

# Multichannel adaptive signal detection: Basic theory and literature review

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**Abstract** Multichannel adaptive signal detection jointly uses the test and training data to form an adaptive detector, and then make a decision on whether a target exists or not. Remarkably, the resulting adaptive detectors usually possess the constant false alarm rate (CFAR) properties, and hence no additional CFAR processing is needed. Filtering is not needed as a processing procedure either, since the function of filtering is embedded in the adaptive detector. Moreover, adaptive detection usually exhibits better detection performance than the filtering-then-CFAR detection technique. Multichannel adaptive signal detection has been more than 30 years since the first multichannel adaptive detector was proposed by Kelly in 1986. However, there are fewer overview articles on this topic. In this paper we give a tutorial overview of multichannel adaptive signal detection, with emphasis on Gaussian background. We present the main design criteria for adaptive detectors, investigate the relationship between adaptive detection and filtering-then-CFAR detection, relationship between adaptive detectors and adaptive filters, summarize typical adaptive detectors, show numerical examples, give comprehensive literature review, and discuss some possible further research tracks.

**Keywords** Constant false alarm rate, multichannel signal, signal mismatch, statistical distribution, subspace signal.

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## 1 Introduction

Signal detection in noise is a fundamental problem in various areas, such as radar, sonar, communications, optical image, hyperspectral imagery, remote sensing, medical imaging, subsurface prospecting, and so on. Taking the radar system for example, the received data for early radar systems are of single channel, and hence, the data are scalar-valued. In contrast, with the applications of pulsed Doppler techniques and/or multiple transmit/receive (T/R) modules, along with the increase in computation power and advances in hardware design, the received data for modern radar systems are usually multichannel, namely, vector-valued or even matrix-valued. Moreover, the frequency diversity, polarization diversity, or waveform diversity can also lead to the multichannel form of the received data. The multichannel data contain more information, compared with the single-channel data. On one hand, using the multichannel data, we have more degrees of freedom (DOFs) to design adaptive processors. On the other hand, using the multichannel data model, it is more convenient to characterize the correlated properties between data in different channels. Using these correlated properties, one can design a filter, whose output signal-to-noise (SNR) is often higher than that for a single-channel data. Similarly, utilizing the data correlation, one can devise a detector, which has superior detection performance to a detector for single-channel data.

Remarkably, noise is ubiquitous, which, in a general sense, usually includes thermal noise and clutter. For multichannel data in the cell under test (also called primary data), the noise covariance matrix is unknown and needs to be estimated. A common strategy is using the training data (also called secondary data) to form appropriate estimator. It is pointed out in [1] that modern strategy for radar

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detection should include the following three features: 1) being adaptive to the noise spectral density or its probability density function (PDF), 2) maintaining constant false alarm rate (CFAR) property, and 3) having a relatively simple processing scheme. Multichannel adaptive signal detection is a kind of this strategy. It jointly utilizes the test data and training data to design adaptive detectors, which usually possess the CFAR property. The resulting adaptive detector is then compared with a certain detection threshold, set to ensure a fixed probability of false alarm (PFA). Finally, a target is declared to be present (absent) if the threshold is exceeded (not exceeded).

Two points are worth to be emphasized. One is that the word “adaptive” in the first feature above indicates that the spectrum character of the noise is unknown in advance or is changing in the operational environment, and hence adaptive techniques are needed. The other is that the CFAR property or the CFARness<sup>1)</sup>, which, for single-channel signal, means that the detection threshold of a detector is independent of the noise power. Equivalently, the statistical property of the detector is functionally independent of the noise power under the signal-absence hypothesis. In contrast, for multichannel signal detection, the CFARness means that the statistical property of the detector is also functionally independent of the structure of the noise covariance matrix under the signal-absence hypothesis. This kind of CFARness is referred to as the matrix CFAR in [2] and covariance matrix-CFAR in [3].

Multichannel adaptive signal detection was first investigated by Kelly in 1986. In the seminal paper [4], Kelly proposed the famous detector, i.e., Kelly’s GLRT (KGLRT) for detecting a rank-one signal in homogeneous environment (HE). The rank-one signal has a known steering vector but unknown amplitude. For the HE model, the noise in the training and test data is both subject to mean-zero circularly complex Gaussian distribution, with the same covariance matrix.

There is more than three decades since Kelly proposed the famous KGLRT in 1986. Multichannel adaptive signal detection has been adopted in various areas. Based on different design criteria, numerous detectors have been proposed for different problems. Recently, an important book is edited by De Maio and Greco [5]. However, there are seldom survey papers on multichannel signal detection. In particular, references [6] and [7] gave overview of signal detection in compound-Gaussian clutter for subspace signals and rank-one signals, respectively. These two references are mainly on known clutter or known noise covariance matrix. Moreover, the target is point-like and no signal mismatch is considered. Different from the above two references, in this paper we give a review of multichannel adaptive signal detection in unknown noise, with emphasis on Gaussian background.

In this paper, we give a tutorial on multichannel adaptive signal detection, and present a brief survey of the state of the art. For brevity, “adaptive detection” always means “multichannel adaptive signal detection” in the following. In Section 2, we present the basic theory for adaptive detection, including data model, main detector design criteria, relationship between adaptive detection and filter-then-CFAR detection, and relationship between adaptive detection and adaptive filtering. In Section 3, we give comprehensive literature review. In Section 4, we analyse and compare the detection performance of some typical adaptive detectors. Finally, Section 5 summarizes this paper and gives some further research tracks in adaptive detection.

## 2 Basic theory

### 2.1 Main detector design criteria

The GLRT, Rao test, and Wald test are three main detector design criteria<sup>2)</sup>. These three criteria are referred to as “the Holy Trinity” in statistical inference [26]. Before listing these criteria, we need to formulate a binary hypothesis mathematically. A binary hypothesis has two possible cases, namely, the null (signal-absence) hypothesis and alternative (signal-presence) hypothesis. Hence, a binary hypothesis test can be written as

$$\begin{cases} H_0 : \mathbf{x} = \mathbf{n}, \mathbf{x}_{e,l} = \mathbf{n}_{e,l}, l = 1, 2, \dots, L, \\ H_1 : \mathbf{x} = \mathbf{s} + \mathbf{n}, \mathbf{x}_{e,l} = \mathbf{n}_{e,l}, l = 1, 2, \dots, L, \end{cases} \quad (1)$$

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1) CFARness is an important property required by an effective detector in practice, because the PFA may be dramatically raised to an unaffordable value if a detector does not maintain CFARness and the noise changes severely.

2) There are also some other often used criteria, such as the gradient test [8], Durbin test [9], test based on maximal invariant statistic [10], multifamily likelihood ratio test [11], and other modifications of the likelihood ratio test [11], which are utilized for adaptive detector design, e.g., [12–25].

where  $H_0$  denotes the null hypothesis,  $H_1$  denotes the alternative hypothesis,  $\mathbf{x}$  is the test data,  $\mathbf{s}$  is the signal to be detected,  $\mathbf{n}$  is the noise in the test data, whose covariance matrix, denoted as  $\mathbf{R}$ , is generally unknown,  $\{\mathbf{x}_{e,l}\}_{l=1}^L$  are  $L$  training data, used to estimate the unknown  $\mathbf{R}$ .

For the detection problem in (1), the GLRT is [27]

$$t_{\text{GLRT}} = \frac{\max_{\Theta_1} f_1(\mathbf{x}, \mathbf{X}_L)}{\max_{\Theta_0} f_0(\mathbf{x}, \mathbf{X}_L)}, \quad (2)$$

where  $\Theta_1$  and  $\Theta_0$  denote the unknown parameters under hypotheses  $H_1$  and  $H_0$ , respectively,  $f_1(\mathbf{x}, \mathbf{X}_L)$  and  $f_0(\mathbf{x}, \mathbf{X}_L)$  are the joint PDFs of the test data  $\mathbf{x}$  and training data  $\mathbf{X}_L = [\mathbf{x}_{e,1}, \mathbf{x}_{e,2}, \dots, \mathbf{x}_{e,L}]$  under hypotheses  $H_1$  and  $H_0$ , respectively.

