roughness. Most often, the value of a product also significantly depends on perceived quality and especially on appearance quality (Legay *et al.*, 2005). This article intends to show how these two product quality criteria may be related.

The study of variations of a surface in relation to an ideal basically implies to define boundaries of acceptability. In this way, a model that provides the most accurate description of the geometry has to be defined. The fact that surface measurement machines are able to record an increasing speed and number of points has meant that models can describe measured surfaces with greater accuracy. In this context, the "modal" parameterization uses the properties of natural dynamic eigenmodes of a plane geometry. Any measured surface can be expressed as a linear combination of a mode vector basis, which means that a prior knowledge of its primary form is not necessary. This decomposition sorts the surface defects by frequency and increasing complexity.

The accuracy of this parameterization opens new fields of analysis that deal with the appearance quality of surfaces. Currently, quality controllers make the visual evaluation of products, and their assessments stand for reference to market the products. However, to overcome the disadvantages of human subjectivity, a more metrological approach is offered. After measuring and filtering by modal decomposition, scratches and other visual defects can be exacted, because of their low periodicity, even if they have small amplitude. The efficiency of this method enables us to create a map of the curvatures of the resulting surfaces. Indeed, visual defects represent a break in the shape continuity or in the reflectivity of the object, and can thus be interpreted as a change of local curvature of the surface (Rose *et al.*, 2009).

This work is part of a European research program, which involves France and Switzerland whose purpose is to show that modal parameterization enables an interesting multi-scale analysis of surfaces. This method is assumed as being a relevant analysis regarding the visual impact of the filtered surfaces. The presented examples are prestige lighters that could have appearance anomalies, which are amplified after the coating step. One difficulty arises from the size and diversity of these anomalies, and also from the variability of the assessments made by human sensory controls. The lighter surfaces were measured by field expanded confocal microscopy. First, the measured surfaces are decomposed into the modal vector basis, and then surfaces are filtered in order to extract form, waviness and roughness defects. Finally the visual impact of these anomalies is investigated.

Parameterization Methods for Surface Geometrical Variations

State of Art

In tolerancing and assemblies, surface geometrical variations generally lead to the definition of acceptability boundaries. Indeed, the ISO 1101 standard (2004) defines and characterizes the primary form defect with the concept of tolerance zones, which are limited volumes characterized by a scalar. In order to make the distinction, which this definition does not, between a slightly conical cylinder and a cylinder with a trefoil defect, a large number of models have been developed to characterize the measured surface more accurately. Moreover, the increasing speed and number of points that surface measurement machines can deal with have extended these parameterizations to a multi-scale analysis of surfaces.

One approach is to decompose the measured data into a set of prior functions. The waviness, or straightness—once circularity has been accounted for (Cho and Tu, 2001, 2002)—can be characterized with the Fourier transform. These studies demonstrate the advantage of using Fourier series to model the relationship between displacements of spindle machining and part circularity. In image processing, JPEG compression is performed using the discrete cosine transform. Capello and Semeraro (2000, 2001a,b) used this decomposition to interpret the geometrical variations of rectangular areas. In the optical field, Wyant and Creath ('92) have described the functional defects of the lens with Zernike polynomials. This work has led to an ISO standard 10110-5 (2001) dealing with the description of aberration defects of optical instruments. Henke *et al.* ('99) worked on the geometrical variations of cylindrical surfaces and matched the Chebyshev polynomials with the Fourier series to evaluate systematic manufacturing defects. Regarding the multi-scale analysis of surfaces, Chen *et al.* ('95) demonstrated that the wavelet decomposition is of particular interest when describing the functional appearances and quality manufacturing of machined surfaces. Lee *et al.* ('98) made the same observations on roughness profiles and concluded that this filtering method is well adapted for applications in tribology.

A second approach is to characterize the geometrical variations of the measured data. The empirical mode decomposition, introduced by Huang *et al.* ('98), expresses a signal as a sum of a finite number of components, called intrinsic mode functions (IMFs). Boudraa *et al.* (2003) used this type of decomposition to characterize seabed variations. The result of the decomposition of simulated bathymetric profiles shows that the obtained IMFs accurately characterize the different kinds of waviness on the seabed. The principal component analysis (PCA) is another method that does not require the elaboration of an a priori base of functions. Summerhays *et al.* (2002) and Henke *et al.* ('99) propose a model to describe defects in specific shapes (eigen shapes) determined by the PCA, highlighting the relevance of this decomposition in the case of complex and discontinuous geometrical variations. These parameterizations are relevant for analyzing specifically one order of defect, but cannot contribute effectively to an efficient multi-scale analysis of a surface.

