Multibeam cellular communication systems with dynamic channel assignment across multiple sectors *

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In cellular communication systems, directional multibeam antennas at cell sites can be used to reduce co-channel interference, increase frequency reuse and improve system capacity. When combined with dynamic channel assignment (DCA), additional improvement is possible. We propose a multibeam scheme using dynamic channel assignment across multiple sectors. A cell is divided into several sectors, each of which is covered by several directional beams. Specific channels are allocated to each sector as in fixed channel assignment (FCA). A channel of a sector is dynamically assigned to a wireless user who communicates through one of the several beams of the sector. The assignment is made so that constraints on the allowable co-channel interference are satisfied. Limitations due to co-channel interference are analyzed. A tractable analytical model for the proposed scheme is developed using multidimensional birth–death processes. Theoretical traffic performance characteristics such as call blocking probability, forced termination probability, hand-off activity, carried traffic and channel rearrangement rate are determined. With the proposed scheme, call blocking probability can be reduced significantly for a fixed offered traffic. Alternatively, system capacity can be increased while blocking probability is maintained below the required level. Smaller forced termination probability is obtainable in comparison with corresponding FCA schemes.

1. Introduction

As the number of wireless users grows rapidly, increased system capacity is sought. Sectorization and cell splitting are used to allow increased system capacity while interference is limited to maintain signal quality. Space Division Multiple Access (SDMA) is a technique which can increase system capacity by using directional multibeam antennas to reduce co-channel interference [18,24]. This allows reduced frequency reuse distances for the same link quality of communication links and results in higher system capacity.

One approach in SDMA uses switched multibeams. Multiple beams are used to cover the service area of a base station and the beam with the strongest signal power is selected to serve the user [12]. Recent work on switched multibeam schemes includes the investigation of the gain improvement achieved with a multibeam antenna compared to the traditional sector configuration [14]. The tradeoffs between hysterisis level, switching time and gain for a multibeam antenna system are considered in [14]. The effects of incorrect beam selection on the average signal-to-noise (SNR) power ratio and signal-to-interference (SIR) power ratio with a switched multibeam antenna system are examined in [16]. The frequency reuse efficiency of multibeam antenna systems is investigated in [13,23]. The possibilities of channel reuse within beams of the same cell are analyzed in [5,20]. The multibeam antenna systems combined with the dynamic channel assignment (DCA) scheme is investigated in [20]. In [20], we have considered a multibeam antenna system in which channels that are assigned to a sector can be reused in *different beams of the same sector* provided the required angular separation between beams is met. DCA within a single sector is used. System capacity can be increased significantly using this approach. Even if channels are not reused in different beams of the same sector, improvement in system capacity is still possible. In this paper, we consider such a case and propose a scheme which combines the advantages of switched multibeam antennas and dynamic channel assignment, in which *DCA is used across multiple sectors*.

DCA is a technique which can improve channel reuse in cellular communications [3,11,25,26]. In DCA, all channels are dynamically assigned to wireless users, subject to the constraints on the allowable co-channel interference. Channel rearrangement can be used to avoid unnecessary blocking of calls and allows the channel resources to be used efficiently. In channel rearrangement, the channel used to serve a particular call is not fixed. Depending on the channel occupancy and interference conditions, a call may switch between several different channels during its lifetime.

We consider a large population of mobile wireless users in a large geographical region covered by cells. A cell is divided into several sectors, each of which is covered by several directional beams. Specific channels are allocated to each sector as in FCA, and DCA is used across multiple sectors. Channels of a sector are dynamically assigned to wireless users in the sector as long as the co-channel interference constraints are satisfied. A wireless user can access any of the channels of a sector without regard to the beam through which it communicates. A 120°-sectored multibeam cellular system is considered. A system layout with two beams in each sector and a cluster size of four is investigated. The use of directional multibeam antennas in

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a 120°-sectored cellular system can reduce the cluster size from seven to four while the link quality is still maintained approximately the same. Similar approaches can be developed for a lager cluster size or a larger number of beams. For 120°-sectored systems with a larger cluster size, the number of interfering co-channel beams is still two because there are still only six co-channel cells in the first tier for hexagonal geometry and still only two beams per sector. The same approach applies. However, the increased cluster size will reduce the spectrum efficiency (which is defined by Erlang/km^2/kHz [1]) and decrease the system capacity significantly. With the increased number of beams, wireless users will experience more hand-offs when they are roaming between beams in the service area. The control signal traffic associated with the hand-offs increases the signaling load of the system at the network level.

Models to compute fundamental traffic performance measures for the proposed scheme are devised. These include call blocking probability, forced termination probability, hand-off activity, carried traffic and channel rearrangement rate. Multidimensional birth–death processes are discussed in [4,21,22]. The global balance equations are determined and solved for the state probabilities, using the framework developed in earlier work [21,22]. Performance characteristics are found from these state probabilities.

System traffic performance can be improved by using a multibeam scheme and DCA technique. The proposed 6-beam multibeam scheme using DCA and traditional 6-sectored scheme using FCA are compared. For a fixed offered traffic, the blocking probability of calls can be reduced significantly. Alternatively, more new call traffic can be supported while the blocking probability is maintained. From a wireless subscriber's point of view, forced termination probability is a major concern. Smaller forced termination probability means a wireless user's call will be less likely to experience interruption during its lifetime. The proposed multibeam scheme has smaller forced termination probability than traditional sectorized FCA schemes.

2. Model description

2.1. System model

We consider a large geographical region covered by cells and traversed by large numbers of wireless platforms, such as vehicles and pedestrians. Each platform can support at most one call. An example model that we consider is a 120°-sectored multibeam cellular system with two beams in each sector and a cluster size of four. This provides a total of six beams per cell as shown in figure 1. The beam in the counterclockwise direction is called the left beam and the other is called the right beam. Specific channels are allocated to each sector as in FCA. There are C channels assigned to each sector. Sectors which have the same angular orientation are allocated the same set of channels. These are co-channel sectors. For example, as shown in

