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## **MULTI-UNIT AUCTIONS IN THE PROCUREMENT OF ELECTRICITY**

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**ABSTRACT:** *This paper reviews some multi-unit auction mechanisms that are used in the procurement of electricity. In ordinary multi-unit auctions bidders compete to buy several units of the same object from the seller, while in procurement or reverse auctions suppliers of electricity compete to sell a certain number of units. Reverse electricity auctions are used in numerous countries and they create a competitive market for electricity, instead of state-owned monopolies providing electricity at administratively set prices.*

*In this paper we will present the most commonly used multi-unit auction forms in electricity markets. Auctions for electricity from renewable energy sources deserve special attention, since these suppliers cannot compete with large-scale producers at the present state of technology.*

**KEY WORDS:** *Electricity auctions; Uniform-price auction; Discriminatory auction; Descending clock auction; Hybrid auctions; Sequential auctions; Combinatorial auctions.*

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## **1. INTRODUCTION**

In this paper we will analyse some forms of multi-unit auction mechanisms that are used in the procurement of electricity. Throughout the paper we will make a distinction between ordinary and reverse or procurement auctions. In ordinary multi-unit auctions bidders compete for a different number of units of the same product. If there are  $K$  objects for sale, bidders with the highest bids obtain the units. In the case of reverse auctions, bidders compete to obtain contracts for supplying a certain number of units. In this case, bidders with  $K$  lowest bids win the auction.

There are many types of multi-unit auctions, but we will focus our attention only on those that are used in the procurement of electricity: uniform-price and discriminatory auctions, descending clock auctions, hybrid auctions, and sequential and combinatorial auctions. In electricity auctions usually one supplier is not able to supply the total amount of electricity and there are several winners of the auction, so electricity auctions could be considered as multi-unit auctions. In ordinary uniform-price auctions bidders with  $K$  highest bids win and pay the price equal to the  $K+1$ -th highest bid (the highest losing bid). In the case of reverse auctions bidders with the  $K$  lowest bid win and obtain the price equal to the  $K+1$ -th lowest bid. In reverse discriminatory auctions bidders with  $K$  lowest bids sell their units and are paid the price equal to their bid. In a descending clock auction the auctioneer starts the auction with a high price and bidders inform the auctioneer about the quantities they want to supply. If the total supply is higher than demand the auctioneer lowers the price until there is no excess supply. Hybrid auctions represent combinations of simple auction forms designed to combine the best features of simple auction mechanisms. Instead of buying all units in reverse auctions at once, the auctioneer might buy these units one by one in sequential auctions. Finally, in combinatorial auctions bidders place bids for packages of items they are willing to supply.

The electricity market has several characteristics that distinguish it from some other markets (Wilson, 2002). First of all, the demand in this market is volatile and influenced by seasonal and cyclical factors. Second, the storage of electricity is prohibitively expensive or almost impossible. Third, due to the presence of

economies of scale, this market is imperfectly competitive. In order to increase the competition between existing suppliers and to attract new production plants some countries organise electricity auctions. This practice is relatively new, and electricity auctions enhance competitive procurement of electricity instead of supplying electricity from state owned monopolies at administratively set prices. This idea of creating markets where they do not exist is in line with the idea of Nobel Prize winner Alvin Roth (2007). He was awarded the Nobel Prize in economics for the creation of markets to facilitate efficient matching between doctors and hospitals and children and schools. In this sense, auctions create markets in cases where markets did not exist before, such as spectrum auctions, electricity auctions, auctions for landing slots at airports, and so on.

In multi-unit auction theory, for ordinary auctions it is assumed that the bidder assigns a value to each unit for sale that is distributed according to some probability distribution, where the probability distribution function for the first unit dominates the probability distribution function for the second unit according to first-order stochastic dominance, the probability distribution function for the second unit dominates the probability distribution function for the third unit according to first-order stochastic dominance, and so on. Bidders can have private or interdependent values. In the case of private values, the value that a bidder assigns to the object is independent of the values of other bidders. In the case of interdependent values a value that a bidder assigns to the object depends on other bidders' values<sup>1</sup>. In this paper we will assume that bidders have private values. The theory of multi-unit auctions with interdependent values is extremely complicated and difficult to analyse<sup>2</sup>.

In procurement electricity auctions, the marginal cost of production of electricity is analogical to the bidder's value in ordinary auctions. The bidder's expected profit in a reverse auction is the difference between the selling price

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<sup>1</sup> For more detailed analysis of single object auctions with private and interdependent values, see Trifunović (2010, 2011)

<sup>2</sup> The most important papers that established the theory of multi-unit auctions with interdependent values are Dasgupta and Maskin (2000) and Jehiel and Moldovanu (2001). The latter paper proved the impossibility theorem, which states that it is impossible to achieve efficient allocation when bidders have multidimensional signals.

and his cost, which is his private information. The two objectives that an auction mechanism in reverse auctions has to achieve are minimisation of expected payment from buyer to bidders and efficiency. Efficient allocation is achieved if bidders with the lowest costs are assigned contracts for providing electricity.

The rest of the paper is organized as follows. In the second part we will analyse uniform and discriminatory auctions. In third and fourth parts we will deal with descending clock and hybrid auctions. The fifth part is dedicated to sequential auctions, and the sixth to combinatorial auctions. The seventh part describes the auctions for renewable energy sources. In all these cases we will start our analysis with ordinary auctions because it is easier to understand the logic of each auction mechanism. Then these results can be extended to reverse auctions, which are a mirror image of ordinary auctions. In the eighth part we will discuss collusive behaviour of bidders in electricity auctions. We will see that some auction mechanisms are more effective in preventing collusive behaviour of participants. The last part concludes the discussion.

## **2. UNIFORM-PRICE AND DISCRIMINATORY AUCTIONS**

We will first explain how ordinary uniform-price and discriminatory auctions operate, and later on we will move to reverse auctions. Assume that there are  $K$  units for sale and  $N$  bidders, with  $N > K$ . Bidders submit sealed bids and bidders with  $K$  highest bids are the winners, and all these bidders pay a price equal to the highest losing bid in a uniform-price auction, while in a discriminatory auction the winners pay their bids. Suppose that there are two bidders and two units for sale. Two bidders submit bids for two units: suppose that first bidder bids 5 and 3 and the second 4 and 2. Each bidder obtains one unit and the highest losing bid is equal to 3 and determines the price that bidders pay for the units won, and the revenue for the seller is 6. In a discriminatory auction the first bidder would pay 5 for the item and the second would pay 4, yielding a total revenue of 9 for the seller.

