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Multi-Vehicles Interaction Graph Model for Cooperative Collision Warning System

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***Abstract*—Cooperative collision warning system for road vehicles, enabled by recent advances in positioning systems and wireless communication technologies, can potentially reduce traffic accident significantly. To improve the system, we propose a graph model to represent interactions between multiple road vehicles in a specific region and at a specific time. Given a list of vehicles in vicinity, we can generate the interaction graph using several rules that consider vehicle's properties such as position, speed, heading, etc. Safety applications can use the model to improve emergency warning accuracy and optimize wireless channel usage. The model allows us to develop some congestion control strategies for an efficient multi-hop broadcast protocol.**

***Index Terms*—cooperative collision warning, graph model, vehicles interaction**

I. Introduction

Road traffic accidents bring about 1.2 million deaths and 50 million injuries each year worldwide, and those casualties are predicted to increase by 65% between 2000 and 2020 [1]. The development and improvement of vehicular safety technologies can significantly reduce the accident rate and improve road safety. One of the important safety technologies is collision warning system, which prevents accidents by providing the driver with early warning of possible crash. A collision warning system in a vehicle monitors the surrounding environment, including the state and motion of neighboring vehicles, and predicts the possibility of collisions.

Earlier development of collision warning systems relied on environment sensors such as radar, lidar, sonar and video camera to obtain information of the neighboring vehicles [2]. These sensors provide information such as relative distance and speed of objects surrounding the vehicle. Recent approach in the development of collision warning is heading toward cooperative driving concept, driven by rapid advances in wireless communication technologies. Cooperative collision warning system incorporates global navigation satellite system, such as GPS, and wireless networking technology, such as Wi-Fi, to realize the concept of "360 degrees driver situation awareness" without expensive equipment [3].

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Cooperative systems have several advantages compared with non-cooperative systems that use environment sensors: GPS and Wi-Fi combination generally costs less compared with the environment sensors, have wider coverage range and are not limited by the line-of-sight by using multi-hop transmission, and can obtain more accurate information of vehicles properties such as speed, acceleration, size, etc. In addition to safety purposes, the wireless networking capability also enables a wide variety of non-safety applications, such as traffic management, electronic payment, driver information and assistance, and general Internet access.

Several cooperative collision warning system concepts and prototypes [4], [5], [6], [7] have been proposed and developed. A typical system employs a GPS and an IEEE 802.11-based wireless network device which will most likely use the IEEE 802.11p DSRC/WAVE protocol standards [8]. However, improvement in robustness and reliability is still required for the system to be widely adopted and deployed. One of the challenges is the capability and performance of the communication system. Safety applications require fast and reliable delivery of information to all relevant vehicles, in a network environment that is highly unreliable and has limited capacity [9]. In a dense traffic environment where every vehicle communicates with each other, channel congestion is a major problem, causing contention, collision, and high bandwidth consumption.

In order for the IEEE 802.11p DSRC/WAVE standard to support delay-critical safety applications and provide reliable communication, we need an efficient higher-level communication protocol that can optimize the wireless channel usage by reducing redundant transmission. For cooperative collision warning applications, the protocol optimization requires contextual information on how vehicles interact with each other. For example, a vehicle should send warning message only to other vehicles that may possibly be endangered. A vehicles interaction model can provide the contextual information that is needed to determine those endangered vehicles.

In this paper we propose a model to represent the interactions between multiple vehicles at a specific region and time. The interactions are modeled using graph theory, where a vehicle is represented by a vertex and an interaction between two vehicles is represented by an edge. Interaction between two vehicles is determined by calculating their motion properties.

The aim is to provide contextual information for cooperative collision warning applications, therefore, improving the overall reliability. To the best of our knowledge, this kind of model has not been explored before. The existing model used in collision warning system [4], [7] currently only represents the interaction between a pair of vehicles and does not consider the aggregation of other pairs. Other related models such as microscopic traffic models have been used in traffic flow simulations, but not in the collision warning system. This interaction model will enable several potential improvements in cooperative collision warning system. It can improve and optimize existing routing algorithms and communication protocols, and in addition, it can also improve vehicles movement prediction and collision warning accuracy.