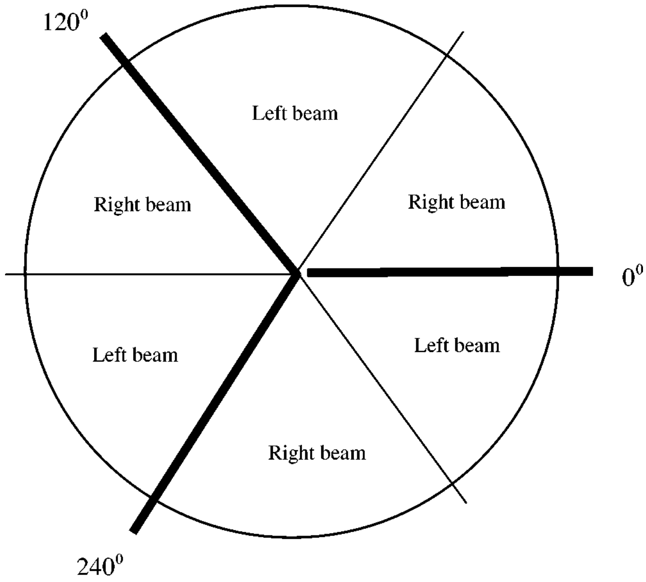


Figure 1. The beam layout of 120°-sectorized multibeam cellular communication systems with 2 beams in each sector.

figure 2a, sectors 0, 1, 2, 3, and so on are co-channel sectors. *We define* ***the interfering co-channel beams of a given sector*** *as the beams in the neighboring co-channel sectors that point to* ***the given sector*** *Here only* ***the first tier*** *co-channel sectors of a given sector are considered.* There are two interfering co-channel beams in the first tier co-channel sectors for a given sector. We can see from figure 2a, *the right beam of sector 0 and the left beam of sector 1* point to the same sector – *sector 7*. These are *interfering co-channel beams of sector 7*. Similarly, *the right beam of sector 1 and the left beam of sector 2* are *the interfering co-channel beams of sector 8*, and so on. Such a pair of interfering co-channel beams is considered at the same time when the channel assignment is made.

The simultaneous use of the same channel in a pair of interfering co-channel beams will create two interfering wireless users to a desired wireless user in a sector of a co-channel cell in the first tier. In a traditional 6-sectored cellular system with FCA there would be only one interfering wireless user. This excessive co-channel interference to a desired wireless user when compared with the traditional 6-sectored cellular system at the same cluster size must be precluded by a constraint on the DCA scheme. The simultaneous use of the same channel in the right beam of sector 0 and the left beam of sector 1 will cause excessive co-channel interference in sector 7; such use must be precluded. Similarly, the simultaneous use of the same channel in the right beam of sector 1 and the left beam of sector 2 will cause excessive co-channel interference in sector 8 and also must be precluded, and so on. In general, the simultaneous use of the same channel in any such a pair of interfering co-channel beams must be precluded, which indicates that the total number of channels in use in any such a pair of interfering co-channel beams cannot exceed C. There is a possible circumstance that the