Modal Parameterization

This method is based on a Discrete Modal Decomposition (DMD) (Formosa *et al.*, 2005). The DMD decomposes a signal into a set of discrete functions, similar to the Fourier transform. In this decomposition, the signal represents measured geometrical elements, for example: measured flat surface, measured cylinder, or measured sphere. This parameterization aims at expressing measured geometrical surfaces as a sum of discrete functions, called modes or modal deformations. These functions are computed from the previous nominal geometrical element. The process of the decomposition consists of projecting (1) the measured surface (vector V) into the modes' vector space (Q):

$$Q^{*} \cdot V = ((Q^{\mathrm{T}} \cdot Q)^{-1} \cdot Q^{\mathrm{T}}) \cdot V = \lambda \tag{1}$$

where the vectors (Q_i) form the modal basis (Q). While λ vector is composed by each mode's contributions, namely the modal coordinates. The projection performed in the basis Q is not orthonormal, so the dual basis Q^{*} is used. The vectors Q_i are scaled with the infinity norm ($\|Q_i\|_{\infty} = 1$), to obtain metric and homogeneous λ_i contribution.

Finally, the measure can be expressed as follows (2), where N_q is the number of modes chosen for the decomposition, and ε is the decomposition error. The modes Q_i come from the resolution of a classic problem of dynamics (3).

$$V = \sum_{i=1}^{N_q} \lambda i Q i + \varepsilon(N_q) \tag{2}$$

$$K \cdot \ddot{q} + M \cdot q = 0 \tag{3}$$

with:

$$\begin{cases} M: & \text{generalized mass matrix} \\ K: & \text{generalized stiffness matrix} \\ q: & \text{displacement vector} \end{cases}$$

This dynamic problem (3) can be solved either analytically (Leissa, '69) or numerically by the finite element method (Zienkiewicz and Taylor, 2002). The problem is usually solved numerically, which provides a DMD.

Compared with the Fourier transform, the modal parameterization operates similarly but it enables any types of surface to be analyzed, without a priori knowledge. Moreover, the proposed method can be adapted to more complex decompositions by inserting pre-defined forms (namely the "technological modes,", see *Modal filtering* section) in the vector basis.

Application: Multi-Scale Modal Analysis of a Lighter

Preamble

Here are some definitions of the vocabulary frequently used in this article:

- Anomaly: change (visual or geometric) in relation to the ideal surface.
- Defect: change (visual or geometric) not acceptable compared with the ideal surface.
- Guilloche (engine-turning): a decorative engraving technique in which a very precise, intricate and repetitive pattern is mechanically engraved into an underlying material. This technique is named after the French engineer "Guillot," who invented a machine "that could scratch fine patterns onto metallic surfaces."

Samples

In this article, our methodology is applied to the surfaces of two lighters. Two samples (Fig. 1) have been selected:

- *Lighter A*: plane geometry
- *Lighter B*: the surface is "guilloched" with vertical lining

The lighters are coated with a precious metal (after the metal is "guilloched" where applicable), and then polished in order to obtain the required surface appearance. Many of the individual operations are completed by hand, sometimes taking up to 100 hr per lighter.

Types of Defects

On lighter A, special attention will be afforded to the flatness defect of the surface (Fig. 1). This defect is of primary form order. On lighter B, the guilloche engraving has produced a peel material defect. This defect in size, compared with the guilloched pattern, is of roughness order (Fig. 2). To the manufacturer, this is of particular concern since it appears during the manufacturing process, and must therefore be detected quickly before coating begins, as this would accentuate the defect. In addition, due to the position of the defect at the trough of the "V" pattern, a polishing operation would be of no use. Currently, these anomalies are detected by visual inspections. This type of object can sometimes be subjected to up to 300 control operations before market approval.

Measurements

Many optical methods in use today, including interferometric techniques, laser triangulation techniques and confocal microscopy, measure the topography of a surface with precision. For this study, a surface measuring machine based on extended field confocal microscopy is used, equipped with a confocal chromatic optical probe with 300 μm depth of field, and 0.06 μm axial accuracy.