In the case of the uniform-price auction in the previous example, the bid for the second unit of the first bidder determines the price. Thus each bidder has an

incentive to shade his bid on the second unit, since it can determine the price that he has to pay for the first unit. In fact a bidder is facing the following trade off: if he lowers his bid on the second unit he decreases the price that he has to pay on the first unit, but at the cost of reducing the probability of obtaining the second unit. For any further unit the bidder would shade his bid to a greater extent, since he becomes more concerned about the price he has to pay on the units won than on the probability of obtaining an additional unit. Engelbrecht-Wiggans and Khan (1998a) call this result demand reduction, and prove that the bidder submits a bid equal to his value for the first unit and shades his bid for the second and all additional units where the amount of bid shading increases for each additional unit. This result is proved in appendix A. Ausubel and Cramton (2002) prove that demand reduction can be so severe that the bidder finds it optimal to place a zero bid for the second unit, yielding zero revenue for the seller.

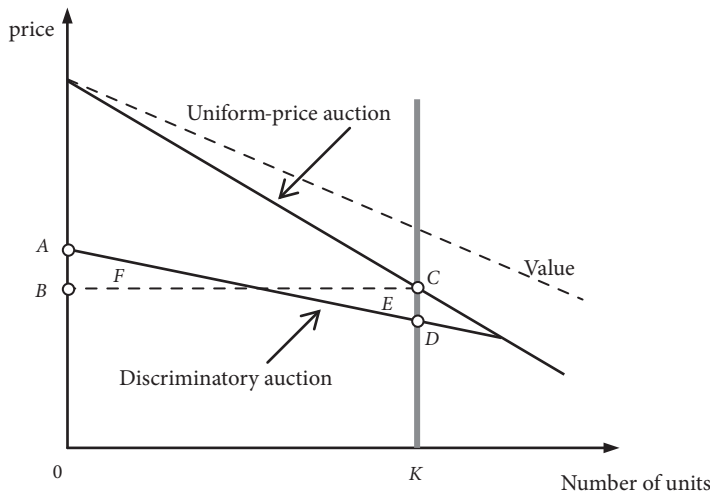
Since the bidder shades his bid and increases the amount of bid shading for each additional unit, Ausubel and Cramton (2002) prove that, in the case of symmetric bidders who want to buy more than one unit, the allocation in a uniform-price auction is inefficient. For example, suppose that the first bidder has higher values than the second bidder for all three units, but due to increased bid shading it is possible that second bidder's bid for the first unit is higher than first bidder's bid for the third unit.

In a first-price auction the bidder bids lower than his value, and Maskin and Riley (2000) prove that in the case of asymmetric bidders the weak bidder bids more aggressively than the strong bidder in a first-price auction. The same logic applies to the case of discriminatory auctions where the value for the first unit dominates the value for the second unit according to first order stochastic dominance, and the bidder bids more aggressively for the second unit than for the first unit. This result is proved in appendix B. Engelbrecht-Wiggans and Khan (1998b) show that due to this effect the amount of bid shading decreases for any additional unit.

However, the comparison of expected revenues between a uniform and a discriminatory auction is not straightforward. Assume that the value function

for the bidder is linearly decreasing. In that case the bidding function for the uniform-price auction is steeper than the value function, and the bidding function for the discriminatory auction is less steep than the value function (Figure 1).

**Figure 1.** *Expected revenue in uniform and discriminatory auctions*



If there are  $K$  units for sale, the expected revenue for the seller in a uniform-price auction is equal to the area BOKC, and in a discriminatory auction to the area AOKD. Thus the discriminatory auction dominates the uniform-price auction in terms of revenue if area F is larger than E.

In electricity auctions reverse uniform-price and discriminatory auctions are used. In a uniform-price auction all winning suppliers are paid the lowest losing bid. The marginal cost of production of electricity is a random variable distributed according to some distribution function. The bidder's expected profit is the difference between the selling price and his cost. We know that in an ordinary uniform auction the bidder shades his bid below his value for the second and all additional units. In a reverse uniform auction the bidder has an incentive to bid higher than his marginal cost for the second and all additional units in order to receive a positive profit if his bid turns out to be the lowest losing bid that determines the market price. Based on data from English

electricity auctions, Wolfram (1998) finds that bidders bid according to this theoretical prediction. Moreover, Parisio and Bosco (2003) prove that the amount of bid increase over marginal cost increases in producer's capacity.

By using the logic of the proof in appendix A, the expected profit of bidder 1 in a reverse uniform-price auction who competes for supplying two units could be written as follows:

$$\Pi(\mathbf{b}) = \int_{\{\mathbf{c}: c_1 > b_2\}} 2c_1 h(\mathbf{c}) d\mathbf{c} - (k_1 + k_2) + \int_{\{\mathbf{c}: c_2 > b_1 \text{ and } c_1 < b_2\}} \min\{b_2, c_2\} h(\mathbf{c}) d\mathbf{c} - k_1, \quad (1)$$

where the vector of marginal costs of production for bidder 1 for these units is  $\mathbf{k} = (k_1, k_2)$ ,  $(b_1, b_2)$  are bids of bidder 1 for the first and second units,  $\mathbf{c} = (c_1, c_2)$  is the vector of competing bids facing bidder 1, and the probability density function of the random variable  $\mathbf{C}$  is  $h(\cdot)$ . The first term is the expected profit of bidder 1 when his bid for the second unit is lower than the highest competing bid when he supplies two units. In this case  $c_1$  is the lowest losing bid that determines the price for bidder 1. The second term is his expected profit when his bid for the first unit is lower than the second highest competing bid but the bid for the second unit is higher than the highest competing bid, and bidder 1 supplies only one unit. In this case the lowest losing bid is  $\min\{b_2, c_2\}$ .

Hortaçsu and Puller (2008) study uniform-price auctions in the Texas electricity spot market. This market is very specific. The supply of electricity is determined by long-term contracts, and depending on weather shocks the auctioneer announces one day ahead whether there will be a need for additional quantities of electricity or if it is necessary to reduce the supply of electricity relative to the schedule in long-term contracts. Bidders then offer to increase or reduce the amount of electricity relative to long term contracts in this balancing market. Hortaçsu and Puller (2008) found that the bidders' profit was lower than optimal, and this stems from high bids rather than from competitive pricing. Moreover, they find that actual bidding strategies of companies with large market shares are close to optimal strategies, whereas bidding strategies of small companies differed considerably from the optimal.