In the following section, we discuss other work related with our proposed model. The proposed model is presented in section III. Section IV discusses the benefits of the proposed model and its potential applications. Finally, section V concludes this paper.

II. Related Work

The use of graph theory to model vehicles interactions and formations has been presented in previous literatures [10], [11], [12], but all of them focus on formation and control. Our model focuses on providing contextual information for cooperative collision warning systems. One of the typical applications is in the scenario of chain collision [13], [14], where a communication protocol is needed to propagate warning messages to prevent collisions.

Biswas et al. [13] proposed an intelligent broadcast protocol that use implicit acknowledgment and random delay to reduce the unnecessary rebroadcast. Yang et al. [14] proposed a communication protocol for emergency warning dissemination which uses a state transition mechanism and limits the forwarding distance to reduce redundant messages. Both protocols only focus on straight road or highway scenario, and assume that all vehicles behind (following) an abnormal vehicle (vehicle in emergency situation) will be endangered and thus need to be warned. There are no further details regarding how to actually identify the endangered vehicles. This paper tries to address this challenge by providing a method to determine the endangered vehicles and generate generic interaction graph that is not limited only to highway scenario.

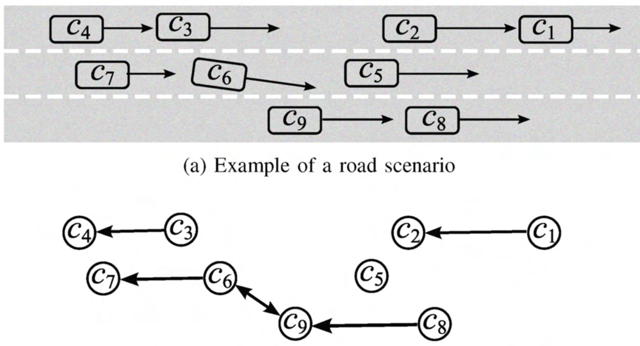
A simple model that calculates route contention between two vehicles [4] can be used to predict the collision at intersection point. This method computes intersection point of two vehicles' trajectories, and then compares the expected time-to-intersection for each vehicle to determine the possibility of collision. We adopt the similar method for determining whether two vehicles are influencing each other or not, in the case when the trajectories of two vehicles are intersecting each other.

The movement of a vehicle depends on the road condition on its desired path. A vehicle reacts based on the obstacles or other vehicles in front of it. If there is obstacles that can cause collision, a vehicle must be slowed down or stopped; otherwise, it can accelerate to its desired speed. This is the basic concept behind microscopic traffic models [15]. One of the microscopic traffic models is safe distance model [16] from car-following theory that defines the minimum distance required between vehicles given their speed and deceleration capability. We adopt the safe distance model to determine if a vehicle is influenced by other vehicle. For example, if a vehicle A is following a vehicle B and the distance between them is less than the calculated safe distance, it means that vehicle A is influenced by vehicle B.

III. Modeling Vehicles Interaction

We model the interactions between multiple vehicles using a directed graph which will be elaborated in this section. A vertex represents a vehicle and a directed edge represents an interaction in the graph. We define an interaction as a state in which two vehicles have an influence upon one another. The possible interaction between two vehicles can be: first vehicle is influenced by second vehicle, vice versa, or both. The main idea is to find out all possible interactions for all pairs of vehicles as illustrated in Figure 1. Given the interaction graph, we can identify which vehicles are influenced by other vehicles, where an arrow from node A to node B means that B is influenced by A.

A vehicle interacts with other vehicle if there is a possibility of collision between them. Possibility of collision is calculated using vehicle kinematics based on the motion properties of the vehicle such as trajectory and speed. The general principle is to compare the actual distance to collision point with the minimum distance required for the vehicle to stop using maximum deceleration. If the actual distance is less than the safe distance, the following vehicle is influenced by the leading vehicle.