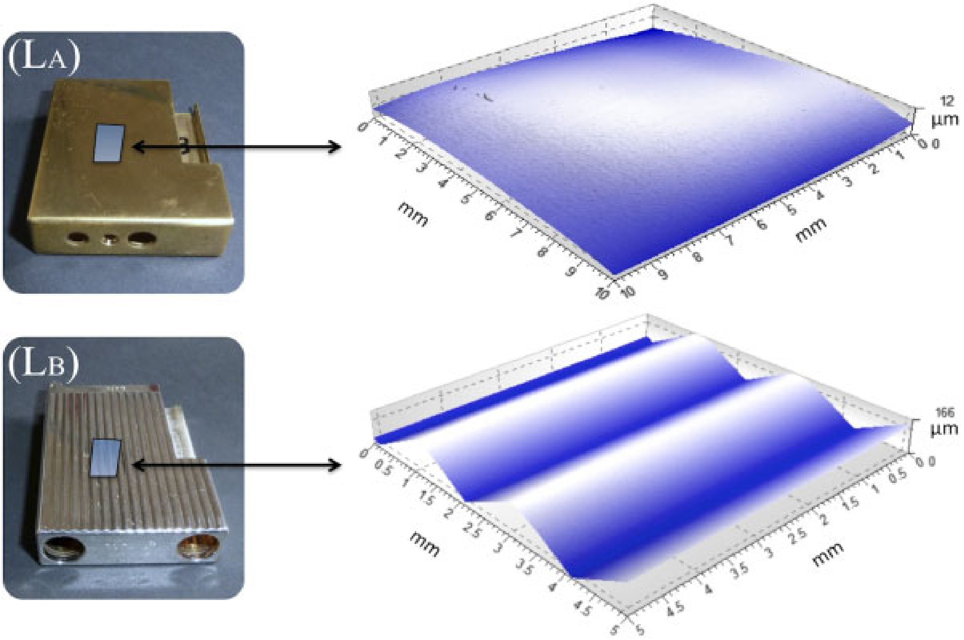

Fig 1. Samples for this study: prestige lighters.

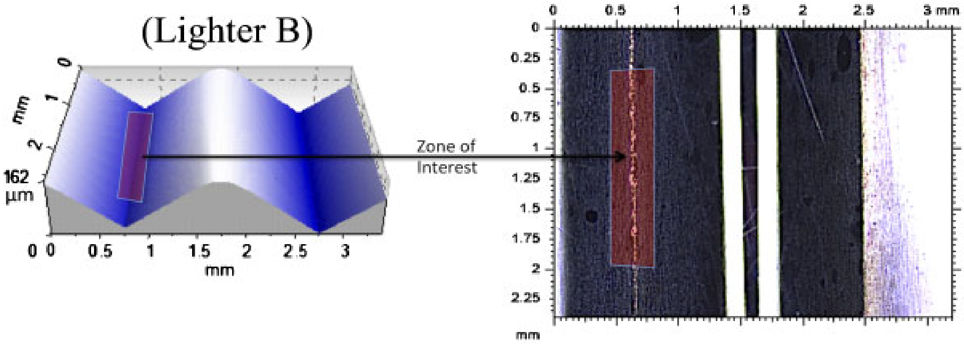

Fig 2. 3D representation of the guilloched pattern, and CCD camera picture of the zone of interest.

Lighter A: This sample has been chosen to highlight the form defect separately from the surface's primary form (Fig. 4). The chosen measurement parameters are as follows:

Axis *X*	Length:	10 mm	Points	40,000
	X Step:	50 µm	Measurement time	4 min
Axis *Y*	Length:	10 mm	ΔZ	12 µm
	Y Step:	10 µm	Axial accuracy (Z)	0.06 µm

Lighter B: With the use of a High Definition CCD camera the location of potential surface anomalies are defined as zones of interest and a precise measurement of the surface can then be performed (Figs. 3 and 4). The chosen measurement parameters are as follows:

Axis *X*	Length:	1 mm	Points	625,000
	X Step:	2 µm	Measurement time	20 min
Axis *Y*	Length:	2.5 mm	ΔZ	96 µm
	Y Step:	2 µm	Axial accuracy (Z)	0.06 µm

Multi-Scale Analysis by the Modal Method

As this article aims to show how modal parameterization can help in the multi-scale analysis of surfaces, this section describes how modal filtering can highlight primary form, waviness and roughness components of the surface.