To derive the Rao and Wald tests, we need the Fisher information matrix (FIM), which, for circularly symmetric random parameters, is defined as [28]

$$\mathbf{I}(\Theta) = \text{E} \left[ \frac{\partial \ln f(\mathbf{x}, \mathbf{X}_L)}{\partial \Theta^*} \frac{\partial \ln f(\mathbf{x}, \mathbf{X}_L)}{\partial \Theta^T} \right]. \quad (3)$$

For convenience, the FIM is usually partitioned as

$$\mathbf{I}(\Theta) = \begin{bmatrix} \mathbf{I}_{\Theta_r, \Theta_r}(\Theta) & \mathbf{I}_{\Theta_r, \Theta_s}(\Theta) \\ \mathbf{I}_{\Theta_s, \Theta_r}(\Theta) & \mathbf{I}_{\Theta_s, \Theta_s}(\Theta) \end{bmatrix}, \quad (4)$$

where

$$\Theta = [\Theta_r^T, \Theta_s^T]^T, \quad (5)$$

$$\mathbf{I}_{\Theta_r, \Theta_r}(\Theta) = \text{E} \left[ \frac{\partial \ln f(\mathbf{x}, \mathbf{X}_L)}{\partial \Theta_r^*} \frac{\partial \ln f(\mathbf{x}, \mathbf{X}_L)}{\partial \Theta_r^T} \right], \quad (6a)$$

$$\mathbf{I}_{\Theta_r, \Theta_s}(\Theta) = \text{E} \left[ \frac{\partial \ln f(\mathbf{x}, \mathbf{X}_L)}{\partial \Theta_r^*} \frac{\partial \ln f(\mathbf{x}, \mathbf{X}_L)}{\partial \Theta_s^T} \right], \quad (6b)$$

$$\mathbf{I}_{\Theta_s, \Theta_r}(\Theta) = \text{E} \left[ \frac{\partial \ln f(\mathbf{x}, \mathbf{X}_L)}{\partial \Theta_s^*} \frac{\partial \ln f(\mathbf{x}, \mathbf{X}_L)}{\partial \Theta_r^T} \right], \quad (6c)$$

$$\mathbf{I}_{\Theta_s, \Theta_s}(\Theta) = \text{E} \left[ \frac{\partial \ln f(\mathbf{x}, \mathbf{X}_L)}{\partial \Theta_s^*} \frac{\partial \ln f(\mathbf{x}, \mathbf{X}_L)}{\partial \Theta_s^T} \right]. \quad (6d)$$

In (5),  $\Theta_r$  is the relevant parameter, such as the signal amplitude,  $\Theta_s$  is the nuisance parameter, e.g., the noise covariance matrix. Note that if  $\ln f(\mathbf{x}, \mathbf{X}_L)$  is twice differential with respect to  $\Theta$ , then the FIM in (3), under the regularity condition, can be calculated by [29]

$$\mathbf{I}(\Theta) = -\text{E} \left[ \frac{\partial^2 \ln f(\mathbf{x}, \mathbf{X}_L)}{\partial \Theta^* \partial \Theta^T} \right], \quad (7)$$

which is often more easier to be derived.

Then, the Rao and Wald tests for complex-valued signals are [29]<sup>3)</sup>

$$t_{\text{Rao}} = \frac{\partial \ln f_1(\mathbf{x}, \mathbf{X}_L)}{\partial \Theta_r} \Big|_{\Theta=\hat{\Theta}_0}^T [\mathbf{I}^{-1}(\hat{\Theta}_0)]_{\Theta_r, \Theta_r} \frac{\partial \ln f_1(\mathbf{x}, \mathbf{X}_L)}{\partial \Theta_r^*} \Big|_{\Theta=\hat{\Theta}_0}, \quad (8)$$

and

$$t_{\text{Wald}} = (\hat{\Theta}_{r1} - \Theta_{r0})^H \left\{ [\mathbf{I}^{-1}(\hat{\Theta}_1)]_{\Theta_r, \Theta_r} \right\}^{-1} (\hat{\Theta}_{r1} - \Theta_{r0}), \quad (9)$$

respectively, where  $\hat{\Theta}_0$  and  $\hat{\Theta}_1$  are the maximum likelihood estimates (MLEs) of  $\Theta$  under hypotheses  $H_0$  and  $H_1$ , respectively,  $\hat{\Theta}_{r1}$  is the MLE of  $\Theta_r$  under hypothesis  $H_1$ ,  $\Theta_{r0}$  is the value of  $\Theta_r$  under hypothesis  $H_0$ , and  $\{[\mathbf{I}^{-1}(\Theta)]_{\Theta_r, \Theta_r}\}^{-1}$  is the Schur complement of  $\mathbf{I}_{\Theta_s, \Theta_s}(\Theta)$ , namely,

$$\{[\mathbf{I}^{-1}(\Theta)]_{\Theta_r, \Theta_r}\}^{-1} = \mathbf{I}_{\Theta_r, \Theta_r}(\Theta) - \mathbf{I}_{\Theta_r, \Theta_s}(\Theta) \mathbf{I}_{\Theta_s, \Theta_s}^{-1}(\Theta) \mathbf{I}_{\Theta_s, \Theta_r}(\Theta). \quad (10)$$

3) The complex-valued Rao test is also given in [30] which is a generalization of the one in (8) and suitable of non-circularly symmetric random parameters.

In some cases the relevant parameter  $\Theta_r$  and/or the nuisance parameter  $\Theta_s$  may be known. Obviously, in these cases we use these true values, and do not need to derive their MLEs.

It is worthy pointing out that the two-step variations of the three design criteria are also adopted. Precisely, the GLRT, Rao test, or Wald test is first derived under the assumption that the noise covariance matrix is known or its structure is known. Then the noise covariance matrix in the corresponding detector is replaced by a proper estimate by using the training data. For example, the two-step GLRT (2S-GLRT) can be mathematically expressed as

$$t_{2S\text{-GLRT}} = \left[ \frac{\max_{\Theta'_1} f_1(\mathbf{x}, \mathbf{X}_L)}{\max_{\Theta'_0} f_0(\mathbf{x}, \mathbf{X}_L)} \right] \bigg|_{\mathbf{R}=\hat{\mathbf{R}}}, \quad (11)$$

where  $\Theta'_1$  and  $\Theta'_0$  denote the unknown parameters except for  $\mathbf{R}$  under hypotheses  $H_1$  and  $H_0$ , respectively, and  $\hat{\mathbf{R}}$  is an appropriate estimation of  $\mathbf{R}$ .

From the three detector design criteria in (2), (8) and (9), we know that one of the key point to how to find the derivatives of scalar real-valued functions, such as the PDFs, with respect to a complex-valued scalar, vector, or matrix. One of the most useful book on this topic may be the one by Hjørungnes [31], which is written in engineering-oriented manner. The theory of finding complex-valued derivatives in [31] is based on the complex differential of the objective function. Using the complex differential is much more easier to find a derivative than using the component-wise approach, such as the famous book by Magnus and Neudecker [32], which mainly focuses on real-valued derivatives.

It is worthy pointing out that the following fact is often used in deriving a detector with simplified detection statistic or in a form whose statistical distribution is easy to be derived. Precisely, if a detector can be expressed as a monotonically increasing function of another one, then these two detectors are equivalent. We try to find a related reference. However, it is not found. Hence, we summarize the above fact in the following theorem.

**Theorem 1:** Let  $t_1$  and  $t_2$  are two detectors, and

$$t_2 = g(t_1) \quad (12)$$

monotonically increases with  $t_1$ . Then  $t_1$  and  $t_2$  have the same detection performance such that they have the identical probability of detection (PD) under the same PFA.