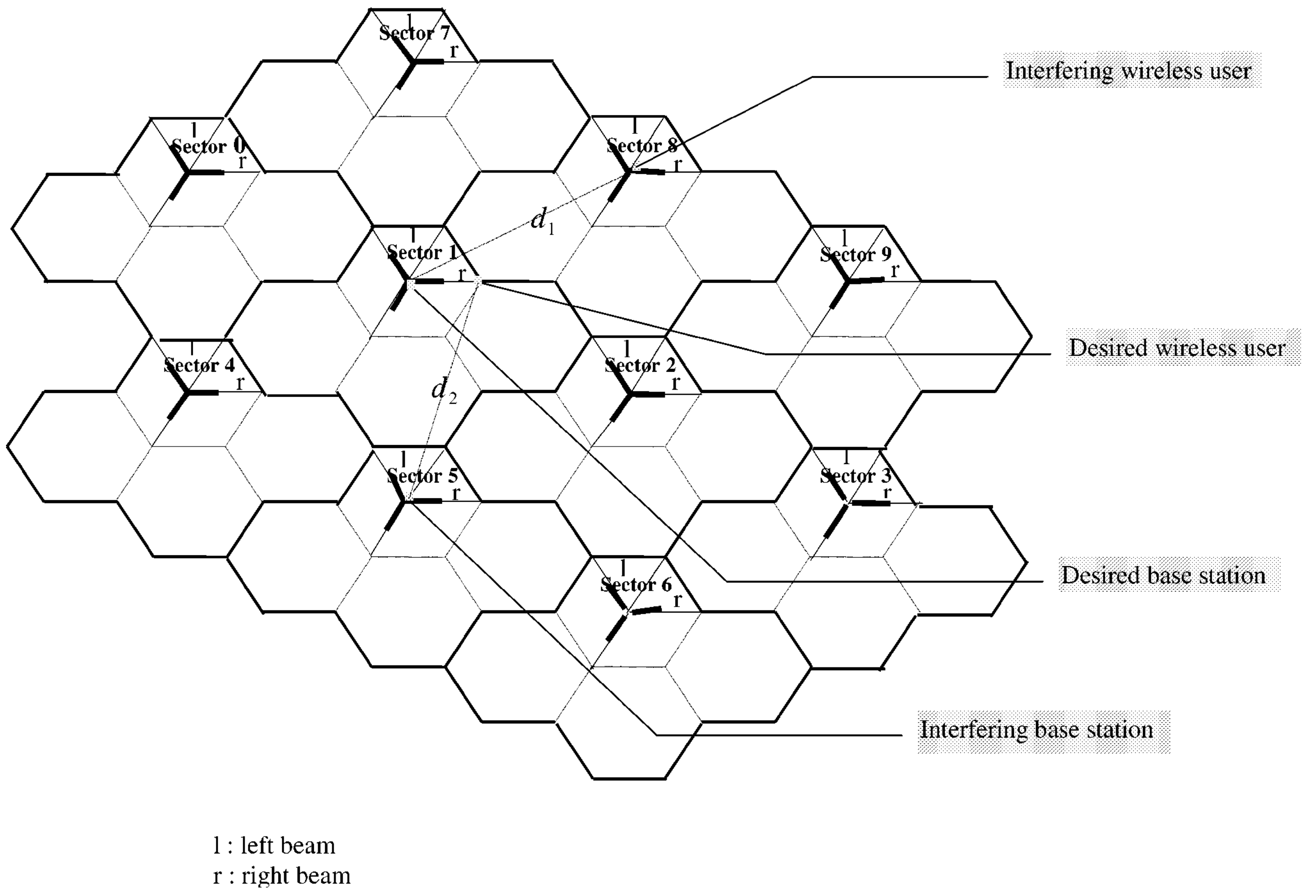


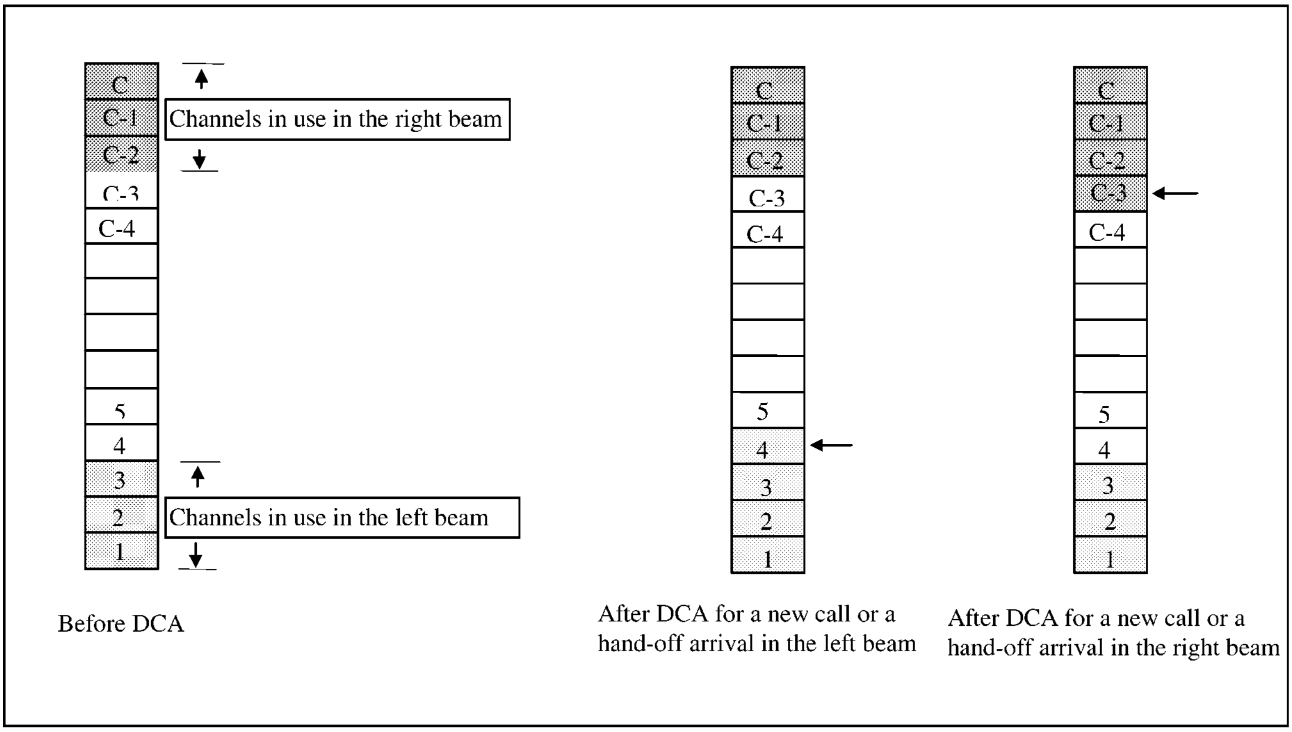
Figure 2a. Analysis of co-channel interference for both up-link and down-link case in 120°-sectored multibeam system with 2 beams in each sector. The cluster size is 4 ($N = 4$).

same channel is not used in the given sector, the simultaneous use of the same channel in the interfering co-channel beams of that sector seems fine. However, such an approach increases the complexity of system implementation and number of database lookups. The use and assignment of channels at a base station is now coupled to the use and assignment of channels of its six neighboring co-channel base stations. Each base station must track the channel usage of its six neighboring co-channel base stations to make an appropriate assignment of channels. In this paper, we will only consider the approach that the simultaneous use of the same channels in the interfering co-channel beams of a given sector is precluded without regard to the channel use condition of the same channel in the given sector. This approach will only require a given base station to track the channel usage of its two adjacent co-channel base stations whose use and assignment of channels is coupled to the use and assignment of channels of the given base station.