Modal decomposition

As mentioned above, DMD is core to the modal method (see *Parameterization methods for surface geometrical variations* section). It consists of decomposing the surface ${}^{\mathrm{meas}}V$ into a set of descriptors derived from mechanical dynamics. These descriptors are natural vibration eigenmodes of the reference plane geometry. They form a vector space (Fig. 5), called the *modal base*.

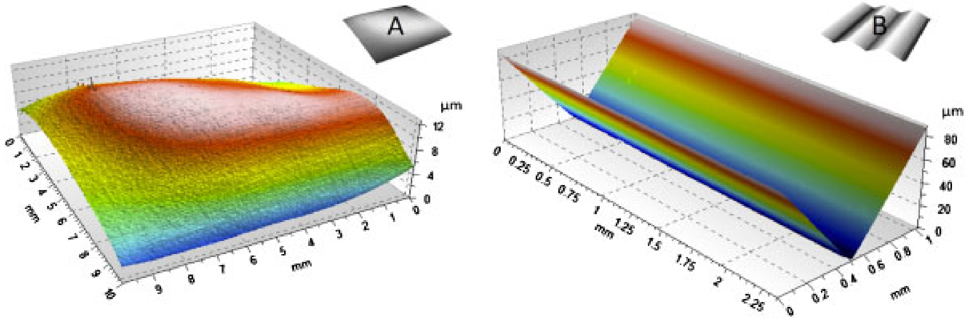

Fig 3. 3D reconstruction of the measured surfaces of the two samples.

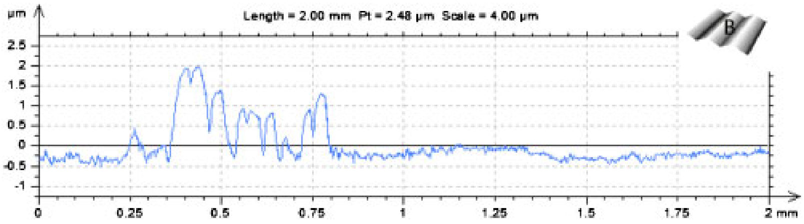

Fig 4. Line extraction at the trough of ≪V≫pattern.

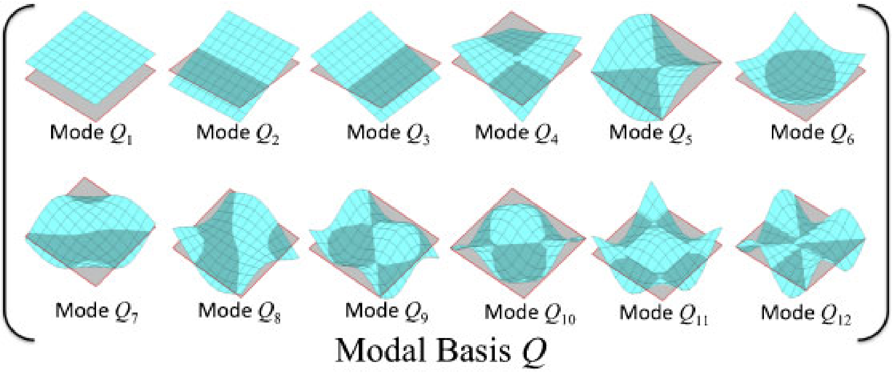

Fig 5. Vector basis of the decomposition (for a plane geometry).

The surface measured is projected in the modal base and the following decomposition is obtained (4):

$$^{\text{meas}}V = \sum_{i=1}^{N_q} \lambda iqi + \epsilon(N_q) \tag{4}$$

where λi is the modal contributions and Qi is vectors of the plane modal basis.

In this application, the surface is projected onto 500 modes to enable the filtering of possible errors with high frequency and low amplitude. Following the DMD calculation on both lighter surfaces, results can be visualized as an amplitude modal spectrum (Fig. 6).

For both the samples, it can be noted that the modes' contributions in the decomposition decrease quickly: the high-frequency components show much lower magnitudes than the low-frequency components of the decomposition. This is a common characteristic when the DMD is applied on "real" surfaces. Indeed, this parameterization is relevant because the vectors of the modal base are precisely those basic primary forms commonly found on surfaces. This enables simple surface defects to be sorted simultaneously both by frequency and importance.