**Proof:** Let the PFAs of  $t_1$  and  $t_2$  be  $\text{PFA}_1$  and  $\text{PFA}_2$ , respectively. Then

$$\text{PFA}_1 = \Pr[t_1 > \eta_1; H_0], \quad (13)$$

$$\text{PFA}_2 = \Pr[t_2 > \eta_2; H_0], \quad (14)$$

where  $\eta_1$  and  $\eta_2$  are detection thresholds of  $t_1$  and  $t_2$ , respectively. According to (12), (14) can be rewritten as

$$\text{PFA}_2 = \Pr[g(t_1) > \eta_2; H_0] = \Pr[t_1 > g^{-1}(\eta_2); H_0], \quad (15)$$

where the second equality is owing to the fact that  $g(t_1)$  is a monotonically increasing function of  $t_1$ , and  $g^{-1}(\cdot)$  denotes the inverse function of  $g(\cdot)$ . Comparing (13) and (15), and using  $\text{PFA}_1 = \text{PFA}_2$ , we have

$$\eta_1 = g^{-1}(\eta_2). \quad (16)$$

The PDs of  $t_1$  and  $t_2$  can be expressed as

$$\text{PD}_1 = \Pr[t_1 > \eta_1; H_1] \quad (17)$$

and

$$\text{PD}_2 = \Pr[t_2 > \eta_2; H_1], \quad (18)$$

respectively. Since  $t_2 = g(t_1)$  is a monotonically increasing function of  $t_1$ , (18) can be recast as

$$\text{PD}_2 = \Pr[t_1 > g^{-1}(\eta_2); H_1] = \Pr[t_1 > \eta_1; H_1] = \text{PD}_1, \quad (19)$$

where the second equality is obtained according to (16). This completes the proof.  $\blacksquare$

Adaptive detection is different from filtering-then-CFAR detection, which is widely adopted in most radar systems. Moreover, adaptive detection is highly related with adaptive filtering, although their purposes are different. In the following two subsections, we investigate the relationship between them.

## 2.2 Relationship between adaptive detection and filtering-then-CFAR detection

Nowadays, the mainly used detection scheme in most radar systems is the filtering-then-CFAR approach. Precisely, the test data are first filtered and then processed by the CFAR techniques. The CFAR processing is a technique which makes the detection threshold of a detector independent of noise covariance matrix. Or, equivalently, through CFAR processing, the statistical characteristics of the detector does not depend on the noise covariance matrix under the signal-absence hypothesis. There are many CFAR technologies, such as cell-averaging CFAR (CA-CFAR), greatest-of-selection CFAR (GO-CFAR), ordered statistic CFAR (OS-CFAR), and so on [33, 34]. It seems that the filtering-then-CFAR detection scheme is a natural approach for detecting a target in noise, since adaptive filtering can obtain high output SNR, which benefits the detection process.

The theoretical basis behind the filtering-then-CFAR detection scheme for multichannel data can be traced back to the classic paper [35]. Precisely, for airborne radar space-time two-dimensional signal processing, the test data, if containing the target signal, can be written as

$$\mathbf{x} = a\mathbf{s} + \mathbf{n}, \quad (20)$$

where  $\mathbf{x}$  is an  $N_a N_p \times 1$  test data vector,  $N_a$  is the number of the antennas,  $N_p$  is the number of the pulses received by each antenna,  $\mathbf{s} = \mathbf{s}_p \otimes \mathbf{s}_a$  is an  $N_a N_p \times 1$  signal space-time steering vector, with  $\mathbf{s}_p$  and  $\mathbf{s}_a$  being an  $N_p \times 1$  time steering vector and an  $N_a \times 1$  space steering vector, respectively,  $\otimes$  denotes the Kronecker product, and  $\mathbf{n}$  is the noise, including clutter and thermal noise, distributed as circularly complex Gaussian distribution with covariance matrix  $\mathbf{R}$ .

In [35], to detect the target in (20), the test data vector  $\mathbf{x}$  is first filtered by an  $N_a N_p \times 1$  weight vector  $\mathbf{w}$ . Hence, the output of the filter can be expressed as

$$y = \mathbf{w}^H \mathbf{x}. \quad (21)$$

For the filtered data  $y$ , the optimum detector, in the Neyman-Pearson sense, is the likelihood ratio test, given by

$$t_{\text{LRT}} = \frac{f_1(y|\mathbf{x} = a\mathbf{s} + \mathbf{n})}{f_0(y|\mathbf{x} = \mathbf{n})}, \quad (22)$$

where  $f_1(\cdot)$  and  $f_0(\cdot)$  are the PDFs under signal-presence and signal-absence hypotheses, respectively. The optimum filter weight  $\mathbf{w}$  can be obtained by maximizing (22), written symbolically as

$$\mathbf{w}_{\text{opt}} = \max_{\mathbf{w}} \frac{f_1(y|\mathbf{x} = a\mathbf{s} + \mathbf{n})}{f_0(y|\mathbf{x} = \mathbf{n})}, \quad (23)$$

which is shown to be equivalent to [35]

$$\mathbf{w}_{\text{opt}} = \max_{\mathbf{w}} \frac{|\mathbf{w}^H \mathbf{s}|^2}{\mathbf{w}^H \mathbf{R} \mathbf{w}}, \quad (24)$$

and the solution to (24) is

$$\mathbf{w}_{\text{opt}} = \mu \mathbf{R}^{-1} \mathbf{s}, \quad (25)$$

where  $\mu$  is an arbitrary non-zero constant.

A well-known equivalent solution to (24) is the minimum variance distortionless response (MVDR), which is mathematically formed as [36]

$$\begin{cases} \min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w}, \\ \text{s.t. } \mathbf{w}^H \mathbf{s} = 1, \end{cases} \quad (26)$$

and the corresponding solution is

$$\mathbf{w}_{\text{MVDR}} = \frac{\mathbf{R}^{-1} \mathbf{s}}{\mathbf{s}^H \mathbf{R}^{-1} \mathbf{s}}. \quad (27)$$

Taking (27) into (21) and performing the norm-squared operation leads to

$$t_{\text{MF}} = \frac{|\mathbf{s}^H \mathbf{R}^{-1} \mathbf{x}|^2}{(\mathbf{s}^H \mathbf{R}^{-1} \mathbf{s})^2}. \quad (28)$$

Gathering the above results indicates that the optimum detection in (22) is equivalent to the optimum filtering in (24), and the optimum filter weight is given in (25). Based on the above results, a technique called space-time adaptive processing (STAP) came into being, which is regarded as one of most effective technology for airborne radar clutter cancellation, and numerous achievements have been obtained [37–39]. Note that STAP is a filtering technique<sup>4)</sup>, whose aim is to maximize the output SNR. To realize the final target detection, CFAR processing is needed.

It is worth pointing out that the above equivalence between optimum detection and optimum filtering holds under *certain processing flow* and *certain assumptions*. The specific processing flow is filtering-then-detection. Precisely, the multichannel test data vector  $\mathbf{x}$  is first filtered by the weight vector  $\mathbf{w}$ , resulting in the scalar-valued data  $y$ . Then, a detector is devised based on the filtered data  $y$ . The assumption is that the noise  $\mathbf{n}$  in the test data is Gaussian distributed, and its covariance matrix  $\mathbf{R}$  is known in advance. Unfortunately, the above assumption is usually not satisfied in practice, since radar system works in varying environment. When the noise covariance matrix  $\mathbf{R}$  is unknown, it is usually replaced by the sample covariance matrix (SCM), formed by using the training data received in the vicinity of the test data. Then the optimum filter in (25) becomes the sub-optimum filter of the sample covariance inversion (SMI) [42]. To complete target detection, it also needs appropriate CFAR processing.

Note that the above filtering-then-CFAR detection scheme adopts adaptive filtering. However, there is another filtering-then-CFAR detection scheme, which performs non-adaptive filtering, such as moving target indication (MTI) and moving target detection (MTD) and pulse Doppler processing. The key point in the MTD and pulse Doppler processing is Doppler filtering using multiple pulses. However, the number of the pulses used in the MTD is much smaller than that used in the pulse Doppler processing. Moreover, the MTD is often used by ground-based radar, while the pulse Doppler processing is mainly used by airborne radar. This non-adaptive filtering-then-CFAR detection scheme usually has lower complexity compared with the adaptive filtering-then-CFAR detection scheme, however, suffers from certain performance loss, since its filtering performance is limited.