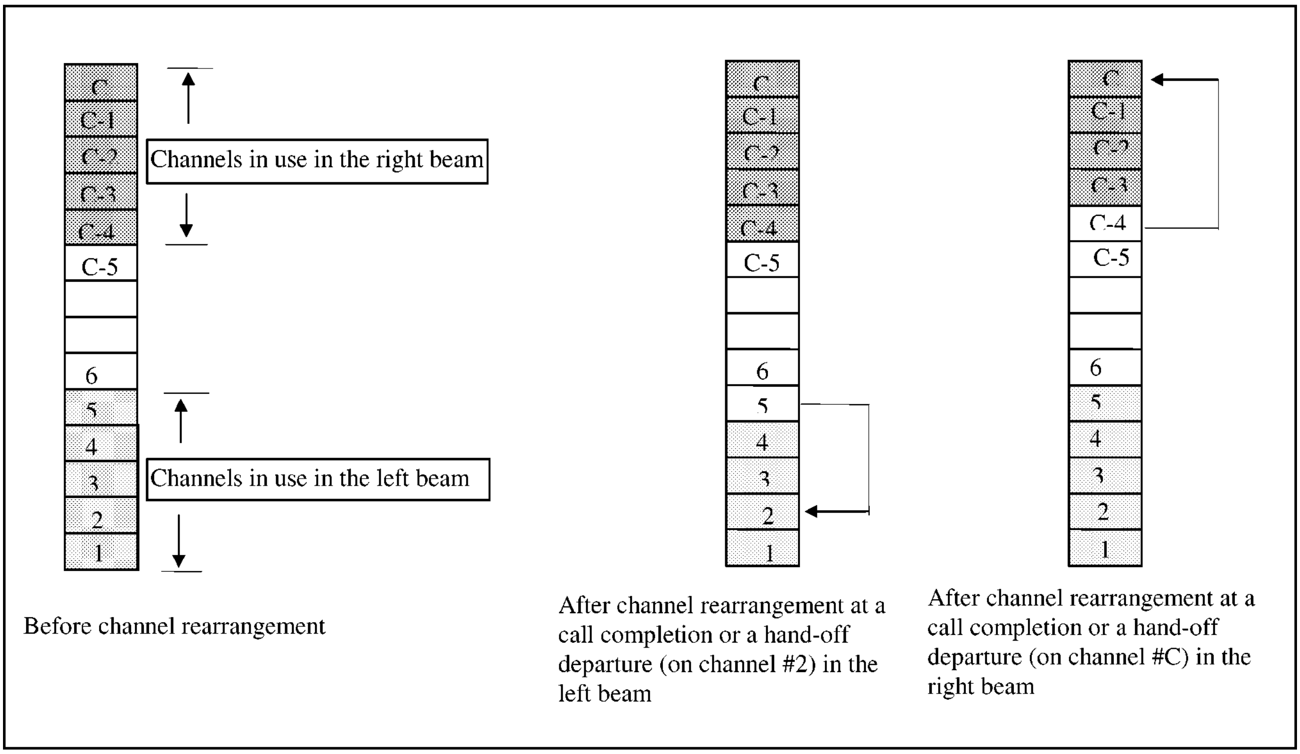
2.2. Cut-off priority scheme

A cut-off priority scheme is used to favor *hand-off calls* with respect to *new calls*. This approach reserves a certain number of channels for use by hand-off calls. Specific channels are not reserved, just the number. Referring to figure 2a, consider sector 1, which has two beams (left and right). The left beam of sector 1 is an interfering co-channel beam of sector 7. We see from figure 2a that the right beam of sector 0 is also an interfering co-channel beam of sector 7. Since the simultaneous use of the same channel in the left beam of sector 1 and the right beam of sector 0 will cause excessive co-channel interference in sector 7, the use and assignment of channels to calls in sector 1 is coupled to the use of channels in sector 0. Succinctly, because the left beam of sector 1 and the right beam of sector 0 are interfering co-channel beams of the same sector (sector 7), the simultaneous use of channels in sectors 1 and 0 must be constrained. Similarly, the right beam of sector 1 and the left beam of sector 2 are interfering co-channel beams of the same sector (sector 8), the simultaneous use of channels in sectors 1 and 2 must be constrained. In order to provide cut-off priority for hand-off calls, we will use the following channel assignment constraints. For clarity we discuss this assignment for sector 1, but the rules are similar for all sectors.

1. The total number of channels in use in sector 1 cannot exceed C. (The channel limit is C.)
2. Sector 1 reserves C_{h1} channels of the C channels allocated to the sector for use by hand-off calls. Thus new calls that arrive in sector 1 will be blocked if the number of channels in use in sector 1 is $C - C_{\mathrm{h1}}$ or greater.



Dynamic channel assignment



Channel rearrangement

Figure 2b. Explanations of dynamic channel assignment and channel rearrangement.

3. The total number of channels in use in the pair of these interfering co-channel beams (the *left beam of sector 1* and the *right beam of sector 0*) cannot exceed C. Similarly, the number of channels in use in the *right beam of sector 1* and the *left beam of sector 2* cannot exceed C (and so forth, in like manner).
4. $C_{\rm h2}$ channels of the C channels are reserved for use by hand-off calls in a pair of the interfering co-channel beams. Thus new calls that arrive in the *left beam of sector 1* will be blocked if the number of channels in use in the *left beam of sector 1* and the *right beam of sector 0* is $C - C_{\rm h2}$ or greater. Similarly, new calls that arrive in the *right beam of sector 1* will be blocked if the number of channels in use in the *right beam of sector 1* and the *left beam of sector 2* is $C - C_{\rm h2}$ or greater. (This pair is the interfering co-channel beams of sector 8.)
5. Hand-off attempts (to sector 1) will fail if the number of channels in use in sector 1 is C. Hand-off attempts (to the left beam of sector 1) will also fail if the number of channels in use in the left beam of sector 1 and the right beam of sector 0 is C, and hand-off attempts (to the right beam of sector 1) will also fail if the number of channels in use in the right beam of sector 1 and the left beam of sector 2 is C.

In this way, hand-off calls have access to more channels than new calls do, and increasing $C_{\rm h1}$ or $C_{\rm h2}$ provides increasing priority for hand-offs at the expense of blocking new call originations. Thus forced termination and blocking performance can be exchanged.

2.3. Teletraffic model

A large population of wireless platforms is considered. The new call origination rate from a non-communicating platform is denoted by Λ. The number of non-communicating platforms in any sector is denoted by ν. Therefore, the total call origination rate in a sector, Λ_n, is $\Lambda_n = \Lambda \cdot \nu$. It is assumed that the number of non-communicating platforms is much larger than the number of channels in a sector so that the call generation rate does not depend on the number of calls in progress. This infinite population model is reasonable for commercial cellular systems.

In earlier work [8,22], the concept of dwell time was used to characterize the mobility of platforms. This is the amount of time that a wireless platform is within the communicating range of a given gateway. The dwell time depends on many factors such as propagation conditions, the path that a wireless platform follows, its velocity profile along the path, and especially the definition of communicating range. The dwell time is modeled by a negative exponential distribution (n.e.d.) random variable having a mean $T_{\rm D} = 1/\mu_{\rm D}$. The unencumbered call session duration of a call is assumed to be random with a n.e.d. and having a mean $T_{\rm C} = 1/\mu_{\rm C}$. The new call originations are assumed to follow a Poisson point process with a mean arrival rate $\Lambda_{nb} = \Lambda_n/2$.

3. Dynamic channel assignment and channel rearrangement

Dynamic channel assignment can improve traffic performance. For DCA in the present context, there is no fixed set of channels allocated to each beam. Subject to the allowable co-channel interference, the channels of a *sector* are dynamically assigned to wireless users in that *sector* without regard to the *beam* through which they communicate. Channel rearrangement is used to avoid unnecessary blocking of calls and allow the channel resources to be used more efficiently. When DCA is combined with channel rearrangement, additional improvement is possible. The mechanism to assign channels dynamically and to rearrange channels is described as follows.

3.1. Dynamic channel assignment

The channels of a sector are numbered by integers from 1 to C. A channel is assigned to a new call or a hand-off call according to the following strategy. At the time of a new call or a hand-off call arrival in the left beam of a sector, an available channel with the lowest number is assigned to serve the call. If a new call or a hand-off call arrives in the right beam of a sector, an available channel with the highest number is assigned to serve the call. The channel that is assigned to a user in a beam must be available in the sector to which the beam belongs. This is because the same channels are not allowed to be used in the same sector. Furthermore, the channel also must be available in the interfering co-channel beams. This is because the simultaneous use of the same channel in these beams will cause excessive co-channel interference. With these constraints channels are dynamically assigned to wireless users without violating the co-channel interference constraints. Examples of channel arrangement are shown in the top of figure 2b. Total number of channels per sector is C. Channels #1 to #3 are in use in the left beam and channels #$C-2$ to #C are in use in the right beam. When a new call or a hand-off call arrives in the left beam, channel #4 is the available channel with the lowest number and thus is assigned to serve the call. When a new call or a hand-off call arrives in the right beam, channel #$C-3$ is the available channel with the highest number and thus is assigned to serve the call.