Surface reconstruction

To assess the relevance of modal decomposition, it is useful to reconstruct the surface with all calculated modes (in this case 500), and to compare the resulting surface with the measured one.

- For sample A: the initial form defect amplitude is 15 µm, and the residual between the measured surface and the modal reconstructed surface is 0.4 µm.

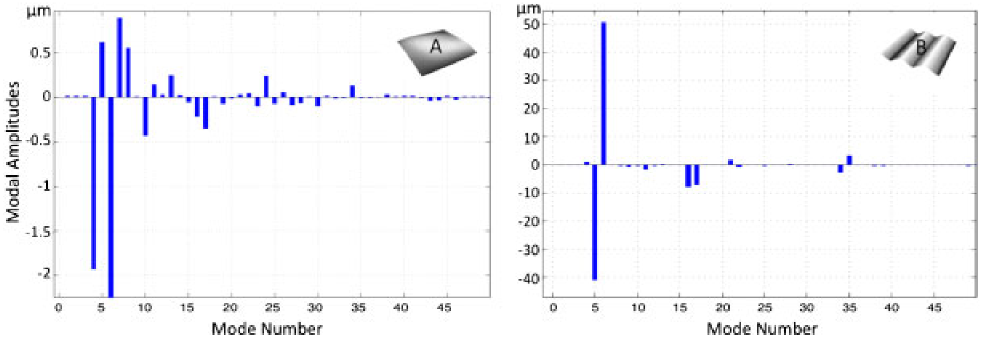

Fig 6. Amplitude modal spectrum, for modes [0, 50].

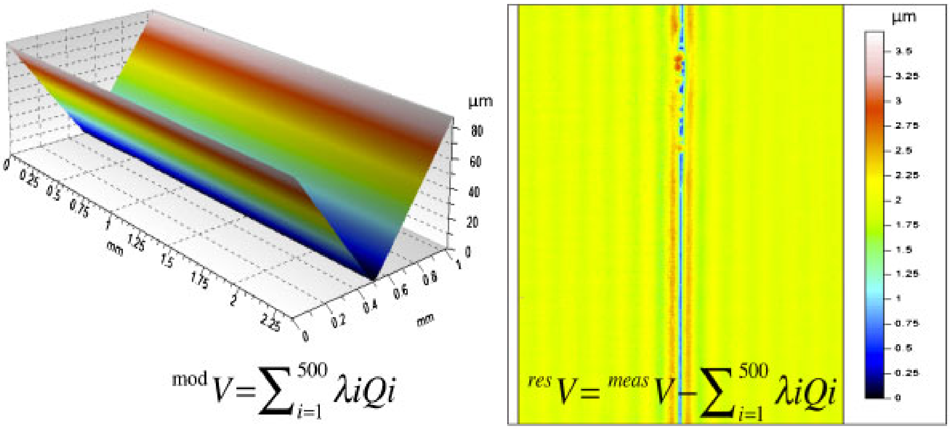

Fig 7. Modal surface reconstruction.

- For sample B: the initial surface has a primary form of between [−40 µm and 40 µm], and the obtained residual is between [−1.5 µm and 1.5 µm] (Fig. 7). This residual is mainly composed of measurement noise, and nonperiodic defects, which are located at the troughs of the guilloched pattern. This means that modal parameterization provides accurate surface characterization, as well as the effective extraction of low frequency primary form anomalies. Moreover, a relatively small number of modal vectors with strong contributions are observed in the spectrum. This can lead to a worthwhile approximation of the measured surface using only a small number of parameters.

Figure 8 shows a 3-D surface reconstruction using the 15 dominant modes, and the corresponding residual.

Modal filtering

The modal method enables partial or complete reconstruction, sorting the modal contributions by frequency or amplitude.

Filtering by frequency makes real-time visualization of the form, waviness and roughness defects of the surface possible, which can be reconstructed, calculation-free, from the beginning, middle or end of the decomposition. Clearly, the thresholds between primary form, waviness, and roughness strongly influence the results. As normalization cannot define these thresholds, the modal method does not shed new light on this issue. However, the DMD user can easily vary the thresholds and process partial reconstructions, which helps when defining them for each surface.