(a) Example of a road scenario

(b) Possible interaction graph of the above scenario

Figure 1: Modeling multiple vehicle interaction

A. System Assumptions

The modeling concept presented in this paper is based on several system assumptions. We assume that all the vehicles are equipped with the positioning and communication systems. A positioning system that consists of an estimator, centimeter-level accuracy GPS, and in-vehicle sensors, is used to obtain vehicle's position, heading, speed, and acceleration. We assume that the positioning system can provide accurate relative distance between vehicles. A wireless communication system is used to exchange those vehicle properties using periodic broadcast, so every vehicle can have up-to-date information on other vehicles. This means that every vehicle maintains a set of other surrounding vehicles and their properties. How to determine the optimum number of vehicles in the set is beyond the scope of this paper. We also assume that no digital map information is available, although the model could be extended to include such information.

B. Graph Definition

A vehicle c is defined by a tuple $c = (x, y, w, l, v, \theta, a_d)$, where x and y are the position coordinates, w is the width, l is the length, v is the speed, a_d is the maximum deceleration, and θ is the heading of the vehicle. We define a list of vehicles in a specific area as a set $C = \{c_1, c_2, c_3 \ldots, c_n\}$.

The vehicle interaction model is defined as a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ that consists of a set of vertices $\mathcal{V} = \{v_1, v_2, v_3 \ldots, v_n\}$ and the set of directed edges $\mathcal{E} \subseteq \{\langle v_i, v_j \rangle : v_i, v_j \in \mathcal{V}, v_i \neq v_j\}$. A vertex v_n represents a vehicle, and an edge $e_{ij} = \langle i, j \rangle$ represents interaction between i and j, where i is influencing j. The interactions between vehicles are identified using rules detailed in the following subsection.

C. Graph Generation Rules

We can construct an interaction graph $\mathcal{G}$ by defining the set of vertices $\mathcal{V}$ and the set of edges $\mathcal{E}$. The set of vertices $\mathcal{V}$ is equivalent to the set of vehicles C . Given a set of vehicles C, we can construct an interaction graph $\mathcal{G}$ by firstly assigning C to $\mathcal{V}$ and generating the set of edges $\mathcal{E}$ by the following procedure. We enumerate all pairs of vehicles from $\mathcal{V}$, and for each pair (v_i, v_j), check whether there is interaction between v_i and v_j. If there is, create an edge e_{ij}, e_{ji}, or both and add it to the set of edges $\mathcal{E}$. Algorithm 1 provides the details of the graph generation procedure.

Algorithm 1 Generating initial interaction graph

input: A set of vehicles C
output: An interaction graph $\mathcal{G}$

1: create a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = C$ and $\mathcal{E} = \emptyset$
2: **for each** pair of vehicles (v_i, v_j) in $\mathcal{V}$ **do**
3:   **if** v_i and v_j are moving in the same direction **and** have overlapped paths **then**
4:     calculate actual longitudinal distance d_a, between v_i and v_j
5:     find the follower f and the leader l
6:     calculate following safe distance d_{sf}
7:     **if** $d_a < d_{sf}$ **then**
8:       $\mathcal{E} = \mathcal{E} \cup \{\langle l, f \rangle\}$
9:     **end if**
10:   **else if** v_i and v_j are moving in the opposite direction **and** have overlapped paths **then**
11:     calculate actual longitudinal distance d_a, between v_i and v_j
12:     calculate opposite safe distance d_{so}
13:     **if** $d_a < d_{so}$ **then**
14:       $\mathcal{E} = \mathcal{E} \cup \{\langle v_i, v_j \rangle, \langle v_j, v_i \rangle\}$
15:     **end if**
16:   **else**
17:     calculate intersection point $P_\times$
18:     calculate time-to-intersection TTX_i and TTX_j
19:     **if** $|TTX_i - TTX_j| < \lambda$ **then**
20:       calculate actual relative distance d_{ai} from v_i to $P_\times$
21:       calculate actual relative distance d_{aj} from v_j to $P_\times$
22:       calculate safe distance d_{si} and d_{sj}
23:       **if** $d_{ai} < d_{si}$ and $d_{aj} < d_{sj}$ **then**
24:         $\mathcal{E} = \mathcal{E} \cup \{\langle v_i, v_j \rangle, \langle v_j, v_i \rangle\}$
25:       **end if**
26:     **end if**
27:   **end if**
28: **end for**