3.2. Channel rearrangement

Channel rearrangement is used at a call completion and a hand-off departure. Channels in use are always rearranged to maintain a *compact pattern*. That is, channels in use in the left beam are always those channels with the lowest numbers and channels in use in the right beam are always those channels with the highest numbers. At the time of a call completion or a hand-off departure, channels in use are rearranged if the call completion or hand-off departure creates a pattern that is not compact. This fosters a high level of channel reuse. Since channels that are currently in use must satisfy the co-channel interference constraints, once they are released due to call completion or hand-off departure, they are good for use by other calls that switch channels due to the channel rearrangement. Examples of channel rearrangement are shown in the bottom of figure 2b. As an example for the left beam, the call using channel #2 in the left beam completes and the call using channel #5 switches to use channel #2 to maintain a *compact pattern*. As an example for the right beam, the call using channel #C completes and the call using channel #$C-4$ switches to use channel #C.

4. Analysis of co-channel interference

We consider the usual hexagonal geometry and a 120°-sectorized multibeam scheme with two beams in each sector. Let R denote the radius of a cell. The reuse distance D is defined as the distance between the base stations of two nearest co-channel cells. Let N denote the cluster size, which is related to the reuse shift parameters (i,j) by $N=i^2+ij+j^2$. The integers i and j determine the reuse pattern and identify co-channel cells. The co-channel reuse factor, Q, is defined as the ratio of D to R. This ratio is

$$\frac{D}{R}=\sqrt{3N}.$$

We consider the carrier-to-interference ratio (CIR) of multibeam cellular communication systems in the worst case. Normal cellular practice specifies CIR to be 18 dB or higher. This is based on subjective tests and the criterion that 75% of the users say voice quality is “good” or “excellent” in 90% of the total covered area on a flat terrain [15]. Let I denote the normalized co-channel interference, which is normalized by the desired power. The CIR of multibeam cellular communication systems can be calculated as

$$\mathrm{CIR}=10\log_{10}\left(\frac{1}{I}\right)\ \mathrm{dB}. \tag{1}$$

Consider a cellular system with cluster size $N=4$. As shown in figure 2a, sectors 0, 1, 2, 3, and so on are co-channel sectors. There are two co-channel beams in the first tier co-channel sectors for a sector. The right beam of sector 4 and left beam of sector 5 point to the same sector – sector 1, and are the co-channel beams of sector 1. Similarly, the right beam of sector 5 and left beam of sector 6 are co-channel beams of sector 2, etc. In the following discussion we use “up-link” to denote wireless user to base station and “down-link” to denote base station to wireless user.

In the worst case of up-link, the desired wireless user is at the vertex of a given sector and the interfering wireless user is at the position that is nearest to the gateway of the given sector in the co-channel sector. In the worst case of down-link, the desired wireless user is at the vertex of a given sector which is closest to the interfering gateway of its co-channel sector. For example, suppose that the desired wireless user is served by the right beam of the sector 1 as shown in figure 2. Consider the up-link case. In the worst case, the distance between the desired wireless user and its serving gateway is R, and the distance between the interfering wireless user in co-channel sector 8 and the desired gateway of sector 1 is d_1. From the geometry of figure 2, for a cluster of 4, d_1 is calculated as $2R\sqrt{3}$. Let I_u denote the value of I for the up-link in the worst case. The normalized co-channel interference on the up-link, I_u, can be calculated as

$$I_\mathrm{u}=\left(\frac{R}{d_1}\right)^{\gamma}, \tag{2}$$

in which $d_1=2R\sqrt{3}$ and γ is a propagation constant that is heavily influenced by the actual terrain environment. The value of γ usually lies between 3 and 5.

Consider the down-link case. In the worst case, the distance between the desired wireless user and its serving gateway is R, and the distance between the desired wireless user and its nearest interfering gateway of sector 5 is d_2. From geometry of figure 2, d_2 is calculated as $\sqrt{13}R$. Let I_d denote the value of I for the down-link in the worst case. The normalized co-channel interference on the down-link, I_d, can be calculated as

$$I_\mathrm{d}=\left(\frac{R}{d_2}\right)^{\gamma}, \tag{3}$$

in which $d_2=\sqrt{13}R$.

The comparisons of CIR between the traditional 6-sector scheme and the proposed 6-beam multibeam scheme are shown in table 1 for both up-link and down-link in the worst case. It can be seen that the CIR on the up-link is worse than the CIR on the down-link for both schemes. Therefore up-link is the dominant one that limits quality of communication links. CIR on the up-link has the same value for both schemes.

Table 1
Comparisons of CIR between the traditional scheme and the proposed scheme in the worst case. Parameters: *cluster size* $N=4$, *propagation constant* $\gamma=4$.

CIR	UP-LINK	DOWN-LINK
Traditional 6-sector scheme	21.58 dB	25.58 dB
Proposed 6-beam multibeam scheme	21.58 dB	22.28 dB

5. State description

For the problem under consideration in this paper (infinite population model, single platform type and single call type), two state variables are needed to describe the status of each sector: one is the number of channels in use in the left beam and the other is the number of channels in use in the right beam. A complete state representation for the whole system will be a string of sector states with two state variables for each sector. However, the huge number of system states precludes pursuing this approach for most cases of interest. A simplified approach is to decouple a sector from others (using average new call arrival rates and average hand-off arrival rates from neighbors) and to model the statistical behavior of a given sector independently from the behavior of other sectors. Therefore, even though we calculate the theoretical performance characteristics from a given sector independently from others, we do consider the effect from its neighbors. This is similar to the approach used in [2,21,22]. Because of the homogeneous property of the system, the behavior of any sector in statistical equilibrium is the same as any other. However, due to the co-channel interference constraints, no two interfering co-channel beams of a given sector can use the same channel simultaneously. These two interfering co-channel beams belong to two adjacent co-channel sectors. Thus the activities of any two adjacent co-channel sectors located in the same line perpendicular to the orientation of the co-channel sectors are not statistically independent. To account for this dependency, we consider two such adjacent co-channel sectors at the same time. That is, we consider two such adjacent co-channel sectors as a basic element. We can define the state of the basic element by a sequence of nonnegative integers, l_1, r_1, l_2, r_2. In this sequence, the state variables $l_i,\ i = 1, 2$, is the number of calls served by the left beam of sector i and $r_i,\ i = 1, 2$, is the number of calls served by the right beam of sector i. Then, for convenience, we order the states using an index $s = 0, 1, 2, \ldots, S_{\text{max}}$. Thereafter, l_i and $r_i,\ i = 1, 2$, can be shown as explicitly dependent on the state. That is, $l_i = l(s, i)$ and $r_i = r(s, i),\ i = 1, 2$, in which $l(s, i)$ is the number of calls served by the left beam of sector i when the basic element is in state s and $r(s, i)$ is the number of calls served by the right beam of sector i when the basic element is in state s.

