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Multivalued Logic, Neutrosophy and Schrodinger equation

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Multi-Valued Logic, Neutrosophy, and Schrödinger Equation

Florentin Smarandache
V. Christianto

For Eighty Years of Quantum Mechanics, 1926-2006

Hexis-Phoenix, 2005

Florentin Smarandache

Vic Christianto

Multi-Valued Logic, Neutrosophy, and Schrödinger Equation

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Abstract

This book was intended to discuss some paradoxes in Quantum Mechanics from the viewpoint of Multi-Valued-logic pioneered by Lukasiewicz, and a recent concept Neutrosophic Logic. Essentially, this new concept offers new insights on the idea of 'identity', which too often it has been accepted as given.

Neutrosophy itself was developed in attempt to generalize Fuzzy-Logic introduced by L. Zadeh. While some aspects of theoretical foundations of logic are discussed, this book is not intended solely for pure mathematicians, but instead for physicists in the hope that some of ideas presented herein will be found useful.

The book is motivated by observation that despite almost eight decades, there is indication that some of those paradoxes known in Quantum Physics are not yet solved. In our knowledge, this is because the solution of those paradoxes requires re-examination of the foundations of logic itself, in particular on the notion of identity and multi-valuedness of entity.

The book is also intended for young physicist fellows who think that somewhere there should be a 'complete' explanation of these paradoxes in Quantum Mechanics. If this book doesn't answer all of their questions, it is our hope that at least it offers a new alternative viewpoint for these old questions.

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Multi-Valued-logic, Neutrosophy, and Schrödinger Equation

Florentin Smarandache & V. Christianto

Dec. 2005

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Foreword

It comes me a bit inappropriate to praise this book, since some of the derivations in it are based on my modest contribution to science. But this is the kind of work that is truly worthy of praise. Florentin Smarandache and V. Christianto are two distinguished scientists and great authors. The idea of linking multi-valued logic to quantum mechanics is an excellent, new idea that, as far as I know, has not been considered before in physics.

Smarandache and Christianto prove that multi-valued logic can lead to the resolution of long standing paradoxes in quantum mechanics, such as the Schrödinger cat paradox, the quantum Sorites paradox, etc. Most importantly, they convincingly explain -with Schrödinger's equation- that the orbits of planets are quantized! (i.e., the same sets of rules that apply to an atomic system also apply to a planetary system). This argument, as they show, is supported by recent astronomical observations.

Throughout the book, the authors make a number of important predictions that can be tested experimentally. For key arguments and conclusions, they reinforce their points of view with a mountain of citations and irrefutable experimental evidence.

The book is written in a refreshing, humorous style that makes it a truly delightful piece of reading. This is a highly recommended book for researchers and graduate students who are looking for potentially new, breakthrough ideas in physics or applied mathematics.

Ezzat G. Bakhoum
Dec. 2005

1 Introduction: Paradoxes, Lukasiewicz Multi-Valued logic

”*This statement is false.”* Supposed we write like this in the beginning of this book, and then can you prove it really false? We doubt it, because the statement reveals contradiction in formal system of logic: If the statement is false, then we cannot believe in its own claim, and *vice versa*. Sometimes it is called ‘self-referential’ statement. In programming language, for instance, a ‘self-referential’ code would mean a program which duplicates itself. For an example, see Appendix I.

It is known that there are limitations to the present accepted axiomatic foundations of logic. Some of these limitations are known in the form of paradoxes. For instance, there are paradoxes [2] known to logicians:

- Russell paradox--- Consider the set of all sets are not members of themselves. Is this set a member of itself?
- Epimenidean paradox--- Consider this statement: “This statement is false.” Is this statement is true?
- Berry paradox --- Consider this statement: “Find the smallest positive integer which to be specified requires more characters then there are in this sentence.” Does this sentence specify an integer?

While there are of course other paradoxes known mostly by mathematicians, including Zeno paradox, Banach-Tarski paradox, etc., those aforementioned paradoxes delimit what can be proved [2]. The first, devised by Bertrand Russell, indicated that informal reasoning in mathematics can yield contradictions, and it led to the creation of formal systems. The second, attributed to Epimenides, was adapted by Gödel to prove that within a formal system there are true statements that are unprovable. The third leads to specify that a specific number cannot be proved random.

In the meantime, it is not an understatement if Weinberg wrote in his book “*The discovery of quantum mechanics in the mid-1920s was the most profound revolution in physical theory since the birth of modern physics in the seventeenth century*”. And it is known that Quantum Mechanics is at the core of more recent theories intended to describe the nature of elementary particles, including Quantum Field Theory, Quantum Electrodynamics, Quantum Chromodynamics, and so forth.

But quantum mechanics in its present form also suffers from the same limitations of the foundations of logic; therefore it is not surprising that there are difficult paradoxes that astonished physicists’ mind for almost eight decades. Some of these known paradoxes are:

- Wigner’s friend;

- Einstein-Podolski-Rosen paradox;
- Schrödinger's cat paradox.

While numerous attempts have been made throughout the past eight decades to solve all these paradoxes, it seems that only few of the present theories take these paradoxes into consideration as an inherent contradiction (or, to be more precise: 'logic inexactness') in the conceptual foundation of physical theories. It is perhaps because most discussions on these paradoxes are only given as an afterthought.

This book was intended to discuss these problems from the viewpoint of Multi-Valued-logic pioneered by Lukasiewicz, and a recent concept Neutrosophy. Essentially, this new concept offers new insights on the idea of 'identity', which too often it has been accepted as given. Neutrosophy itself was developed in attempt to generalize Fuzzy-Logic introduced by L. Zadeh. While some aspects of theoretical foundations of logic are discussed, this book is not intended solely for pure mathematicians, but instead for physicists in the hope that some of ideas presented herein will be found useful. Therefore we have made our best attempt to present our arguments in an understandable manner. But in some chapters, we should have to introduce a new language to make our presentation clear. Of course, it is recommended to verify our propositions outlined herein via proper experiments [8].

The book is motivated by observation that despite almost eight decades, there is strong indication that some of those paradoxes known in quantum physics are not yet solved. In our knowledge, this is because the solution of those paradoxes requires re-examination of the foundations of logic itself, in particular on the notion of identity and multi-valuedness of entity. Therefore, by elucidating this axiomatic basis of logic, perhaps we could offer a new viewpoint for rethinking those paradoxes, in the same way Frege had to rethink his own propositions on the concept of 'extension' after reading Russell's book [3].

In Chapter 2, we discuss history of Multi-Valued logic and Lukasiewicz's historical contribution on this issue. We also discuss previous attempts to include Multi-Valued-logic in Quantum Mechanics.

In Chapter 3, we will introduce Neutrosophic Logic and its elementary notations. This chapter will be useful for subsequent discussion of solution of Quantum Paradox (Chapter 5). For a sense of balance we also discuss, albeit only in introduction manner, a few other present theories to describe those quantum paradoxes.

In Chapter 4 we discuss Schrödinger's equation and its interpretations.

In Chapter 6 we discuss Quantum Sorites Paradox.

In Chapter 7 we discuss Epistemological aspects of Multi Valued logic and how Neutrosophic Logic treats the real sets.

In Chapter 8 we discuss how Schrödinger equation could be generalized to describe quantization of celestial systems.

Here and there, along this book we put some known paradoxes and humours in order to elucidate the concepts discussed. Our conviction is that paradoxes and humours serve not only for delight, but also to illustrate limitation of the human language in particular in the context of ‘multi-valuedness’ [5a].

As we know, some people think that paradoxes are like 'curse' to human language, while others think they are blessing. This is somewhat similar to 'imperfection' in geometry. It is natural to suppose that imperfection is 'defect' that human does while doing something practical, but we know that great architects like Le Corbusier made some great design based on the notion of '*imperfection*' [11]. In the same way Neutrosophy takes 'indeterminacy' seriously into the truth-value system, therefore extends Lukasiewicz’s trivalent logic [12].

Interestingly, via Smarandache’s paradox we can even prove that humours are also serious things. Let’s write Smarandache’s paradox: “*Let A be some attribute (e.g., possible, present, perfect, etc.). If all is A, then the non-A must also be A.*” Therefore we can deduce that non-serious things (humours) are also serious. Using similar approach we can prove some nontrivial results, such as: Non-language is also part of language (watch your gesture); Non-spirit is also part of spirit (hence body-mind-spirit is unity). To summarize, we can re-phrase Wheeler’s paradox: “The question is: What is the question?” to become “The question is: What is non-What?” The reader will also find numerous examples of non-trivial assertions in this book.

All in all, this book is also intended for young physicist fellows who think that somewhere there should be a ‘complete’ explanation of these paradoxes in Quantum Mechanics. If this book doesn’t answer all of their questions, it is our hope that at least it offers a new alternative viewpoint for these old questions.

December 2005

FS, VC

The mathematician may be compared to a designer of garments, who is utterly oblivious of the creatures whom his garments may fit. To be sure, his art originated in the necessity for clothing such creatures, but this was long ago; to this day a shape will occasionally appear which will fit into the garment as if the garment had been made for it. Then there is no end of surprise and delight.
D'Alembert, Jean Le Rond (1717-1783)
[French mathematician and encyclopedist]

2 Lukasiewicz Multi-Valued-logic: History and Introduction to Multi-Valued Algebra

2.1 Introduction to trivalent logic and plurivalent logic

We all have heard of typical binary logic, Yes or No. Or in a famous phrase by Shakespeare: “To be or not to be.” In the same way all computer hardwares from early sixties up to this year are built upon the same binary logic.

It is known that the Classical Logic, also called Bivalent Logic for taking only two values {0, 1}, or Boolean Logic from British mathematician George Boole (1815-64), was named by the philosopher Quine (1981) “*sweet simplicity*.” [57] But this typical binary logic is not without problems. In the light of aforementioned ‘garment analogue’, we can compare this binary logic with a classic black-and-white tuxedo. It is timeless design, but of course you will not wear it for all occasions. Aristotle himself apparently knew this problem; therefore he introduced new terms ‘contingency’ and ‘possibility’ into his modal logic [5]. And then American logician Lewis first formulated these concepts of logical modality. [5].

Throughout history, numereous mathematicians attempted to improve this ‘classical’ logic, but the most prominent logician for introducing non-binary logic was Lukasiewicz. He developed the $\{0, a_1, ..., a_n, 1\}$ Multi-Valued, or Plurivalent Logic, while Post originated the m-valued calculus. In simple words, Lukasiewicz introduced trivalent truth using his truth-value tables. Furthermore he used the symbols 1, ½ and 0 to denote the three truth-values of his trivalent logic. Unlike Boolean binary digits, these symbols are numerical, but not algebraic. Therefore one cannot perform operations with them. They are only used to display truth-values: 1=true, 0=false, $1/2 = a$ third truth-value, equidistant from both (perhaps true and perhaps false). [5]

For introduction, it is worth to summarize here other approaches known in history.

Peirce [56], before 1910, developed semantics for three-valued logic in an unpublished note, but Emil Post's dissertation (1920s) is cited for originating the three-valued logic [12]. Here "1" is used for truth, "1/2" for indeterminacy, and "0" for falsehood. Also, Reichenbach, leader of the logical empiricism, studied it.

Halldon [52], Korner [53], Tye [58] make use of three-valued logic to solve Sorites Paradoxes. They used truth tables, such as Kleene's, but everything depended on the definition of validity.

A three-valued paraconsistent system has the values: 'true', 'false', and 'both true and false' [12]. The ancient Indian metaphysics considered four possible values of a statement: 'true (only)', 'false (only)', 'both true and false', and 'neither true nor false'; J. M. Dunn [50] formalized this in a four-valued paraconsistent system as his First Degree Entailment semantics; The Buddhist logic added a fifth value to the previous ones, 'none of these' (called *catushkoti*).

The idea of tripartition (truth, falsehood, indeterminacy) appeared in 1764 when J. H. Lambert investigated the credibility of one witness affected by the contrary testimony of another [43]. He generalized Hooper's rule of combination of evidence (1680s), which was a Non-Bayesian approach to find a probabilistic model. Koopman in 1940s introduced the notions of lower and upper probability, followed by Good, and Dempster (1967) gave a rule of combining two arguments [38]. Shafer [46] extended it to the Dempster-Shafer Theory of Belief Functions by defining the Belief and Plausibility functions and using the rule of inference of Dempster for combining two evidences proceeding from two different sources. Belief function is a connection between fuzzy reasoning and probability. The Dempster-Shafer Theory of Belief Functions is a generalization of the Bayesian Probability (Bayes 1760s, Laplace 1780s); this uses the mathematical probability in a more general way, and it is based on probabilistic combination of evidence in artificial intelligence.

In Lambert "there is a chance p that the witness will be faithful and exact, a chance q that he will be mendacious, and a chance 1-p-q that he will simply be careless" (according to Shafer [46]). Therefore three components add up to 1.

Van Fraassen [47] introduced the supervaluation semantics in his attempt to solve the sorites paradoxes, followed by Dummett [41] and Fine [42]. They all tripartitioned, considering a vague predicate which having border cases is undefined for these border cases. Van Fraassen took the vague predicate 'heap' and extended it positively to those objects to which the predicate definitively applies and negatively to those objects to which it definitively doesn't apply. The remaining objects border was called penumbra. A sharp boundary between these two extensions does not exist for a soritical predicate. Inductive reasoning is no longer valid too; if S is a Sorites predicate, the proposition "$\exists n(Sa_n \& \neg Sa_{n+1})$" is false. Thus, the predicate Heap (positive extension) = true, Heap (negative extension) = false, Heap (penumbra) = indeterminate.

Narinyani (1980) used the tripartition to define what he called the "indefinite set" [44], and Atanassov (1982) continued on tripartition and gave five generalizations of the fuzzy set, studied their properties and applications to the neural networks in medicine [43]:

a) *Intuitionistic Fuzzy Set* (IFS): Given an universe E, an IFS A over E is a set of ordered triples <universe_element, degree_of_membership_to_A(M), degree_of_non-membership_to_A(N)> such that $M+N \leq 1$ and $M, N \in [0, 1]$. When $M + N = 1$ one obtains the fuzzy set, and if $M + N < 1$ there is an indeterminacy $I = 1-M-N$.

b) *Intuitionistic L-Fuzzy Set* (ILFS): Is similar to IFS, but M and N belong to a fixed lattice L.

c) *Interval-valued Intuitionistic Fuzzy Set* (IVIFS): Is similar to IFS, but M and N are subsets of [0, 1] and $\sup M + \sup N \leq 1$.

d) *Intuitionistic Fuzzy Set of Second Type* (IFS2): Is similar to IFS, but $M^2 + N^2 \leq 1$. M and N are inside of the upper right quarter of unit circle.

We will discuss a generalization of fuzzy set in the form of Neutrosophy in the subsequent chapter. Now it seems worth to take a look first at Lukasiewicz's initial method, and a few subsequent developments.

2.2 History of Lukasiewicz and Multi-Valued logic

The research of Polish thinker Lukasiewicz can be regarded as a fresh departure from Aristotelian binary logic [5]. He was a leading member of Warsaw school of logic, and published his paper "0 logice trojwartoscioweJ" ("On Trivalent Logic") in 1920.

In fact, our *intuition* perhaps has told us numerous times that something is not quite right with binary logic, Yes or No. It seems like there is no sunrise or sunset, because all we observe is only 'pure dark' in the middle of the night, or 'pure bright' like in the high noon without cloud. Yet, we find sunset and sunrise always looks beautiful. In the same way, the ordinary use of Aristotelian logic seems to be too restrictive. This is why *Lukasiewicz* argued that the traditional Aristotelian logic goes "against all our intuitions." Various subsequent developments after Lukasiewicz are also called 'intuitionistic logic'.

It would be better to begin with a short biography of Jan Lukasiewicz [10a], one of the most notable pioneers of Multi-Valued-logic. Jan Lukasiewicz was born at 21 December 1878 in Lvov, Austrian Galicia (now Ukraine). He died at 13 February 1956 in Dublin, Ireland. Jan's father, Luke Lukasiewicz was a captain in the Austrian army, because Lvov was attached to Austria in the 1772 partition of Poland. Jan's mother, Leopoldine Holtzer, was the daughter of an Austrian civil servant.

Young Lukasiewicz was interested in mathematics at school and entered the University of Lvov where he studied mathematics and philosophy. Then he continued to work for his doctorate, which was awarded in 1902 with the highest distinction possible. Wishing to lecture in universities, Lukasiewicz continued to study, submitting his thesis to the University of Lvov in 1906. Then in 1911 he was promoted to an extraordinary professor at Lvov. And there were large changes, which would open new possibilities to Lukasiewicz. At the outbreak of World War I, Germany and Austrian-Hungary took control of most of his country. And it resulted in refounding of the University of Warsaw, which began to operate as a Polish University in November 1915.

Lukasiewicz was invited to the new University of Warsaw when it reopened in 1915. It was an exciting time in Poland, and a new Kingdom of Poland was declared on 5 November 1916. Lukasiewicz became Polish Minister of Education in 1919 and a professor at Warsaw University from 1920-1939; during this period Lukasiewicz was twice rector of Warsaw University.

During this time Lukasiewicz and Lesniewski founded the Warsaw School of Logic. Tarski, who was a student of Lesniewski, would make this school internationally famous. Lukasiewicz published his famous text *Elements of Mathematical Logic* in Warsaw in 1928 (the English translation appeared in 1963).

In 1930, Lukasiewicz published his paper "*Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagenkalkuels*" (Philosophical Observations on Polyvalent Systems of Propositional Logic). In this paper the author explains his ideas, from the point of view of both logic and philosophy.[5] In order to give illustrations of Lukasiewicz's own arguments, let use quote his paragraphs from this cited paper:

"*When I became aware of the incompatibility of traditional theorems of modal propositions in 1920, I was in the process of establishing a normal bivalent propositional calculus based on the matrix method. At that time I was convinced that it was possible to demonstrate all the thesis of the ordinary propositional calculus assuming that propositional variables could take on only two values, "0" (false), and "1" (true). This assumption corresponds to the basic theorem that every proposition is either true or false. For brevity's sake, I will refer to it as the law of bivalence. Although it is sometimes referred to as the law of excluded middle, I prefer to restrict this latter term to the well known principle of classical logic which states that two contradictory propositions cannot both be false at the same time."*

"Our whole system of logic is based on the law of bivalence, though it has been fiercely disputed since ancient times. Aristotle knew this law, but he questioned its validity as it referred to future contingent propositions. The law of bivalence was flatly rejected by the Epicureans. Chrysippus and the Stoics were the first ones to develop it fully and incorporate it as a principle of their dialectic, the equivalent of modern day propositional calculus. The arguments regarding the law of bivalence have metaphysical overtones: its supporters are resolute determinists; whereas its opponents generally have an indeterministic Weltanschauung. Thus, we are once again in the area of concepts of possibility and necessity."

Lukasiewicz's trivalent logic is known as infinite-value logic Ł, and it has been developed further in fuzzy-logic. It has definitions and axioms that can be summarized as follows [48]:

D~ $\sim A = A \supset \perp$ (1.1)

Dt $t = \sim\perp$ (1.2)

Ł1 $A \supset (B \supset A)$ (1.3)

Ł2 $(A \supset B) \supset ((B \supset C) \supset (A \supset C))$ (1.4)

$D\vee$ $A \vee B = (A \supset B) \supset B$ (1.5)

$D\oplus$ $A \oplus B = \sim A \supset B$ (1.6)

Ł3 $((A \supset B) \supset B) \supset ((B \supset A) \supset A)$ (1.7)

Ł4 $((A \supset \perp) \supset (B \supset \perp)) \supset (B \supset A)$ (1.8)

For description and discussion of some notations used above, the reader is referred to [49]. Of course, it is not the only possible approach, as we shall see subsequently.