We define three distinct cases that need to be considered in order to determine whether there is any interaction between any pair of vehicles. First case is *following* (line 3), where both vehicles are traveling in the same direction. Figure 2 illustrates this first case. Second case is *opposite* (line 10), where both vehicles are traveling in the opposite direction. Figure 3 illustrates this second case. Third case is *intersects* (line 16), for any other conditions besides those previous two cases. Figure 4 illustrates this third case. These three cases can cover all the possible traffic scenarios without road information from a digital map.

The absolute position of vehicle n is in Cartesian coordinate (x_n, y_n), referenced as the center point of the vehicle. Cartesian coordinate can be calculated from GPS coordinates. The heading of the vehicle θ_n is in radian where $0 \leq \theta_n < 2\pi$, and $\theta_n =$ 0means heading north. Let A and B be a pair of vehicles to be processed. For each case, we perform vector geometry and kinematics calculations to find if exists interaction between A and B.

Case 1 (Following): This is a case where two vehicles are following each other (vehicle A is following B or vice versa). Figure 2 shows a general following scenario to illustrate the concept. First we need to determine if A and B are moving in the same direction by comparing the heading angle where $\theta_A \approx \theta_B$, using a condition $|\theta_A - \theta_B| < \delta$, where δ is a parameter to accommodate for some small difference. Next, determine if A and B have overlapped paths using following procedure.
Find the distance between point B and P, d_P using following equations:

$$x_{A'} = \sin\theta_A + x_A \ , \quad y_{A'} = \cos\theta_A + y_A \tag{1}$$

$$u = \frac{(x_B - x_A)(x_{A'} - x_A) + (y_B - y_A)(y_{A'} - y_A)}{\sqrt{(x_{A'} - x_A)^2 + (y_{A'} - y_A)^2}} \tag{2}$$