If C denotes the number of channels in each sector, we can specify the constraints on permissible states as

$$l(s, i) + r(s, i) \leqslant C, \qquad \text{for all } i, \tag{4}$$

$$r(s, i) + l(s, i + 1) \leqslant C, \quad \text{for all } i. \tag{5}$$

The inequality (4) means that the number of channels in use in any sector when the basic element is in state s cannot be larger than C. The inequality (5) means that the number of channels in use in two interfering co-channel beams of any given sector when the basic element is in state s cannot be larger than C.

6. Driving processes and state transition flow

The state probabilities $p(s)$ in statistical equilibrium are required to determine the performance measures of interest. To calculate the state probabilities, the state transitions and the corresponding transition rates must be identified and calculated. There are four relevant driving processes. These are: $\{\text{n}\}$ the generation of new calls in the beams of interest; $\{\text{c}\}$ the completion of calls in the beams of interest; $\{\text{h}\}$ the arrival of communicating wireless users at the beams of interest from other sectors; $\{\text{d}\}$ the departure of communicating wireless users from the beams of interest to other sectors; $\{\text{hd}\}$ the call hand-off of communicating wireless users within beams of the same sector. The transition rates into state s from predecessor state x due to these driving processes are denoted by $r_{\text{n}i}, r_{\text{c}i}, r_{\text{h}i}, r_{\text{d}i},\ i = 1, 2, 3, 4$, and $r_{\text{hd}i},\ i = 1, \ldots, 6$, respectively. All driving processes and corresponding state transition flow are explained in the appendix of [19].

7. Flow balance equations

From the equations given above, the total transition flow into s from any permissible predecessor state x can be found using

$$q(s, x) = \sum_{i=1}^{4} \left[\gamma_{\text{n}i}(s, x) + \gamma_{\text{c}i}(s, x) + \gamma_{\text{h}i}(s, x) + \gamma_{\text{d}i}(s, x)\right] + \sum_{i=1}^{6} \gamma_{\text{hd}i}(s, x), \tag{6}$$

in which $s \neq x$, and flow into a state has been taken as a positive quantity. The total flow out of state s is denoted as $q(s, s)$, and is given by

$$q(s, s) = -\sum_{\substack{k=0\\k\neq s}}^{S_{\text{max}}} q(k, s). \tag{7}$$

To find the statistical equilibrium state probabilities for a sector, we write the flow balance equations for the states. These are a set of $S_{\text{max}} + 1$ simultaneous equations for the unknown state probabilities $p(s)$. They are of the form

$$\begin{aligned} &\sum_{j=0}^{S_{\text{max}}} q(i, j)p(j) = 0, \quad i = 0, 1, 2, \ldots, S_{\text{max}} - 1, \\ &\sum_{j=0}^{S_{\text{max}}} p(j) = 1, \end{aligned} \tag{8}$$

in which, for $i \neq j,\ q(i, j)$ represents the net transition flow into state i from state j, and $q(i, i)$ is the total transition flow out of state i. These equations express that in statistical equailibrium, the net probability flow into any state is zero, and the sum of the probabilities is unity.

8. Performance measures

There are six performance measures of interest:

(1) call blocking probability,

(2) hand-off failure probability,

(3) forced termination probability,

(4) hand-off activity,

(5) carried traffic, and

(6) channel rearrangement rate.

Once the statistic equilibrium state probabilities and transition flows are found, the required performance measures can be calculated.

8.1. Blocking probability

The blocking probability for a call is the average fraction of new calls that are denied access to a channel. Blocking of new calls occurs if there are no channels to serve the call. We define the following set of states:

$$B = \{s\colon\ l(s,1) + r(s,1) \geqslant C - C_{\mathrm{h1}} \cup r(s,1) + l(s,2) \geqslant C - C_{\mathrm{h2}}\}. \tag{9}$$

Then the blocking probability is

$$P_B = \sum_{s \in B} p(s). \tag{10}$$

8.2. Hand-off failure probability

The hand-off failure probability for calls is the average fraction of hand-off attempts that are denied a channel. We note that hand-off attempts have potential access to all channels of a sector and the co-channel beams of any given site without regard to C_{h1} or C_{h2}. We define the following sets of states, in which hand-off attempts will fail:

$$H_1 = \{s\colon\ l(s,1) + r(s,1) = C \cup r(s,1) + l(s,2) = C\}, \tag{11}$$

$$H_2 = \{s\colon\ l(s,1) + r(s,1) = C\}. \tag{12}$$

Then the hand-off failure probability due to call hand-offs across sectors can be written as

$$P_{H_1} = \sum_{s \in H_1} p(s) \tag{13}$$

and the hand-off failure probability due to call hand-offs within beams of the same sector can be written as

$$P_{H_2} = \sum_{s \in H_2} p(s). \tag{14}$$

If F_1 denote the fraction of hand-off departures from a beam of a sector to other sectors and F_2 denote the fraction of hand-off departures from a beam to other beam of the same sector, we have $F_1 + F_2 = 1$. Then the average hand-off failure probability is

$$P_{\mathrm{H}} = F_1 \cdot P_{H_1} + F_2 \cdot P_{H_2}. \tag{15}$$

8.3. Forced termination probability

The forced termination probability is defined as the probability that a call that is not blocked is interrupted due to hand-off failure during its lifetime. Let p be the probability that a non-blocked call satisfactorily completes before the hand-off attempt occurs, and q be the probability that hand-off attempt occurs first. Because of the negative exponential assumption, we have

$$p = \mu_{\mathrm{C}}/(\mu_{\mathrm{C}} + \mu_{\mathrm{D}}), \tag{16}$$

$$q = \mu_{\mathrm{D}}/(\mu_{\mathrm{C}} + \mu_{\mathrm{D}}). \tag{17}$$

The probability that a non-blocked call is forced to terminate on its kth hand-off attempt is

$$Y(k) = P_{\mathrm{H}} \cdot q^k \cdot (1 - P_{\mathrm{H}})^{k-1}. \tag{18}$$

The forced termination probability is therefore

$$P_{\mathrm{FT}} = \sum_{k=1}^{\infty} Y(k). \tag{19}$$

This can be compactly written in closed form as

$$P_{\mathrm{FT}} = \frac{q \cdot P_{\mathrm{H}}}{1 - q \cdot (1 - P_{\mathrm{H}})}. \tag{20}$$

8.4. Hand-off activity

Hand-off activity H_{A} is the expected number of hand-off attempts that a non-blocked call will experience during its lifetime. There will be exactly k hand-off attempts if (1) the call fails at the kth hand-off attempt, or (2) the call succeeds at the kth hand-off attempt but successfully completes before the $(k+1)$th hand-off attempt.