Crawford, Crespo, and Tauchen (2007) analyse bidding strategies in British uniform-price electricity auctions. In contrast to the previous model of asymmetric information where a Bayesian-Nash equilibrium was sought, they assume the bidding model of perfect information where bidders' costs are common knowledge and determine the Nash equilibrium for this game. They justify this assumption with the fact that costs of generating electricity from different generators are known in the majority of cases in the British market. In equilibrium bidders have asymmetric strategies. Bidders who determine the market price in a uniform-price auction are called price setters, and other bidders are called non-price setters. In the Nash equilibrium price setters bid significantly over their marginal costs for marginal units, while non-price setters bid close to their marginal costs. This model confirms the conclusion of uniform-price auction theory with incomplete information, but only for price setting bidders. Non-price setters behave differently than the theory predicts. These theoretical predictions are supported by the data on British electricity auctions between 1993 and 1995.

The advantage of the uniform-price auction is that it encourages the participation of small bidders. Federico and Rahman (2003) determined that, despite the fact that new suppliers entered the electricity market in England after the introduction of the auction market in 1990, the real price of electricity rose during the first ten years. They speculate that this fact was caused by the use of a uniform-price auction where bidders bid higher than their marginal cost and where they can make cartel agreements.<sup>3</sup> They propose the use of a discriminatory auction to overcome these problems. But we know that in a reverse discriminatory auction bidders also bid higher than their cost. On the other hand, Kahn, Cramton, Porter, and Tabors (2001) claim that a discriminatory auction can lead to inefficient allocation, since more efficient producers who are concerned about their profit would calculate a higher profit margin over their costs than less efficient producers, whose primary concern is the probability of winning. Besides, to submit a bid in a discriminatory auction the bidder needs more information than in a uniform-price auction, and larger

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<sup>3</sup> In the eighth part we will discuss in more detail the possibilities of bidders' collusive behaviour and issues related to the protection of competition in electricity auctions.



suppliers have easier access to all the information that could possibly reduce the level of competition. In repeated auction games as in the electricity market, where the same players interact a cartel agreement could be enforced regardless of the type of auction. Empirical research by Fabra and Toro (2005) aims to determine whether the periods of low prices in the Spanish highly concentrated uniform-price electricity auction market is triggered by collusive agreements. By using experimental analysis Bower and Brunn (2001) determined that prices are higher in discriminatory than in uniform-price electricity auctions. Fabra and von der Fehr (2006) find that for certain characteristics of demand, the cost and capacity of a suppliers' discriminatory auction dominates the uniform-price auction, while for other characteristics of the same factors the uniform-price auction dominates the discriminatory auction. In the same fashion, Federico and Rahman (2003) find that in the competitive electricity market the discriminatory auction reduces welfare compared to the uniform-price auction. In the case of a monopolistic electricity supplier, if the demand for electricity is not too volatile and the marginal cost of the supplier is not too steep, the discriminatory auction increases social welfare. Otherwise, if demand is too volatile and the marginal cost is too steep, the uniform-price auction increases social welfare compared to the discriminatory auction. The main disadvantage of these sealed-bid uniform-price and discriminatory auctions is that there is no price discovery, which increases the possibility of inefficient allocation.

### **3. DESCENDING CLOCK AUCTIONS**

As we said before, auctions created markets where they did not exist before, as in spectrum and electricity markets. In the sale of spectrum rights the simultaneous ascending auction is often used. This auction mimics the *tâtonnement* process of the Walrasian auctioneer. In the first stage bidders submit sealed bids for different packages. After that the auctioneer determines the highest bid for every item. In the next stage bidders can place higher bids on any item and the auctioneer again determines the highest standing bid. This process continues until there are no new highest bids for any item at a certain stage and all items are sold at the standing price. Ausubel and Milgrom (2002) have studied this auction extensively and claim that demand reduction exists in this auction.

In the procurement of electricity a similar form is used, called the descending clock auction. In a descending clock auction the auctioneer starts the reverse auction with a high price and bidders provide quantities they want to supply. If supply is larger than demand, the auctioneer lowers the price and bidders again provide quantities they wish to supply. The auctioneer lowers the price until supply equals demand. The main advantage of this auction form is that it allows price discovery, which reduces the winner's curse. Moreover, bidders are not supposed to reveal the lowest price at which they are willing to provide electricity, which is an important issue in repeated electricity auctions with the same participants. The main disadvantage of this auction mechanism is that it allows bidders to coordinate their decisions during the auction.

#### **4. HYBRID AUCTIONS**

Hybrid auctions represent combinations of simple auction forms, which are designed to combine the best features of standard auctions. We will first discuss some popular ordinary hybrid auctions - Anglo-Dutch, Dutch-Anglo, and Amsterdam auction - and after that we will present hybrid auctions used in the procurement of electricity.

The Anglo-Dutch auction was studied by Klemperer (1998, 2004) and this mechanism was employed in spectrum auctions in the UK. This auction operates in two stages. Suppose, for simplicity, that there is a single object for sale. Bidders compete in an English auction in the first stage. This stage is finished when two bidders remain. The remaining two bidders enter the second stage and submit sealed bids in a first-price auction, at least as high as their bid in the first stage. This auction combines the best features of first-price and English auctions. The second stage gives weak bidders an opportunity to win against strong bidders and encourages the entry of weak bidders. The main advantage of the English auction is efficient allocation. Moreover, if bidders have interdependent values they can infer the signals of other bidders in an English auction and reduce the winner's curse. Abbink et al. (2005) examined the properties of this auction in an experimental study.