Nowadays, trivalent logic has been applied to various fields, including for practical purposes. While the impact of Multi-Valued Logic to hardware design remains sluggish, there are already some attempts to bring this concept of multi-valued logic into circuit design, resulting in a few US patents. [125]

2.3 Introduction to Multi-Valued-Algebra, Chang's Notation

A natural path for development of Lukasiewicz's trivalent logic leads us to Multi-Valued-algebra. Of course, it is not the only possible approach, as we shall see subsequently. Multi-Valued-algebra is to Boolean algebra parallel as Lukasiewicz to bivalent logic. It was devised in order to introduce non-standard binary logic into Boolean algebra. In the end we find a much more profound system of algebra.

Multi-Valued-algebra was developed by Chang [9] as the algebraic counterpart of Lukasiewicz logic. They are extensions of Boolean algebras just as Lukasiewicz logic is an extension of classical logic: boolean algebras coincide with idempotent Multi-Valued-algebras. As a summary of Multi-Valued-algebra, we wrote here a few theorems developed by Chang [9][48]. A Multi-Valued-algebra is a structure $A = (A, \oplus, \neg, 0, 1)$ satisfying the following equations [9]:

$$x \oplus (y \oplus z) = (x \oplus y) \oplus z \tag{1.9}$$

$$x \oplus y = x \oplus y \tag{1.10}$$

$$\perp \oplus x = x \tag{1.11}$$

$$x \oplus \neg \perp = \neg \perp \tag{1.12}$$

$$\neg\neg x = x \tag{1.13}$$

$$\neg(\neg x \oplus y) \oplus y = \neg(\neg y \oplus x) \oplus x \tag{1.14}$$

There are of course, other possible approaches to extend trivalued logic, like 'syntactic theory' [5a]. Some authors argue that syntactic theory offers advantages over modal logic. But we will discuss in the next chapter another recent development, which is intended as generalization of fuzzy-logic, i.e. Neutrosophic logic.

2.4 Linkage between Multi-Valued Logic and Quantum Mechanics

At this point a physicist fellow could waive his hand and ask a question: "Hello, anybody here? Why after all should we include this messy Multi-Valued-logic into our theories? I could predict elementary particles without any reference to this multi-valuedness, and we could live happily ever after."[132a][132b] Excellent question, it's great to hear that at least some of our fellows could realize how deep the problem is. Now, here's our comment. (As Gauss would say: "Let's forget clamour of Boetians for a while.")

First of all, the use of Multi-Valued-logic to describe quantum systems has been proposed by at least dozen of quantum physicists (including Heisenberg, as we will describe in subsequent chapter). Other physicists have noted that conventional statistical theories cannot describe quantum phenomena sufficiently. This is because of some reasons:

(i) Conventional statistic theories are inadequate to describe more realistic systems, because it is mainly based on Boolean logic. Primas wrote [17d]:

> "…we summarize the traditional axiomatization of calculus of probability in terms of Boolean algebras and its set-theoretical realization in terms of Kolmogorov probability spaces. Since the axioms of mathematical probability theory say nothing about the conceptual meaning of "randomness" one considers probability as property of the generating conditions of a process so that one can relate randomness with predictability (or retrodictability). In the measure-theoretical codification of stochastic processes genuine chance processes can be defined rigorously as so-called regular processes which do not allow a long-term prediction."

(ii) Conventional statistic theories cannot describe all aspects of 'uncertainty'. Therefore it is required to extend this theory to generalized concept, using trivalent logic, such as Fuzzy-Logic and Neutrosophy. For instance, Zwick noted:

> "…set theory and probability theory are inadequate frameworks to capture the full scope of the concept of "uncertainty." Uncertainty in set theory means nonspecificity; in probability theory it derives from conflicting likelihood claims. Generalizations of set and probability theories, for example, fuzzy set theory and fuzzy measure theory, expand the concept of uncertainty while encompassing these standard connotations; they are thus potentially of great value for science." [17a, p.1]

(iii) Quantum phenomena are quite different from other phenomena we used to know in daily experience.

Nonetheless, the use of Multi-Valued-logic remains unpopular, because thus far only very few physicists seem to care if Multi-Valued-logic offers any real advantage toward better understanding of quantum phenomena. But let us point out here a few arguments in favour of the use of Multi-Valued-logic to describe quantum systems.

For instance Zwick noted [17a, p. 7]:

> "Quantum mechanics is important for Nicolescu beyond its implications concerning levels of reality. Nicolescu advocates a "logic of included middles," … and illustrates his position by pointing to the wave-particle duality, in which a contradiction at a classical level is reconciled at the quantum level. If A is a wave and non-A a particle, then the [A, non-A] contradic-

> tion at the classical (lower) level is resolved by T, the system at the quantum higher level. Nicolescu argues that this logical structure is open-ended, that new contradictions emerge at the upper level which can be resolved by a still higher level. This implies the *existence of a "level of reality" higher than quantum theory*, but no suggestion is given about what this higher level might be. Contradictory pairs abound in quantum theory, but examples from domains other than physics are unfortunately not offered.
> For systems theorists, the view that classical 2-valued logic may not be adequate to describe reality needs no justification from quantum theory, as it is well-articulated in the systems -- especially the fuzzy -- literature, though this is not mentioned in the essay…, which goes back at least to Lukasiewicz (1930)."

Furthermore, Slavnov [17b] has argued that it is possible to describe Quantum Mechanical systems without the use of quantum logic, i.e. by using Multi-Valued-logic. This is quite similar with Jammer's discussion of Reichenbach's theory [118]. Reichenbach tackled probability as an approach and weighed its value dealing with indeterminacy and pondered its massive philosophical issues. He attempted to apply probability to logical propositions. Essentially, Reichenbach innovated this**:** trichon(true, mu, false). His critics responded, quintessentially, "tertium non datur." Von Weizsäcker's infinite valued logic is better than any three valued approach, but von Weizsäcker, like many of his colleagues, via his apparent assumption of analytic stoppability, seemed to fail to see quantum reality as emergent and unstoppable process [118].

In the mean time, Burkhard Heim also considered Multi-Valued-logic in his syntrometry/quantum geodynamics [17c].

Therefore it could be summarized here, while the notion of Multi-Valued-logic is not so common yet, at least there are some fundamental reasons to consider its use seriously, in particular when describing paradoxical aspects of Quantum Phenomena. We will discuss implications of using Multi-Valued-logic if subsequent chapters, in particular in our discussion of solution of Schrödinger's cat paradox.

2.5 Exercise

This exercise section is in particular intended for physicist fellows who keep on thinking that a revision to classical logic is merely a matter of choice (like the 'Axiom of Choice') or only another mathematical technique.

Some of these examples will illustrate clearly that somehow the basic logic we use to derive conclusions has inherent 'imperfection.' These examples are where mathematics becomes look bizarre. Perhaps someday mathematicians would call this subject as different category like *'The art of absurd mathematics and weird algebra*."

If you can find out where the logic slippery is in these examples, then you can go on to the next chapters. If you don't find it, please take a look again more carefully.

The most part of these examples is from scijokes forum [13].

No.	**Example**	**Source**
1	<u>Theorem:</u> All numbers are equal to zero. <u>Proof:</u> Suppose that a=b. Then $a = b$ $a^2 = ab$ $a^2 - b^2 = ab - b^2$ $(a + b)(a - b) = b(a - b)$ $a + b = b$ $a = 0$	Benjamin Tilly
2	<u>Theorem :</u> 3=4 <u>Proof:</u> Suppose: $a + b = c$ This can also be written as: $4a - 3a + 4b - 3b = 4c - 3c$ After reorganizing: $4a + 4b - 4c = 3a + 3b - 3c$ Take the constants out of the brackets:	Michael Ketzlick

No.	Example	Source
	4 * (a+b-c) = 3 * (a+b-c) Remove the same term left and right: 4 = 3	
3	<u>Theorem:</u> 1$ = 1c. <u>Proof:</u> 1$ = 100c = (10c)^2 = (0.1$)^2 = 0.01$ = 1c Here $ means dollars and c means cents.	Benjamin Tilly
4	<u>Theorem:</u> e=1 <u>Proof:</u> 2*e = f 2^(2*pi*i)e^(2*pi*i) = f^(2*pi*i) e^(2*pi*i) = 1 so: 2^(2*pi*i) = f^(2*pi*i) 2=f thus:	

No.	**Example**	**Source**
	$e=1$	
5	Theorem: $1 + 1 = 2$ Proof: $n(2n - 2) = n(2n - 2)$ $n(2n - 2) - n(2n - 2) = 0$ $(n - n)(2n - 2) = 0$ $2n(n - n) - 2(n - n) = 0$ $2n - 2 = 0$ $2n = 2$ $n + n = 2$ or setting $n = 1$ $1 + 1 = 2$ (Comment: what if $n > 1$?)	John Baez
6	Theorem: $1 = 0$ Proof: $x=1$ $x^2=x$ $x^2-1=x-1$ $(x+1)(x-1)=(x-1)$ $(x+1)=(x-1)/(x-1)$	

No.	Example	Source
	x+1=1	
	x=0	
	0=1	
7	<u>Theorem:</u> All numbers are equal.	J. Jamison
	<u>Proof:</u>	
	Choose arbitrary a and b, and let t = a + b. Then	
	a + b = t	
	(a + b)(a - b) = t(a - b)	
	a^2 - b^2 = ta - tb	
	a^2 - ta = b^2 - tb	
	a^2 - ta + (t^2)/4 = b^2 - tb + (t^2)/4	
	(a - t/2)^2 = (b - t/2)^2	
	a - t/2 = b - t/2	
	Therefore:	
	a = b	
8	<u>Theorem:</u> n=n+1	
	<u>Proof:</u>	
	(n+1)^2 = n^2 + 2*n + 1	
	Bring 2n+1 to the left:	
	(n+1)^2 - (2n+1) = n^2	

No.	Example	Source
	Substract n(2n+1) from both sides and factoring, we have:	
	(n+1)^2 - (n+1)(2n+1) = n^2 - n(2n+1)	
	Adding 1/4(2n+1)^2 to both sides yields:	
	(n+1)^2 - (n+1)(2n+1) + 1/4(2n+1)^2 = n^2 - n(2n+1) + 1/4(2n+1)^2	
	This may be written:	
	[(n+1) - 1/2(2n+1)]^2 = [n - 1/2(2n+1)]^2	
	Taking the square roots of both sides:	
	(n+1) - 1/2(2n+1) = n - 1/2(2n+1)	
	Add 1/2(2n+1) to both sides:	
	n+1 = n	
9	Theorem: It is possible to square the circle.	
	Proof:	
	No mathematician has squared the circle.	
	Therefore: No one who has squared the circle is a mathematician.	
	Therefore: All who have squared the circle are non-mathematicians.	
	Therefore: Some non-mathematician has squared	

No.	Example	Source
	the circle. Therefore: It is possible to square to circle. [QED]	
10	<u>Theorem:</u> ln(2) = 0 <u>Proof:</u> Consider the series equivalent of ln 2: ln 2 = 1 - 1/2 + 1/3 - 1/4 + 1/5 - 1/6 ... Rearange the terms: ln 2 = (1 + 1/3 + 1/5 + 1/7 ...) - (1/2 + 1/4 + 1/6 + 1/8 ...) Thus: ln 2 = (1 + 1/3 + 1/5 + 1/7 ...) + (1/2 + 1/4 + 1/6 + 1/8 ...) - 2 * (1/2 + 1/4 + 1/6 + 1/8 ...) Combine the first to series: ln 2 = (1 + 1/2 + 1/3 + 1/4 + 1/5 ...) - (1 + 1/2 + 1/3 + 1/4 + 1/5 ...) Therefore: ln 2 = 0	
11	All dogs are animals. All cats are animals. Therefore, all dogs are cats.	

No.	Example	Source
12	Alternative to #11: "…all dogs are pets and all dogs bark, and cats are pets, too, and therefore cats bark."	From Umberto's book, *Foucault Pendulum*, p. 50
13	A statistician can have his head in an oven and his feet in ice, and he will say that on the average he feels fine.	Comment: An example, which could be regarded as (much better) simplified version of Schrödinger 's cat paradox.
14	Feynman's joke: "*We decide that trivial mean 'proved.' So we start making joke with mathematician colleagues: You mathematicians could only prove 'trivial' theorems*."	From G. Johnson's book '*Strange beauty' (1999).*

A logical theory may be tested by its capacity for dealing with puzzles, and it is a wholesome plan, in thinking about logic, to stock the mind with as many puzzles as possible, since these serve much the same purpose as is served by experiments in physical science.
---Bertrand Russell

3 Neutrosophy

In this chapter we discuss how Neutrosophy offers a better alternative than Lukasiewicz's trivalent logic, in particular because it offers freedom in the value of 'indeterminacy'. Essentially, Neutrosophy is an attempt to generalize Fuzzy Logic introduced by L. Zadeh.

The multi-valued logic of Lukasiewicz was replaced by Goguen [51] and Zadeh [59][60] with an Infinite-Valued Logic (of continuum power, as in the classical mathematical analysis and classical statistics) called Fuzzy Logic, where the truth-value can be any number in the closed unit interval [0, 1]. The Fuzzy Set was introduced by Zadeh in 1975.

Therefore, we could generalize the fuzzy logic to a transcendental logic, called "*neutrosophic logic*" [12]: where the interval [0, 1] is exceeded, i.e. , the percentages of truth, indeterminacy, and falsity are approximated by non-standard subsets – not by single numbers, and these subsets may overlap and exceed the unit interval in the sense of the non-standard analysis; also the superior sums and inferior sum, $n_{sup} = \sup T + \sup I + \sup F \in\ ]^-0, 3^+[$, may be as high as 3 or 3^+, while $n_{inf} = \inf T + \inf I + \inf F \in\ ]^-0, 3^+[$, may be as low as 0 or $^-0$.

This chapter starts with an introduction and formalization of Neutrosophy concepts, and then we discuss *non-standard analysis* because it is necessary in defining non-standard real subsets and especially the non-standard unit interval $]^-0, 1^+[$, all used by neutrosophic logic. Thereafter the neutrosophic logic components are introduced followed by the definition of neutrosophic logic and neutrosophic logic connectors, which are based on set operations. Original work consists in the definition of neutrosophic logic and neutrosophic connectors as an extension of intuitionistic fuzzy logic and the comparison between NL and other logics, especially the IFL.

3.1 Introduction to Neutrosophy [12a]

Neutrosophy studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. It considers that every idea <A> tends to be neutralized, balanced by <Non-A> ideas - as a state of equilibrium. Neutrosophy is the base of *neutrosophic logic*, *neutrosophic set* *that* generalizes the fuzzy set, and of *neutrosphic probability* and *neutrosophic statistics*, which generalize the classical and imprecise probability and statistics respectively. From etymology viewpoint, Neutro-sophy comes from [French *neutre* < Latin *neuter*, neutral, and Greek *sophia*, skill/wisdom] means knowledge of neutral thought.

Neutrosophy is a branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophic Logic is a multiple-valued logic in which each proposition is estimated to have the percentages of truth, indeterminacy, and falsity in T, I, and F respectively, where T, I, F are standard or non-standard subsets included in the non-standard unit interval $]^{-}0, 1^{+}[$. It is an extension of fuzzy, intuitionistic, paraconsistent logics.

This mode of thinking:

- proposes new philosophical theses, principles, laws, methods, formulas, movements;

- reveals that world is full of indeterminacy;
- interprets the uninterpretable { i.e. to deal with paradoxes [55] and paradoxism [61] };
- regards, from many different angles, old concepts, systems: showing that an idea, which is true in a given referential system, may be false in another one, and vice versa;
- attempts to make *peace in the war of ideas*;
- measures the stability of unstable systems, and instability of stable systems.

3.2 Introduction to Non-Standard Analysis [12a]

In 1960s Abraham Robinson has developed the *non-standard analysis*, a formalization of analysis and a branch of mathematical logic, which rigorously defines the infinitesimals. Informally, an infinitesimal is an infinitely small number. Formally, x is said to be infinitesimal if and only if for all positive integers n one has $|x| < 1/n$. Let $\varepsilon>0$ be a such infinitesimal number. The *hyper-real number set* is an extension of the real number set, which includes classes of infinite numbers and classes of infinitesimal numbers. Let's consider the non-standard finite numbers $1^{+} = 1+\varepsilon$, where "1" is its standard part and "ε"

its non-standard part, and $^-0 = 0\text{-}\varepsilon$, where "0" is its standard part and "ε" its non-standard part.

Then, we call $]\,^-0, 1^+[$ a non-standard unit interval. Obviously, 0 and 1, and analogously non-standard numbers infinitely small but less than 0 or infinitely small but greater than 1, belong to the non-standard unit interval. Actually, by "^-a" one signifies a monad, i.e. a set of hyper-real numbers in non-standard analysis:

$(^-a) = \{a\text{-}x: x \in \mathbb{R}^*, x \text{ is infinitesimal}\}$,

and similarly "b^+" is a monad:

$(b^+) = \{b+x: x \in \mathbb{R}^*, x \text{ is infinitesimal}\}$.

Generally, the left and right borders of a non-standard interval $]\,^-a, b^+[$ are vague, imprecise, themselves being non-standard (sub)sets (^-a) and (b^+) as defined above. Combining the two before mentioned definitions one gets, what we would call, a binad of "$^-c^+$":

$(^-c^+) = \{c\text{-}x: x \in \mathbb{R}^*, x \text{ is infinitesimal}\} \cup \{c+x: x \in \mathbb{R}^*, x \text{ is infinitesimal}\}$, which is a collection of open punctured neighborhoods (balls) of c.

Of course, $^-a < a$ and $b^+ > b$. No order between $^-c^+$ and c.

Addition of non-standard finite numbers with themselves or with real numbers:

$^-a + b = {}^-(a + b)$

$a + b^+ = (a + b)^+$

$^-a + b^+ = {}^-(a + b)^+$

$^-a + {}^-b = {}^-(a + b)$ (the left monads absorb themselves)

$^-a + {}^-b = {}^-(a + b)$ (the left monads absorb themselves)

$a^+ + b^+ = (a + b)^+$ (analogously, the right monads absorb themselves).

Similarly for subtraction, multiplication, division, roots, and powers of non-standard finite numbers with themselves or with real numbers.

By extension let $\inf\,]\,^-a, b^+[\, = {}^-a$ and $\sup\,]\,^-a, b^+[\, = b^+$.

3.3 Definition of Neutrosophic Components [12a]

Let T, I, F be standard or non-standard real subsets of $]\,^-0, 1^+[$,

with $\sup T = t\_sup$, $\inf T = t\_inf$,

$\sup I = i\_sup$, $\inf I = i\_inf$,

$\sup F = f_{sup}$, $\inf F = f_{inf}$,

and $n_{sup} = t_{sup}+i_{sup}+f_{sup}$,

$n_{inf} = t_{inf}+i_{inf}+f_{inf}$.

The sets T, I, F are not necessarily intervals, but may be any real sub-unitary subsets: discrete or continuous; single-element, finite, or (countably or uncountably) infinite; union or intersection of various subsets; etc.

They may also overlap. The real subsets could represent the relative errors in determining t, i, f (in the case when the subsets T, I, F are reduced to points).

In the next stage of development, T, I, F, called *neutrosophic components*, will represent the truth value, indeterminacy value, and falsehood value respectively referring to neutrosophy, neutrosophic logic, neutrosophic set, neutrosophic probability, and neutrosophic statistics.

This representation is closer to the human mind reasoning. It characterizes/catches the *imprecision* of knowledge or linguistic inexactitude received by various observers (that's why T, I, F are subsets - not necessarily single-elements), *uncertainty* due to incomplete knowledge or acquisition errors or stochasticity (that's why the subset I exists), and *vagueness* due to lack of clear contours or limits (that's why T, I, F are subsets and I exists; in particular for the appurtenance to the neutrosophic sets).

One has to specify the superior (x_sup) and inferior (x_inf) limits of the subsets because in many problems arises the necessity to compute them.

3.4 Formalization [12a]

Let's note by <A> an idea, or proposition, theory, event, concept, entity, by <Non-A> what is not <A>, and by <Anti-A> the opposite of <A>. Also, <Neut-A> means what is neither <A> nor <Anti-A>, i.e. neutrality in between the two extremes. And <A'> a version of <A>.