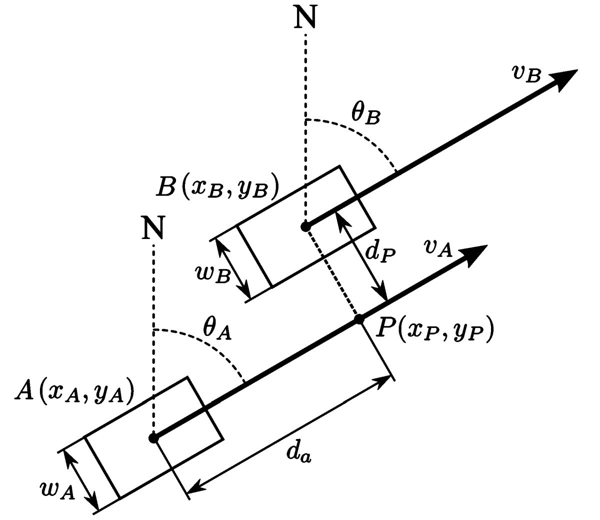


Figure 2: Two vehicles are following each other

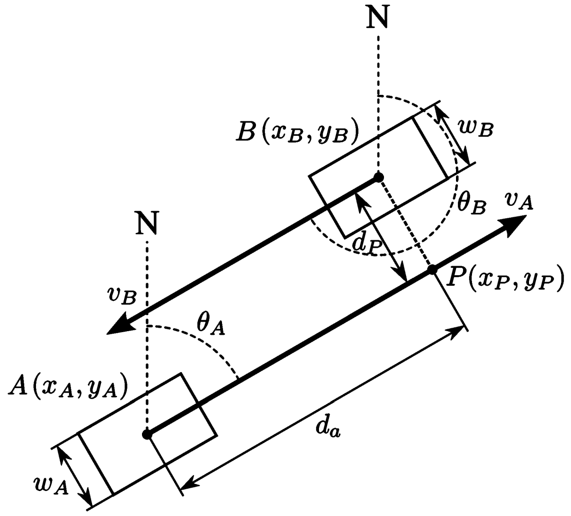


Figure 3: Two vehicles are heading toward each other

$$x_P = x_A + u\,(x_{A'} - x_A) \tag{3}$$

$$y_P = y_A + u\,(y_{A'} - y_A) \tag{4}$$

$$d_P = \sqrt{(x_P - x_B)^2 + (y_P - y_B)^2} \tag{5}$$

Equation 1 calculates point A' coordinates, where $\vec{AA'}$ is the normalized vector of v_A. To calculate point P coordinates we use Equations 2, 3, and 4. Distance between B and P then can be calculated using Equation 5.

If $d_P < \left(\frac{w_A}{2} + \frac{w_B}{2} + d_{gap}\right)$ then A and B have overlapped paths. w_A and w_B are the width of vehicles A and B respectively. d_{gap} is a parameter for the acceptable lateral distance between vehicles. Longitudinal distance d_a can be calculated using Equation 6:

$$d_a = \sqrt{(x_P - x_A)^2 + (y_P - y_A)^2} \tag{6}$$

To find the follower f and the leader l, find the angle α of vector $\vec{AP}$. If $\alpha \approx \theta_A$ then $f = A$ and $l = B$, else $f = B$ and $l = A$. To calculate the safe distance d_{sf}, we use a function of velocities and decelerations defined in the formula below [17]:

$$d_{sf} = d_{min} + v_f \cdot t_r + \frac{1}{2}\left(\frac{v_f^2}{a_f} - \frac{v_l^2}{a_l}\right) \tag{7}$$

where d_{min} is a parameter of expected minimal distance between vehicles, t_r is a parameter of driver reaction time, v_f is the following vehicle speed, a_f is the following vehicle maximum deceleration, v_l is the leading vehicle speed, a_l is the leading vehicle maximum deceleration. A simplified formula to calculate the vehicle maximum deceleration a_d is given in Equation 8 [18], which requires coefficient of friction between tire and roadway f, road grade G, and gravity constant g.

$$a_d = g\,(f \pm G) \tag{8}$$

Case 2 (Opposite): This is a case where two vehicles are heading toward each other and there is a collision possibility. Figure 3 shows a general opposite scenario to illustrate the concept. First we need to determine if A and B are moving in the opposite direction by comparing the heading angle using a condition $||\theta_A - \theta_B| - \pi| < \delta$, where δ is a parameter to accommodate for some small difference. Next, determine if A and B have overlapped paths using following procedure.
Find the distance between a point and a line d_P using Equations 5. If $d_P < \left(\frac{w_A}{2} + \frac{w_B}{2} + d_{gap}\right)$ then check if vehicles A and B are moving closer to each other, instead of moving further, by finding the angle α of vector $\vec{AP}$. If $\alpha \approx \theta_A$ then A and B have overlapped paths. . Longitudinal distance d_a can be calculated using Equation 6. Opposite safe distance d_{so} can be calculated using Equation 9.

$$d_{so} = d_{min} + v_A \cdot t_{rA} + \frac{1}{2}\left(\frac{v_A^2}{a_A}\right) + v_B \cdot t_{rB} + \frac{1}{2}\left(\frac{v_B^2}{a_B}\right) \tag{9}$$

Case 3 (Intersects): This is a case where two vehicles have intersecting paths and there is a collision possibility. Figure 4 shows a general intersection scenario to illustrate the concept. This case checks if the two vehicle's predicted paths do not form two parallel lines, thus can have an intersection point. The first two cases check for the conditions resulted in parallel lines. We adopt the collision prediction method from intersection collision warning concept proposed in previous literature [4]. Given two non-parallel infinite line, an intersection point $P_\times\,(x_\times, y_\times)$ can be computed using the following formulas:

$$x_\times = \frac{(y_B - y_A) - (x_B \cot\theta_B - x_A \cot\theta_A)}{\cot\theta_A - \cot\theta_B} \tag{10}$$

$$y_\times = \frac{(x_B - x_A) - (y_B \tan\theta_B - x_A \tan\theta_A)}{\tan\theta_A - \tan\theta_B} \tag{11}$$