A Dutch-Anglo auction reverses the order of the two stages and the second stage is contingent. In the first stage bidders participate in a sealed bid auction. The bidder with the highest bid wins if his bid is sufficiently higher than the second highest bid. In other words, he wins if the difference between his bid and the second highest bid is higher than a certain threshold. Otherwise the highest bidder and all bidders with bids sufficiently close to the highest bid enter the second stage, where they compete in an English auction with the reserve price equal to the highest bid in the first stage. Dutra and Menezes (2002) analyse equilibrium strategies for the Dutch-Anglo auction for the discrete distribution of values that contain a private and common value component. They show that due to the fact that the second stage of a Dutch-Anglo auction could be regarded as an English auction with an endogenously set reserve price in the first stage, the seller obtains higher expected revenue in a Dutch-Anglo auction than in standard auctions with exogenously set reserve price. This auction was used on several occasions in Brasil. Dutra (2001) discusses the success of the Dutch-Anglo auction in the privatisation of the telecommunications company Telebras and the Banestado bank in Brasil.

The Amsterdam auction is similar to the Anglo-Dutch auction, with one important difference. The first stage consists of an English auction that finishes when two bidders remain. The price at which the last bidder drops out of the first stage is called the bottom price. The remaining two bidders who enter the second stage submit sealed bids higher than the bottom price. The bidder with the highest bid wins, and in the first-price Amsterdam auction he pays his bid, whereas in the second-price Amsterdam auction the winner pays the runner-up's bid. In contrast to the Anglo-Dutch auction, both bidders receive a premium. This premium is some fraction of the difference between the losing bid in the second stage and the bottom price. Goeree and Offerman (2004) analysed Amsterdam auctions from a theoretical and experimental point of view and they determined that these auctions perform well with asymmetric bidders. Hu, Offerman, and Onderstal (2011) found that the second-price Amsterdam auction is more successful in fighting collusion than the first-price and the English auction. In the case of asymmetric bidders they predict that in an 'aggressive equilibrium', where weak bidders bid aggressively to obtain the

premium, collusion is less likely to occur in an Amsterdam auction than in the other two simple auction forms. These theoretical results are supported by experimental findings, where weak bidders were motivated to bid aggressively in a second-price Amsterdam auction to obtain the premium and this auction mechanism triggered less collusion than the first-price and the English auction<sup>4</sup>.

In electricity auctions two new hybrid forms were invented, according to Maurer and Barroso (2011). The first is a descending clock auction with a discriminatory auction. In the first stage bidders compete in a descending clock auction until supply is higher than demand multiplied by a certain parameter unknown to bidders. This stage is used as the price-discovery stage. In the second stage qualified bidders compete in a discriminatory auction. This stage reduces the price that needs to be paid for procurement of electricity and the possibility of collusion.

We will illustrate how this auction operates by using the example from Dutra and Menezes (2005). At the beginning of the auction the auctioneer sets the starting prices for procurement of electricity for different years and the reserve prices (the highest acceptable prices). The auctioneer determines the reference supply (the forecasted demand multiplied by some constant unknown to bidders). In the first stage bidders know only the starting price. Suppose that there are three contracts for providing electricity in the next three years, 2014, 2015, and 2016. Suppose for simplicity that the forecasted demand in the next three years is 150 MW, that the reference supply is equal to 110% of forecasted demand, that starting prices are higher for more distant future contracts, and that starting prices are equal to reserve prices. These data are shown in Table 1.

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<sup>4</sup> By analysing the data from Serbian privatisation, Trifunović and Ristić (2012) determined that the first-price auction was more effective in increasing competition and preventing collusion than the English auction. However, hybrid auctions would give even better results.

**Table 1.** Descending clock and discriminatory auction: starting prices, demand, and reference supply

Contract	2014	2015	2016	Total
Demand	150	150	150	450
Reference supply	165	165	165	495
Starting price	65	70	75	

**Source:** Dutra and Menezes, (2005).

In the first stage bidders observe the starting prices and inform the auctioneer of their supply at these prices. This is shown in Table 2.

**Table 2.** Descending clock and discriminatory auction: stage 1

Contract	2014	2015	2016	Total
Current price	65	70	75	
Bidder 1	50	50	50	150
Bidder 2	40	50	40	130
Bidder 3	50	70	100	220
Total Supply	140	170	190	500
Demand	150	150	150	450

**Source:** Dutra and Menezes, (2005).

The table shows us that the total supply is higher than the reference supply, and the auctioneer lowers prices for the 2015 and 2016 contracts, which are in excess supply. After that bidders inform the auctioneer of their supply at new prices, but the total supply of each bidder cannot be higher than in the previous phase. This is shown in Table 3.

**Table 3.** Descending clock and discriminatory auction: stage 1a

Contract	2014	2015	2016	Total
Current price	65	68	73	
Bidder 1	55	45	45	145
Bidder 2	50	40	35	125
Bidder 3	75	60	70	205
Total Supply	180	145	150	475
Demand	150	150	150	450

Source: Dutra and Menezes, (2005).

At this point the total supply is lower than the reference supply, the first stage is finished, and the second stage begins. The contract for 2015 is closed since it has excess demand, while the other two contracts are open. This information is common knowledge. In this stage bidders place bids for the quantity they wanted to supply in the previous stage and winners are paid their bids in this discriminatory auction stage. However, bidders can place additional bids for the closed 2015 contract if they are not able to sell part of their supply in the opened contracts. Suppose that bidder 2 submits a bid of 65 for the option to supply electricity in the 2015 contract, in the case that part of his quantity for the 2014 contract is not accepted. The auctioneer first accepts the lowest offers and then the higher offers. This is illustrated in Table 4, where the quantity that each bidder obtains is given in bold.

**Table 4.** Descending clock and discriminatory auction: stage 2

Contract	2014		2015		2016	
Current price	65		68		73	
	Bid	Quantity	Bid	Quantity	Bid	Quantity
Bidder 1	60	55 ( <b>55</b> )	67	45 ( <b>20</b> )	70	45 ( <b>45</b> )
Bidder 2	64	50 ( <b>20</b> )	65	40 ( <b>40+30</b> )	69	35 ( <b>35</b> )
Bidder 3	62	75 ( <b>75</b> )	63	60 ( <b>60</b> )	71	70 ( <b>70</b> )
Supply		180		145		150
Demand		150		150		150

Source: Dutra and Menezes, (2005).

In this example bidder 2 had the highest bid for the 2014 contract and only part of his quantity is accepted. He manages to obtain an additional 30 MW for the 2015 contract at the expense of bidder 1. But bidder 1 is not allowed to place an additional bid for other contracts as the 2015 contract was closed in the first stage. This explains why this auction design can lead to inefficient allocation, as bidder 1 might have placed lower bids for the other two contracts than the existing bids.