<Non-A> is different from <Anti-A>.

For example:

If <A> = white, then <Anti-A> = black (antonym),

but <Non-A> = green, red, blue, yellow, black, etc. (any color, except white),

while <Neut-A> = green, red, blue, yellow, etc. (any color, except white and black),

and <A'> = dark white, etc. (any shade of white).

<Neut-A> η <Neut-(Anti-A)>, neutralities of <A> are identical with neutralities of <Anti-A>.

<Non-A> includes <Anti-A>, and <Non-A> includes <Neut-A> as well,

also

<A> intersected with <Anti-A> is equal to the empty set,

<A> intersected with <Non-A> is equal to the empty set.

<A>, <Neut-A>, and <Anti-A> are disjoint two by two.

<Non-A> is the completeness of <A> with respect to the universal set.

Fundamental Theory of Neutrosophy can be summarized as follows:

> Every idea <A> tends to be neutralized, diminished, balanced by <Non-A> ideas (which includes, besides Hegel's <Anti-A>, the <Neut-A> too) - as a state of equilibrium. In between <A> and <Anti-A> there are infinitely many <Neut-A> ideas, which may balance <A> without necessarily <Anti-A> versions.

To neuter an idea one must discover all its three sides: of sense (truth), of non-sense (falsity), and of undecidability (indeterminacy) - then reverse/ combine them. Afterwards, the idea will be classified as neutrality.

Furthermore, Neutrosophy can be viewed as delimitation of other philosophical theories:

a) Neutrosophy is based not only on analysis of oppositional propositions, as *dialectic* does, but on analysis of neutralities in between them as well.

b) While *epistemology* studies the limits of knowledge and justification, neutrosophy passes these limits and takes under magnifying glass not only defining features and substantive conditions of an entity <E> - but the whole <E'> derivative spectrum in connection with <Neut-E>. *Epistemology studies philosophical contraries, e.g. <E> versus <Anti-E>, neutrosophy studies <Neut-E> versus <E> and versus <Anti-E> which means logic based on neutralities.*

c-d) *Neutral monism* asserts that ultimate reality is neither physical nor mental. Neutrosophy considers a more than pluralistic viewpoint: infinitely many separate and ultimate substances making up the world.

e) *Hermeneutics* is the art or science of interpretation, while neutrosophy also creates new ideas and analyzes a wide range ideational field by balancing instable systems and unbalancing stable systems.

f) *Philosophia Perennis* tells the common truth of contradictory viewpoints; neutrosophy combines with the truth of neutral ones as well.

g) *Fallibilism* attributes uncertainty to every class of beliefs or propositions, while neutrosophy accepts 100% true assertions, and 100% false assertions as well - moreover, checks what referential systems the percent of uncertainty approaches zero or 100.

3.5 Evolution of an idea [12a]

<A> in the world is not cyclic, but discontinuous, knotted, boundless:

<Neut-A> = existing ideational background, before arising <A>;

<Pre-A> = a pre-idea, a forerunner of <A>;

<Pre-A'> = spectrum of <Pre-A> versions;

<A> = the idea itself, which implicitly gives birth to

<Non-A> = what is outer <A>;

<A'> = spectrum of <A> versions after (mis)interpretations

(mis)understanding by different people, schools, cultures;

<A/Neut-A> = spectrum of <A> derivatives/deviations, because <A> partially mixes/melts first with neuter ideas;

<Anti-A> = the straight opposite of <A>, developed inside of <Non-A>;

<Anti-A'> = spectrum of <Anti-A> versions after

(mis)interpretations (mis)understanding by different people, schools, cultures;

<Anti-A/Neut-A> = spectrum of <Anti-A> derivatives/deviations,

which means partial <Anti-A> and partial <Neut-A> combined in various percentage;

<A'/Anti-A'> = spectrum of derivatives/deviations after mixing

<A'> and <Anti-A'> spectra;

<Post-A> = after <A>, a post-idea, a conclusiveness;

<Post-A'> = spectrum of <Post-A> versions;

<Neo-A> = <A> retaken in a new way, at a different level, in new conditions, as in a non-regular curve with inflection points, in evolute and involute periods, in a recurrent mode; the life of <A> restarts.

<Neo-A> has a larger sphere (including, besides parts of old <A>, parts of <Neut-A> resulted from previous combinations), more characteristics, is more heterogeneous (after combinations with various <Non-A> ideas). But, <Neo-A>, as a whole in itself, has the tendency to homogenize its content, and then to de-homogenize by mixture with other ideas.

And so on, until the previous <A> gets to a point where it paradoxically incorporates the entire <Non-A>, being indistinct of the whole. And this is the point where the idea dies, cannot be distinguished from others. The Whole breaks down, because the motion is characteristic to it, in a plurality of new ideas (some of them containing grains of the original <A>), which begin their life in a similar way.

Thus, in time, <A> arrives to mix with <Neut-A> and <Anti-A>.

3.6 Definition of Neutrosophic Logic [12][22]

As an alternative to the existing logics we propose a non-classical one, which represents a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction.

A logic in which each proposition is estimated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F, where T, I, F are defined above, is called *Neutrosophic Logic*.

We use a subset of truth (or indeterminacy, or falsity), instead of a number only, because in many cases we are not able to exactly determine the percentages of truth and of falsity but to approximate them: for example a proposition is between 30-40% true and between 60-70% false, even worst: between 30-40% or 45-50% true (according to various analyzers), and 60% or between 66-70% false.

The subsets are not necessary intervals, but any sets (discrete, continuous, open or closed or half-open/half-closed interval, intersections or unions of the previous sets, etc.) in accordance with the given proposition.

A subset may have one element only in special cases of this logic.

Constants: (T, I, F) truth-values, where T, I, F are standard or non-standard subsets of the non-standard interval $]\,{}^-0, 1^+[$, where $n_{inf} = \inf T + \inf I + \inf F \geq {}^-0$, and $n_{sup} = \sup T + \sup I + \sup F \leq 3^+$.

Atomic formulas: a, b, c, …..

Arbitrary formulas: A, B, C, …

The neutrosophic logic is a formal frame trying to measure the truth, indeterminacy, and falsehood. There are many neutrosophic rules of inference [39].

Let's make use from the modal logic the notion of "world", which is a semantic device of what the world might have been like. Then, one says that the neutrosophic truth-value of a statement A, $NL_t(A) = 1^+$ if A is 'true in all possible worlds' (*syntagme* first used by Leibniz) and all conjunctures, that one may call "absolute truth" (in the modal logic it was named *necessary truth,* Dinulescu-Campina [40] names it '*intangible absolute truth'*), whereas $NL_t(A) = 1$ if A is true in at least one world at some conjuncture, we call this "relative truth" because it is related to a 'specific' world and a specific conjuncture (in the modal logic it was named *possible truth*).

Similarly for absolute and relative falsehood and absolute and relative indeterminacy. The neutrosophic inference [38], especially for plausible and paradoxist information, is still a subject of intense research today.

3.7 Differences between Neutrosophic Logic (NL) and Intuitionistic Fuzzy Logic (IFL)

The differences between IFL and NL (and the corresponding intuitionistic fuzzy set and neutrosophic set) are [12]:

a) Neutrosophic Logic can distinguish between *absolute truth* (truth in all possible worlds, according to Leibniz) and *relative truth* (truth in at least one world), because NL(absolute truth)=1^+ while NL(relative truth)=1. This has application in philosophy (see the neutrosophy). That's why the unitary standard interval [0, 1] used in IFL has been extended to the unitary non-standard interval $]^-0, 1^+[$ in NL. Similar distinctions for absolute or relative falsehood, and absolute or relative indeterminacy are allowed in NL.

b) In NL there is no restriction on T, I, F other than they are subsets of $]^-0, 1^+[$, thus:

$$^-0 \leq \inf T + \inf I + \inf F \leq \sup T + \sup I + \sup F \leq 3^+.$$

This non-restriction allows paraconsistent, dialetheist, and incomplete information to be characterized in NL {i.e. the sum of all three components if they are defined as points, or sum of superior limits of all three components if they are defined as subsets can be >1 (for paraconsistent information coming from different sources) or < 1 for incomplete information}, while that information can not be described in IFL because in IFL the components T (truth), I (indeterminacy), F (falsehood) are restricted either to t+i+f=1 or to

$t^2 + f^2 \leq 1$, if T, I, F are all reduced to the points t, i, f respectively, or to sup T + sup I + sup F = 1 if T, I, F are subsets of [0, 1].

c) In NL the components T, I, F can also be *non-standard* subsets included in the unitary non-standard interval $]^-0, 1^+[$, not only *standard* subsets included in the unitary standard interval [0, 1] as in IFL.

d) NL, like dialectism, can describe paradoxes, NL(paradox) = (1, I, 1), while IFL can not describe a paradox because the sum of components should be 1 in IFL([11],[12],[13]).

3.8 Operations with Sets [12]

We will present these set operations in order to be able to introduce the neutrosophic connectors.

Let S_1 and S_2 be two (unidimensional) real standard or non-standard subsets, then one defines:

3.8.1 Addition of Sets:

$S_1 \oplus S_2 = \{x \mid x=s_1+s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\}$,

with $\inf (S_1 \oplus S_2) = \inf S_1 + \inf S_2$, $\sup (S_1 \oplus S_2) = \sup S_1 + \sup S_2$;

and, as some particular cases, we have

$\{a\} \oplus S_2 = \{x \mid x=a+s_2, \text{ where } s_2 \in S_2\}$

with $\inf (\{a\} \oplus S_2) = a + \inf S_2$, $\sup (\{a\} \oplus S_2) = a + \sup S_2$.

3.8.2 Subtraction of Sets:

$S1 \ominus S2 = \{x \mid x=s_1-s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\}$.

For real positive subsets (most of the cases will fall in this range) one gets

$\inf (S_1 \ominus S_2) = \inf S_1 - \sup S_2$, $\sup (S_1 \ominus S_2) = \sup S_1 - \inf S_2$;

and, as some particular cases, we have

$\{a\} \ominus S_2 = \{x \mid x=a-s_2, \text{ where } s_2 \in S_2\}$,

with $\inf (\{a\} \ominus S_2) = a - \sup S_2$, $\sup (\{a\} \ominus S_2) = a - \inf S_2$;

also $\{1^+\} \ominus S_2 = \{x \mid x=1^+-s_2, \text{ where } s_2 \in S_2\}$,

with $(\inf \{1^+\} \ominus S_2) = 1^+ - \sup S_2$, $\sup (\{1^+\} \ominus S_2) = 1 - \inf S_2$.

3.8.3 Multiplication of Sets:

$S_1 \odot S_2 = \{x \mid x=s_1 \cdot s_2$, where $s_1 \in S_1$ and $s_2 \in S_2\}$.

For real positive subsets (most of the cases will fall in this range) one gets

$\inf(S_1 \odot S_2) = \inf S_1 \cdot \inf S_2$, $\sup(S_1 \odot S_2) = \sup S_1 \cdot \sup S_2$;

and, as some particular cases, we have

$\{a\} \odot S_2 = \{x \mid x=a \cdot s_2$, where $s_2 \in S_2\}$,

with $\inf(\{a\} \odot S_2) = a \cdot \inf S_2$, $\sup(\{a\} \odot S_2) = a \cdot \sup S_2$;

also $(\{1^+\} \odot S_2) = \{x \mid x=1^+ \cdot s_2$, where $s_2 \in S_2\}$,

with $\inf(\{1^+\} \odot S_2) = 1^+ \cdot \inf S_2$, $\sup(\{1^+\} \odot S_2) = 1^+ \cdot \sup S_2$.

3.8.4 Division of a Set by a Number:

Let $k \in \mathbb{R}^*$, then $S_1 \oslash k = \{x \mid x=s_1/k$, where $s_1 \in S_1\}$.

3.9 Generalizations [12]

When all neutrosophic logic set components are reduced to one element, then $t_{sup} = t_{inf} = t$, $i_{sup} = i_{inf} = i$, $f_{sup} = f_{inf} = f$, and $n_{sup} = n_{inf} = n = t+i+f$, therefore neutrosophic logic generalizes:

- the *intuitionistic logic*, which supports incomplete theories (for $0 < n < 1$ and $i=0$, $0 \le t, i, f < 1$);
- the *fuzzy logic* (for $n = 1$ and $i = 0$, and $0 \le t, i, f \le 1$); from "CRC Concise Encyclopedia of Mathematics" [36], the fuzzy logic is "*an extension of two-valued logic such that statements need not to be True or False, but may have a degree of truth between 0 and 1*";
- the *intuitionistic fuzzy logic* (for $n=1$);
- the *Boolean logic* (for $n = 1$ and $i = 0$, with t, f either 0 or 1);
- the *multi-valued logic* (for $0 \le t, i, f \le 1$).

  Definition of <many-valued logic> from "The Cambridge Dictionary of Philosophy" [editor R. Audi, 1995, p. 461]: "*propositions may take many values beyond simple truth and falsity, values functionally determined by the values of their components*". As we have discussed in the preceding chapter, Lukasiewicz considered three values (1, 1/2, 0). Post considered m values, etc. But they varied in between 0 and 1 only. In the neutrosophic logic a proposition may take values even greater than 1 (in percentage greater than 100%) or less than 0.

- the *paraconsistent logic*, which support conflicting information (for $n > 1$ and $i = 0$, with both $t, f < 1$);
- the *dialetheism*, which says that some contradictions are true (for $t = f = 1$ and $i = 0$;
- some paradoxes can be denoted this way too);
- the *faillibilism*, which says that uncertainty belongs to every proposition (for $i > 0$);

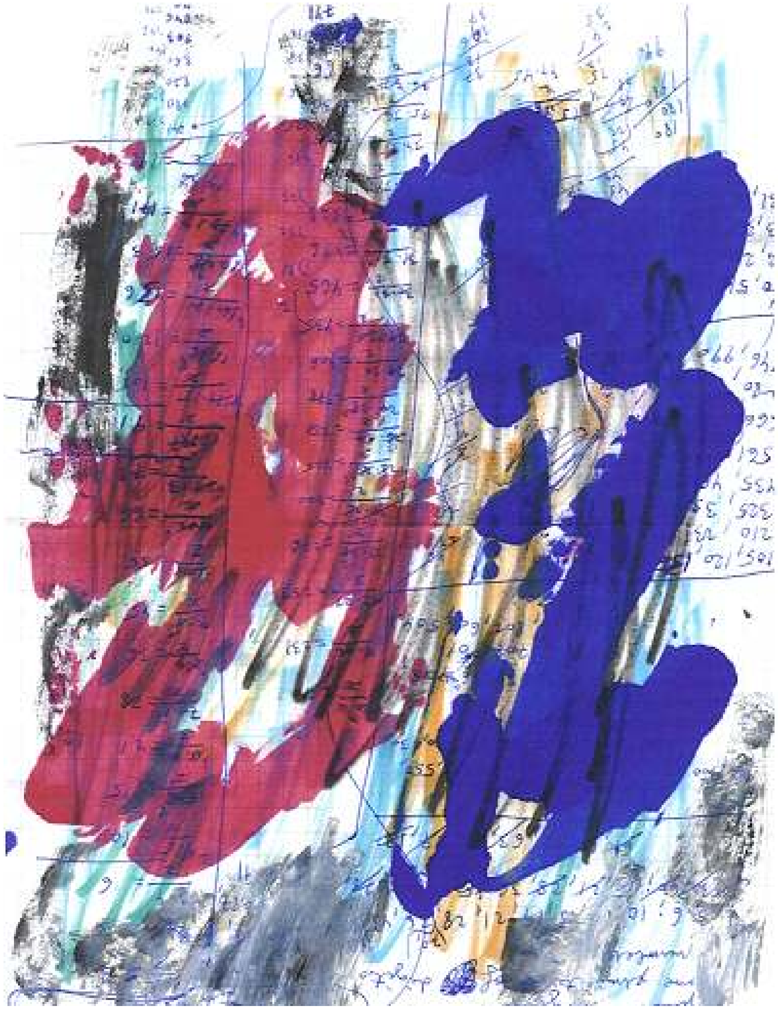
Compared with all other logics, the neutrosophic logic and intuitionistic fuzzy logic introduce a percentage of "*indeterminacy*" - due to unexpected parameters hidden in some propositions, or unknowness, but neutrosophic logic let each component t, i, f be even boiling *over 1* (overflooded), i.e. be 1^+, or freezing *under 0* (underdried), i.e. be $^-0$ in order to be able to make distinction between relative truth and absolute truth, and between relative falsity and absolute falsity in philosophy.

To summarize this chapter, this Neutrosophic Logic is an attempt to unify many logics in a single field. Yet, a too large generalization may sometimes have no practical impact. But we have discussed throughout this chapter that such unification attempts have been done in the history of sciences.

In recent years, Neutrosophy has impacts in various different fields, including artworks like poetry, drama, and painting. A few examples of Smarandache's painting (or perhaps 'non-painting' term is more appropriate) are given below [132]:

Picture 3.1. Example 1 of Smarandache's Outer Art

Picture 3.2. Example 2 of Smarandache's Outer Art.



Picture 3.3. Example 3 of Smarandache's Outer Art.

The essence of Smarandache's non-Art can be summarized as follows: "*non-A is also part of A, therefore non-Art is also Art*." More or less, perhaps this could be related to the logic behind Zen's humble teaching: daily life is also part of doing Zen. (‘Non-Zen is also part of Zen’, if daily life can be described as part of ‘non-Zen’ activities.).

In the subsequent chapter, we will discuss how Neutrosophic Logic could offer a plausible solution for paradoxes in Quantum Mechanics, in particular Schrödinger’s paradox. But in order to do so, first we should introduce a review on the Schrödinger wave equation, and discuss where the problem begins.

"The law of the excluded middle either rules or does not rule, O.K.?"

4 Schrödinger equation

4.1 Introduction

As we all perhaps already know, Schrödinger equation is the most well-known equation to describe non-relativistic quantum systems. Its relativistic version was developed by Klein-Gordon and Dirac, but Schrödinger equation has wide applicability in particular because it resembles classical wave dynamics. For an introduction to non-relativistic quantum mechanics, see [62].

As we all know, Schrödinger equation begins with definition of total energy $E = \vec{p}^2 / 2m$. Then, by using a known substitution:

$$E = i\hbar.\partial / \partial t , \quad p = \hbar\nabla / i , \tag{4.1}$$

one gets [4a]:

$$\left[i\hbar\partial / \partial t + \hbar\overline{\nabla}^2 / 2m - U(x) \right] \Psi = 0 \tag{4.2}$$

or

$$(i\partial / \partial t)\Psi = H.\Psi \tag{4.2a}$$

Historically, this equation began when Schrödinger was giving a lecture on wave interpretation of quanta. At the end of presentation Debye made a short remark: "If there is wave, then there should be wave equation." Thereafter, Schrödinger came up with equation mentioned above. His equation appeared for the first time in a paper published in 1926 in *Ann. Phys.* Vol. 79 (See Picture 4.1). The paper began with a remarkable note [113]:

> "In this article I should like to show, first of all for the simplest case of the (non-relativistic and unperturbed) hydrogen atom, that the usual rule for quantization can be replaced by another requirement in which there is no longer any mention of 'integers'. The integral property follows, rather, in the same natural way that, say, the *number of nodes* of a vibrating string must be an integer. The new interpretation …strikes very deeply into the true nature of the quantization rules."[1]

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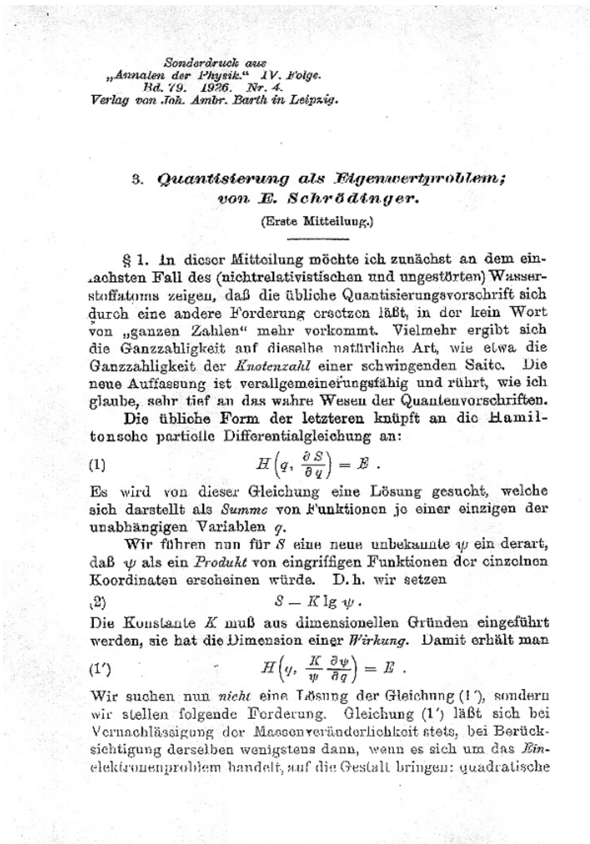
[1] Some string theorists perhaps would find this remark interesting. Equally likely, as part of 21st century theory, perhaps we could expect some improved versions of the present string theories, like banjo theories, violin theories, cello theories, guitar theo-



Sonderdruck aus
„Annalen der Physik." IV. Folge.
Bd. 79. 1926. Nr. 4.
Verlag von Joh. Ambr. Barth in Leipzig.