(a) *Form defect*

Owing to the fact that the modes are naturally ordered according to their frequency, the primary form of the measured surface is obtained from a combination of the modes up to the first predefined threshold (5), allowing for a reconstructed form defect to be visualized.

$$
{}^{\mathrm{form}}V=\sum_{i=1}^{Nf}\lambda iQi \tag{5}
$$

The fact that sample A is geometrically plane means that the form defect is obtained directly from a combination of the first set of (low frequency) modes. A reconstruction from the first 20 modes of the decomposition is shown in Figure 9. As is

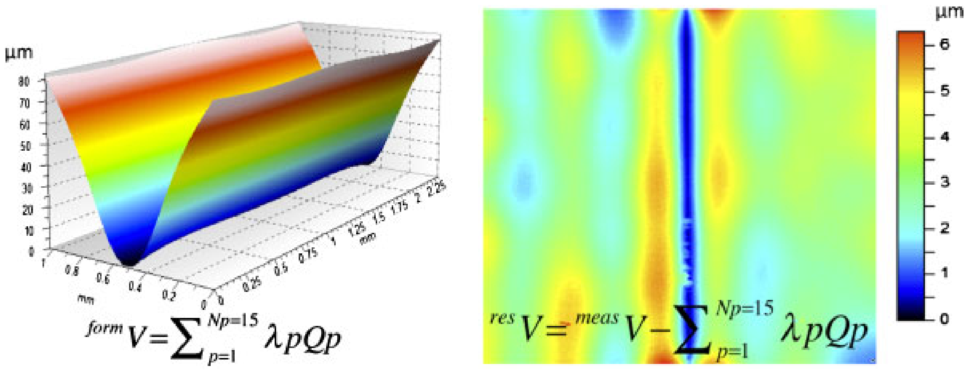

Fig 8. Modal reconstruction from dominant modes of the surface.

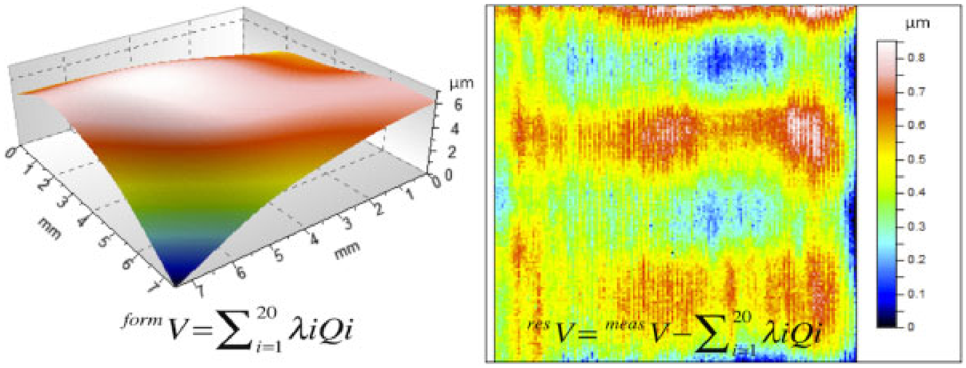

Fig 9. 3D modal reconstruction of the first 20 modes of the surface.

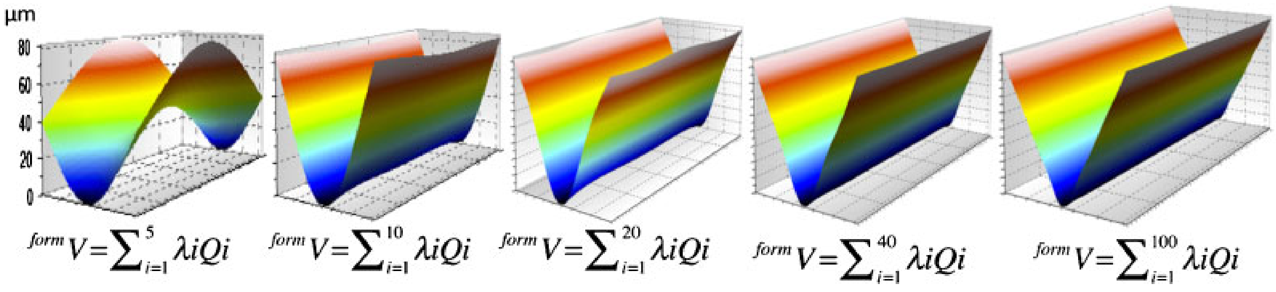

Fig 10. Primary form reconstruction through the first 5, 10, 20, 40, 100 modes of the surface.

apparent, the corresponding residual only contains waviness modes, and has small amplitude.