The second hybrid form is a multi-unit extension of the Dutch-Anglo auction. This is a first-price auction followed by an iterative descending auction. In the first stage bidders submit sealed bids, and bidders with the lowest bid and other bidders with bids no more than 5% higher than the lowest bid enter the second stage where they compete in a descending auction. This auction is used in cases where price discovery is not so important. These two types of auctions were used in Brasil but their success depended on the optimality of chosen reserve prices. Auctions with very low reserve prices resulted in a suboptimal level of contracting.

## **5. SEQUENTIAL AUCTIONS**

In ordinary sequential auctions the seller sells different units in separate first-price or second-price auctions. Milgrom and Webber (2000) analyse sequential first-price auctions where each bidder wants to buy at most one unit. They prove that a bid is an increasing function of value and in each subsequent auction the bidder with the highest value among active bidders wins. But the winner in one auction has a lower value than the winner in the previous auction, and this effect decreases bids. On the other hand in subsequent rounds bidders bid more aggressively due to the fact that fewer objects are left for sale. In equilibrium these two effects exactly offset each other and the expected price in the current auction is equal to the realized price in the previous auction (Martingale property). In other words, prices should have no trend in sequential auctions. In the case of interdependent values the second effect dominates and prices should have an increasing trend. The same result holds for sequential second-price auctions with private and interdependent values. Milgrom and Webber (2000) prove that the bidder bids more aggressively in second-price than in first-price

auctions, since the bidder pays the price equal to the second highest bid. However, Ashenfelter (1989) observed that in sequential auctions for bottles of wine, prices exhibited a decreasing trend. The same result was obtained by Beggs and Graddy (1997) in art auctions. There were several attempts to resolve this declining price anomaly in theoretical models. McAfee and Vincent (1993) postulate that the declining price anomaly stems from bidders' risk aversion, which induces more aggressive bidding in first rounds, while Jeitschko (1999) claims that this phenomenon stems from uncertain supply.

Milgrom and Weber (2000) prove that in the case of interdependent values the first-price sequential auction yields higher revenue accruing to the seller than the discriminatory auction, and that the sequential second-price auction yields higher revenue than the uniform-price auction. This effect is related to signal inference in sequential auctions, which reduces the winner's curse for active bidders and leads to more aggressive bidding than in sealed bid auctions.

In the case of electricity auctions, the auctioneer might prefer to sell four three-month contracts in sequential auctions instead of a single contract for the whole year. Maurer and Barroso (2011) analyse when it is optimal for an auctioneer to use sequential auctions. If transaction costs for bidders are high, then it is better to organise a single auction than a sequence of auctions in order to boost competition. If price discovery is an important issue, then sequential auctions are better than a single auction since the bidder can learn the costs of other bidders after each round. Some external factors such as weather conditions might influence the price of electricity in a single auction, and the price can be too low or too high depending on the nature of external influences. Bidders might end up in a bad position if the price in a single auction happens to be too low due to unexpected events. In this case sequential auctions reduce the risk that bidders face.

## **6. COMBINATORIAL AUCTIONS**

In ordinary package auctions bidders can place bids on different packages that are predetermined by the auctioneer. If objects for sale are complements, then the value of the bundle of objects for a bidder is higher than the sum of values of



individual objects. Palfrey (1983) and Chakraborty (1999) analyse when it is optimal for a seller to bundle. In package auctions bidders need to have substantial financial resources to participate, which lowers the level of competition.

In contrast to package auctions, in combinatorial auctions bidders choose packages or individual items on which they place bids. These auctions are more complicated than package auctions since an individual item can belong to different packages that different bidders wish to buy, and the determination of the optimal bid and the winning bids is a very challenging task that can be solved with combinatorial optimisation. The first theoretical papers dealing with combinatorial auctions were by Rassenti, Smith, and Bullfin (1982), who applied combinatorial auctions to the sale of landing slots at airports, and Berenheim and Whinston (1986), dealing with first-price combinatorial auctions. Combinatorial auctions were first employed in the sale of spectrum licenses that exhibited complementarity. Milgrom (2004) gives an example of the sale of 12 spectrum licenses in the USA where the Federal Communication Commission (FCC) accepted bids for any of 4,095 possible packages. The FCC conducted laboratory experiments to determine how this auction would operate and organised training for bidders. A detailed discussion concerning combinatorial auctions can be found in Cramton, Shoam, and Steinberg (2006).

In combinatorial electricity auctions bidders place bids for different types of contracts for providing electricity. It is possible that there are many bids for some contracts and that one contract belongs to different packages that bidders are interested in. The auctioneer then has to determine the winning bids that minimise the cost of providing electricity, with the constraint that an individual contract can be awarded at most once. For example, the auctioneer might be auctioning off four contracts for providing electricity for 6 hours in a single day. If it is optimal for a plant to generate electricity for at least 12 consecutive hours, these contracts are clearly complements from the point of view of that bidder, and in this case it is optimal to organise a combinatorial auction.

## **7. AUCTIONS FOR RENEWABLE ENERGY**

Renewable sources of energy deserve special attention, since they cannot compete with other sources of energy such as thermal and nuclear plants because the cost of producing electricity from renewable sources is considerably higher. However, if the cost of externalities is internalised for thermal and nuclear plants, then their cost advantage is considerably lower or even disappears. Energy from renewable sources is provided by wind power, biomass, geothermal energy, wave power, and hydro plants. Large hydro plants can compete with other types of plants, while small hydro plants deserve special attention.

There are two main approaches for securing the provision of electricity and the development of renewable sources of energy. The first approach is non-competitive and it is based on feed-in tariffs. In this case long-term contracts are offered to suppliers at prices that are higher than market prices and are different for different forms of production. These administratively set prices decrease over time with improvements in technology.

The other approach is to organise auctions for electricity from renewable sources. There might be auctions that include all types of technologies or only technology-specific auctions like wind, solar, small hydro plants, etc. In this market-based approach the price of electricity is determined on a competitive base. However, Maurer and Baroso (2011) provide examples from auctions for wind farms in China and Brasil where winning bidders offered too low prices, which were below their long-run marginal costs. Some of these auctions turned out to be unsuccessful, since these plants were not able to provide electricity at such a low price.