3. *Quantisierung als Eigenwertproblem;*
von E. Schrödinger.

(Erste Mitteilung.)

§ 1. In dieser Mitteilung möchte ich zunächst an dem einfachsten Fall des (nichtrelativistischen und ungestörten) Wasserstoffatoms zeigen, daß die übliche Quantisierungsvorschrift sich durch eine andere Forderung ersetzen läßt, in der kein Wort von „ganzen Zahlen" mehr vorkommt. Vielmehr ergibt sich die Ganzzahligkeit auf dieselbe natürliche Art, wie etwa die Ganzzahligkeit der *Knotenzahl* einer schwingenden Saite. Die neue Auffassung ist verallgemeinerungsfähig und rührt, wie ich glaube, sehr tief an das wahre Wesen der Quantenvorschriften.

Die übliche Form der letzteren knüpft an die Hamiltonsche partielle Differentialgleichung an:

$$H\left(q, \frac{\partial S}{\partial q}\right) = E \, . \tag{1}$$

Es wird von dieser Gleichung eine Lösung gesucht, welche sich darstellt als *Summe* von Funktionen je einer einzigen der unabhängigen Variablen q.

Wir führen nun für S eine neue unbekannte ψ ein derart, daß ψ als ein *Produkt* von eingriffigen Funktionen der einzelnen Koordinaten erscheinen würde. D. h. wir setzen

$$S = K \lg \psi \, . \tag{2}$$

Die Konstante K muß aus dimensionellen Gründen eingeführt werden, sie hat die Dimension einer *Wirkung*. Damit erhält man

$$H\left(q, \frac{K}{\psi}\frac{\partial \psi}{\partial q}\right) = E \, . \tag{1'}$$

Wir suchen nun *nicht* eine Lösung der Gleichung (1'), sondern wir stellen folgende Forderung. Gleichung (1') läßt sich bei Vernachlässigung der Massenveränderlichkeit stets, bei Berücksichtigung derselben wenigstens dann, wenn es sich um das *Einelektronenproblem* handelt, auf die Gestalt bringen: quadratische

Picture 4.1. Original paper by Schrödinger in 1926 edition of Ann.Phys.[115]

Other physicists found this equation (4.2) is very near to what they knew from classical dynamics; therefore they could embrace to the new quantum dynamics quickly. The basic assertion here is the use of Operator Schema [70]:

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ries etc. See also: http://www.newscientist.com/channel/fundamentals/quantum-world/mg18825293.700

$$Position: x, y, z \rightarrow x\bullet, y\bullet, z\bullet : (\psi)$$
$$Momentum: p_{x,}p_{y,}p_{z\rightarrow}(ih/2\pi)\partial()/\partial x,(ih/2\pi)\partial()/\partial y,$$
$$(ih/2\pi)\partial()/\partial z : (\psi)$$
$$Energy: E \rightarrow (-ih/2\pi)\partial()/\partial t : (\psi)$$
(4.3)

While this equation seems quite clear to represent quantum dynamics, the physical meaning of the wavefunction itself is not clear. Soon thereafter Born came up with hypothesis that the square of the wavefunction has the meaning of chance to find the electron in the region defined by dx (Copenhagen School). While so far his idea was quickly adopted as 'standard interpretation', his original 'guiding field' interpretation has been dropped after criticism by Heisenberg over its physical meaning [4b]. Nonetheless, a definition of 'Copenhagen interpretation' is that it gives the wavefunction a role in the actions of something else, namely of certain macroscopic objects, called 'measuremet instruments', therefore it could be related to phenomenological formalism [4b]. This phenomenological viewpoint is so typical in Bohr's argument, which asserted that nothing but what we could measure should be considered.

Schrödinger apparently also didn't like this statistical interpretation of his equation, and also phenomenological viewpoint, so he came up with his Cat paradox in 1935 (see the next chapter). In a formal and complete rebuttal, Schrödinger once was quoted to have remarked, "*I am opposing, as it were, the whole of quantum mechanics*." [68] This problem apparently has not yet been solved up to this time. And this is perhaps why he didn't contribute anymore to the subsequent development of (Copenhagen's) Quantum Mechanics.

It is not difficult to imagine why Schrödinger did not accept easily the statistical interpretation of his wave mechanics equation. Like Einstein (which inspired him with EPR paradox in 1935 paper), Schrödinger saw clearly that accepting statistical interpretation would not only make God as the player of quantum dice, but in world scale it would mean an assertion that God is running a *quantum Las Vegas* [70]. In his later lectures, documentation showed that Schrödinger apparently expected to extend his wave theory to a complete theory where the notion of 'particle' in classical sense is abandoned completely in favor of his 'wave-function' [70].

Nonetheless, we should also note here that there are other approaches different from Born hypothesis which was accepted as 'the standard interpretation', including:

- The square of the wavefunction represents a measure of the density of matter in region defined by dx (Determinism school). Schrödinger apparently preferred this argument, albeit his attempt to prove this idea has proved to be wrong;
- The square of the wavefunction of Schrödinger equation as the vorticity distribution (including topological vorticity defects) in the fluid [69];
- The wavefunction in Schrödinger equation represents tendency to make structures;
- The wavemechanics can also be described in terms of topological Aharonov effect, which then it could be related to the notion of topological quantization [4].

We will discuss solution of this paradox in the subsequent chapter. But first let us discuss how the classical dynamics could also be treated in 'quantum mechanical' way, so it could serve to illustrate how Schrödinger equation resembles neatly the classical dynamics equation. Meanwhile, other physicists may argue that there is clear departure from Classical Mechanics to Quantum Mechanics.

4.2 Quantum wave dynamics and classical dynamical system

One of the deep questions related to the physical meaning of wavefunction of the Schrödinger equation is whether there is neat linkage between Schrödinger equation and classical wave dynamics. In other words, whether there is coherent picture to describe electron from these different approaches: quantum wave dynamics and classical electrodynamics. There are opponents of this 'reconciliation' program, of course, who emphasize that these two camps are so different like the sky from the earth. Some proponents of quantum mechanics could go so far by noting that quantum mechanics defy any attempt to describe physical meaning in classical sense. Bohr himself summarized Copenhagen viewpoint as follows:[2]

> "In quantum mechanics we are not dealing with an arbitrary renunciation of a more detailed analysis of of atomic phenomena, but with recognition that such an analysis is in principle excluded."

Nonetheless, some proponents of 'realism' interpretation of Quantum Mechanics predict that there should be a complete 'realism' description of

[2] Goldstein, S., Quantum Theory without Observers – Part One, *Physics Today*, March 1998, p. 42-46

physical model of electron, where non-local hidden variables could be included [4][1a]. We consider that this question remains open for discussion, in particular in the context of plausible analogue between classical electrodynamics and non-local quantum interference effect, in particular via Aharonov effect [8]. Hofer has also argued in the same direction, noting that it is possible to find physical meaning of wavefunction in classical electrodynamics sense [8a].

Similar program has been proposed recently by 't Hooft using term '*Determinism beneath Quantum Mechanics*.' [8b]. His argument was based on dissipative system. Interestingly, Blasone *et al*. also argued that it is possible to derive quantum mechanical wave from classical dissipative system, via damped harmonic oscillators [8c].

In this section we will only discuss 't Hooft's deterministic argument [8b], which emphasize that there is neat connection between the quantum harmonic oscillator and the classical particle moving along a circle. When time is assumed to be discrete (with step τ), then the evolution operator can be written as [8b]:

$$U(\Delta t = \tau) = e^{-iH\tau} \tag{4.4}$$

After introducing a few assumptions, one gets conventional quantum harmonic oscillator [8b]:

$$[\hat{x}, \hat{p}] \to i: \qquad\qquad H \to \omega^2 \hat{x}^2 / 2 + \hat{p}^2 / 2 \tag{4.5}$$

Alternatively, it is possible to find a generalization of Schrödinger equation from Nottale's approach [103]. In order to do this, first we could rewrite Nottale's generalized Schrödinger equation via diffusion approach [103][107]:

$$\begin{aligned} &i2m\gamma\left[-\left(i\gamma + a(t)/2\right)\left(\partial\psi/\partial x\right)^2 \psi^{-2} + \partial \ln\psi/\partial t\right] \\ &+ i\gamma a(t).\left(\partial^2\psi/\partial x^2\right)/\psi = \Phi + a(x) \end{aligned} \tag{4.6}$$

where ψ , $a(x)$, Φ , γ each represents classical wave function, an arbitrary constant, scalar potential, and a constant, respectively. If the function f(t) is such that

$$a(t) = -i2\gamma , \quad \alpha(x) = 0 , \quad \gamma = \hbar / 2m \tag{4.7}$$

then one recovers the Schrödinger equation (10.1).

In other words, one could expect to find a neat link between Schrödinger equation and classical wave dynamics. Another way to put forth the idea is to preserve that 'particles' mean particles, regardless we use classical dynamics method or Schrödinger equation; this would lead us to introduce 'quantum potential' term [4b].

At this point, suffice it to say that perhaps one of the great questions in order to answer quantum paradoxes is to define the relationship between classical dynamics and quantum wave dynamics. We will return to this issue in Postscript chapter.

4.3 A new derivation of Schrödinger-type equation

In this section we will make an attempt to re-derive a Schrödinger-type equation, but with a new definition of total energy.

In this regard, it seems worthnoting here that it is more proper to use Noether's expression of total energy in lieu of standard derivation of Schrödinger's equation ($E = \vec{p}^2 / 2m$). According to Noether's theorem [111], the total energy of the system corresponding to the time translation invariance is given by:

$$E = mc^2 + (cw/2).\int_0^\infty \left(\gamma^2.4\pi r^2.dr\right) = k\mu c^2 \tag{4.8}$$

where k is *dimensionless* function. It could be shown, that for low-energy state the total energy could be far less than $E = mc^2$. In this regard, interestingly Bakhoum [79] has also argued in favor of using $E = mv^2$ for expression of total energy, which expression could be traced back to Leibniz. Therefore it seems possible to argue that expression $E = mv^2$ is more generalized than the standard expression of special relativity, in particular because the total energy now depends on actual velocity [111].

We start with Bakhoum's assertion $E = mv^2$, instead of more convenient form $E = mc_s^{\ 2}$. This notion would imply [79]:

$$H^2 = p^2.c^2 - m_o^{\ 2}.c^2.v^2 . \tag{4.9}$$

Therefore, for phonon speed in the limit p→0, we write [112]:

$$E(p) \equiv c_s.|p| . \tag{4.10}$$

In the first approximation, we could derive Klein-Gordon-type relativistic equation from equation (4.9), as follows. By introducing a new parameter:

$$\zeta = i(v/c) , \tag{4.11}$$

then we can rewrite equation (4.9) in the known procedure of Klein-Gordon equation:

$$E^2 = p^2.c^2 + \zeta^2 m_o^{\ 2}.c^4 , \tag{4.12}$$

where $E = mv^2$. [79] By using known substitution:

$$E = i\hbar.\partial / \partial t , \quad p = \hbar \nabla / i ,$$

(4.13) and dividing by $(\hbar c)^2$, we get Klein-Gordon-type relativistic equation:

$$- c^{-2}.\partial \Psi / \partial t + \nabla^2 \Psi = k_o^{'2}.\Psi , \tag{4.14}$$

where

$$k_o^{'} = \zeta m_o c / \hbar . \tag{4.15}$$

One could derive Dirac-type equation using similar method, but we leave this problem as an exercise to the reader. Nonetheless, the use of new parameter (4.11) seems to be indirect solution, albeit it simplifies the solution, because here we can use the same solution from Klein-Gordon equation.

Alternatively, one could derive a new quantum relativistic equation, by noting that expression of total energy $E = mv^2$ is already relativistic equation. We will derive here an alternative approach using Ulrych's [7] method to get relativistic wave function from this expression of total energy [111].

$$E = mv^2 = p.v \tag{4.16}$$

Taking square of this expression, we get [111a]:

$$E^2 = p^2.v^2 \tag{4.17}$$

or

$$p^2 = E^2 / v^2 \tag{4.18}$$

Now we use Ulrych's substitution [7]:

$$\left[(P - qA)_\mu (\overline{P} - qA)^\mu \right] = p^2 , \tag{4.19}$$

and introducing standard substitution in Quantum Mechanics (4.13), one gets:

$$\left[(P - qA)_\mu (\overline{P} - qA)^\mu \right] \Psi = v^{-2}.(i\hbar.\partial / \partial t)^2 \Psi , \tag{4.20}$$

or

$$\left[(- i\hbar \nabla_\mu - qA_\mu)(- i\hbar \nabla^\mu - qA^\mu) - (i\hbar / v.\partial / \partial t)^2 \right] \Psi = 0 . \tag{4.21}$$

This equation is comparable to Schrödinger equation for a charged particle interacting with an external electromagnetic field [69]:

$$\left[(- i\hbar \nabla_\mu - qA_\mu)(- i\hbar \nabla^\mu - qA^\mu) \Psi \right] = \left[- i2m.\partial / \partial t + 2mU(x) \right] \Psi . \tag{4.22}$$

In other words, we could re-derive Schrödinger-type equation for a charged particle from Ulrych's approach [7].

> Goso said, "When you meet a Man of the Way on the road, greet him not with words, nor with silence. Tell me, how will you greet him?"
> --A Zen teaching

5 Solution to Schrödinger's Cat paradox

5.1 Standard interpretation

It is known that Quantum Mechanics could be regarded more as a 'mathematical theory' rather than a physical theory [62, p. 2]. It is wave mechanics allowing a corpuscular duality. Already here one could find problematic difficulties: i.e. while the quantity of wavefunction itself could be computed, the physical meaning of wavefunction itself remains '*indefinable*' [62]. Furthermore, this notion of wavefunction corresponds to another fundamental indefinable in euclidean geometry: the point [62, p.2]. It is always a baffling question for decades, whether the electron could be regarded as wave, a point, or we should introduce a non-zero finite entity [8a]. Attempts have been made to describe wave equation in such non-zero entity but the question of the physical meaning of wavefunction itself remains mystery.

The standard Copenhagen interpretation advertised by Bohr and colleagues (save DeBroglie, Einstein, Schrödinger who advocated 'realistic' interpretation) asserts that it is practically *impossible* to know what really happens in quantum scale. The quantum measurement itself only represents reading in measurement apparatus, and therefore it is difficult to separate the object to be measured and the measurement apparatus itself. Bohr's phenomenological viewpoint perhaps could be regarded as pragmatic approach, starting with the request not to attribute a deep meaning to the wave function but immediately go over to statistical likelihood [67]. Consequently, how the process of 'wave collapse' could happen remains mystery. In the end, one could say that Copenhagen interpretation rejects the notion of objective reality.

Heisenberg himself once emphasized this viewpoint when asked directly the question: '*Is there a fundamental level of reality?"* He replied as follows:

> "This is just the point: I do not know what the words fundamental reality mean. They are taken from our daily life situation where they have a good meaning, but when we use such terms we are usually extrapolating from our daily lives into an area very remote from it, where we cannot expect the words to have a meaning. This is perhaps one of the fundamental difficulties of philosophy: that our thinking hangs in the language. Anyway, we are

> forced to use the words so far as we can; we try to extend their use to the utmost, and then we get into situations in which they have no meaning."[3]

A modern version of this interpretation suggests that at the time of measurement, the wave collapses instantaneously into certain localized object corresponding to the action of measurement. In other words, the measurement processes define how the wave should define itself. At this point, the wave ceases to become coherent, and the process is known as 'decoherence.' Decoherence may be thought of as a way of making real for an observer in the large scale world only one possible history of the universe which has a likelihood that it will occur. Each possible history must in addition obey the laws of logic of this large-scale world. The existence of the phenomenon of decoherence is now supported by laboratory experiments. [63] It is worth-noting here, that there are also other versions of decoherence hypothesis, for instance by Tegmark [65] and Vitiello [64].

In the meantime, the 'standard' Copenhagen interpretation emphasizes the role of observer where the 'decoherence viewpoint' may not. In fact, one could say that Copenhagen's viewpoint could be interpreted as a variation of Raven's paradox: "*Observing a red apple increases the likelihood of all ravens being black.*" See Appendix 3 for more complete list of known paradoxes.

The problem becomes more adverse because the axioms of standard statistical theory themselves are not fixed forever [62][124, chapter 17]. And here is perhaps the source of numerous debates concerning the interpretation and philosophical questions implied by Quantum Mechanics. From this viewpoint, Neutrosophic Logic offers a new viewpoint to problems where indeterminacy presents. We will discuss this subsequently.

5.2 Schrödinger's cat paradox

To make the viewpoint on this paradox a bit clearer, let us reformulate the paradox in its original form. According to Uncertainty Principle, any measurement of a system must disturb the system under investigation, with a resulting lack of precision in the measurement. Soon after reading Einstein-Podolsky-Rosen's paper discussing incompleteness of Quantum Mechanics, Schrödinger in 1935 came up with a series of papers in which he used 'the

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[3] Buckley, P., & F.D. Peat, *A question of Physics: Conversations in Physics and Biology*, Routledge and Kegan Paul, London and Henley (1979) 9.

Cat paradox' to give an illustration of the problem of viewing these particles in a "thought experiment" [113]:

> "One can even set up quite ridiculous cases. A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter there is a bit of radioactive substance, *so* small, that *perhaps* in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives *if* meanwhile no atom has decayed. The first atomic decay would have poisoned it. The wave-function of the entire system would express this by having in it the living and the dead cat (pardon the expression) mixed or smeared into equal parts."

In modern version, we can re-phrase this problem as follows:

> "Let suppose we put a cat in a cage with a radioactive atom, a Geiger counter, a hammer, and a poison bottle; further suppose that the atom in the cage has a half-life of one hour, a fifty-fifty chance of decaying within an hour. If the atom decays, the Geiger counter will tick; the triggering of the counter will activate the hammer, which will break the poison bottle, which will kill the cat. If the atom doesn't decay, none of the above things happen, and the cat will be alive. Now the question: What is the state of the cat after an hour?"

In principle, Schrödinger's thought experiment asks whether the cat is dead or alive after an hour. The most logical solution would be to wait an hour, open the box, and see if the cat is still alive. However once you open the box to determine the state of the cat you have viewed and hence disturbed the system and introduced a level of uncertainty into the results. The answer, in quantum mechanical terms, is that before you open the box the cat is in a state of being half-dead and half-alive.[67]

Of course, at this point one could ask whether it is possible to find out the state of the cat without having to disturb its wavefunction via action of 'observation'. If the meaning of word 'observation' here is defined by '*to open the box and see the cat*', and then it seems that we could argue whether it is possible to propose another equally possible experiment where we introduce a pair of twin cats, instead of only one. A cat is put in the box while another cat is located in a separate distance, let say 1 meter from the box. If the state of the cat inside the box altered because of poison reaction, it is likely that we could also observe its effect to its twin, perhaps something like 'sixth sense' test (perhaps via monitoring frequency of the twin cat's brain). This plausible experiment could be viewed as a 'thought experiment' an alternative of Bell-Aspect-type experiment.