The probability that the call fails at the kth hand-off attempt is

$$Z_1(k) = q^k \cdot (1 - P_{\mathrm{H}})^{k-1} \cdot P_{\mathrm{H}}. \tag{21}$$

The probability that the call succeeds at the kth hand-off attempt but successfully completes before the $(k+1)$th hand-off attempt is

$$Z_2(k) = q^k \cdot (1 - P_{\mathrm{H}})^k \cdot p. \tag{22}$$

Consequently, the hand-off activity is

$$H_{\mathrm{A}} = \sum_{k=1}^{\infty} k \cdot [Z_1(k) + Z_2(k)]. \tag{23}$$

This can be simplified to

$$H_{\mathrm{A}} = q \cdot [P_{\mathrm{H}} + (1 - P_{\mathrm{H}}) \cdot p] / [1 - q \cdot (1 - P_{\mathrm{H}})]^2. \tag{24}$$

8.5. Carried traffic

The carried traffic in a beam is the average number of channels occupied by the calls in that beam. The carried traffic in a beam is

$$A_{\mathrm{c}} = \sum_{s=0}^{S_{\max}} r(s,1) \cdot p(s). \tag{25}$$

8.6. Channel rearrangement rate

Channel rearrangement rate is the average rate of channels that have to be rearranged. We define the following set of states:

$$W = \{s\colon\ r(s,1) > 0\}. \tag{26}$$

The channel rearrangement rate in a beam is

$$R = \mu_c \cdot \sum_{s \in W} \bigl(r(s,1) - 1\bigr) \cdot p(s). \tag{27}$$

9. Numerical results and discussion

Numerical results were generated using the approach described in this paper. For all figures, an unencumbered call duration of 100 seconds was assumed. The fraction of hand-off departures from a beam of a sector to *other sectors* was assumed to be $2/3$ and the fraction of hand-off departures from a beam to *another beam of the same sector* was assumed to be $1/3$. Sectors have $C = 20$ channels each, which corresponds to 10 channels per beam. The total number of platforms per beam is 200. For figures 3–5, the new call origination rate per platform was varied from 1.7×10^{-4} to 3.3×10^{-4} calls/sec. For figures 6 and 7, the dwell time of platforms was varied from 25 to 150 seconds.

Figure 3 shows the dependence of call blocking probability on demand. The proposed 6-beam multibeam DCA scheme performs better than the traditional 6-sector FCA scheme. System capacity can be improved by using a multibeam scheme and DCA technique because of more channel reuse. For a fixed offered traffic, the blocking probability of calls can be reduced significantly. Alternatively, more new call traffic can be accommodated while the blocking probability is maintained below the required threshold.

Figure 4 shows the dependence of forced termination probability on demand. The proposed 6-beam multibeam scheme has smaller forced termination probability than the traditional 6-sector scheme for a fixed offered traffic. Alternatively, more new call traffic can be supported while the forced termination probability is maintained. Increasing the value of C_h (or C_{h1}, C_{h2}) reduces the forced termination probability at the cost of increasing blocking probability.

Figure 5 shows how carried traffic depends on demand. The figures show that (for given offered traffic) blocking and forced termination probabilities are greatly reduced by the use of the proposed scheme. At the same time carried

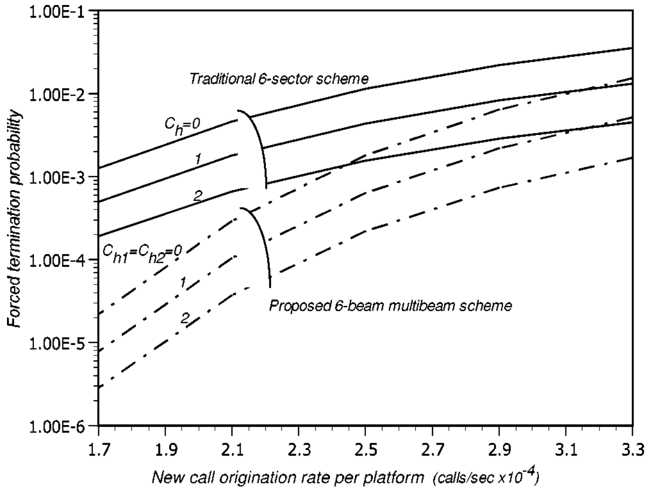


Figure 4. Forced termination probability depends on demand. Parameters: total number of platforms per beam = 200, number of channels per beam = 10, T_C = 100 seconds, T_D = 150 seconds, $F_1 = 2/3$, $F_2 = 1/3$.

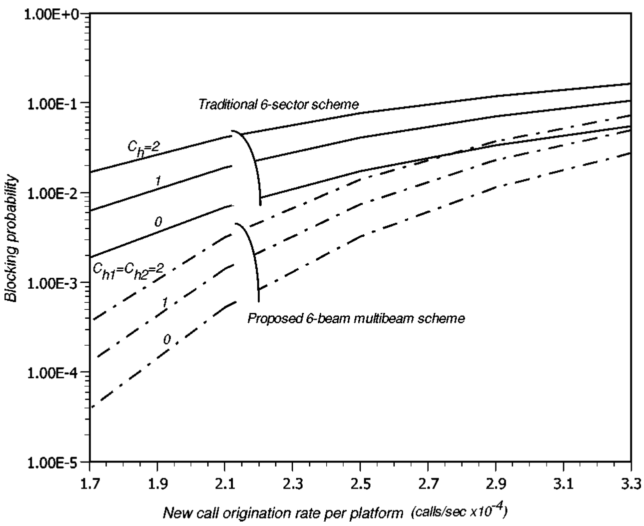


Figure 3. Blocking probability depends on demand. Parameters: total number of platforms per beam = 200, number of channels per beam = 10, T_C = 100 seconds, T_D = 150 seconds, $F_1 = 2/3$, $F_2 = 1/3$.