## **8. COLLUSION IN ELECTRICITY AUCTIONS**

One of the most important issues in auction design is the prevention of collusive agreements between bidders. The collusive behaviour in reverse auctions results in a lower level of competition, which increases the price for the bid-taker and reduces efficiency. In electricity auctions explicit or tacit agreements increase

the price of electricity well above the marginal cost of production. This outcome reduces the whole population's consumer surplus and is naturally a topic of interest to the competition commission.

There are three possible ways of fighting collusion in electricity auctions. The first approach is based on the policies and procedures of the competition commission, which is authorised by law to protect competition in partial markets. The second approach aims to prevent collusive behaviour *ex ante* with the choice of auction mechanisms that are less prone to collusion. We dealt with this issue earlier when we analysed different auction mechanisms. The third approach is based on restrictions that a bid-taker can impose, independently of the auction mechanism and the competition commission, to diminish the negative effects of collusive behaviour. For instance, imposing the maximal acceptable price in reverse auctions is equivalent to imposing the minimal acceptable price in ordinary auctions.

The standard problem facing the competition commission is to find evidence of collusive behaviour, due to the low quality of information that is available to the commission<sup>5</sup>. This fact increases the incentives of bidders to make collusive agreements to increase the price of electricity. More intensive competition of bidders resulting from the absence of collusion would reduce the markup over the marginal cost. Klemperer (2008) insists that, due to the specific position of auction markets, the competition commission should have a stricter rather than a more tolerant position. The tolerant approach is based on the assumption that the market share of a company participating in an auction does not imply market power, and that the winner's curse is sufficient to assure a non-collusive outcome to the auction. This approach considers only one-shot single-object auctions where only one winner exists. This is certainly not the case with repeated electricity auctions with several winners, which constitute an

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<sup>5</sup> Besides material proof of illegal collusive behaviour, the competition commission can use quantitative methods based on empirical data that could provide evidence of the existence of cartel agreements. Harrington (2008) discusses the use of empirical analysis for detection of cartel agreements. Porter and Zona (1993) developed empirical methodology for the detection of bid rigging in procurement auctions.

environment with strong incentives for bidders to coordinate decisions in order to maximise their profit by collusion.

Based on examples from England, New Zealand, California, and in some western states in the USA, Rothkopf (2002) finds that the introduction of electricity auctions resulted in high electricity prices. He claims that this phenomenon stems from the collusive behaviour of bidders and from the abuse of the dominant suppliers' market power. The abuse of market power is related to the highly inelastic demand for electricity. Moreover, in real time and on a daily basis demand is completely inelastic, which is related to the characteristics of this market that we discussed in the first part of the paper. Rothkopf (2002) illustrates this fact by stating that an unexpected loss of one KWh of electricity that in normal circumstances costs ten cents, in other circumstances can cost consumers ten dollars. The low level of demand elasticity in a situation where the demanded quantity of electricity is almost equal to production capacity can induce the pivotal supplier to raise the price of electricity by making artificial deficits. The control of this type of market power based on standard measures of market concentration is not appropriate to electricity markets, and Rothkopf (2002) suggests that the bid-taker should directly regulate this type of market power. One possible strategy is to set bid caps or maximal acceptable prices, which should be proportional to the estimated level of production costs. The other option is to exclude the bids of all bidders that possess sufficient capacity and economic incentives to make artificial deficits. In this case the bid-taker has the role of regulator.

Rothkopf (1999) emphasised that the repeat character of electricity auctions was neglected by economists and that they were inappropriately analysed by using the theory of single auctions. The repetition of electricity auctions provides high incentives for collusive behaviour. As in one-shot auctions, the cartel agreement is more stable in open than in sealed-bid repeat auctions. In repeated sealed-bid auctions bidders have less information concerning the behaviour of other participants in previous auctions than in open auctions. This fact prevents members of a cartel from obtaining all the necessary information about a cartel member's cheating. On the other hand, in open auctions there is less possibility

of cheating because the misbehaviour of one member in the agreement is obvious.

Fabra (2003) also analyses how the choice between repeated uniform-price and discriminatory electricity auctions influences the probability of collusive behaviour. By using infinitely repeated games she finds that collusive behaviour is more probable in uniform-price auctions than in discriminatory auctions. In discriminatory auctions collusive behaviour results in symmetric equilibria. On the other hand, in uniform-price auctions collusion results in asymmetric equilibria and bid rotating. Fabra (2003) proves that cartel agreement is more stable in asymmetric equilibria in uniform-price auctions. Moreover, the profit from collusive behaviour is higher in uniform-price than in discriminatory auctions.

The applicability of any approach for preventing collusion in isolation is limited. Only the coordinated use of all the strategies will be effective. The repeat nature of electricity auctions makes it very difficult for the competition commission to prevent this phenomenon by itself, and the combination of clever auction design and bid-takers' strategies against collusion are also necessary.

## **9. CONCLUSION**

In this paper we have analysed the most commonly used auction forms in the procurement of electricity. Some forms were specially designed to increase efficiency and decrease the price of electricity. We have also seen that prevention of collusion is an important issue in choosing the optimal auction design. We can expect that in the future more sophisticated auction mechanisms will be developed for procurement of electricity.

Electricity auctions are widely used in developed countries to determine the price of an important resource such as energy. These auctions have also been successfully implemented in many countries in South America, resulting in substantial investment in new plants and technologies.

It is almost certain that in developing countries where as yet there are no electricity auctions, a market-oriented approach to determine the price of an increasingly scarce resource will be necessary. This is also the case in Serbia, where inevitably the present state-owned monopoly will have to be replaced with competitive generating units. At that point the problem of this state-owned monopoly's low level of efficiency and corruption will be resolved.

## APPENDIX

### A. Demand reduction in a uniform-price auction<sup>6</sup>

Suppose that there are two units for sale and that bidder 1 assigns values  $\mathbf{v} = (v_1, v_2)$  to these units. Denote by  $(b_1, b_2)$  the bids of bidder 1 for the first and second units, and by  $\mathbf{c} = (c_1, c_2)$  the vector of the competing bids facing bidder 1, and denote the density function of the random variable  $\mathbf{C}$  by  $h(\cdot)$ . The expected profit of bidder 1 is:

$$\Pi(\mathbf{b}) = \int_{\{\mathbf{c}: c_1 < b_2\}} (v_1 + v_2 - 2c_1)h(\mathbf{c})d\mathbf{c} + \int_{\{\mathbf{c}: c_2 < b_1 \text{ and } c_1 > b_2\}} (v_1 - \max\{b_2, c_2\})h(\mathbf{c})d\mathbf{c} .(A1)$$

The first term represents the expected profit of bidder 1 when his bid for the second unit is higher than the highest competing bid when he obtains two units and the highest losing bid is  $c_1$ . The second term represents his expected profit when his bid for the first unit is higher than the second highest competing bid, but the bid for the second unit is lower than the highest competing bid, and the first bidder obtains one unit. The highest losing bid in this case is  $\max\{b_2, c_2\}$ .