5.3 Hidden-variable hypothesis

It would be incomplete to discuss quantum paradoxes, in particular Schrödinger's cat paradox, without mentioning hidden-variable hypothesis. There are various versions of this argument, but it could be summarised as an assertion that there is 'something else' which should be included in the Quantum Mechanical equations in order to explain thoroughly all qauntum phenomena. Sometimes this assertion can be formulated in question form: "*Can quantum mechanics be considered complete*?"[17f] Interestingly, however, the meaning of 'complete' itself remains quite abstract (fuzzy).

An interpretation of this cat paradox suggests that the problem arises because we mix up the macroscopic systems (observer's wavefunction and apparatus' wavefunction) from microscopic system to be observed. In order to clarify this, it is proposed that "*the measurement apparatus should be described by a classical model in our approach, and the physical system eventually by a quantum model*." [114]

5.4 Hydrodynamic viewpoint and diffusion interpretation

In attempt to clarify the meaning of wave collapse and decoherence phenomenon, one could consider the process from (dissipative) hydrodynamic viewpoint [66].

Historically, the hydrodynamic/diffusion viewpoint of Quantum Mechanics has been considered by some physicists since the early years of wave mechanics. Already in 1933, Fuerth showed that Schrödinger equation could be written as a diffusion equation with an imaginary diffusion coefficient [62]:

$$D_{qm} = i\hbar / 2m \tag{5.1}$$

But the notion of imaginary diffusion is quite difficult to comprehend. Alternatively, one could consider a classical Markov process of diffusion type to consider wave mechanics equation. Consider a continuity equation:

$$\partial\rho / \partial t = -\nabla.(\rho v)\,, \tag{5.2}$$

where [62]:

$$v = v_0 - D\nabla.\ln\rho\,, \tag{5.3}$$

which is a Fokker-Planck equation. Then the expectation value for the energy of particle can be written as [62]:

$$< E >= \int \left[mv^2/2 + D^2 m/2 (D \ln \rho)^2 + eV \right] \rho . d^3 x \qquad (5.4)$$

Alternatively, it could be shown that there is exact mapping between Schrödinger equation and viscous dissipative Navier-Stokes equations [69], where the square of the wavefunction of Schrödinger equation as the vorticity distribution (including topological vorticity defects) in the fluid [69]. This Navier-Stokes interpretation differs appreciably from more standard Euler-Madelung fluid interpretation of Schrödinger equation [62], because in Euler method the fluid is described only in its inviscid limit.

5.5 Other less known interpretations

A slightly different interpretation could be described as follows. One could say that Copenhagen interpretation asserts that what does exist is only wave function, which then collapses because of an action of observer (to observe). Therefore it is equally possible to say that once the box is closed, the cat disappears and becomes wave function again until an observer opens the box to see the cat's state. In this way the cat's wave function ceases to be coherent, decoherence process happens and the cat pops out to our 'window of observation.' To extrapolate this question to our daily observation, there are academic discussions suggesting that ontological implications of this Copenhagen interpretation would mean that 'the moon is not there when nobody looks at it.' [17e] Now, one could imagine an hour, let suppose for an hour United Nations declare a 'Moon disappearance' hour, when no body on Earth should look at the Moon. The question is: "*Will the Moon remain there or not?*"

One could also mention here an alternative interpretation called 'multiverses hypothesis,' which says (more or less) that all the different possibilities happen, each in a separate "parallel" universe, i.e. there are at least 2 parallel universes, one with the particle emitted and detected and one were it has not. The multiverse hypothesis could be viewed as an attempt to generalize Feynman's path integral method to large-scale implications. In other words, it implies that the entire Universe splits into multiple copies of itself when the entangled particles are emitted. *The moment the first measurement is made, a particular universe is selected*. Now we can ask another question: Let suppose an observer opens the box twice, does he observe the same cat, or he observes different cats from different part of multiverses?

While it's too early to conclude that Multiverse hypothesis does not work, it seems that there are ontological questions awaiting to be clarified, if one demands this hypothesis should be considered as a serious alternative. In the mean time, there is a somewhat better alternative of 'multiverse viewpoint', i.e. to use trivalent logic similar to Lukasiewicz's hypothesis, but this time the trivalent logic is used to describe the 'potentia' of wave-function. In other words, instead of describing 'perhaps false and perhaps true', we suppose there is a state of '*half real and half not-real'*. Apparently, Heisenberg proposed this hypothesis for the first time, i.e. "he took quantum theory's vibratory possibilities literally: the attributes of unobserved objects exist, according to Heisenberg, exactly as represented in the theory--as possibilities, not actualities. The unobserved atom does not really have a definite position, for instance, but only a tendency, an inclination, to be in several possible positions all at the same time. In Heisenberg's view an atom is certainly real, but its attributes dwell in an existential limbo '*halfway between an idea and a fact*', a quivering state of attenuated existence that Heisenberg called "potentia", a world devoid of single-valued actuality but teeming with unrealized possibilities." [135]. It shall be noted here that throughout the book, we discuss multi-valuedness to describe likelihood without the use of the Principle of Excluded Middle. But whether this state of '*halfway between an idea and a fact*' could be ascribed physical meaning, remains another philosophical question.

5.6 How Neutrosophy could offer solution to Schrödinger's paradox

Neutrosophic Logic finds an interesting application in the context of Schrödinger's cat paradox. It could explain how the 'mixed' state could be.

For example the Schrödinger's Cat Theory says that the quantum state of a photon can basically be in more than one place in the same time which, translated to the neutrosophic set, means that an element (quantum state) belongs and does not belong to a set (a place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a question of "alternative worlds" theory very well represented by the neutrosophic set theory.

In Schrödinger's equation on the behavior of electromagnetic waves and "matter waves" in quantum theory, the wave function Ψ, which describes the superposition of possible states, may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).

Interpretation in our daily experience could be somewhat more practical. Here are a few examples:

1. The likelihood that tomorrow it will rain is say 50-54% true according to meteorologists who have investigated the past years' weather, 30 or 34-35% false according to today's very sunny and droughty summer, and 10 or 20% undecided (indeterminate).
2. The likelihood that Yankees will win tomorrow versus Cowboys is 60% true (according to their confrontation's history giving Yankees' satisfaction), 30-32% false (supposing Cowboys are actually up to the mark, while Yankees are declining), and 10 or 11 or 12% indeterminate (because of the hazard: sickness of players, referee's mistakes, atmospheric conditions during the game). These parameters act on players' psychology.
3. The likelihood that candidate C will win an election is say 25-30% true (percent of people voting for him), 35% false (percent of people voting against him), and 40% or 41% indeterminate (percent of people not coming to the ballot box, or giving a blank vote – not selecting anyone, or giving a negative vote).
4. From a pool of refugees, waiting in a political refugee camp in Turkey to get the American visa, a% have the chance to be accepted - where a varies in the set A, r% to be rejected – where r varies in the set R, and p% to be in pending (not yet decided) - where p varies in P. Say, for example, that the chance of someone Michael in the pool to emigrate to USA is (between) 40-60% (considering different criteria of emigration one gets different percentages, we have to take care of all of them), the chance of being rejected is 20-25% or 30-35%, and the chance of being in pending is 10% or 20% or 30%. Then the neutrosophic probability that Michael emigrates to the Unites States is NP(Michael) = ((40-60), (20-25)U(30-35), {10,20,30}), closer to the life's thinking. This is a better approach than the classical probability, where 40 [P(Michael) [60, because from the pending chance - which will be converted to acceptance or rejection - Michael might get extra percentage in his will to emigration, and also the superior limit of the subsets sum 60+35+30 > 100 and in other cases one may have the inferior sum < 0, while in the classical fuzzy set theory the superior sum should be 100 and the inferior sum m 0. In a similar way, we could say about the element Michael that Michael((40-60), (20-25)U(30-35), {10,20,30}) belongs to the set of accepted refugees.

As we have shown, Neutrosophic probability is useful to those events, which involve some degree of indeterminacy (unknown) and more criteria of evaluation - as quantum physics. This kind of probability is necessary because it provides a better representation than classical probability to uncertain events.

Now let's return to our cat paradox. Let's consider a neutrosophic set of a collection of possible locations (positions) of particle x. And let A and B be two neutrosophic sets. One can say, by language abuse, that any particle x neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between - 0 and 1 + . For example: x(0.5, 0.2, 0.3) belongs to A (which means, with a probability of 50% particle x is in a position of A, with a probability of 30% x is not in A, and the rest is undecidable); or y(0, 0, 1) belongs to A (which normally means y is not for sure in A); or z(0, 1, 0) belongs to A (which means one does know absolutely nothing about z's affiliation with A). More general, x((0.2-0.3), (0.40-0.45)4[0.50-0.51], {0.2, 0.24, 0.28}) belongs to the set A, which mean:

- with a likelihood in between 20-30% particle x is in a position of A (one cannot find an exact approximate because of various sources used);
- with a probability of 20% or 24% or 28% x is not in A;
- the indeterminacy related to the appurtenance of x to A is in between 40-45% or between 50-51% (limits included).
- The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and n_sup = 30%+51%+28% > 100% in this case.

To summarize our proposition, given the Schrödinger's cat paradox is defined as a state where the cat can be dead, or can be alive, or it is undecided (i.e. we don't know if she is dead or alive), then herein the Neutrosophic Logic, based on three components, truth component, falsehood component, indeterminacy component (T, I, F), works very well. In Schrödinger's cat problem the Neutrosophic Logic offers the possibility of considering the cat neither dead nor alive, but undecided, while the fuzzy logic does not do this. Normally indeterminacy (I) is split into uncertainty (U) and paradox (conflicting) (P).

We could expect that someday this proposition based on Neusotrophic Logic could be transformed into a useful guide for experimental verification of quantum paradox [67][68].

The french scientist Ampere was on his way to an important meeting at the Paris Academy. In the carriage he got a brilliant idea which he immediately wrote down ... on the wand of the carriage: dH=ipdl/r^2. As he arrived he payed the driver and ran into the building to tell everyone. Then he found out his notes were on the carriage and he had to hunt through the streets of Paris to find his notes on wheels.

6 Sorites Quantum paradox and Quantum Quasi-paradox

There can be generated many paradoxes or quasi-paradoxes that may occur from the combination of quantum and non-quantum worlds in physics. Even the passage from the micro-cosmos to the macro-cosmos, and reciprocally, can generate unsolved questions or counter-intuitive ideas. We define a quasi-paradox as a statement which has a *prima facie* self-contradictory support or an explicit contradiction, but which is not completely proven as a paradox.

We present herein four elementary quantum quasi-paradoxes and their corresponding quantum Sorites paradoxes, which form a class of quantum quasi-paradoxes.

6.1 Introduction

Dictionary of Mathematics (Borowski & Borwein, 1991), the **paradox** is "an apparently absurd or self-contradictory statement for which there is *prima facie* support, or an explicit contradiction derived from apparently unexceptionable premises". Some paradoxes require the revision of their intuitive conception (Russell's paradox, Cantor's paradox), others depend on the inadmissibility of their description (Grelling's paradox), others show counter-intuitive features of formal theories (Material implication paradox, Skolem Paradox), others are self-contradictory [Smarandache Paradox: "All is <A> the <Non-A> too!", where <A> is an attribute and <Non-A> its opposite; for example "All is possible the impossible too!" [36]. Paradoxes are normally true and false in the same time.

The **Sorites paradoxes** are associated with Eubulides of Miletus (fourth century B.C.) and they say that there is not a clear frontier between visible and invisible matter, determinist and indeterminist principle, stable and unstable matter, long time living and short time living matter. Generally, between <A> and <Non-A> there is no clear distinction, no exact frontier.

Where does <A> really end and <Non-A> begin? One extends Zadeh's "fuzzy set" concept to the "neutrosophic set" concept.

Let's now introduce the notion of quasi-paradox: A **quasi-paradox** is a statement which has a *prima facia* self-contradictory support or an explicit contradiction, but which is not completely proven as a paradox. A quasi-paradox is an *informal* contradictory statement, while a paradox is a *formal* contradictory statement.

Some of the below quantum quasi-paradoxes can later be proven as real quantum paradoxes.

6.2 Quantum Paradox and Quantum Sorites Paradox

It is the interaction of the quantum world with this "environment", associated with the large-scale world, which is thought to cause wave function collapse. For this reason we do not perceive the quantum behavior of every particle inside Schrödinger's cat; the presence of such an "environment" (the body of the cat) is thought to cause the cat to be seen to be either dead or alive, even though it may be poisoned as a result of a quantum phenomenon.

The below quasi-paradoxes and Sorites paradoxes are based on the antinomies: visible/invisible, determinist/indeterminist, stable/unstable, long time living/short time living, as well as on the fact that there is not a clear separation between these pairs of antinomies.

6.2.1.1. **Invisible Quasi-Paradox:** Our visible world is composed of a totality of invisible particles

6.2.1.2. **Invisible Sorites Paradox:** There is not a clear frontier between visible matter and invisible matter.

a) An invisible particle does not form a visible object, nor do two invisible particles, three invisible particles, etc. However, at some point, the collection of invisible particles becomes large enough to form a visible object, but there is apparently no definite point where this occurs.

b) A similar paradox is developed in an opposite direction. It is always possible to remove a particle from an object in such a way that what is left is still a visible object. However, repeating and repeating this process, at some point, the visible object is decomposed so that the left part becomes invisible, but there is no definite point where this occurs.

6.2.2.1. This **Uncertainty Quasi-Paradox**: Large matter, which is at some degree under the 'determinist principle', is formed by a totality of elementary particles, which are under Heisenberg's 'indeterminacy principle'.

6.2.2.2. **Uncertainty Sorites Paradox:** Similarly, there is not a clear frontier between the matter under the 'determinist principle' and the matter under 'indeterminist principle'.

6.2.3.1. **Unstable Quasi-Paradox**: 'Stable' matter is formed by 'unstable' elementary particles (elementary particles decay when free).

6.2.3.2. **Unstable Sorites Paradox:** Similarly, there is not a clear frontier between the 'stable matter' and the 'unstable matter'.

6.2.4.1. **Short-Time-Living Quasi-Paradox**: 'Long-time-living' matter is formed by very 'short-time-living' elementary particles.

6.2.4.2. **Short-Time-Living Sorites Paradox:** Similarly, there is not a clear frontier between the 'long-time-living' matter and the 'short-time-living' matter.

More such quantum quasi-paradoxes and paradoxes can be designed, all of them forming a class of Smarandache quantum quasi-paradoxes." (Dr. M. Khoshnevisan, Griffith University).

The truth, as always, will be far stranger
Arthur C. Clarke. 2001 - A Space Odyssey.

7 Epistemological Aspects of Multi-Valued Logic

It seems worth to consider here an introduction to epistemelogical aspects of Multi-Valued logic and Neutrosophic Logic, albeit for more precise discussions the readers are recommended to find literature on Logic.

As Gell-Mann often remarked, being physicists is more like a sailor who has to sail between Scylla and Charybdis.[4] In other words it always requires a great effort to find a balanced comprehension between *abstraction* (pure thought, logical formalism) and *observation* (experimental data).

Similarly, throughout history mankind has attempted to find an absolute and objective truth, which does not suffer from either relative views or subjective biases. In the meantime, the experimental data often displays something different than what he expects via pure comprehension; despite of course Nature should be in essence the same truth. In other words, if deduction belongs to set of Theories (T), and data belongs to set of Observation (O), and then ideally one expects that someday they would be the same:

$a = b$, where $a \in T$ and $b \in O$

The problem begins when one recognizes that our theories are always bounded by some categorization, like mathematical truth, religious truth, political truth, therefore it becomes much more difficult to find out whether a=b for all subjects of interests. It is more likely that we could define a=b for a limited scope of interests. But more problems arise from the fact that most of these categorizations of truth are also bounded by confusion [119]. This is why we should say that for most cases we could only deal with a 'partial truth'.

Another problem may arise, for instance if a group of smart but 'mad' scientists attempt to create a new purely-idealistic kind of reality, and make

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[4] G. Johnston, *Strange Beauty*, (1999). In Greek Myth: Odysseus spent nine years returning home. Along his voyage by sea, he came upon Scylla and Charybdis. Scylla was an enormous sea monster with numerous hands and six dog heads sprouting from her body; she ate men alive. Charybdis was a tremendous whirlpool that digested ships whole. Since the only way to get home was to choose either route, Odysseus had to decide on one horror or the other. He chose Scylla, losing six crewmen to Scylla's hunger. (Source: http://www.areopagus.net/grkterms.htm)

specific observations which are designed such that they will support their idea. Such a typically Berkeleian- theory while sounds strange could happen (or as Sherlock Holmes would say: "It is a capital mistake to theorise before one has data. Insensibly one begins to twist facts to suit theories instead of theories to suit facts."). This possibility is also discussed in Borges' story of a new planet which cannot be observed, and the only remnant could be found is an encyclopaedia [123]. Therefore it is required that our scientific discourse is intended to describe the phenomena corresponding to the Universe of discourse (U), i.e. a=b=c, where c belongs to the set U.

Furthermore, a part of categorization process is how to make distinction. By distinction here we mean "*Distinction is a process that separates a unity into 'something that is' and 'something that isn't the something that is*." [119]. Therefore a fundamental principle of logic is to make distinction: A 'is', then not-A is not A. Not-A is usually denoted as ~A.

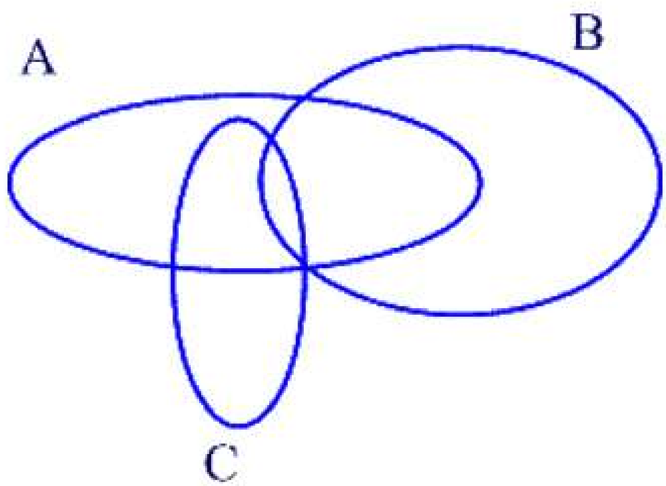
Now we come to the notion of The Principle of Contradiction, which states that a proposition is necessarily true if its negation entails a contradiction. The negation of A != ~A is to say that A = ~A but A is not ~A, hence we have a contradiction. In other words, this principle excludes the possibility of blurred area where ~A merges with A. This basic assertion of this exclusion is known as principle of *Excluded Middle*.

Because in our daily life, normally we can only find incomplete information, therefore our theories could have some level of indeterminacy. (This is not the same with uncertainty, if uncertainty is defined as random part of the experimental data). In Neutrosophic Logic this indeterminacy becomes part of truth-value. Neutrosophic Logic also offers significant advantage over the previous 'classical' notion set, by allowing breaking of principle of *Excluded Middle*.

A criticism could arise at this point: Does the broken Excluded Middle really happen? Or is it merely the result of our own perception?