Using the intersection point, calculate the expected time-to-intersection (TTX) for both vehicles using the following

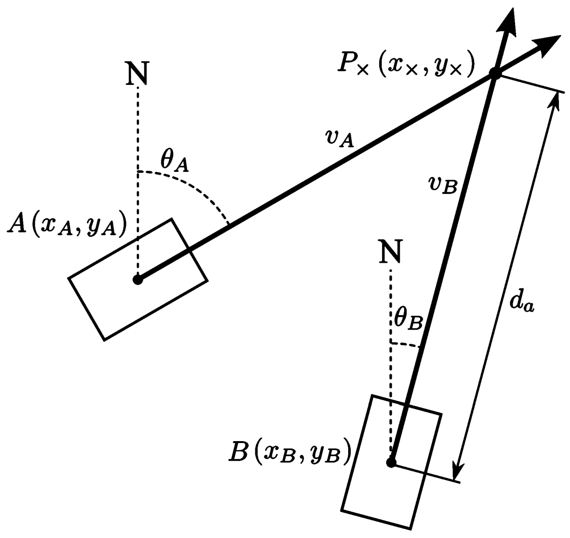


Figure 4: Two vehicles with intersecting paths

formulas:

$$TTX_A = \frac{|\vec{r}_\times - \vec{r}_A|}{|\vec{v}_A|}\mathbf{sign}\left((\vec{r}_\times - \vec{r}_A)\cdot\vec{v}_A\right) \quad (12)$$

$$TTX_B = \frac{|\vec{r}_\times - \vec{r}_B|}{|\vec{v}_B|}\mathbf{sign}\left((\vec{r}_\times - \vec{r}_B)\cdot\vec{v}_B\right) \quad (13)$$

where $\vec{v}_A$, $\vec{v}_B$ are the velocities of vehicles A and B, $\vec{r}_n$ is the vector representation of coordinate (x_n, y_n), and **sign**() is a sign function to identify if a vehicle has passed through the intersection. If both vehicles are expected to arrive at the intersection point around the same time, in which $|TTX_i - TTX_j| < \lambda$, then there is a route contention. We use λ as a parameter that depends on the vehicles' size, velocity, angle of the paths, and other uncertainties. If there is a route contention then we calculate the actual distance d_{ai} and d_{aj}using Equation 6. The safe distance d_{si} and d_{sj} is calculated using the following formula:

$$d_{sn} = d_{min} + v_n \cdot t_r + \frac{1}{2}\left(\frac{v_n^2}{a_n}\right) \quad (14)$$

D. Graph Generation Illustration

As an example on how to generate the interaction graph, Figure 5 illustrates the process presented in the Algorithm 1. The input is a set of four vehicles $\{c_1, c_2, c_3, c_4\}$ where each vehicle has properties defined as a tuple $c_n = (x, y, w, l, v, \theta, a_d)$. We use the same width $w = 2$ m, length $l = 4.5$ m, maximum deceleration $a_d = 6$ m/s^2 for all vehicles. Details of the vehicles' properties are given in Table I. Figure 5(a) illustrates the vehicles formation.

Given the input as described above, the process described in Algorithm 1 will be illustrated as follow. We process all of the possible pair from the set of vehicles. Without losing generality, in this example we begin with the pair c_1 and c_2. This pair is an example of two vehicles with intersecting paths. Using procedures given in Case 3, we add edges (c_1, c_2) and (c_2, c_1) to the graph. This step will produce a graph as shown in Figure 5(b). The next pair is c_1 and c_3. We know c_1 is influenced by c_3 using the procedures given in Case 1. Therefore, we add an edge (c_3, c_1) as shown in Figure 5(c). The next step is shown in Figure 5(d), where we add an edge (c_4, c_1) using Case 2 to process the pair c_1 and c_4. Pair c_2 and c_3 does not fulfill the conditions given in all of the cases, which means there is no interaction between them. Figure 5(e) shows the graph after pair c_2 and c_4 has been processed using procedures in Case 3. Finally, Figure 5(f) shows the completed interaction graph after the pair c_3 and c_4 is processed using Case 2.