Denote by  $H_1$  the marginal distribution function for the highest competing bid  $c_1$ , and by  $H_2$  the marginal distribution function for the second highest competing bid  $c_2$ . The distribution function  $H_1(\cdot)$  dominates the distribution function  $H_2(\cdot)$  according to first order stochastic dominance,  $H_1(\cdot) \leq H_2(\cdot)$ . Denote the associated density functions by  $h_1$  and  $h_2$ . Bidder 1 wins two units if his bid for the second unit is higher than the highest competing bid,

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<sup>6</sup> This proof is given in Englbrecht-Wiggans and Khan (1998a), but we will use the notation from Krishna (2009).

$\Pr[C_1 < b_2] = H_1(b_2)$ . If the bid of bidder 1 for the first unit is higher than the second highest competing bid ( $c_2 < b_1$ ), bidder 1 will get at least one unit and he can obtain two units if  $c_1 < b_2$ . The probability that bidder 1 obtains at least one unit is  $\Pr[C_2 < b_1] = H_2(b_1)$ . The probability that bidder 1 obtains exactly one unit is the difference between the probability of obtaining at least one unit and the probability of obtaining exactly two units  $H_2(b_1) - H_1(b_2)$ . If  $b_2 < c_1$  bidder 1 will not obtain the second unit, and in that case if  $c_2 < b_2$ ,  $b_2$  is the highest losing bid and this is the market price for all units. The probability of this event is  $\Pr[C_2 < b_2 < C_1] = H_2(b_2) - H_1(b_2)$ . Therefore, the expected profit of bidder 1 is:

$$\begin{aligned}
 \Pi(\mathbf{b}) = & H_1(b_2)(v_1 + v_2) - 2 \int_0^{b_2} c_1 h_1(c_1) d c_1 + [H_2(b_1) - H_1(b_2)] v_1 - [H_2(b_2) - H_1(b_2)] b_2 - \\
 & - \int_{b_2}^{b_1} c_2 h_2(c_2) d c_2. \tag{A2}
 \end{aligned}$$

The first two terms represent the expected profit when bidder 1 gains two units. The other three terms represent the expected profit of bidder 1 if he obtains exactly one unit, where the fourth term is expected payment if the highest losing bid is  $b_2$ , and the fifth term is expected payment if the highest losing bid is  $c_2$ .

Diferentiating the last expression with respect to  $b_2$  we obtain:

$$\begin{aligned}
 \frac{\partial \Pi}{\partial b_2} = & h_1(b_2) v_1 + h_1(b_2) v_2 - 2 b_2 h_1(b_2) - h_1(b_2) v_1 - [H_2(b_2) - H_1(b_2)] - \\
 & - h_2(b_2) b_2 + h_1(b_2) b_2 + h_2(b_2) b_2. \tag{A3}
 \end{aligned}$$

$$\frac{\partial \Pi}{\partial b_2} = h_1(b_2)[v_2 - b_2] - [H_2(b_2) - H_1(b_2)]. \tag{A4}$$

When  $b_2 = v_2$  we have that:

$$\left. \frac{\partial \Pi}{\partial b_2} \right|_{b_2=v_2} = -[H_2(b_2) - H_1(b_2)] < 0, \tag{A5}$$

where the last inequality follows from the fact that distribution function  $H_1(\cdot)$  dominates distribution function  $H_2(\cdot)$  according to first order stochastic dominance,  $H_1(\cdot) \leq H_2(\cdot)$ . Since the partial derivative of the profit function at  $b_2 = v_2$  is negative, this implies that the equilibrium bid for the second unit has to be lower than the value,  $b_2 < v_2$ .

From (A4) we can analyse the trade-off for bidder 1 when he determines his bid for the second unit. If he raises the bid for the second unit, he increases the probability of obtaining that unit. This increase in probability is captured by the term  $h_1(b_2)$ , and the bidder's profit on the second unit won is  $v_2 - b_2$ . Therefore, the first term in (A4) is the expected profit from the incremental increase in  $b_2$ . However, the marginal increase in  $b_2$  increases the expected payment on the first unit if  $b_2$  turns out to be the highest losing bid. The probability of this event is  $H_2(b_2) - H_1(b_2)$ , and the second term in (A4) is the expected loss due to the incremental increase in  $b_2$ . ■

#### **B. Bidding strategies for asymmetric bidders and uniform distribution and optimal strategies in discriminatory auctions<sup>7</sup>**

We will determine the bidding strategies in the case of two asymmetric bidders with uniformly distributed private values. The first bidder has a private value  $v_1$  uniformly distributed on the interval  $[0, \omega_1]$  while the second bidder has a value  $v_2$  uniformly distributed on the interval  $[0, \omega_2]$ , where  $\omega_1 > \omega_2$ . We will denote the distribution function by  $F$ . The first bidder is called the strong bidder and the second is called the weak bidder.

We will denote the bidding strategies of the first and second bidder by  $b_1(\cdot)$  and  $b_2(\cdot)$ , respectively. These strategies are strictly increasing and differentiable. The inverse bidding strategies assign value to any bid  $v_1(\cdot) \equiv b_1^{-1}(\cdot)$  and  $v_2(\cdot) \equiv b_2^{-1}(\cdot)$ . We have two boundary conditions  $b_1(0) = b_2(0) = 0$  and

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<sup>7</sup> This model of asymmetric bidders was studied by Maskin and Riley (2000), but the proof is given according to Krishna (2009).



$b_1(\omega_1) = b_2(\omega_2) = \bar{b}$ . If  $b_1(\omega_1) > b_2(\omega_2)$ , bidder 1 would certainly win and he can reduce the price he pays by bidding lower.