For unknown noise, if the test and training data are directly utilized to devise multichannel adaptive detectors, then better detection performance can be obtained, compared with the above filtering-then-CFAR detection scheme. Adaptive detection is just a kind of this detection scheme. Precisely, for adaptive detection, the test and training data are jointly utilized to design an adaptive detector, and then it is compared with a detection threshold, set according to a pre-assigned PFA. If the value of a detector is greater than the threshold, a target is claimed. Otherwise, no target is claimed.

The block diagrams of filtering-then-CFAR detection and adaptive detection are summarized in Figure 1. It can be concluded that the filtering-then-CFAR detection approach (adaptive or non-adaptive) needs two independent processing procedures, as its name indicates, i.e., filtering and CFAR processing. In contrast, independent filtering processing is not needed for adaptive detection, which achieves the function of filtering and CFAR processing simultaneously, both embedded in the detection statistic of the adaptive detection.

### 2.3 Relationship between adaptive detectors and adaptive filters

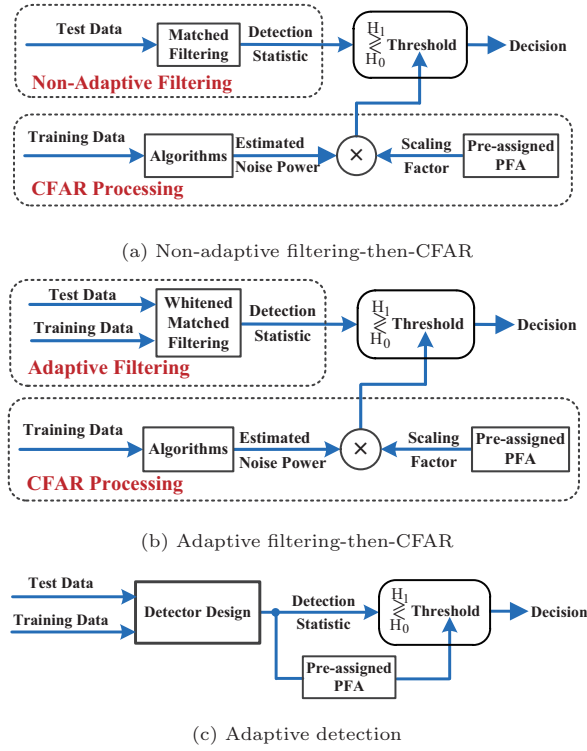
As explained above, adaptive filters and adaptive detectors have different purposes, since the former tries to maximize the output SNR, while the latter tries to maximize the PD with a fixed PFA. However, adaptive filters and adaptive detectors have some common feature. They both adopt adaptivity. Precisely, they use training data to adaptively estimate the unknown noise covariance matrix. This is the essential point in adaptive processors. Moreover, adaptive detectors have the function of adaptive filtering, which, however, is not achieved in an independent procedure, as pointed above.

As an example, Figure 2 shows the block diagrams of one adaptive filter, namely, the SMI [42], and three adaptive detectors, namely, the KGLRT [4], adaptive matched filter (AMF) [43, 44], and De Maio's Rao (DMRao) [45]<sup>5)</sup>. The SMI can be obtained by replacing  $\mathbf{R}$  with the SCM  $\mathbf{S}$  in (28), resulting in

$$t_{\text{SMI}} = \frac{\tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{s}}} \tilde{\mathbf{x}}}{\tilde{\mathbf{s}}^H \tilde{\mathbf{s}}}. \quad (29)$$

4) Strictly speaking, the STAP technique is much less than its literal meaning. Precisely, STAP is a filtering technique to reject the clutter and jammer (if present) for airborne radar [40, 41].

5) The SMI is proposed based on the idea of filtering-then-CFAR detection. Mathematically, it can be written as  $\left[ \max_{\mathbf{w}} \frac{|\mathbf{w}^H \mathbf{s}|^2}{\mathbf{w}^H \mathbf{R} \mathbf{w}} \right]_{\mathbf{R} = \frac{1}{L} \mathbf{S}}$ . The KGLRT, AMF and DMRao are proposed for the detection problem in (1) according to the criteria of GLRT, 2S-GLRT and Rao test, respectively. The AMF can also be obtained according to the Wald test.



**Figure 1** Block diagrams for filtering-then-CFAR detection and adaptive detection

Moreover, the detection statistics of the KGLRT, AMF, and DMRao are

$$t_{\text{KGLRT}} = \frac{\tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{s}}} \tilde{\mathbf{x}}}{1 + \tilde{\mathbf{x}}^H \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{s}}} \tilde{\mathbf{x}}}, \quad (30)$$

$$t_{\text{AMF}} = \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{s}}} \tilde{\mathbf{x}}, \quad (31)$$

and

$$t_{\text{DMRao}} = \frac{\tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{s}}} \tilde{\mathbf{x}}}{(1 + \tilde{\mathbf{x}}^H \tilde{\mathbf{x}})(1 + \tilde{\mathbf{x}}^H \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{s}}} \tilde{\mathbf{x}})}, \quad (32)$$

respectively, where  $\tilde{\mathbf{x}} = \mathbf{S}^{-\frac{1}{2}} \mathbf{x}$ ,  $\tilde{\mathbf{s}} = \mathbf{S}^{-\frac{1}{2}} \mathbf{s}$ ,  $\mathbf{x}$  is the test data vector,  $\mathbf{s}$  is the signal steering vector,  $\mathbf{S} = \mathbf{X}_L \mathbf{X}_L^H$  is the SCM<sup>6)</sup>, and  $\mathbf{P}_{\tilde{\mathbf{s}}} = \frac{\tilde{\mathbf{s}} \tilde{\mathbf{s}}^H}{\tilde{\mathbf{s}}^H \tilde{\mathbf{s}}}$  is the orthogonal projection matrix of  $\tilde{\mathbf{s}}$ .

The SMI and AMF can be taken as the outputs of certain adaptive filters, and then their corresponding weight vectors are<sup>7)</sup>

$$\mathbf{w}_{\text{SMI}} = \frac{\mathbf{S}^{-1} \mathbf{s}}{\mathbf{s}^H \mathbf{S}^{-1} \mathbf{s}} \quad (33)$$

and

$$\mathbf{w}_{\text{AMF}} = \frac{\mathbf{S}^{-1} \mathbf{s}}{\sqrt{\mathbf{s}^H \mathbf{S}^{-1} \mathbf{s}}}, \quad (34)$$

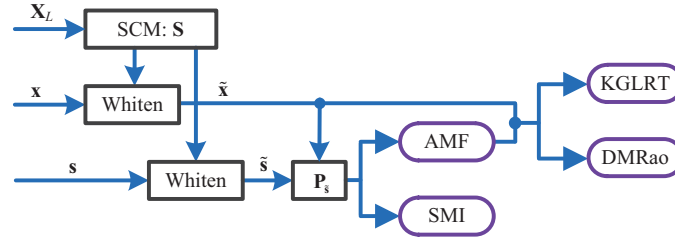
respectively. However, the KGLRT and DMRao cannot be expressed as the output of a filter.

Two key functions of adaptive filtering are clutter rejection and signal integration. The former is achieved by the “whiten” model, accomplished by the matrix  $\mathbf{S}^{-\frac{1}{2}}$ , while the latter is achieved by the orthogonal projection matrix “ $\mathbf{P}_{\tilde{\mathbf{s}}}$ ”. It is seen from Figure 2, along with (29)-(32), that the SMI, KGLRT, AMF, and DMRao all have the function of adaptive filtering. Moreover, the AMF and SMI have the same filtering performance, since they have the same output SNR. This can be verified by substituting

6) A more common SCM in adaptive filtering is defined as  $\mathbf{S}' = \frac{1}{L} \mathbf{X}_L \mathbf{X}_L^H$ . However, for adaptive detection it is usually more convenient to use the SCM defined as  $\mathbf{S} = \mathbf{X}_L \mathbf{X}_L^H$ .