Id	x(m)	y(m)	v(m/s)	θ(rad)
c_1	3	3	10	0
c_2	8	4	8	$\frac{17}{9}\pi$
c_3	3	15	15	0
c_4	3.5	30	5	π

Table I: Example of vehicle properties

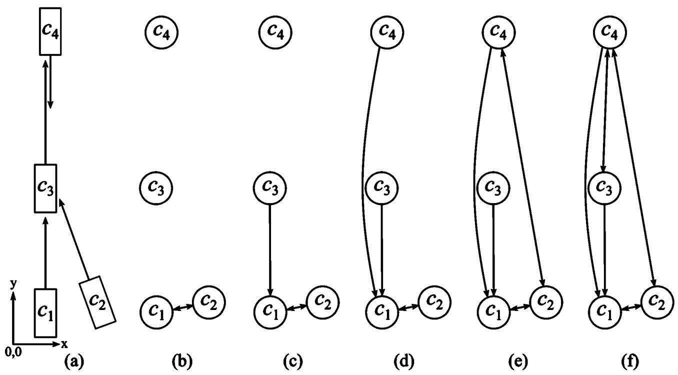


Figure 5: Example on how to generate the interaction graph

To summarize, the interaction graph models the interactions between a set vehicles by assessing every possible pairs of vehicle from the set. In the next section, we discuss the potential applications of the model.

IV. Discussion

Cooperative collision warning system requires contextual information of the surrounding environment of the vehicle to enable 360-degree awareness. This information can be shared by several subsystems or domains within the system. Our proposed model provides the contextual information that can be used to improve communication, prediction, and warning systems, and therefore has potential to improve the overall system reliability.

One of the expected application is to use the model in the communication protocol. In the case where a warning message needs to be sent using multi-hop transmission, the protocol needs to make decision of where to forward and which nodes should forward the message. Using the graph we can easily identify which vehicles that are endangered by a specific vehicle. Using this information, we can determine the message recipients and devise an algorithm to select the most effective forwarder. In this way, we can reduce the unnecessary rebroadcast of the same message.

We use Figure 1 as an example on how to use the model to optimize communication. Based on the diagram, we can identify the groups of each connected nodes, which can be used to form a multicast group. Instead of using broadcast to send warning messages to all vehicles, we use multicast to send warning messages only to relevant vehicles. For example, vehicle c_8 only needs to send warning message to c_9, c_6, and c_7. By sending messages only to relevant vehicles, we can reduce redundant transmission, and therefore, minimize the wireless channel usage and reduce channel congestion. An efficient communication protocol also can reduce the average delay of warning messages. By sending message only to relevant vehicles and having lower average delay, we can improve the accuracy and effectiveness of the warning system.

V. Conclusions and Future Work

We have proposed a graph model to represent vehicles interaction in a specific region and at a specific time. We have presented the graph model and an algorithm to construct it, given a list of vehicles and their properties. The model provides contextual information for cooperative collision warning system, especially useful to determine the target vehicles that are required to receive warning messages in an event of emergency. As discussed in previous section, our model can be used to improve cooperative collision warning generally, and specifically to improve emergency warning accuracy and to optimize wireless channel usage by improving the communication protocol and routing algorithms.

We have also discussed the prospective benefits of the model for multi-hop broadcast protocol optimization with some example cases, in which further research is needed to develop the initial ideas. Currently, we are investigating an efficient communication protocol that uses this model to minimize communication. In the future we will look into several enhancements to improve accuracy and reliability of the model. Road geometry and topology are important factors that can improve accuracy and reliability of the model. We can process road information obtained from digital map, and incorporate the result to the model. Using non-straight-line path or trajectory in the motion calculations can bring the model closer to reality. Another interesting problem is data aggregation, which involves how to represent and distribute the graph in an efficient manner. The improved model aims to bring more reliable and robust collision warning systems.

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