For an ideal surface that is not flat, the first modes of the decomposition contain the primary form, as well as the form defect. The guilloched pattern in sample B and the form defect are combined in the primary form reconstruction (Fig. 10). In order to separate the form defect from the primary (ideal) form of the surface, it is possible to define "technological" modes, which correspond to the nominal known form of the surface. Thus, the primary form and the form defects can be visualized separately.

For example on Lighter B, the nominal guilloched pattern would be defined as a technological mode, corresponding to the nominal primary form of the lighter surface. Then, decomposing the measured surface in the modal basis will enable to highlight the form defect of the surface separately.

In addition, reconstructions from the 5, 10, 15, 20, 40 and 100 first modes of the DMD show that a small number of modes is sufficient to provide a good primary form description of the surface.

(b) *Waviness and roughness defects*

Similarly, DMD enables the reconstruction of surface waviness components by combining the modes between the predefined primary form and waviness thresholds of the decomposition (6).

The roughness defect (7) is that which contains neither the primary form nor the waviness. This defect gathers the remaining modes of decomposition, the nonperiodic components of the analyzed surface, and the measurement noise.

$$^{\text{waviness}}V = \sum_{i=Nf}^{Nw} \lambda i Q i \tag{6}$$

$$\begin{aligned} ^{\text{roughness}}V &= {}^{\text{meas}}V - {}^{\text{form}}V - {}^{\text{waviness}}V \\ &= {}^{\text{meas}}V - \sum_{i=1}^{Nw} \lambda i Q i \end{aligned}$$

(c) *Summarization: modal filtering*

As we have seen, DMD enables the simultaneous, multi-scale analysis of all defect orders of a surface. The thresholds between primary form, waviness and roughness are defined according to the visualization of reconstructed surfaces, which gives an interesting and useful flexibility to surface analysis. These thresholds can be defined by considering the dimensions of the surface, the object's part typology, the surface reference and the types of defects that are to be extracted. Figure 11 shows an example of multi-scale analysis made by DMD of sample B, highlighting what is of primary form, waviness and roughness order.

Appearance defects analysis

Nowadays, in industries such as horology or jewelry, the product value depends significantly on the perceived quality of parts, and especially on appearance. One principal preoccupation is to achieve a better control of the sensory perception of high added value products and to provide a significant advance in surface quality.

To perform control and internal monitoring of the products' appearance, companies hire controllers specially trained to locate and evaluate defects. These specialists have a higher sensitivity for detection and evaluation of defects than customers. This expertise is supplemented with a weighting of defects intensity depending on their location. This purely sensory approach is the most commonly used nowadays, and constitutes the quality reference for market approval.

To overcome the disadvantages associated with human subjectivity, a more metrological approach is offered. Our investigations focus on two core ideas:

- Appearance anomalies generally have low periodicity. This applies to ''geometric'' anomalies, such as strokes, marks, scratches and impacts, but not to heterogeneity on different areas of the same surface. Despite this, it is assumed that this feature covers the majority of appearance anomalies that can be found.
- Appearance anomalies represent a break in the continuity of the shape and/or reflectivity of the surface. The approach developed above, based on the modal method, opens up interesting research opportunities with respect to these two aspects.

Indeed, the low periodicity of appearance defects enables us to isolate them from other components of the surface, using modal decomposition. Figure 12, which shows the DMD residual of Lighter B, highlights the peel material defects at the trough of the guilloched pattern isolated from the primary form of the surface.

Finally, DMD facilitates extremely precise filtering and provides noiseless surfaces, which correspond well with the measured one. The quality of this filtering allows us to investigate surface curvature mappings in order to quantify the visual impact of anomalies.

Figure 12 shows a curvature map, computed after modal filtering. This approach requires subsequent statistical work to investigate the correlation between levels of calculated curvatures on surfaces, and levels of acceptability defined by sensory experts.