7) Note that the SMI weight in (33) satisfies the constraint  $\mathbf{w}_{\text{SMI}}^H \mathbf{s} = 1$ .





**Figure 2** Block diagrams for one adaptive filter and three adaptive detectors

(33) and (34) into the quantity to be maximized in the right-hand side of (24). However, their detection performance is different, since the AMF has the CFAR property, whereas the SMI does not<sup>8)</sup>.

In summary, adaptive detectors use the test and training data to form specific structures, which are CFAR and have the function of filtering, embedded in the detection statistics.

### 3 Literature Review

According to different criteria, the problem of adaptive detection can be sorted into different types. For example, according to the extension of a target, adaptive detection can be sorted into point target detection and distributed (spread) target detection; according to the fact that whether the signal is mismatched or not, adaptive detection can be sorted as detection in the absence of signal mismatch and detection in the presence of signal mismatch; according to statistical property of the noise, adaptive detection can be sorted into detection in Gaussian noise and detection in non-Gaussian noise; according to the characters of the test and training data, adaptive detection can be sorted into detection in HE and detection in non-homogeneous (heterogeneous) environment; etc. However, the above classifications are too rough. Hence, we review the literature in the following six categories<sup>9)</sup>. For convenience, in each subsection we summarize the corresponding taxonomies in a table.

#### 3.1 Adaptive detection for point targets in the absence of signal mismatch

**Table 1** Related Taxonomy in Subsection 3.1

Taxonomy	Meaning
HE	A scenario that test and training data have the same noise covariance matrix.
PHE	A scenario that test and training data have the same noise covariance matrix upon to unknown scaling factor.
Nonhomogeneity	A scenario that the data in the collection of test and training data do not have the same noise covariance matrix.
Compound-Gaussian process	A random process which is in the form of a product of two components. One is the square root of a positive scalar random process (called texture, accounting for local power change), while the other is a complex Gaussian process (called speckle, accounting for local scattering).
Rank-one signal	A kind of signal, modelled by the product of a known vector and an unknown scaling factor.
Subspace signal	A kind of signal, modelled by the product of a known matrix and an unknown vector. That is to say, a subspace signal lies in a known subspace but with unknown coordinates.

In the seminal paper [4], Kelly considered the detection problem for a point target in HE. Precisely, the point target has a known signal steering vector, embedded in Gaussian noise with unknown covariance matrix. To estimate the unknown noise covariance matrix, a set of IID training data was used, which is signal-free and shares the same noise covariance matrix with the test data. Then Kelly proposed the famous KGLRT. According to the 2S-GLRT, Chen *et al.* [43] and Robey *et al.* [44] independently derived the well-known AMF, which has small complexity compared with the KGLRT. The corresponding Rao test was obtained by De Maio [45], i.e., the DMRao, which has lower PD than the KGLRT and AMF. However, the DMRao has better performance in terms of rejecting mismatched signals. The corresponding

<sup>8)</sup> The statistical performance analysis of the multi-band generalization of the SMI, called the modified SMI (MSMI), in [46] indicates that the detection threshold of the SMI depends on the noise covariance matrix  $\mathbf{R}$ .

<sup>9)</sup> We are sorry to any researcher whose work is overlooked or otherwise not discussed.



Wald test was also derived by De Maio [47], which coincides with the AMF. Noticeably, in 1994, Gerlach proposed the nonconcurrent mean level adaptive detector (N-MLAD) [48] and concurrent mean level adaptive detector (C-MLAD) [49]. The N-MLAD and C-MLAD are essentially the AMF and DMRao, respectively; see also [50, 51]. Moreover, the AMF was utilized in [52] for simultaneous detection and parameter estimation (i.e., target's Doppler and bearing).

The three detector KGLRT, DMRao, and AMF were all devised under the assumption of the HE. However, the data may have different statistical properties, owing to rapidly changed environmental factors or instrumental factors, such as adaptation of conformal array, bistatic radar, or multisite radar. Partially homogeneous environment (PHE) is a widely used nonhomogeneity model, which well characterizes the environment for airborne radars with low number of training data [53] and also suitable for wireless communications with fades over multiple sources of interference [54]. The GLRT for point target detection in PHE was derived by Kraut *et al.*, denoted as the adaptive coherent estimator (ACE) [55]. It was found in [56] that the Rao and Wald tests in PHE coincide with the ACE. In [57], a simple approach for the threshold setting of ACE, as well as the AMF, was provided. An invariance property of the ACE was given in [58], and it was shown to be uniformly most powerful invariant (UMPI) in [54]. More recently, it was shown in [59] that the ACE using the fixed-point covariance estimate [60] coincides with a maximal invariant component<sup>10</sup>. It is worth to pointing out that the ACE is effective in two kinds of non-homogeneous environment. One is spherically invariant noise [64] or compound-Gaussian noise [61]. The other is Bayesian heterogeneity. Precisely, the covariance matrix of the training data is subject to inverse complex Wishart distribution, and is proportional to the covariance matrix in the test data [65]. Moreover, the ACE is also called the adaptive normalized matched filter (ANMF) [64, 66] or normalized AMF (NAMF) [67]. In [68] the CFAR behavior using experimentally measured data was investigated for the KGLRT, AMF, and two variations of the ACE, namely, recursive ANMF (R-ANMF) [69] and persymmetric (RP-ANMF) [70]. It was shown in [68] that all these detectors exhibit a false alarm rate higher than the preassigned value, and the RP-ANMF is most robust among them. More recently, The problem of target separation detection (TSD) was considered in [71], where TSD tests were designed according to the GLRT. It was shown therein that the TSD tests can effectively monitor the event of target separation.

The above detectors are for rank-one signals, which have a known steering vector. However, a signal may naturally lie in a subspace, but with unknown coordinates, such as polarimetric target detection [72–76]. This type of signal is called subspace signal, which can be mathematically expressed as the product of a full-column-rank matrix and a vector. Under the background of polarimetric target detection, references [77] and [78] generalized the KGLRT and AMF to the case of 2-dimensional subspace. Then, references [79, 80] generalized the KGLRT to the case of subspace with dimension greater than 2, and the detector can be named as the subspace-based GLRT (SGLRT). Similarly, the AMF was generalized to the case of subspace with dimension greater than 2 in [81], and the detector was referred to as the subspace-based AMF (SAMF). The subspace versions of the DMRao and ACE were given in [82] and [83], respectively, and the resulting detectors can be denoted as the subspace-based Rao (SRao) test and adaptive subspace detector (ASD), respectively. The statistical properties of the SGLRT was given in [79, 84], the statistical properties of the SAMF was given in [81], the statistical properties of the ASD was given in [85, 86], and the statistical properties of the SRao was given in [87].

### 3.2 Adaptive detection for distributed targets in the absence of signal mismatch

For a high-resolution radar (HRR), a target may be spread in range, especially a big target, such as a large ship. It was shown in [88] that a properly designed HRR can provide improved detection performance. This is mainly due to two factors. One is that increasing the capability of range resolution of the radar can reduce the amount of energy per range bin backscattered by the clutter. The other is that a distributed target is usually less fluctuated than an unresolved point target.

It was assumed in [53] that the echoes reflected by the distributed target all came from the same direction, and the GLRT and 2S-GLRT for distributed target detection in HE and PHE were derived. The corresponding Rao and Wald tests in HE were derived in [89], while the Rao and Wald test in PHE were given in [90]. The 2S-GLRT in HE in [53] was known as the generalized AMF (GAMF). Similarly,

<sup>10</sup> It is observed that the upper-bound performance of the ACE is provide by the normalized matched filter (NMF), which was given in [61, 62]. Moreover, the NMF was shown in [63] to be the UMPI detector in spherically invariant random vector (SIRV) disturbance with a specific texture.