Now suppose that bidder 2 follows the strategy  $b_2(\cdot)$ . We will derive the optimal strategy for bidder 1. The expected profit of bidder 1 who has a value  $v_1$  and bids  $b$  is:

$$\Pi_1(b, v_1) = F(v_2(b))(v_1 - b). \quad (\text{B1})$$

In the case of uniform distribution  $F(v_2(b)) = \frac{v_2(b)}{\omega_2}$  and first order condition w.r. to  $b$  is:

$$v_2'(b) = \frac{v_2(b)}{v_1(b) - b}, \quad (\text{B2})$$

and the same differential equation could be obtained for bidder 2. The system of two differential equations can be written in the following fashion:

$$(v_1'(b) - 1)(v_2(b) - b) = v_1(b) - v_2(b) + b \quad (\text{B3})$$

$$(v_2'(b) - 1)(v_1(b) - b) = v_2(b) - v_1(b) + b \quad (\text{B4})$$

We can write the sum of these two differential equations in the following form:

$$\frac{d}{db}((v_1(b) - b)(v_2(b) - b)) = 2b. \quad (\text{B5})$$

Integration of both sides gives

$$((v_1(b) - b)(v_2(b) - b)) = b^2 \quad (\text{B6})$$

By using the boundary conditions  $v_1(\bar{b}) = \omega_1$  and  $v_2(\bar{b}) = \omega_2$ , the last equation becomes:

$$((\omega_1 - \bar{b})(\omega_2 - \bar{b})) = \bar{b}^2. \quad (\text{B7})$$

By solving (B7) we obtain that:

$$\bar{b} = \frac{\omega_1 \omega_2}{\omega_1 + \omega_2}. \quad (\text{B8})$$

Replacing (B6) in (B2), we obtain:

$$v_2'(b) = \frac{v_2(b)(v_2(b) - b)}{b^2}. \quad (\text{B9})$$

We will define the new variable:

$$\chi_2(b) = \frac{(v_2(b) - b)}{b}, \quad (\text{B10})$$

$$(v_2(b) - b) = \chi_2(b)b, \quad (\text{B11})$$

Differentiating the last equation, we obtain:

$$v_2'(b) - 1 = \chi_2'(b)b + \chi_2(b). \quad (\text{B12})$$

Using (B12) on the left hand side of (B9) and (B10) on the right hand side, we have:

$$\chi_2'(b)b + \chi_2(b) + 1 = \chi_2(b)(\chi_2(b) + 1), \quad (\text{B13})$$

$$\frac{\chi_2'(b)}{\chi_2(b)^2 - 1} = \frac{1}{b}. \quad (\text{B14})$$

The solution of this differential equation is:

$$\chi_2(b) = \frac{1 - k_2 b^2}{1 + k_2 b^2}, \quad (\text{B15})$$

for some constant of integration  $k_2$ . By inserting this result in (B11) we obtain:

$$v_2(b) - b = \frac{1 - k_2 b^2}{1 + k_2 b^2} \cdot b, \quad (\text{B16})$$

$$v_2(b) = \frac{2b}{1 + k_2 b^2}, \quad (\text{B17})$$

$$b_2(v) = \frac{1}{k_2 v_2} \left( 1 - \sqrt{1 - k_2 v_2^2} \right). \quad (\text{B18})$$

Now we use the boundary condition  $v_2(\bar{b}) = \omega_2$  and (B8) to obtain:

$$\frac{2\bar{b}}{1 + k_2 \bar{b}^2} = \omega_2 \quad (\text{B19})$$

$$k_2 = \frac{1}{\omega_2^2} - \frac{1}{\omega_1^2}. \quad (\text{B20})$$

In the same fashion, the bidding strategy for bidder 1 is:

$$b_1(v) = \frac{1}{k_1 v_1} \left( 1 - \sqrt{1 - k_1 v_1^2} \right), \quad (\text{B21})$$

$$k_1 = \frac{1}{\omega_1^2} - \frac{1}{\omega_2^2}. \quad (\text{B22})$$

By using the fact that  $k_1 = -k_2$  and assuming that  $v_1 = v_2 = v$ , the bidding strategies can be written as:

$$b_1(v) = \frac{\sqrt{1 + k_2 v^2} - 1}{k_2 v}, \quad b_2(v) = \frac{1 - \sqrt{1 - k_2 v^2}}{k_2 v}. \quad (\text{B23})$$

It is obvious that weak bidder 2 bids more aggressively than strong bidder 1 if:

$$1 - \sqrt{1 - k_2 v^2} > \sqrt{1 + k_2 v^2} - 1, \quad (\text{B24})$$

which is equivalent to  $k_2^2 v^4 > 0$ . Thus, if both bidders have the same value  $v_1 = v_2 = v$ , the weak bidder bids more aggressively than the strong bidder. This result can be extended in a straightforward manner to discriminatory auctions. In this case the bidder bids more aggressively for the second unit than for the first unit since his distribution function of value for the second unit dominates his distribution function for the first unit.

We will now determine first-order conditions for optimal bidding strategies in a discriminatory auction. Suppose that the bidder has values  $v_1$  and  $v_2$  for the first and second units and that he bids  $b_1$  and  $b_2$ . By using the same logic from appendix A, the bidder wins two units with probability  $H_1(b_2)$  and he wins exactly one unit with probability  $H_2(b_1) - H_1(b_2)$ . The expected profit of bidder 1 in a discriminatory auction is:

$$\begin{aligned} \Pi(\mathbf{b}) = & H_1(b_2)(v_1 + v_2 - b_1 - b_2) + [H_2(b_1) - H_1(b_2)](v_1 - b_1) = \\ & H_2(b_1)(v_1 - b_1) + H_1(b_2)(v_2 - b_2). \end{aligned} \quad (\text{B25})$$

If the bidder submits a higher bid for the first unit than for the second,  $b_1 > b_2$ , then we have two first order conditions:

$$h_2(b_1)(v_1 - b_1) = H_2(b_1), \quad (\text{B26})$$

$$h_1(b_2)(v_2 - b_2) = H_1(b_2). \quad (\text{B27})$$

This result implies that bidding strategies for the first and second units are separable, which means that the bid for the first unit is independent of the bidder's value for the second unit, and vice versa.

However, due to more aggressive bidding for the second unit, the bidder can submit the same bids for the first and second units,  $b_1 = b_2 = b$ . In this case the first order condition is:

$$h_2(b)(v_1 - b) + h_1(b)(v_2 - b) = H_2(b) + H_1(b). \quad \blacksquare \quad (\text{B28})$$

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