Conclusion

This article has underlined the relevance of choosing modal parameters for the multi-scale

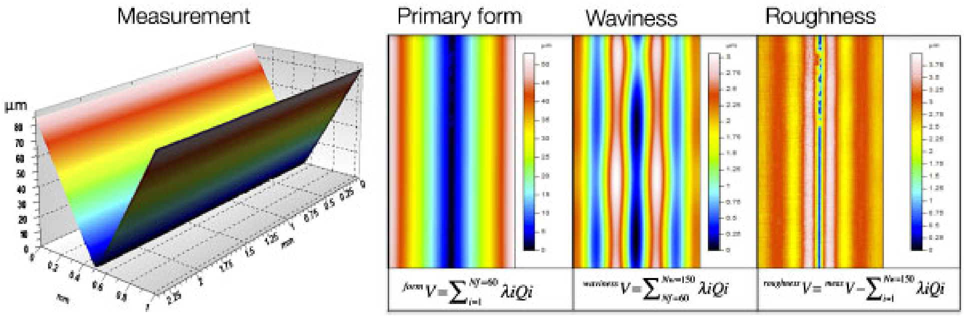

Fig 11. Multi-scale analysis on a prestige lighter.

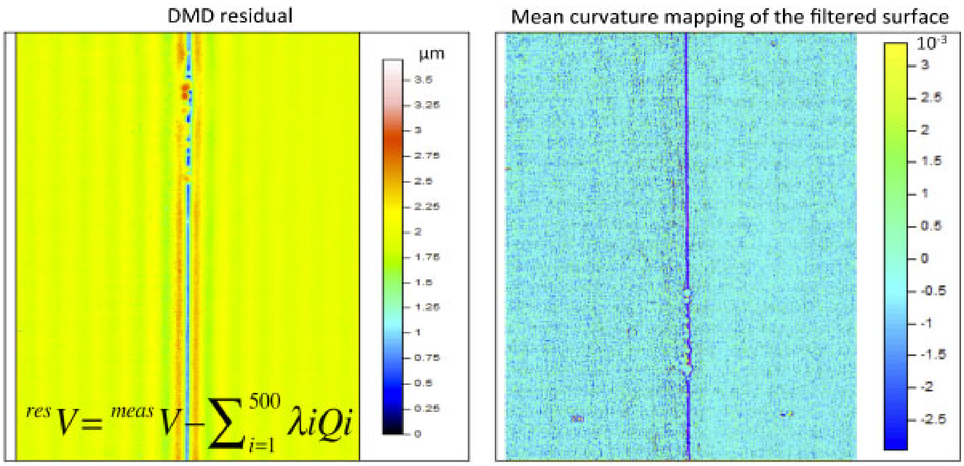

Fig 12. DMD residual and curvature mapping of a Lighter surface.

analysis of surfaces. The modal parameters can accurately characterize a surface and no prior knowledge of its characteristics is needed. It can describe any shape and is therefore exhaustive.

Through this parameterization, the DMD has been developed and enables the decomposition of a surface into a base whose vectors are the dynamics eigenmodes of the analyzed geometry. Projecting the measured surface onto the modal vector base carries out the modal decomposition. The contributions of the various modes are then visualized as an amplitude modal spectrum. The form, waviness or roughness defects of the surface are obtained by a partial reconstruction of the decomposition. An interesting feature of this method, which has been emphasized, is that the highest magnitudes correspond to the low frequency modes. Higher complexity defects (those of higher frequencies) have much smaller amplitudes. This second feature allows the DMD to provide an accurate approximation of the measured surface; what's more, using few parameters. This methodology was applied to two samples for which geometric and appearance qualities are of primary concern.

The accuracy of this parameterization has been shown to be more than sufficient to analyze the appearance aspect of a surface by a metrological approach. This study focuses on two appearance defect features: the low periodicity of appearance anomalies, and the discontinuities in shape or in reflectivity of the surface, which can be interpreted as a change of local curvature. The results rendered by this approach should be validated by further statistical analysis.

This is an advanced method, and the quality of the surface characterization opens up opportunities of application to various fields. DMD is not restricted by the scale of the surface in question, which lends itself to a great number of possible applications, such as Earth surface modeling, or in the fields of tolerances and assemblies.

Acknowledgements

These works have been made in the *Symme Lab.* of the Savoy University, and applied to samples of the *St. Dupont* Company, partner of the Program.

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