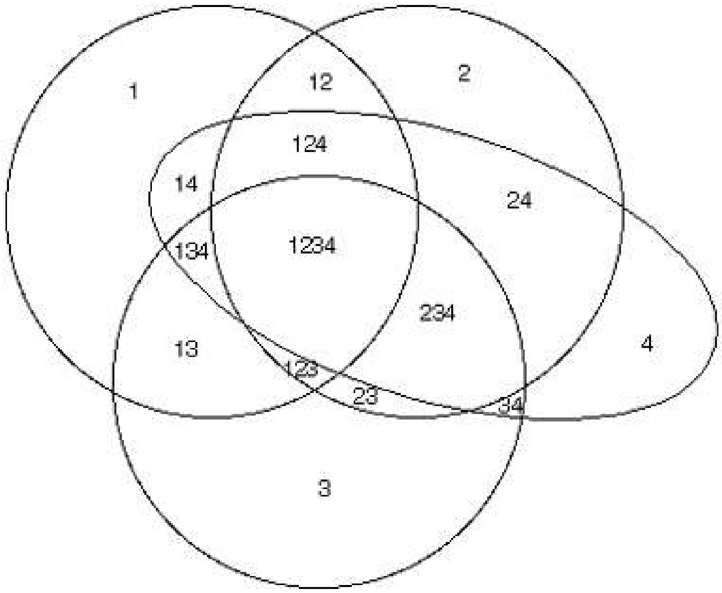
While at first glance it is not so easy to find examples of daily condition where the Excluded Middle is broken, you don't have to be a Tibetan monk to understand that sometimes the identity of some members of a particular set is not too strictly defined. Sometimes the problem comes from the use of language to define the set itself. For example, have you heard someone says something like this phrase: "*You have your mother's eyes, your father's hair, and your grandfather's brain."* Doesn't it indicate that our identity is also 'defined' by other's identity (set of family members)? Similarly we can say that the phrase: "*The set of Indian people*," is quite blurred, because one could always argue whether someone who was born in New Delhi but then moved to Europe could be counted as 'Indian people'. Therefore, to reduce indeterminacy level, it is required to define the set more specifically, for example: ‘The set of Indian people who were born in India, and now live in India.”

Similarly the set of “*all young people who often wear blue T-shirt*,” has a large degree of indeterminacy. First, the notion ‘young’ is not clearly defined, and also we could always argue ‘how often’ he/she wears T-shirt (once in a week, or each day), and also ‘how blue’ is the T-shirt. Should it be light-blue, dark-blue etc. To speak more precise, in standard ‘set’ theory, the set of A (the set of young people), B (the set of people wear T-Shirt), C (the set of people wear blue clothes) are defined as follows:



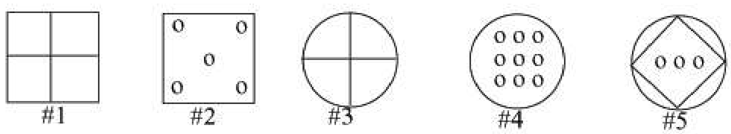
Picture 7.1. Standard sets.

But in daily life most real sets are not clearly defined. Let suppose we define the set of “*all young Indian people who wear blue T-shirt often*,” which has four sets, and then the set diagram should be as follows (see Picture 7.2).



Picture 7.2. Real sets diagram.

Similarly, a common question in daily life is to make association between obscure groupings of data, such as: “Which one of these pictures does not belong to the others?”



Picture 7.3. A question to make association

For other example to comparison in genome data, see [132c].

In the following sections, we consider some basic assertions of real sets in Neutrosophic Logic. This analysis belongs to non-standard analysis. For other possible non-standard analysis, see Appendix 2.

7.1 Non-standard real numbers and real sets [121]

Let T, I, F be standard or non-standard real subsets $]^-0, 1^+[$,

with sup T = t_sup, inf T = t_inf,
sup I = i_sup, inf I = i_inf,
sup F = f_sup, inf F = f_inf,
and n_sup = t_sup + i_sup + f_sup,
n_inf = t_inf + i_inf + f_inf.
Obviously: t_sup, i_sup, f_sup $\leq 1^+$, and t_inf, i_inf, f_inf $\geq {}^-0$,
whereas n_sup $\leq 3^+$ and n_inf $\geq {}^-0$.

The subsets T, I, F are not necessarily intervals, but may be any real subsets: discrete or continuous; single-element, finite, or (either countably or uncountably) infinite; union or intersection of various subsets; etc. They may also overlap. These real subsets could represent the relative errors in determining t, i, f (in the case when the subsets T, I, F are reduced to points).

This representation is closer to the human mind reasoning. It characterizes/catches the *imprecision* of knowledge or linguistic inexactitude received by various observers (that's why T, I, F are subsets - not necessarily single-elements), *uncertainty* due to incomplete knowledge or acquisition errors or stochasticity (that's why the subset I exists), and *vagueness* due to lack of clear contours or limits (that's why T, I, F are subsets and I exists; in particular for the appurtenance to the neutrosophic sets).

One has to specify the superior (x_sup) and inferior (x_inf) limits of the subsets because in many problems arises the necessity to compute them.

It is also possible to describe how to make addition of non-standard finite numbers.

Addition of non-standard finite numbers with themselves or with real numbers:

${}^-a + b = {}^-(a + b)$

$a + b^+ = (a + b)^+$

${}^-a + b^+ = {}^-(a + b)^+$

${}^-a + {}^-b = {}^-(a + b)$ (the left monads absorb themselves)

$a^+ + b^+ = (a + b)^+$ (analogously, the right monads absorb themselves)

Similarly for subtraction, multiplication, division, roots, and powers of non-standard finite numbers with themselves or with real numbers.

7.2 Epimenidean Paradox and Grelling Paradox [121]

Lukasiewicz, together with Kotarbinski and Leniewski from the Warsaw Polish Logic group (1919-1939), questioned the status of truth: eternal, sempiternal (everlasting, perpetual), or both?

Let's use the notion of "world" from the modal logic, which is a semantic device of what the world might have been like. Then, one says that the neutrosophic truth-value of a statement A, $NL_t(A) = 1^+$ if A is 'true in all possible worlds' (syntagme first used by Leibniz) and all conjunctures, that one may call "absolute truth" (in the modal logic it was named *necessary truth,* Dinulescu-Campina [128] names it 'intangible absolute truth'), whereas $NL_t(A) = 1$ if A is true in at least one world at some conjuncture, we call this "relative truth" because it is related to a 'specific' world and a specific conjuncture (in the modal logic it was named *possible truth*). Because each 'world' is dynamic, depending on an ensemble of parameters, we introduce the sub-category 'conjuncture' within it to reflect a particular state of the world.

How can we differentiate <the truth behind the truth>? What about the <metaphoric truth>, which frequently occurs in the humanistic field?

One attempts to formalize. For n µ1 one defines the "n-level relative truth" of the statement A if the statement is true in at least n distinct worlds, and similarly "countably-" or "uncountably-level relative truth" as gradual degrees between "first-level relative truth" (1) and "absolute truth" (1^+) in the monad $\mu(1^+)$. Analogue definitions one gets by substituting "truth" with "falsehood" or "indeterminacy" in the above.

In *largo sensu* the notion "world" depends on parameters, such as: space, time, continuity, movement, modality, (meta-)language levels, interpretation, abstraction, (higher-order) quantification, predication, complement constructions, subjectivity, context, circumstances, etc. Pierre d'Ailly upholds that the truth-value of a proposition depends on the sense, on the metaphysical level, on the language and meta-language; the auto-reflexive propositions (with reflection on themselves) depend on the mode of representation (objective/subjective, formal/informal, real/mental).

In a formal way, let's consider the world W as being generated by the formal system FS. One says that statement A belongs to the world W if A is a well-formed formula (*wff*) in W, i.e. a string of symbols from the alphabet of W that conforms to the grammar of the formal language endowing W. The grammar is conceived as a set of functions (formation rules) whose inputs are symbols strings and outputs "yes" or "no". A formal system comprises a formal language (alphabet and grammar) and a deductive apparatus (axioms and/or rules of inference). In a formal system the rules of inference are syntactically and typographically formal in nature, without reference to the meaning of the strings they manipulate.

Similarly for the neutrosophic falsehood-value, $NL_f(A) = 1^+$ if the statement A is false in all possible worlds, we call it "absolute falsehood", whereas $NL_f(A) = 1$ if the statement A is false in at least one world, we call it "relative falsehood". Also, the neutrosophic indeterminacy-value $NL_i(A) = 1^+$ if the statement A is indeterminate in all possible worlds, we call it "absolute indeterminacy", whereas $NL_i(A) = 1$ if the statement A is indeterminate in at least one world, we call it "relative indeterminacy".

On the other hand, $NL_t(A) = {}^-0$ if A is false in all possible world, whereas $NL_t(A) = 0$ if A is false in at least one world; $NL_f(A) = {}^-0$ if A is true in all possible world, whereas $NL_f(A) = 0$ if A is true in at least one world; and $NL_i(A) = {}^-0$ if A is indeterminate in no possible world, whereas $NL_i(A) = 0$ if A is not indeterminate in at least one world.

The ${}^-0$ and 1^+ monads leave room for degrees of super-truth (truth whose values are greater than 1), super-falsehood, and super-indeterminacy.

Here there are some corner cases:

There are tautologies, some of the form "B is B", for which $NL(B) = (1^+, {}^-0, {}^-0)$, and contradictions, some of the form "C is not C", for which $NL(B) = ({}^-0, {}^-0, 1^+)$.

While for a paradox, P, $NL(P) = (1,1,1)$. Let's take the Epimenides Paradox, also called the Liar Paradox, "This very statement is not true". If it is true then it is false, and if it is false then it is true. But the previous reasoning, due to the contradictory results, indicates a high indeterminacy too. The paradox is the only proposition true and false in the same time in the same world, and indeterminate as well!

Let's take the Grelling's Paradox, also called the heterological paradox [Suber, 1999], "If an adjective truly describes itself, call it 'autological', otherwise call it 'heterological'. Is 'heterological' heterological?" Similarly, if it is, then it is not; and if it is not, then it is.

For a not well-formed formula, nwff, i.e. a string of symbols which do not conform to the syntax of the given logic, NL(nwff) = n/a (undefined). A proposition which may not be considered a proposition was called by the logician Paulus Venetus *flatus voci*. NL(*flatus voci*) = n/a.

7.3 Neutrosophic Statistics and Neutrosophic Probability Space

Neutrosophic Probability is a generalization of the classical probability in which the chance that an event A occurs is t% true - where t varies in the subset T, i% indeterminate - where i varies in the subset I, and f% false - where f varies in the subset F.

One notes NP(A) = (T, I, F).

It is also a generalization of the imprecise probability, which is an interval-valued distribution function.

Neutrosophic Statistic is the analysis of the events described by the neutrosophic probability. This is also a generalization of the classical statistics and imprecise statistics.

Neutrosophic Probabilistic Space is the universal set, endowed with a neutrosophic probability defined for each of its subset, forms a neutrosophic probability space.

Let A and B be two neutrosophic events, and $NP(A) = (T_1, I_1, F_1)$, $NP(B) = (T_2, I_2, F_2)$ their neutrosophic probabilities. Then we define:

$NP(A\cap B) = NP(A) \odot NP(B)$.

$NP(\neg A) = \{1\} \ominus NP(A)$.

$NP(A\cup B) = NP(A) \oplus NP(B) \ominus NP(A) \odot NP(B)$.

1. NP(impossible event) = $(T_{imp}, I_{imp}, F_{imp})$, where $\sup T_{imp} \leq 0$, $\inf F_{imp} \geq 1$; no restriction on I_{imp}.

NP(sure event) = $(T_{sur}, I_{sur}, F_{sur})$,
where $\inf T_{sur} \geq 1$, $\sup F_{sur} \leq 0$; no restriction on I_{sur}.
NP(totally indeterminate event) = $(T_{ind}, I_{ind}, F_{ind})$;
where $\inf I_{ind} \geq 1$; no restrictions on T_{ind} or F_{ind}.

2. $NP(A) \in \{(T, I, F)$, where T, I, F are real subsets which may overlap$\}$.
3. $NP(A\cup B) = NP(A) \oplus NP(B) \ominus NP(A\cap B)$.
4. $NP(A) = \{1\} \ominus NP(\neg A)$.

Neutrosophic probability is useful to those events, which involve some degree of indeterminacy (unknown) and more criteria of evaluation - as quantum physics. This kind of probability is necessary because it provides a better representation than classical probability to uncertain events.

7.4 Generalization of other Probabilities

In the case when the truth- and falsity-components are complementary, i.e. no indeterminacy and their sum is 1, one falls to the classical probability. As, for example, tossing dice or coins, or drawing cards from a well-shuffled deck, or drawing balls from a turn.

An interesting particular case is for n=1, with $0\leq t,i,f\leq 1$, which is closer to the classical probability.

For n=1 and i=0, with $0\leq t,f\leq 1$, one obtains the classical probability.

From the intuitionistic logic, paraconsistent logic, dialetheism, faillibilism, paradoxism, pseudoparadoxism, and tautologism we transfer the "adjectives" to probabilities, i.e. we define the intuitionistic probability (when the probability space is incomplete), paraconsistent probability, faillbilist probability, dialetheist probability, paradoxist probability, pseudoparadoxist probability, and tautologic probability respectively.

Hence, the neutrosophic probability generalizes:

- the intuitionistic probability, which supports incomplete (not completely known/determined) probability spaces (for $0<n<1$ and $i=0$, $0[t,f[1$) or incomplete events whose probability we need to calculate;
- the classical probability (for $n=1$ and $i=0$, and $0 \leq t,f \leq 1$);
- the paraconsistent probability (for $n>1$ and $i=0$, with both $t,f<1$);
- the dialetheist probability, which says that intersection of some disjoint probability spaces is not empty (for $t=f=1$ and $i=0$; some paradoxist probabilities can be denoted this way);
- the faillibilist probability (for $i>0$);
- the pseudoparadoxism (for $n\_sup>1$ or $n\_inf<0$);
- the tautologism (for $t\_sup>1$).

Compared with all other types of classical probabilities, the neutrosophic probability introduces a percentage of "indeterminacy" - due to unexpected parameters hidden in some probability spaces, and let each component t, i, f be even boiling *over* 1 to 1^+ (overflooded) or freezing *under* 0 (underdried) to $^-0$.

For example: an element in some tautological probability space may have $t>1$, called "overprobable" (i.e. $t = 1^+$). Similarly, an element in some paradoxist probability space may be "overindeterminate" (for $i>1$), or "overunprobable" (for $f>1$, in some unconditionally false appurtenances); or "underprobable" (for $t<0$, i.e. $t = {}^-0$, in some unconditionally false appurtenances), "underindeterminate" (for $i<0$, in some unconditionally true or false appurtenances), "underunprobable" (for $f<0$, in some unconditionally true appurtenances).

This is because we should make a distinction between unconditionally true ($t>1$, and $f<0$ or $i<0$) and conditionally true appurtenances ($t \leq 1$, and $f \leq 1$ or $i \leq 1$).

Mathematics is inadequate to describe the universe, since mathematics is an abstraction from natural phenomena. Also, mathematics may predict things, which don't exist, or are impossible in nature.

-- Ludovico delle Colombe criticizing Galilei.

8 Postscript: Schrödinger equation, quantization of celestial systems

In the preceding chapters, we have found the neat linkage between interpretation of Schrödinger equation and Multi-Valued-logic of Lukasiewicz and Neutrosophy.

Now it will be shown that we can expect to use Schrödinger equation to describe quantization of celestial sytems. While this notion of macro-quantization is not widely accepted yet, as we will see the logarithmic nature of Schrödinger equation could be viewed as a support of its applicability to larger systems. As an alternative, we will also discuss an outline for how to derive Schrödinger equation from simplification of Ginzburg-Landau equation. It is known that Ginzburg-Landau equation exhibits fractal character, which implies that quantization could happen at *any scale*.

Before we start our discussion, we think it is compelled to include above a quote intended as a critics to Galilei's work. Somehow we also think that the method described herein has not been widely accepted yet, in particular because a part within our logic system thinks it would take too much deliberation in mathematical part to accept such notion of quantization of celestial systems. But as we have shown before using 't Hooft's argument, we could come to quantum-mechanical type description even from standard classical dynamical system. Therefore it seems that it should not impose too much baggage to accept the use of Schrödinger equation to describe also classical systems, including celestial quantization. After all, the use of Schrödinger equation has proved itself to help in finding new objects known as extrasolar planets [85][86]. And we could be sure that new extrasolar planets are to be found in the near future.

First, let us rewrite Schrödinger equation in its common form:

$$\left[i\partial/\partial t + \overline{\nabla}^2/2m - U(x)\right]\Psi = 0 \tag{8.1}$$

or

$$(i\partial/\partial t)\Psi = H.\Psi \tag{8.2}$$

Now, it is worthnoting here that Englman & Yahalom [4a] argues that this equation exhibits logarithmic character:

$$\ln\Psi(x,t) = \ln\left(\left|\Psi(x,t)\right|\right) + i.\arg(\Psi(x,t)) \tag{8.3}$$

Schrödinger already knew this expression in 1926, which then he used it to propose his equation called '*eigentliche Wellengleichung'* [4a]. Therefore equation (8.1) can be rewritten as follows:

$$2m(\partial \ln|\Psi|/\partial t) + 2\overline{\nabla}\ln|\Psi|.\overline{\nabla}\arg[\Psi] + \overline{\nabla}.\overline{\nabla}\arg[\Psi] = 0 \qquad (8.4)$$

Interestingly, Nottale's scale-relativistic method [85][86] was also based on generalization of Schrödinger equation to describe quantization of celestial systems. It is known that Nottale-Schumacher's method [87] could predict new exoplanets in good agreement with observed data. Nottale's scale-relativistic method is essentially based on the use of first-order scale-differentiation method defined as follows [85][86]:

$$\partial V / \partial(\ln \delta t) = \beta(V) = a + bV + \ldots \qquad (8.5)$$

Now it seems clear that the logarithmic derivation, which is essential in scale-relativity approach, also has been described properly in Schrödinger's original equation [4a]. In other word, its logarithmic form ensures applicability of Schrödinger equation to describe macroquantization of celestial systems.

In order to emphasize this assertion of the possibility to describe quantization of celestial systems, let us quote Fischer' description [71] of relativistic momentum from superfluid dynamics. Fischer [71] argues that the circulation is in the relativistic dense superfluid, defined as the integral of the momentum:

$$\gamma_s = \oint p_\mu dx^\mu = 2\pi.N_v\hbar\,, \qquad (8.6)$$

and is quantized into multiples of Planck's quantum of action. This equation is the covariant Bohr-Sommerfeld quantization of γ_s. And then Fischer [71] concludes that the Maxwell equations of ordinary electromagnetism can be cast into the form of conservation equations of relativistic perfect fluid hydrodynamics [71a]. Furthermore, the topological character of equation (8.6) corresponds to the notion of topological electronic liquid, where compressible electronic liquid represents superfluidity [83]. For the plausible linkage between superfluid dynamics and various cosmological phenomena, see [73]-[78].

It is worthnoting here, because here vortices are defined as elementary objects in the form of stable topological excitations [71], then equation (8.6) could be interpreted as signatures of Bohr-Sommerfeld quantization from topological quantized vortices. Fischer [71] also remarks that equation (8.6) is quite interesting for the study of superfluid rotation in the context of gravitation. Interestingly, application of Bohr-Sommerfeld quantization to celestial systems is known in literature [85][86], which here in the context of

Fischer's arguments it seems plausible to suggest that quantization of celestial systems actually corresponds to superfluid-quantized vortices at large-scale [83]. In our opinion, this result supports known experiments suggesting neat correspondence between condensed matter physics and various cosmology phenomena [73]-[76].

To make the conclusion that quantization of celestial systems actually corresponds to superfluid-quantized vortices at large-scale a bit conceivable, let us consider the problem of quantization of celestial orbits in solar system.