**Table 2** Related Taxonomy in Subsection 3.2

Taxonomy	Meaning
Distributed target	A target which occupies more than one range bins for a radar system.
DD	A detection problem, for which the received echoes all come from the same direction. However, the corresponding signal steering vector is only known to lie in a given subspace.
GDD	A detection problem, for which both the column and row components of a rank-one matrix-valued signal are constrained to lie in known subspaces, but with unknown coordinates.
DOS signal	A kind of signal, which is matrix-valued and its row and column elements both lie in known subspaces but with unknown coordinates.

we can name the GLRT in HE in [53] as generalized KGLRT (GKGLRT), since it is a generalization of the KGLRT. It is observed that the GLRT in HE proposed in [53] shares the same form as the multiband GLR (MBGLR) in [91]<sup>11)</sup>.

Reference [92] investigated the problem of detecting a distributed target, whose signal steering vector was unknown. The GLRT, 2S-GLRT, modified 2S-GLRT (M2S-GLRT), and spectral norm test (SNT) were proposed. It was shown in [93] that the 2S-GLRT and M2S-GLRT can be obtained according to the Wald test and Rao test, respectively. Some intuitive interpretations about the detectors were also given in [93]. Recently, reference [94] considered the case when the test data matrix was of rank two, and a generalization of ACE was proposed and its analytical performance was given.

In [95] it was assumed that the echoes backscattered by the distributed target all came from the same direction. However, the corresponding signal steering vector was only known to lie in a given subspace. This correspond detection problem was referred to as the direction detection (DD) therein, and the so-called generalized adaptive direction detector (GADD) was proposed according to the 2S-GLRT in PHE. From a mathematical point of view, for the problem of direction detection, the matrix-valued signal to be detected is of rank one, and its column components are constrained to a known subspace, while its row components are completely unknown. A more general signal model was adapted in [96], where both the column and row components of a rank-one matrix-valued signal are constrained to lie in known subspaces, but with unknown coordinates. This kind of problem can be taken as a generalized direction detection (GDD). However, it did not use the training data in [96]. Instead, it was assumed that the dimension of the test data satisfied certain constraint. Then a set of virtual training data can be obtained by using a unitary matrix transformation to the test data. As a consequence, the row structure of the signal was lost. Then the corresponding GLRT and 2S-GLRT were proposed therein. Essentially, the data model in [96] was equivalent to that in [95], but the environments were homogeneous. The Wald test for the DD in HE was proposed in [97], and it was shown that there is no reasonable Rao test for the problem of direction detection. The problem of GDD in HE was exploited in [98], where the corresponding GLRT and 2S-GLRT were proposed. Moreover, the 2S-GLRT in PHE for GDD was given in [99].

For the problem of detecting a distributed target, a systematic and comprehensive investigation was the report by Kelly and Forsythe in 1989 [100], where the solid mathematical background for adaptive signal detection was given. In [100] the signal to be detected is matrix-valued and its row and column elements both lie in known subspaces but with unknown coordinates. This kind of signal model is referred to as the double subspace (DOS) signal in [82, 101]. The DOS signal model is very general and includes many types of point targets and distributed targets as the special cases. In [100], no training data set was utilized. In contrast, a dimension constraint was posed on the test data. Then after a unitary matrix transformation on the test data, a set of virtual training data was obtained. Unfortunately, the row structure of the DOS signal is lost after the unitary matrix transformation. The problem of detecting a DOS signal was generalized in [82, 101], where true training data were assumed available, and many detectors were proposed and compared.

Compared with the detectors for point targets, the statistical performance of the detectors designed for distributed targets is difficult to be derived. In particular, the statistical performance of the GLRT and 2S-GLRT for distributed target in HE, proposed in [53], was given in [91] and [102], respectively. Moreover, the result in [91] was generalized in [103] to the case of signal mismatch. Signal mismatch will be explained detailed in the next subsection.

**Table 3** Related Taxonomy in Subsection 3.3

Taxonomy	Meaning
Signal mismatch	The phenomenon that the actual signal steering vector is not aligned with the nominal one adopted by the radar system.
Robustness	A property that the detection performance of a detector does not decrease severely with the increase of signal mismatch.
Selectivity	A property that the detection performance of a detector decreases rapidly with the increase of signal mismatch.
Directivity	The property (including robustness and selectivity) of a detector when detecting a mismatch signal.
Tunable detector	A kind of detector, which is parameterized by one or more positive scaling factors, called the tunable parameters. By adjusting the tunable parameters, the directivity property of the detector can be changed.
Cascaded detector	A kind of detector, formed by cascading a robust detector and a selective detector.
Weighted detector	A kind of detector, formed by weighting a robust detector and a selective detector.

### 3.3 Adaptive detection in the presence of signal mismatch

In practice, there often exists signal mismatch [104]. Precisely, the actual signal steering vector is not aligned with the nominal one adopted by the radar system. The statistical performance analysis for adaptive detectors in the presence of signal mismatch was first dealt with in [105], where it is shown that a key quantity controlling the detection performance of the KGLRT with mismatched signals is the generalized cosine-squared between the actual signal and the nominal signal in the whitened space. Based on the result in [105], the statistical performance of the AMF and ACE was given in [106], while the performance of the DMRao was dealt with in [45]. The statistical performance of the subspace-based detectors was addressed in [107] for the case of mismatched subspace signals, which is a generalization of the rank-one signal.

Signal mismatch can be caused by array error or target maneuvering. Moreover, signal mismatch can also be caused by jamming signals coming from the radar sidelobe, due to electronic countermeasures (ECM). For different sources of signal mismatch, different types of detectors are needed. For the first case, a robust detector is preferred, which achieves satisfied detection performance when signal mismatch occurs. In contrast, for the second case, a selective detector is preferred, whose detection performance decreases rapidly with the increase of signal mismatch.

One method to design a robust detector for mismatched signals is adopting subspace signal model (for rank-one signals) [79] or enlarging the signal subspace (for subspace signals) [108, 109]. Another method is constraining the actual angle or Doppler frequency lie in an compact interval [110, 111]. Then, maximization of the concentrated likelihood function over the actual angle or Doppler can be formulated as a semidefinite programming (SDP) convex problem, and hence easily solved. A third method is to assume that the actual signal lies in a convex cone, whose axes coincide with the nominal signal steering vector. Then a robust detector is designed by using second-order cone (SOC) programming [112–116]. A fourth method is to adding a random component in the test data under the signal-presence hypothesis. This makes the hypothesis more plausible when signal mismatch happens [117].

To design a selective detector, one approach is to modify the original hypothesis test by adding a determinant unknown fictitious signal (or jammer) under the null hypothesis. The fictitious signal satisfies certain constraints. A useful constraint is that the fictitious signal is orthogonal to the nominal signal in the quasi-whitened space [118] or whitened space [119]. Then the resulting detector will be inclined to choose the null hypothesis when there is no target in the nominal direction but in other directions. Under this idea, many selective detectors have been proposed, such as the adaptive beamformer orthogonal rejection test (ABORT) [118], whitened ABORT (W-ABORT) [119], their Bayesian variations [120], and other modifications [121–123]. The proposed selective detectors in the aforementioned references were mainly under the assumption of the HE. In contrast, a selective detector was proposed in [124] for distributed target detection in PHE. However, the selectivity property of the proposed detector is limited. In [125] a detector with improved selectivity was proposed for distributed target detection in PHE.

Another approach to design selective detector is adding a random unknown fictitious signal under both the null and alternative hypotheses. An intuitive interpretation may be lack. However, it works in certain parameter setting, such as the double-normalized AMF (DN-AMF) [126].

Note that the directivity (robustness or selectivity) of the above detectors cannot be adjusted. In other words, for a given detector, it either works as a robust detector or a selective detector, not both.

11) The MBGLR was proposed for point target detection when a radar system has multiple frequency bands.

This limits the flexibility of the detectors in detecting mismatched signals. Tunable detectors, cascaded detectors, weighted detectors, as well as their combinations, can overcome the above limitation.

