Multi-variable Gould-Hopper and Laguerre polynomials

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ABSTRACT

The idea of monomiality traces back to the early forties of the last century, when J.F. Steffensen, in a largely unnoticed paper [1], suggested the concept of *poweroid*.

A new interest in this subject was created by the work of G. Dattoli and his collaborators [2], [3]

It turns out that all polynomial families, and in particular all special polynomials, are essentially the same, since it is possible to obtain each of them transforming a basic monomial set by means of suitable operators (called the *derivative* and *multiplication* operator of the considered family). This result was theoretically proved in refs. [4], [5], and is closely connected with the achievements of umbral calculus [6] - a term invented by Sylvester - since the exponent, for example in x^n, is transformed into his "shadow" in $p_n(x)$.

Unfortunately the derivative and multiplication operators for a general set of polynomials are given by formal series of the ordinary derivative operator. It is therefore impossible to obtain sufficiently simple formulas to work with. However, for particular polynomials sets, related to suitable classes of generating functions, the above mentioned formal series reduce to finite sums, so that the relevant properties can be easily derived. The leading set in this field is given by the Hermite-Kampé de Fériet (shortly H-KdF) also called Gould-Hopper polynomials [7], [8].

Many sets of multi-variable or multi-index polynomials have been constructed starting from this important polynomial set [9], [10], [11].

The main results are obtained by using operational techniques [12], [2], which have wide applications in the solution of BVP for PDE (see e.g. [13], [14], [15], [16], [17], [18], [19], [20]).

In this article we give a survey of results connected with this subject.

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