In order to obtain planetary orbit prediction from this hypothesis we could begin with the Bohr-Sommerfeld's conjecture of quantization of angular momentum. This conjecture may originate from the fact that according to BCS theory, superconductivity can exhibit macroquantum phenomena [73][101]. In principle, this hypothesis starts with observation that in quantum fluid systems like superfluidity, it is known that such vortexes are subject to quantization condition of integer multiples of 2π, or $\oint v_s.dl = 2\pi.n\hbar / m_4$. As we know, for the wavefunction to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition:

$$\oint_{\Gamma} p.dx = 2\pi.n\hbar \tag{8.6a}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is [90]:

$$\int_0^T v^2.d\tau = \omega^2.T = 2\pi.\omega \tag{8.7}$$

where T=2π/ω is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\omega = n\hbar$. Then we can write the force balance relation of Newton's equation of motion [90]:

$$GMm / r^2 = mv^2 / r \tag{8.8}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum, a new constant g was introduced:

$$mvr = ng / 2\pi \tag{8.9}$$

Just like in the elementary Bohr theory (before Schrödinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form [90]:

$$r = n^2.g^2 / (4\pi^2.GM.m^2) \tag{8.10}$$

which can be rewritten in the known form [85][86]:

$$r = n^2.GM / v_o^2 \tag{8.11}$$

where r, n, G, M, v_o represents orbit radii, quantum number (n=1,2,3,…), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In this equation (8.11), we denote:

$$v_o = (2\pi / g).GMm \tag{8.12}$$

The value of m is an adjustable parameter (similar to g). [85][86]

Using this equation (8.11), we could predict quantization of celestial orbits in the solar system, where for Jovian planets we use least-square method and use M in terms of reduced mass $\mu = (M_1 + M_2)/(M_1.M_2)$. From this viewpoint the result is shown in Table 1 below [90]:

Table 8.1. Comparison of prediction and observed orbit distance of planets in Solar system (in 0.1AU unit) [90]

Object	No.	Titius	Nottale	CSV	Observed	Δ (%)
	1		0.4	0.428		
	2		1.7	1.71		
Mercury	3	4	3.9	3.85	3.87	0.52
Venus	4	7	6.8	6.84	7.32	6.50
Earth	5	10	10.7	10.70	10.00	-6.95
Mars	6	16	15.4	15.4	15.24	-1.05
Hungarias	7		21.0	20.96	20.99	0.14
Asteroid	8		27.4	27.38	27.0	1.40
Camilla	9		34.7	34.6	31.5	-10.00
Object	No.	Titius	Nottale	CSV	Observed	Δ (%)
Jupiter	2	52		45.52	52.03	12.51
Saturn	3	100		102.4	95.39	-7.38
Uranus	4	196		182.1	191.9	5.11
Neptune	5			284.5	301	5.48
Pluto	6	388		409.7	395	-3.72
2003EL61	7			557.7	520	-7.24
Sedna	8	722		728.4	760	4.16
2003UB31	9			921.8	970	4.96
Unobserved	10			1138.1		
Unobserved	11			1377.1		

For comparison purpose, we also include some recent observation by M. Brown *et al*. from Caltech [91][92][93][94]. It is known that Brown *et al*. have reported not less than four new planetoids in the outer side of Pluto orbit, including 2003EL61 (at 52AU), 2005FY9 (at 52AU), 2003VB12 (at 76AU, dubbed as *Sedna.)* And recently Brown and his team reported a new planetoid finding, called 2003UB31 (97AU). This is not to include *Quaoar* (42AU), which has orbit distance more or less near Pluto (39.5AU), therefore this object is excluded from our discussion. It is interesting to remark here that all of those new 'planetoids' are within 8% bound from our prediction of celestial quantization based on the above Bohr-Sommerfeld quantization hypothesis (Table 1). While this prediction is not so precise compared to the observed data, one could argue that the 8% bound limit also corresponds to the remaining planets, including inner planets. Therefore this 8% uncertainty could be attributed to macroquantum uncertainty and other local factors.

While our previous prediction only limits new planet finding until n=9 of Jovian planets (outer solar system), it seems that there are reasons to suppose that more planetoids are to be found in the near future. Therefore it is recommended to extend further the same quantization method to larger n values. For prediction purpose, we include in Table 1 new expected orbits based on the same quantization procedure we outlined before. For Jovian planets corresponding to quantum number n=10 and n=11, our method suggests that it is likely to find new orbits around 113.81 AU and 137.71 AU, respectively. It is recommended therefore, to find new planetoids around these predicted orbits.

As an interesting alternative method supporting this proposition of quantization from superfluid-quantized vortices (8.6), it is worthnoting here that Kiehn has argued in favor of re-interpreting the square of the wavefunction of Schrödinger equation as the vorticity distribution (including topological vorticity defects) in the fluid [69]. From this viewpoint, Kiehn suggests that there is *exact mapping* from Schrödinger equation to Navier-Stokes equation, using the notion of quantum vorticity [69]. Interestingly, de Andrade & Sivaram [98] also suggest that there exists formal analogy between Schrödinger equation and the Navier-Stokes viscous dissipation equation:

$$\partial V / \partial t = \nu . \nabla^2 V \tag{8.13}$$

where ν is the kinematic viscosity. Their argument was based on propagation torsion model for quantized vortices [98]. While Kiehn's argument was intended for ordinary fluid, nonetheless the neat linkage between Navier-Stokes equation and superfluid turbulence is known in literature [99][100].

At this point, it seems worth noting that some criticism arises to the use of quantization method for describing the motion of celestial systems. These criticism proponents usually argue that quantization method (wave mechanics) is oversimplifying the problem, and therefore cannot explain other phenomena, for instance planetary migration etc. While we recognize that there are phenomena which do not correspond to quantum mechanical process, at least we can argue further as follows:

(i) Using quantization method like Nottale-Schumacher did, one can expect to predict new exoplanets (extrasolar planets) with remarkable result [85][86];

(ii) The ‘conventional’ theories explaining planetary migration normally use fluid theory involving *diffusion* process;

(iii) Alternatively, it has been shown by Gibson *et al*. [130] that these migration phenomena could be described via Navier-Stokes approach;

(iv) As we have shown above, Kiehn’s argument was based on *exact-mapping* between Schrödinger equation and Navier-Stokes equations [69];

(v) Based on Kiehn’s argument one these authors published prediction of some new planets in 2004 [90];

(vi) In March 2004 Brown *et al*. reported their finding of a planetoid in Kuiper belt;

(vii) There is other prediction of planetoid in Kuiper Belt object [127], nonetheless the writers don’t mention other planetoids apart of Sedna (outside the Kuiper belt);

(viii) In July 2005, Brown *et al*. reported again a few number of new planetoids, in the Oort Cloud;

(ix) Our subsequent analysis (Table 8.1) seems to suggest that Brown’s report is in good agreement with our previous prediction, therefore it is *not a retro-diction* [90];

(x) To conclude: while our method as described herein may be interpreted as an oversimplification of the real planetary migration process which took place sometime in the past, at least it could provide us with useful tool for prediction;

(xi) Now we also provide new prediction of other planetoids which are likely to be observed in the near future (around 113.8 AU and 137.7 AU);

(xii) It is recommended to use this prediction as guide to finding new objects (in the inner Oort Cloud);

(xiii) There are of course other theories which have been developed to explain planetoids and exoplanets [126].

(xiv) Therefore quantization method could be seen as merely a 'plausible' theory between others (i.e. It could be regarded only as 'partial truth', see the previous chapter).

All in all, what we would like to emphasize here is that the quantization method does not have to be the *true* description of reality with regards to celestial phenomena. As always this method could explain some phenomena, while perhaps lacks explanation for other phenomena. But at least it can be used to predict something which is measurable (exoplanets, and new planetoids in the outer solar system etc.) It appears that this problem is somewhat similar with what happened to Copernican theory, which once was described as: "*absurd philosophically and formally heretical inasmuch as it expressly contradicts the doctrines of the Holy Scripture*."[131] Therefore, it is our conviction that someday a more profound theory will appear which perhaps could reconcile the quantization viewpoint and 'planetary migration' viewpoint. As to the method described herein which could be viewed as merely consists of pure guessing (initially), allow us to quote R. Feynman here:

> "First you guess. Don't laugh; this is the most important step. Then you compute the consequences. Compare the consequences to experience. If it disagrees with experience, the guess is wrong. In that simple statement is the key to science. It doesn't matter how beautiful your guess is or how smart you are or what your name is. If it disagrees with experience, it's wrong." [134]

Picture 8.1. Book cover of Dialogo [131].

In the meantime, it seems also interesting here to consider a plausible generalization of Schrödinger equation in particular in the context of viscous dissipation method. First, we could write Schrödinger equation for a charged particle interacting with an external electromagnetic field [69] in the form of Ulrych's unified wave equation [7]:

$$\left[\left(-i\hbar\nabla_{\mu}-qA_{\mu}\right)\left(-i\hbar\nabla^{\mu}-qA^{\mu}\right)\Psi\right]=\left[-i2m.\partial/\partial t+2mU(x)\right]\Psi . \quad (8.14)$$

In the presence of electromagnetic potential [105], one could include another term into the LHS of equation (8.14):

$$\left[\left(-i\hbar\nabla_{\mu}-qA_{\mu}\right)\left(-i\hbar\nabla^{\mu}-qA^{\mu}\right)+eA_{o}\right]\Psi=2m\left[-i\partial/\partial t+U(x)\right]\Psi . \quad (8.15)$$

This equation has the physical meaning of Schrödinger equation for a charged particle interacting with an external electromagnetic field, which takes into consideration Aharonov effect [105]. Topological phase shift becomes its immediate implication, as already considered by Kiehn [69].

As described above, one could also derived equation (8.11) from scale-relativistic Schrödinger equation [85][86]. It should be noted here, however, that Nottale's method [85][86] differs appreciably from the viscous dissipative Navier-Stokes approach of Kiehn [69], because Nottale only considers his equation in the Euler-Newton limit [103][104].

Alternatively, with respect to our superfluid dynamics interpretation [71], one could also get Schrödinger equation from simplification of Ginzburg-Landau equation. This method will be discussed subsequently. It is known that Ginzburg-Landau equation can be used to explain various aspects of superfluid dynamics [73][74][75]. For alternative approach to describe superfluid dynamics from Schrödinger-type equation, see [96a]-[96b].

According to Gross, Pitaevskii, Ginzburg, wavefunction of N bosons of a reduced mass m* can be described as [96]:

$$-(\hbar^{2}/2m^{*}).\nabla^{2}\psi+\kappa|\psi|^{2}\psi=i\hbar.\partial\psi/\partial t \quad (8.18)$$

For some conditions, it is possible to replace the potential energy term in equation (8.18) with Hulthen potential. This substitution yields:

$$-(\hbar^{2}/2m^{*}).\nabla^{2}\psi+V_{Hulthen}.\psi=i\hbar.\partial\psi/\partial t \quad (8.19)$$

where

$$V_{Hulthen}=-Ze^{2}.\delta.e^{-\delta r}/(1-e^{-\delta r}) \quad (8.20)$$

This equation (8.20) has a pair of exact solutions. It could be shown that for small values of δ, the Hulthen potential (8.20) approximates the effective Coulomb potential, in particular for large radius:

$$V_{Coulomb}^{eff} = -e^2 / r + \ell(\ell+1).\hbar^2 /(2mr^2)$$

(8.21)

Therefore equation (8.21) could be rewritten as:

$$-\hbar^2\nabla^2\psi / 2m^* + \left[-e^2 / r + \ell(\ell+1).\hbar^2 /(2mr^2)\right]\psi = i\hbar.\partial\psi / \partial t \qquad (8.22)$$

For large radii, second term in the square bracket of LHS of equation (8.22) reduces to zero [95],

$$\ell(\ell+1).\hbar^2 /(2mr^2) \to 0 \qquad (8.23)$$

so we can write equation (8.22) as:

$$(-\hbar^2\nabla^2\psi / 2m^* + U).\psi = i\hbar.\partial\psi / \partial t \qquad (8.24)$$

where Coulomb potential can be written as:

$$U = -e^2 / r \qquad (8.25)$$

This equation (8.24) is nothing but Schrödinger equation (8.1). In other words, we have re-derived Schrödinger equation from simplification of Ginzburg-Landau equation, in the limit of small screening parameter. Calculation shows that introducing this Hulthen effect (8.20) into equation (8.19) will yield different result only at the order of 10^{-39} m compared to prediction using equation (8.24), which is of course negligible. Therefore, we conclude that for most celestial quantization problems the result of TDGL-Hulthen (8.22) is *essentially* the same with the result derived from equation (8.1). Now, to derive equation (8.11) from Schrödinger equation, the reader is advised to see Nottale's scale-relativistic method [85][86].

What we would emphasize here is that this derivation of Schrödinger equation from Ginzburg-Landau equation is at good agreement with our previous conjecture that equation (8.6) implies macroquantization corresponding to superfluid-quantized vortices. This conclusion is the main result of this Postscript chapter.

It is also worthnoting here that there is recent attempt to introduce Ginzburg-Landau equation in the context of microtubule dynamics [108], which implies wide applicability of this equation. Furthermore, because Ginzburg-Landau equation represents superfluid dynamics at low-temperature, the fact that we can derive quantization of celestial systems from this equation could be interpreted as signature of Bose-Einstein condensate cosmology [109][110].

One day Chuang-tzu and a friend were walking along a riverbank.
'How delightfully the fishes are enjoying themselves in the water!' Chuang-tzu exclaimed.
'You are not a fish,' his friend said. 'How do you know whether or not the fishes are enjoying themselves?'
'You are not me,' Chuang-tzu said.
'How do you know that I do not know that the fishes are enjoying themselves?'
A Zen koan [134a]

Epilogue

Throughout this book, we have argued that using Multi-Valued-logic viewpoint, in particular Neutrosophy, some known paradoxes in Quantum Physics could be solved in unique way. And it is our hope that some predictions derived in this book will find their way in experiments.

It is also known that there are other numerous applications of Multi-Valued-logic, which have become part of daily numerical tools for hardware designers and programmers alike. It is not difficult to expect that in the near future, applications of Neutrosophic Logic will also be found in the same way now electronic designers have made use Fuzzy Logic of L. Zadeh.

In recent years, a few physicists have suggested that biological systems could be represented using Multi-Valued-logic [17a]. Therefore, it is very likely that study of Quantum Physics of biological systems will also find Neutrosophic Logic useful. Furthermore, it is also likely that Multi-Valued-logic in particular Neutrosophy will improve various other branches of science, which have used mathematical methods extensively, including perhaps econometrics [133a].

Now, this paragraph is intended for physicist fellows who find themselves remain undecided as to whether Neutrosophic Logic is worth serious consideration or not. Let us rewrite again Smarandache's paradox: "Let A be some attribute (e.g., possible, present, perfect, etc.). If all is A, then the non-A must also be A." This statement implies that sometimes impossible things could happen. For a daily example of this paradox, let us quote Douglas Adams: "*The major difference between a thing that might go wrong and a thing that cannot possibly go wrong is that when a thing that cannot possibly go wrong goes wrong, it usually turns out to be impossible to get at and repair*." In other words, sometimes what is supposed to not go wrong can go wrong. Or try another quote by British physicist Ernest Rutherford: "*All of physics is either impossible or trivial. It is impossible until you understand it, and then it becomes trivial*." [134]

After all, allow us to quote C.N. Yang at the end of this book: "*There are only two kinds of math books. Those you cannot read beyond the first sentence, and those you cannot read beyond the first page.*" [134] Therefore if you can manage yourself to read up to this page, we believe that at least we already write a book which could exclude itself from the above C.N. Yang's statement.

And if you find this book improves your comprehension of parts of your own research, so you could become not only smarter but perhaps also wiser (*sophos*), we would consider it as an extra gratuity.

FS & VC

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The cover of this book was adapted from 'Underspin' image created by Cognitive Distortion. Thanks to Mike Fifield (www.cognitivedistortion.com) for his kind permission for the use of his image for this book. In our opinion, this image could represent in beautiful way, this poem by Blake:

"To see a World in a Grain of Sand
And a Heaven in a Wild Flower,
Hold Infinity in the palm of your hand
And Eternity in an hour."

After all, we should also thank to the mysterious cat, which has baffled physicists for almost eight decades.

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There is interesting remark: "*Evoking wabi-sabi, the sense of temporality and imperfection, the concrete surfaces are not flat and mechanical, but instead deliberately billow and strain against the forms to create the illusion of paper, of a worked and natural surface.*"

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Appendix 1.
An example of self-referential code: Quine

This is subject for computer nerds. There are plenty of examples of such 'self-generating' code, but the most popular one is known as Quine.[5] It is possible to write a program which outputs another program which is itself a quine.

:**quine**: /*kwi:n*/ /*n.*/ [from the name of the logician Willard van Orman Quine, via Douglas Hofstadter] A program that generates a copy of its own source text as its complete output. Here is one classic quine:

```
    ((lambda (x)
  (list x (list (quote quote) x)))
 (quote
   (lambda (x)
    (list x (list (quote quote) x)))))
```

This one works in LISP or Scheme. Another example using JavaScript(Scheme-style) described by: E. Barzilay (eli@CS.Cornell.EDU)

```
(function (x) { return unescape(x)+"("+"\""+x+"\""+")"})
  (escape("(function (x) { return unescape(x)+\"(\"+\"\\\"\"+x+\"\\\"\"+\")\"})"))
```

And an example written in C, given by K. Thompson (inventor of UNIX):

```
char s[] = {
        '\t',
        '0',
        '\n',
        '}',
        ';',
```

[5] **http://www.nyx.net/~gthompso/bibliography.htm**

```
    '\n',
'\n',
'm',
'a',
'i',
'n',
'(',
')',
'\n',
'{',
'\n',
'\t',
'i',
'n',
't',
' ',
'i',
';',
'\n',
'\n',
'\t',
'p',
'r',
'i',
'n',
't',
'f',
'(',
'\"',
'c',
'h',
'a',
'r',
' ',
'\\',
't',
's',
'[',
']',
' ',
```

```
'=',
' ',
'{',
'\\',
'n',
'\"',
')',
';',
'\n',
'\t',
'f',
'o',
'r',
'(',
'i',
'=',
'0',
';',
's',
'[',
'i',
']',
';',
'i',
'+',
'+',
')',
'\n',
'\t',
'\t',
'p',
'r',
'i',
'n',
't',
'f',
'(',
'\"',
'\\',
'r',
```

```
        '%',
      'd',
      ',',
      '\\',
      'n',
      '\"',
      ',',
      's',
      '[',
      'i',
      ']',
      ')',
      ';',
      '\n',
      '\t',
      'p',
      'r',
      'i',
      'n',
      't',
      'f',
      '(',
      '\"',
      '%',
      's',
      '\"',
      ',',
      's',
      ')',
      ';',
      '\n',
      '}',
      0
};

main() {
int i;

    printf("char \ts[] = {\n");
    for(i=0;s[i];i++)
```

```
            printf("\r%d,\n",s[i]);
        printf("%s",s);
    }
```

These examples are described here merely to put emphasis on self-referential code and also that self-referential statements are perhaps more ubiquitous than what we've imagined before.

Appendix 2
Summary of bibliography of Non-Standard Logics [120]

The followings summarize a few non-standard logics known in literature.