Tunable detectors are mainly obtained by comparing the similarities in the detection statistics of two or more detectors with different directivity properties, and they, with specific tunable parameters, usually contain conventional detectors as their special cases. Directivity property of a tunable detector for mismatched signals can be smoothly changed by adjusting one or two parameters, called tunable parameters. The first tunable detector was proposed by Kalson in 1992 [127], which contains the KGLRT and AMF as two special cases. However, the selectivity of this tunable detector cannot exceed the KGLRT. Another tunable detector was proposed by Hao *et al.* in [128], termed as KRAO, which contains the KGLRT and DMRao as two special cases. The KRAO has enhanced selectivity but its robustness is limited. In [129] a tunable detector termed as KMABORT, was proposed, which contains the KGLRT, AMF, and ABORT as three special cases. The KMABORT is characterized by two tunable parameters, and hence it has more freedoms in detecting mismatched signals. However, its best robust property for mismatched signals is tantamount to that of the AMF. Fortunately, the AMF is very robust for mismatched signals, although it is not designed specially for robust detection of mismatched signals. A tunable detector, called KWA, was proposed in [130], which contains the KGLRT, W-ABORT, and adaptive energy detector (AED) [131] as its special cases. The KWA can provide even more robust property than the AMF. As a special case of the KWA, the AED does not need the nominal signal steering vector, instead, it only tests whether there exists a signal with sufficient energy. In other words, it does not differentiate between matched signals and mismatched signals. As a result, the AED is most robust. There are some other tunable detectors, such as the ones in [132–135].

A cascaded detector is forming by cascading a robust detector and a selective detector, and hence it has numerous pairs of detection thresholds. By changing the pair of detection thresholds, it can change the directivity property for mismatched signals. This type of cascaded detector is also called two-stage detector. A two-stage detector, referred to as 2SGLRT, cascading the KGLRT and AMF was proposed in [136]. In [106], a two-stage detector, called adaptive sidelobe blanker (ASB), was proposed, which cascades the AMF and ACE. In [45], a two-stage detector, denoted as AMF-Rao, which cascades the AMF and DMRao. In [137], a two-stage detector, called WAS-ASB was proposed, which cascades the SGLRT and W-ABORT. In [138], a two-stage detector, called S-ASB was proposed, which cascades the SGLRT and ACE. In [130], a two-stage detector called KWA-ASB was proposed, which cascades the KWA and SGLRT. In [128] two two-stage detectors were proposed, named as the KRAO-ASB and SKRAO-ASB. The former cascades the AMF and KRAO, while the latter cascades the SGLRT and KRAO. In [139], a two-stage detector, called SD-RAO was proposed, which cascades the SGLRT and DMRao. The above two-stage detectors were all designed for rank-one signals. In contrast, a two-stage detector, named AESD, was proposed in [140] for mismatched subspace signal by cascading the AED and ASD. The useful lecture [141] summarized the selective detectors ABORT and W-ABORT, the tunable detector KWA, the two-stage detectors ASB, AMF-Rao, S-ASB and WAS-ASB. Recently, a survey on the two-stage detector was given in [142].

A weighted detector is constructed by weighting a robust detector and a selective detector. By adjusting the weight, the directivity can be smoothly changed. A weighted detector, called SAMF-ASD, was proposed in [143].

All the tunable detectors, two-stage detectors, and weighted detectors above are designed for point target in HE. The ABORT was generalized in [124] for the distributed target detection both in HE and PHE. For distributed target detection, the W-ABORT was generalized in [144] and [125] in HE and PHE, respectively. Moreover, a tunable detector for distributed target detection in PHE was proposed in [125], called tunable GLRT in PHE (T-GLRT-PHE).

Note that the capabilities of robustness or selectivity of the two-stage detector and weighted detector cannot exceed their corresponding cascaded detectors and weighted detectors, respectively. In contrast, the tunable detector usually has much more freedoms to change the directivity for mismatched signals.

### 3.4 Adaptive detection in interference

Most of the aforementioned detectors are designed without taking into account the presence of interference. In practice, however, there usually exists interference, besides noise and possible signal of interest. Interference can be caused by the intentional ECM or unintentional industrial production.

Masking and deception are two main effects of interference on radar system. Noise interference has

**Table 4** Related Taxonomy in Subsection 3.4

Taxonomy	Meaning
Noise interference	A type of random interference, having the effect of thermal noise or clutter.
Coherent interference	A type of interference, having the effect of deceiving the radar system, which only lies in a direction and occupies a Doppler bin.
Subspace interference	A type of coherent interference, which can be modelled by a subspace model.
Orthogonal interference	A type of coherent interference, which is orthogonal to the signal in some manner.

the effect of masking the radar system, while coherent interference has the effect of deceiving the radar system. Noise interference plays the role of thermal noise or clutter. Hence, it raises the level of the noise. As a result, in order to maintain CFAR property, the radar system has to raise the detection threshold, which reduces the radar sensitivity for target detection [145, pp. 114-115]. Coherent interference usually imitates a real target, and hence it can deceive the radar system. This requires the interference works coherent to the radar system. Coherent interference can also be called false-target interference, including false-range interference, false-velocity interference, and false-direction interference.

From the point of view of data model, coherent interference is usually constrained to lie in a known subspace, and hence is often referred to as subspace interference in the field of adaptive detection. Much work was done by Scharf *et al.* [146–148] for detecting a multichannel signal in subspace interference and thermal noise (or colored noise with known covariance matrix). Some other relative work in subspace interference and colored noise with known covariance matrix was given in [149–152].

In practical applications, the noise covariance matrix is usually unknown, and needed to be estimated. For distributed target detection in subspace interference, it was assumed in [153] that the noise covariance matrix was unknown. To estimate the noise covariance matrix, a set of sufficient training data was used. The GLRT and 2S-GLRT were derived both in HE and PHE therein. The PFA of the GLRT in HE was given in [154]. The corresponding Rao test and two-step Rao (2S-Rao) tests in HE and PHE were derived in [155]. The Wald test and two-step Wald (2S-Wald) tests for point target detection in subspace interference were derived in [156]. Moreover, a modified Rao test was given in [157], which took both the signal coordinate matrix and interference coordinate matrix as the relative parameter. It is shown in [156] that in HE the 2S-GLRT, 2S-Rao, and Wald test (the other detectors all strongly related with these three detectors) whiten the noise (or equivalently reject the clutter) in the same manner. However, they reject the subspace interference in different manners. Recently, the statistical performance of the GLRT for subspace interference was analyzed in [158] for the case that the signal was of rank one. Moreover, the statistical performance of the GLRT-based detectors for point target detection in subspace interference was analysed in [159] for the case of signal mismatch, including the signal match as a special case. It was shown in [159] that the coherent interference and signal mismatch affect the detection performance of the GLRT-based detectors through two generalized angles. One is the angle between the whitened actual signal and the whitened interference subspace. The other is the angle of the actual signal and nominal signal matrix after they are both projected onto the interference-orthogonalized subspace. Reference [160] investigated the detection problem in subspace interference when signal mismatch happens. Two selective detectors and a tunable detector were proposed, and their statistical performance was also given therein. The detection problem in subspace interference was addressed in [161–165] in the framework of invariance principle. When the subspace interference lies in both the test and training data, it was pointed that in [23] that there is no effective GLRT, and a modified GLRT was proposed based on the method of sieves therein.

For the DD problem in the presence of subspace interference in HE, the GLRT and 2S-GLRT were developed in [166], while the Wald test and 2S-Wald test were obtained in [167]. The corresponding 2S-GLRT and 2S-Wald tests in PHE were derived in [168].

In the above references, sufficient information about the coherent interference is assumed available. However, this is not always the case in practice. It was assumed in [169] that the interference subspace was unknown except for its dimension, and a GLRT-like detector was proposed therein. In [87], it was assumed that the coherent interference was unknown but it was orthogonal to the signal in the whitened space. This type of interference was called orthogonal interference therein<sup>12)</sup>. Then three

12) The orthogonal interference satisfies the generalized eigenrelation (GER) defined in [170], which can be approximately met in practice, especially for the out-of-mainbeam interference [171]. It is pointed out in [171] that using secondary data selection strategies, e.g., the power selected training [172], results in the orthogonality of the signal and interference in the whitened space.



detectors were proposed, according to the criteria of GLRT, Rao test, and Wald test. Remarkably, the resulting three detectors share the same forms as the SGLRT, SRao, and SAMF, respectively. However, statistical performance analysis indicated that the orthogonal interference can degrade the detection performance [87]. Moreover, it was assumed in [173–175] that there were uncertainties in signal and coherent interference. To account for these uncertainties, the signal and interference were constrained to certain proper cones. Then effective detectors were proposed by using convex optimization.