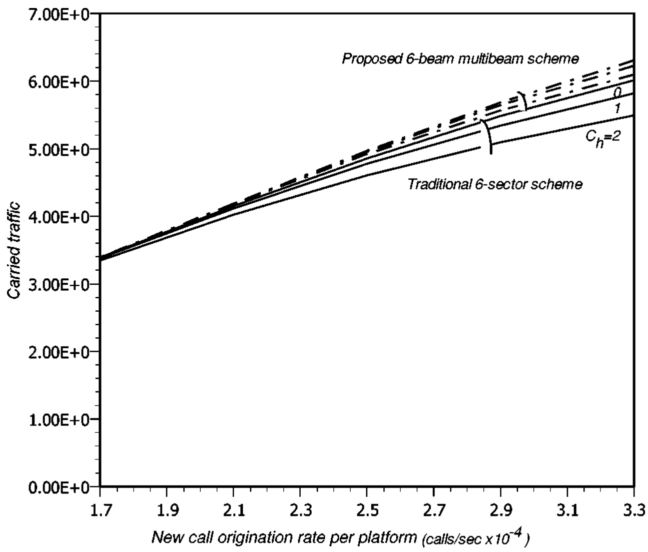


Figure 5. Carried traffic depends on demand. Parameters: total number of platforms per beam = 200, number of of channels per beam = 10, T_C = 100 seconds, T_D = 150 seconds, $F_1 = 2/3, F_2 = 1/3$.

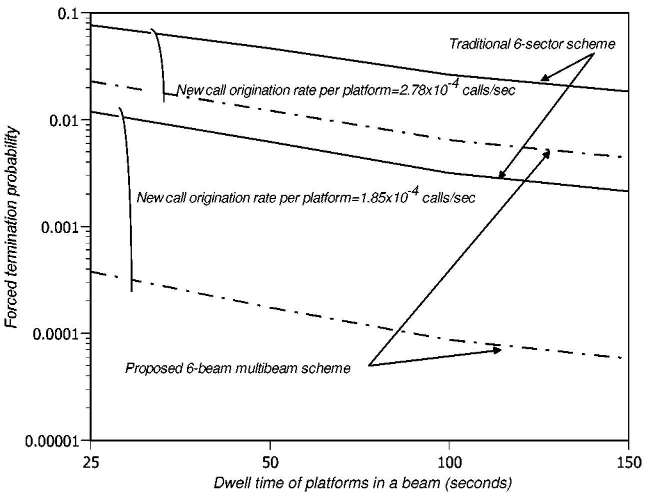


Figure 6. Forced termination probability depends on mobility. Parameters: total number of platforms per beam = 200, number of channels per beam = 10, $T_C = 100$ seconds, $C_{h1} = C_{h2} = 0$, $F_1 = 2/3$, $F_2 = 1/3$. —: forced termination probability for traditional 6-sector scheme, –·–·: forced termination probability for peoposed 6-beam multibeam scheme.

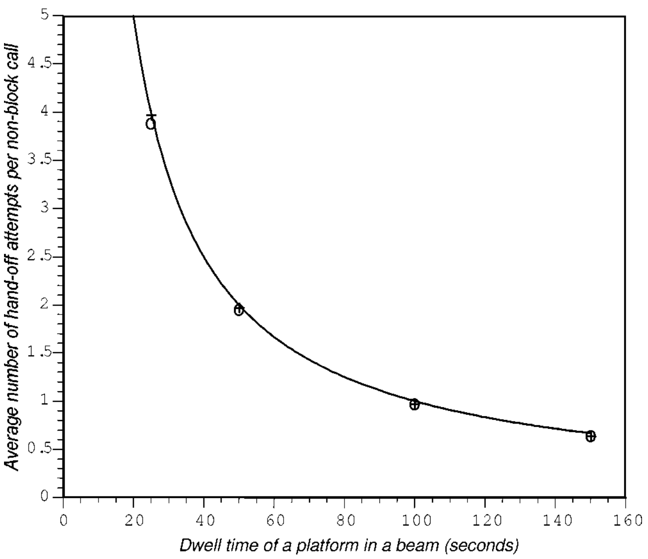


Figure 7. Hand-off activity depends on mobility. Parameters: total number of platforms per beam = 200, number of channels per beam = 10, $T_C = 100$ seconds, $C_{h1} = C_{h2} = 0$, $F_1 = 2/3$, $F_2 = 1/3$. ○: hand-off activity for new call origination rate per platform is 1 call per hour (or 2.78×10^{-4} calls/sec), +: hand-off activity for new call origination rate per platform is 1 call per one and half an hour (or 1.85×10^{-4} calls/sec).

traffic is relatively the same (but increases slightly). This is an advantage of the proposed scheme which allows overall traffic improvement of the major performance metrics. The cost is increased complexity of implementation in comparison with pure fixed channel assignment. For low demand, the carried traffic increases linearly with increasing demand. For higher demand, the increase in traffic is less than the proportional increase in demand. This is especially true for larger C_h (or C_{h1}, C_{h2}) since blocking performance is sacrificed to accommodate hand-offs.

Figure 6 shows how forced termination probability depends on platform mobility. For a fixed dwell time, the proposed 6-beam multibeam scheme has better forced termination performance than the traditional 6-sector scheme under the same offered traffic. Alternatively, higher mobility of wireless platforms can be supported while the forced termination probability is maintained.

Figure 7 shows the dependence of hand-off activity on platform mobility. When the blocking and hand-off failure probabilities are small, the average number of hand-off attempts per non-blocked call is approximately equal to the ratio of unencumbered call session duration of a call to its dwell time – that is, T_C/T_D. This ratio is plotted as a solid curve in figure 7. The average number of hand-off attempts per non-blocked call calculated using the multidimensional birth–death model is also shown. These are indicated by points using symbols "○" and "+" for two different new call origination rates per platform (1 call per hour per platform and 1 call every one and half an hour, respectively). It can be seen that these values are quite close to each other. This is because the range of interest for blocking and hand-off failure probabilities are small values, usually around 0.01 or less. Hand-off activity is insensitive to the offered traffic, but is sensitive to the mobility. Wireless platforms with higher mobility cross cell boundaries more often. The average number of hand-off attempts per non-blocked call decreases as the dwell time increases.

10. Conclusions

We proposed a multibeam scheme using dynamic channel assignment across multiple sectors. The combined approach enhances channel reuse. The framework using a state description and multidimensional birth–death processes was used to compute theoretical traffic performance characteristics for the scheme. In comparison with traditional schemes using FCA, blocking probability of calls can be reduced significantly for a fixed offered traffic. Alternatively, system capacity can be increased while the blocking probability is maintained below the required threshold. The smaller forced termination probability means wireless users will less likely experience interruption during the lifetime of calls. The proposed multibeam scheme has smaller forced termination probability than the traditional scheme.

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