No	Name	Description
1	Combinatory Logic	Logics that replace variables with functions in order to clarify intuitive operations on variables such as substitution. Systems of arithmetic built from combinatory logic can contain all partial recursive functions and avoid Gödel incompleteness.
2	Conditional Logic	Logics that deal with the truth of conditional sentences, particularly in the subjunctive mood. The logic of counterfactual assertions.
3	Constructive Logic	Logics in which a wff is true iff it is provable. Therefore, undecidable truths (like Gödel's G) are ruled out by definition.
4	Cumulative Logic	A logic extending the theory of types. Predicates are true of objects of all lower types, not (as in the simple theory of types) only of objects of the immediately preceding type.
5	Deontic Logic	Logics of permission and obligation (derived from modal logics of possibility and necessity); hence the logic of norms and normative systems.
6	Dynamic Logic	Logics for reasoning about computer programs, especially for proving that a program is "correct" or lacks semantic bugs or does what it is intended to do without error. In dynamic logics, the truth-values of wffs can change according to the rules or functions of a program.
7	Epistemic Logic	The logic of non-truth-functional operators such as "believes" and "knows". For example, let *p mean that I know proposition p. If *p and p $\supset$q are given, then what must

		we add in order to infer *q?
8	Erotetic Logic	The logic of questions and answers. When does a proposition answer a question (correctly or incorrectly)? What's wrong with questions that presuppose false propositions (such as "Have you stopped beating your spouse?")? Do questions bear truth-values? What is the most efficient strategy of asking questions to get an answer from a database?
9	Free Logic	Standard logic without any existence assumptions. While quantifiers do have existential import, singular terms may sometimes denote no existing object or not denote at all. Logical truths must be true for the empty domain as well as all non-empty domains. One motive is to make logic purer by eliminating some remaining metaphysical implications; another is to make translations from natural languages more direct.
10	Fuzzy Logic	Logics in which the underlying set theory is fuzzy set theory. In fuzzy set theory, set membership is not a binary predicate (yes/no, or in/out), but a continuous quantity from 1 to 0. Fuzzy logic introduces a similar gradation of truth-values.
11	Higher-Order Logic	Predicate logics in which quantifers bind predicate variables, and predicates can take other predicates as arguments. In first-order predicate logic, by contrast, quantifiers bind only individual variables, and predicates take only individual terms as arguments.
12	Infinitary Logic	Logics permitting infinitely long wffs, especially disjunctive strings to replace existential quantifiers and conjunctive strings to replace universal quantifiers, or permitting rules of inference that take infinitely many premises. Spurred by Gödel's proof of the incompleteness of *finitary* logic and arithmetic.
13	Intensional Logic	Logics that include apparatus for signifying

		when two meanings (as opposed to two wffs, truth-values, sets, predicates, functions) are identical, and that analyzes inferences involving meanings. (Non-intensional logics are called extensional.)
14	Intuitionistic Logic	Propositional logics (and their predicate logic extensions) in which neither "p $\vee$~p" nor "~~p $\supset$p" are provable. They accept disjunctions A $\vee$B as theorems only if one of the disjuncts is separately provable: i.e. if either $\vdash$A or $\vdash$B. They have the same rules of inference as classical logic. Propositional connectives are undefined primitives.
15	Many-sorted Logic	Logics in which variables are "typed" as they are in many computer programming languages.
16	Many-valued logic	Logics in which there are more than the two standard truth-values "truth" and "falsehood". Motivated by semantic paradoxes like the liar ("this statement is false") and by future contingents ("tomorrow there will be a sea-battle"), that don't easily take either standard truth-value, and by attempts to deal with uncertainty, ignorance, and "fuzziness".
17	Modal Logic	The logic of the modal categories (possibility, actuality, and necessity) and their use in reasoning, for example, in "strict" implication.
18	Neutrosophic Logic	Neutrosophy is a branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. In mathematical terms, it is a multiple-valued logic in which each proposition is estimated to have the percentages of truth, indeterminacy, and falsity in T, I, and F respectively, where T, I, F are standard or non-standard subsets included in the non-standard unit

		interval] $^{-}0, 1^{+}$[. It is an extension of fuzzy, intuitionistic, paraconsistent logics.
19	Non-monotonic Logic	Logics in which the set of implications determined by a given group of premises does not necessarily grow, and can shrink, when new wffs are added to the set of premises.
20	Paraconsistent Logic	Logics in which it is generally false that contradictions imply any and every proposition. Contradictory statements (p·~p) are both true and false, as opposed to simply false. Hence the principle of excluded middle is affirmed, while the principle of non-contradiction denied. Paraconsistent logics can be "lived" if one vows to accept all truths, but does not insist on rejecting all falsehood. Paraconsistent logics do not hold that all paradoxes can be "solved" and urges that they be recognized as contradictions.
21	Partial Logic	Logics in which wffs need not be either true or false, or in which singular terms need not denote anything, or both. Logics that can cope with "truth-value gaps" and "denotation failures".
22	Prehairetic Logic	The logic of preference. For example, if someone prefers A to B and B to C, must she prefer A to C? Must preference be transitive?
23	Quantum Logic	To reflect quantum indeterminacy and uncertainty, quantum logic adds a third truth-value ("indeterminate"); hence the metatheory denies the principle of excluded middle (PEM). Nevertheless, for every p, "p ∨~p" is logically valid in systems of quantum logic. That is, PEM is true in the theory, false in the metatheory. Because both disjuncts of a true disjunction can be false, disjunction and conjunction behave asymetrically; hence the distribution laws generally fail. Motivated to capture the queerness

		of quantum-mechanics; in quantum logic this queerness shows up on the propositional level, in redefined connectives.
24	Relevant Logic	Logics in which "p implies q" only if p is *relevant* to q. Designed to prevent the paradoxes of material implication from arising; p should never imply q simply because p is false or because q is true. The advantage is that implication claims in natural language are better translated; the disadvantage is that implication loses truth-functionality.
25	Stoic Logic	The logic of the ancient Stoics, marked by the introduction of tense operators.
26	Substance Logic	The logic of entities related to one another by such indices as time, space, and possible worlds. Complex entities can model situations normally modelled by n-place relations.

Appendix 3
List of some known paradoxes

The followings summarize some known paradoxes in science, physics, and mathematics.

No	Paradox	Description	URL
1	Russell paradox	Consider the set of all sets are not members of themselves. Is this set a member of itself? (Let R be the set of all sets which are not members of themselves. Then R is neither a member of itself nor not a member of itself. Responses to it include Zermelo's axiomization of set theory, and its own discoverer's theory of types.)	http://www.geocities.com/mathimoh/Essay/essay4.htm
2	Epimenidean paradox	Consider this statement: "This statement is false." Is this statement is true?	
3	Berry paradox	What is "The first number not nameable in fewer than ten words"? (And hasn't it just been named in nine?)	
4	Wigner's friend	It is a thought experiment to highlight how he believed consciousness is necessary to the quantum mechanical measurement process. If a material device is substituted for the conscious friend, the linearity of the wave function implies that the state of the system is in a linear sum of possible states. It is simply a larger indeterminate system.	
5	Einstein-Podolski-	Can far away events influence each other in quantum mechan-	

	Rosen paradox	ics?	
6	Schrödinger's cat paradox	A sealed vessel containing a live cat and a device triggered by a quantum event such as the radioactive decay of the nucleus. If the quantum event occurs, cyanide is released and the cat dies; if the event does not occur the cat lives.	
7	Zeno paradox	The slower when running will never be overtaken by the quicker; for that which is pursuing must first reach the point from which that which is fleeing started, so that the slower must necessarily always be some distance ahead. (Achilles cannot win over the turtle)	http://plus.maths.org/issue17/xfile/index.html
8	Banach-Tarski paradox	Split a ball into 5 pieces; reassemble the pieces to get two balls, both of equal size to the first.	
9	Wheeler's paradox (John Archibald Wheeler)	"The question is: What is the question?"	
10	Olber paradox	Why is the night sky black if there is infinity of stars? (Formulated in 1826, it stated that the night sky should be uniformly illuminated if the universe were infinite and homogeneous with stars in every direction. It was resolved with the discovery of the Red Shift and the realization that stars have finite lifetimes.)	
11	Ehrenfest	Formulated by an Austrian	

	paradox	physicist, it examines a rapidly rotating disc. Since any radial segment of the disc is perpendicular to the direction of motion, the radius should not undergo length contraction. Since, however, the circumference of the disc is parallel to the direction of motion, the circumference should contract.	
12	Richard's paradox	A complete list of definitions of real numbers doesn't exist. (We appear to be able to use simple English to define a decimal expansion in a way which is self-contradictory.)	http://www.dpmms.cam.ac.uk/~wtg10/richardsparadox.html
13	Barber paradox	The barber who shaves all men who don't shave themselves, and no-one else. Should he shave himself?	http://home.att.net/~numericana/answer/sets.htm#barber
14	Ship of Theseus paradox	When every component of the ship has been replaced at least once, is it still the same ship? (Similarly we can ask: If each component of human body has been replaced by equivalent robotic part, is HE the same human?)	
15	Abilene paradox	A group of people often has to decide against its own interests. (People can make decisions based not on what they actually want to do, but on what they think that other people want to do, with the result that everybody decides to do something that nobody really wants to do, but only what they thought that	

		everybody else wanted to do.)	
16	Bertrand Paradox	Two players reaching a state of Nash equilibrium both find themselves with no profits.	
17	Diamond-water paradox	Why is water cheaper than diamonds, when humans need water to survive, not diamonds?	
18	Jevons paradox	Increases in efficiency lead to even larger increases in demand.	
19	St.Petersburg paradox	The so-called "St. Petersburg Game" is played with a fair coin, which is tossed until heads appears. If the game lasts for n+1 tosses, the player receives 2^n dollars. What's a decent price to pay for the privilege of playing this game? (People will only offer a modest fee for a reward of infinite value.)	http://home.att.net/~numericana/answer/utility.htm#petersburg
20	Moore's paradox	"It's raining but I don't believe that it is."	
21	Nihilist paradox	If truth does not exist, the statement "truth does not exist" is a truth, thereby proving itself incorrect.	
22	Mere addition paradox	Is a large population living barely tolerable lives better than a small happy population?	
23	Fermi paradox	If there are many other sentient species in the Universe, then where are they? Shouldn't their presence be obvious?	
24	Mpemba paradox	Hot water can under certain conditions freeze faster than cold water, even though it must pass the lower temperature on the way to freezing.	
25	Raven paradox	Observing a red apple increases the likelihood of all ravens be-	

		ing black.	
26	Socratic paradox	One can never knowingly choose the lesser good or willingly do the wrong thing.	
27	Hausdorff paradox	There exists a countable subset C of the sphere S such that SC is equidecomposable with two copies of itself.	
28	Gabriel's horn	A simple object with finite volume but infinite surface area	
29	Grelling paradox	Is the word "heterological", meaning "not applicable to itself," a heterological word?	
30	Petronius paradox	Moderation in all things. Including moderation	
31	Sorites paradox	At what point does a heap stop being a heap as I take away grains of sand? Alternately, at what point does someone become bald?	
32	D'Alembert paradox	An inviscid liquid produces no drag	
33	Smarandache paradox [136][137]	Let A be some attribute (e.g., possible, present, perfect, etc.). If all is A, then the non-A must also be A. For example, "Nothing is perfect, not even the perfect."	http://mathworld.wolfram.com/SmarandacheParadox.html
34	Smarandache's Invisible Quasi Paradox	Our visible world is composed of a totality of invisible particles	http://www.gallup.unm.edu/~smarandache/eBook-neutrosophics2.pdf
35	Invisible Sorites Paradox	There is not a clear frontier between visible matter and invisible matter.	http://www.gallup.unm.edu/~smarandache/eBook-neutrosophics2.pdf

Source: Some paradoxes are mentioned in http://www.reference.com

Appendix 4
A few basic notations

Merely as refreshment, here’s a few basic notations used in this book.

Notation	Description
∅	The empty set
∈	Element of
∉	Not an element of
⊆	Contained in
⊇	Contains
⊄	Not contained in
∩	Intersection
∪	Union

Terminology. [133]

A *set* is any collection of objects.

The *empty set* is the set containing no elements, denoted by Æ.

If *A* is a set and *x* is a member of *A*, we say *x is an element of A* and denote this by *x* Î *A*.

If *A* and *B* are sets and every element of *A* is also an element of *B* (that is, *x* Î *A* implies *x* Î *B*), then we say *A is a subset of B* or *A is contained in B* and we denote this by *A* Í *B*.

The *intersection* of two sets is the elements they have in common. For example, if A={1, 2, 3, 4} and B={2, 4, 6, 8}, then A Ç B = {2, 4}.

The *union* of two sets is the set of elements that are in at least one of the two sets. For example, if A={1, 2, 3, 4} and B={2, 4, 6, 8} then A È B = {1, 2, 3, 4, 6, 8}.

Appendix 5
A few historical achievements in foundations of set theory

Date	Description
March 3, 1845	Georg Ferdinand Ludwig Philip Cantor was born of Danish parents in St. Petersburg, Russia, in 1845, and moved with his parents to Frankfurt, Germany, in 1352. Georg gave up his father's suggestion of preparing for a career in engineering. Rather, Cantor concentrated on philosophy, physics, and mathematics. He studied at Zurich, Gottingen, and Berlin (where he took his doctorate in 1867). He then spent a long teaching career at the University of Halle from 1869 until 1905. In 1874 he commenced his revolutionary work on set theory and the theory of the infinite. With this work he created a whole new field of mathematical research. Today, Cantor's set theory has penetrated into almost every branch of mathematics, and it has proved to be of particular importance in the foundations of real function theory. Cantor died in a mental hospital in Halle in 1918. Well known is his famous aphorism: "The essence of mathematics lies in its freedom."
March 18, 1871	What mathematician, when asked his age answered "I was x years old in the year x2.? A. De Morgan died. Augustus De Mor6an was born (blind in one eye) in 1806 in Madras, where his father was associated with the East India Company. He was educated at Trinity College, Cambridge, graduating as fourth wrangler, and in 1828 became a professor in the newly established University of London (later renamed University College), where, through his works and his students, he exercised a wide influence in English mathematics. He was well read in philosophy and the history of mathematics, and wrote works on the foundations of algebra, differential calculus, logic, and the theory of probability. He was a highly lucid expositor. His witty and amusing book, A Budget of Paradoxes, still makes entertaining reading. He continued George Boole's work on the algebra of sets, enunciating the principle of duality of set theory, of which the so-called De Morgan Laws are an illustration: If A and B are subsets of a universal set, then the complement of the union of A and B is the intersection of the complements of A and B, and the complement of the intersection of A and B is the union of the complements of A and B (in symbols: (A È B). = A. Ç B. and (A Ç B). = A. È B., where prime denotes complement). Like Boole, De

	Morgan regarded mathematics as an abstract study of symbols subjected to sets of symbolic operations. De Morgan was an outspoken champion of academic freedom and of religious tolerance. He performed beautifully on the flute and was always jovial company, and he was a confirmed lover of big-city life. He had a fondness for puzzles and conundrums, and when asked either his age or his year of birth would reply, "I was x years old in the year x2."
April 20, 1932	The Italian mathematician Giuseppe Peano, who was born in 1858, died. In 1889 Peano brought the searchlight of modern criticism on Euclid's treatment of geometry, revealing a number of defects. Perhaps the gravest of these defects is that Euclid's axiomatic basis is insufficient for a rigorous development of the subject. Very popular with mathematicians is Peano's concise postulate set for the natural number system, which can be used employing no more assumptions, to obtain, first, the rational number system, then the real numbers, and finally the complex numbers. The work of Peano and that of the German logician Gottlieb Frege (1848-1925) originated the modern approach to symbolic logic. Peano's work was motivated by a desire to express all of mathematics as a manipulation of logical symbols. The resulting *Fornulaire de mathematiques* by Peano and his co-workers began its appearance in 1894. In 1890 Peano constructed the first continuous curve passing through every point of a square, thus showing that a continuous curve need not be one-dimensional, but can be area-filling. The curve is defined as the limit of an infinite sequence of curves, and Peano had one of the curves of the sequence put on the terrace of his home, by means of black tiles on white. Peano's curve has (an infinite number of) multiple points. Continuous area-filling curves without this fault were constructed later by David Hilbert (1891), Waclaw Sierpinski (1912), and others.
18 May, 1872	Birthdate of Bertrand Arthur William Russell. Bertrand Russell, descendent of an aristocratic family, was born near Trellack, Wales. The winner of an open scholarship at Trinity College, Cambridge, he took high honours in mathematics and philosophy, and studied under Alfred North Whitehead (1861-1947). In addition to lecturing, largely at universities in the United States, he wrote over forty books on mathematics, logic, philosophy, sociology, and education. He received many awards, such as both the Sylvester and de Morgan medals of the Royal Society (1934), the Order of Merit (1940), and the Nobel Prize for Literature (1950). His out-

	spoken views often embroiled him in controversies. During World I & II, he was dismissed from Cambridge University and imprisoned for four months because of his pacifist views and his opposition to conscription. In the early 1960s, he led pacifist moves to ban nuclear weapons and was again briefly imprisoned. A man of remarkable mind and ability, he died in 1970, mentally alert to the end, at the advanced age of ninety-eight. His most influential work, written with Whitehead, is the monumental Principia mathematics (1910-1913). The basic idea of this work is the identification of much of mathematics with logic by the deduction of the natural numbers, and hence the great bulk of existing mathematics, from a set of promises or postulates for logic itself.
5 September, 1667	Girolamo Saccheri was born. Little is know about Saccheri's life. He is famous for is work with Euclid.s fifth postulate as given in Euclid.s Elements. Many individuals in mathematics have questioned whether it is actually a postulate and should not be replaced as a proposition, derived from the remaining postulates. It is this that Saccheri set out to do. While working on his proof using a quadrilateral, parts of the proof came easy; however other sections of the proof became quite difficult; finally finishing in a rather unconvincing manner. Unfortunately, if Saccheri had simply accepted that there was no proof for the remaining parts, he would have been credited with the discovery of non-Euclidean geometry. His work went little noticed until 1889, when it was resurrected by Eugenio Beltrami (1835-1900).
5 October, 1781	Bernard Bolzano was born in Prague, Czechoslovakia. Bolzano was a priest but was defrocked for heresy and dismissed from his religion teaching post at the University of Prague. He had a leaning toward logic and mathematics, especially analysis, and can be considered a forerunner of the arithmetization of analysis, a program wherein the real number system is first rigorized and then all the basic concepts of analysis derived from this number system. In 1843, he produced a continuous non-differentiable function. His function didn't become known, and it is Weierstrass, about 40 years later, who is usually credited for the first example of this kind. There is a famous theorem in analysis that bears both mathematicians, named the Bolzano-Weirstrass Theorem, which states that every bounded infinite set of points contains at least one accumulation point. The Intermediate-Value Theorem of calculus is often referred to as Bolzano's theorem. There is an amusing story told about Bolzano when he

	once was suffering from an illness manifested by bodily aches and chills. To take his mind off his troubles, he picked up Euclid's Elements and, for the first time read the masterly exposition of the Eudoxian doctrine of ratio and proportion set out in Book V. Lo and behold his pain vanished. It has been said that after that, when anyone became similarly discomforted, Bolzano recommended that the ill one read Euclid's Book V.
2 November, 1815	George Boole was born. Largely self-taught in mathematics, he became interested in formal logic. In 1847, he published a pamphlet entitled The Mathematical Analysis of Logic, which De Morgan praised as epoch-making. Boole maintained that the essential character of mathematics lies in its form rather than in its content; mathematics is not (as some dictionaries today still assert) merely "the science of measurement and number," but, more broadly, any study consisting of symbols along with precise rules of operation upon those symbols, the rules being subject only to the requirement of inner consistency. In 1854, Boole expanded and clarified his earlier work into a book entitled Investigation of the Laws of Thought, in which he established both a formal logic and a new algebra. The algebra of sets is known today as Boolean algebra. Boolean algebra in recent times, has been found to have a number of applications, such as in the theory of electric switching circuits.

Source: Prof. H. Eves, http://pegasus.cc.ucf.edu/~mathed/eves (With kind permission from D. Brumbaugh)

This book was intended to discuss some paradoxes in Quantum Mechanics from the viewpoint of Multi-Valued-logic pioneered by Lukasiewicz, and a recent concept Neutrosophic Logic. Essentially, this new concept offers new insights on the idea of 'identity', which too often it has been accepted as given.

Neutrosophy itself was developed in attempt to generalize Fuzzy-Logic introduced by L. Zadeh. While some aspects of theoretical foundations of logic are discussed, this book is not intended solely for pure mathematicians, but instead for physicists in the hope that some of ideas presented herein will be found useful.

The book is motivated by observation that despite almost eight decades, there is indication that some of those paradoxes known in Quantum Physics are not yet solved. In our knowledge, this is because the solution of those paradoxes requires re-examination of the foundations of logic itself, in particular on the notion of identity and multi-valuedness of entity.

The book is also intended for young physicist fellows who think that somewhere there should be a 'complete' explanation of these paradoxes in Quantum Mechanics. If this book doesn't answer all of their questions, it is our hope that at least it offers a new alternative viewpoint for these old questions.

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