The adaptive detection in completely unknown coherent interference was dealt with in [176]. At the stage of detector design, the unknown interference was assumed to lie in a subspace orthogonal to the signal. According to the GLRT and Wald test, two detectors were proposed, and the detector derived according to the GLRT was called adaptive orthogonal rejection detector (AORD). It was shown that the AORD has better detection performance than others in completely unknown interference. Another distinctive feature of the AORD is that it can even provide significantly performance improvement, compared with the KGLRT and AMF in the absence of interference. This was shown in [177], where the statistical performance of the AORD was also given.

The above references mainly deal with coherent interference. It was assumed in [178] that there was a completely unknown noise interference, and the corresponding GLRT for rank-one signals was shown to be equivalent to the ACE. The corresponding Rao test was given in [126], i.e., the DN-AMF, mainly adopted for mismatched signal detection, as explained in Subsection 3.3. The above results were generalized in [179] when there existed additional coherent interference, and the GLRT, Rao test, and Wald test were derived for subspace signals. In [180] the noise interference was constrained by the GER, and the GLRT was shown to be the same as the KGLRT. Moreover, the corresponding Rao and Wald tests were shown to be the DMRao and AMF, respectively [181]. The results in [180, 181] were generalized in [182] for subspace signals. It was assumed in [183] that the noise interference lies in a subspace orthogonal to the signal subspace, and a detector was proposed according to the 2S-Rao test, named as two-step orthogonal SAMF (2S-OSAMF). Numerical examples shew that the 2S-OSAMF has better detection performance than its competitors even the noise interference is completely unknown.

In [184] the authors considered the problem of determining whether the test data contained a noise interference or not. This problem was solved by formulating the problem as a binary hypothesis test, and a detector was designed according to GLRT criterion. In [185] the authors considered the problem of detecting a signal in the presence of noise interference, which only occupied parts of training data. Two GLRT-related detectors were proposed, which were shown to have better performance than the existing detectors. In [186] the authors considered two scenarios for the signal detection problem in interference. One was that only noise interference existed, and the other is that both noise interference and coherent interference existed. For the first scenario, an effective estimate for the interference covariance was proposed and then utilized in the AMF, which can mitigate the deleterious effects of the noise interference. For the second scenario, a compressive sensing-based GLRT was proposed. Some other detection problems involved in noise interference were given in [187–189].

### 3.5 Adaptive detection with limited training data

**Table 5** Related Taxonomy in Subsection 3.5

Taxonomy	Meaning
Low-rank structure	Noise covariance matrix is a sum of a scaled identity matrix and a low-rank matrix, with eigenvalues much greater than unity.
Persymmetry	Noise covariance matrix is persymmetric about its cross diagonal and Hermitian about its diagonal.
Spectral symmetry	Ground clutter has a symmetric PSD centred around the zero-Doppler frequency.

For adaptive processing, e.g., adaptive detection or adaptive filtering, it usually needs sufficient training data to estimate the unknown noise covariance matrix. In particular, it was shown in [42] that the adaptive filter SMI needs at least  $2N - 3$  IID training data to maintain 3 dB SNR loss, compared with the optimum filter (with known noise covariance matrix), with  $N$  being the dimension of the test data. This is known as the Reed-Mallett-Brenann (RMB) rule [42]<sup>13</sup>. However, this requirement may not be

<sup>13</sup> Recently, a simple proof of the RMB rule has been given in [190]. It is worth pointing out for adaptive detection, more than  $2N - 3$  IID training data are required to maintain 3 dB SNR loss, compared with the optimum detector, as shown in Figure 3 in the following.

always satisfied in practice. Taking an example of the STAP filtering for airborne radar, if the number of antenna elements is 30, the number of pulses is 40, and the system bandwidth is 10 MHz, in order to meet the requirement of the RMB rule, each filter needs received data within a range of roughly 36 km. The IID assumption usually cannot be guaranteed in such a wide range<sup>14</sup>).

*A priori* information-based method and dimension reduction are two main kinds of approach to alleviate the requirement of sufficient IID training data.

### 3.5.1 *A priori information-based methods*

*A priori* information-based method includes several sub-kinds, namely, Bayesian methods, parametric methods, special structure-based methods, etc.

For Bayesian methods<sup>15</sup>), the noise covariance matrix is ruled by a certain statistical distribution [196], and the distribution parameters can be obtained by using limited training data. In [197] the noise covariance matrix was assumed to be subject to a given inverse Wishart distribution, and the Bayesian one-step GLRT (B1S-GLRT) and Bayesian 2S-GLRT (B2S-GLRT) were proposed. It was shown by simulated and experiment data that these two Bayesian detectors can provide better detection performance than the conventional ones with low sample support. Notice that the B1S-GLRT and B2S-GLRT can be taken as the Bayesian generalizations of the KGLRT and AMF, respectively. The Bayesian version of the ACE was derived in [198, 199]. The Bayesian method was also adopted in [120, 200] to devise selective detectors with limited training data. Noticeably, the Bayesian method can be used even no training data are available [201].

Parametric (or model-based) method approximates the interference spectrum with a low-order multichannel autoregressive (AR) model [202]. In other words, the noise covariance matrix can be well characterized by using only a few parameters. Hence, this method largely reduces the required training data. At the same time, it also reduces the computational complexity. In [202], the parametric AMF (PAMF) was proposed. The PAMF was shown to be equivalent to the parametric Rao test in [203], where the asymptotic (in the case of large sample case) statistical distribution was also derived. The corresponding parametric GLRT was obtained in [204], which was shown to have better detection performance than the PAMF. In [205], the nonstationary PAMF (NS-PAMF) and nonstationary normalized PAMF (NS-PAMF) were proposed for adaptive signal detection in hyperspectral imaging. There are many other parametric detectors, e.g., [206–221].

The “structure” for special structure-based methods is for the noise covariance matrix, which may have different kinds of special structures for different antenna configurations or different radar operating environments. The special structures for the noise covariance matrix include low-rank structure, Toeplitz [222], Kronecker [223, 224], persymmetry, spectral symmetry, etc.

For the low-rank structure, which is data-dependent, the noise covariance matrix is a sum of a scaled identity matrix (corresponding to weak thermal noise) and a low-rank matrix (corresponding to strong clutter), with eigenvalues much greater than unity. Then, with limited training data, the principal component approximation of the SCM is usually a better estimation for the noise covariance matrix than the SCM itself [225]. Under this guideline, many reduced-rank approaches have been developed. Precisely, the reduced-rank versions of the KGLRT, AMF, and ACE were exploited in [226] for the problem of space-time adaptive detection (STAD) in airborne radar with the data received by multiple sensors under different pulses. There are many other well-known reduced-rank detectors or filters, such as the principal component analysis (PCA) [225], cross-spectral metric (CSM) [227], multistage Wiener filter (MWF) [228, 229], auxiliary-vector filter (AVF) [230], joint iterative optimization (JIO) [231], conjugate gradient (CG)-based AMF (CG-AMF) [232],<sup>16</sup> and some others [239–243]. Moreover, the diagonally loaded versions of the KGLRT, AMF and ACE were investigated in [12, 244]. Diagonal loading can be often taken as a kind of reduced-rank method, since it uses the low-rank structure information of the noise covariance matrix.

14) In other words, in many applications only a few number of data are IID. There are many approaches to choose qualified data, such as reiterative censored fast maximum likelihood (CFML) [191], generalized inner product (GIP) [192], approximate maximum likelihood (AML) [193], etc.

15) The Bayesian methods were also used to model the detection problem in non-homogeneous environment, e.g., [194, 195].

16) It is worthy pointing out that the MWF, AVF, and CG are equivalent to each other [233, 234], and they all belong to the Krylov subspace technique, which was originally used in numerical calculation [235] and have been recently successfully used in signal processing [236]. Remarkably, the Krylov subspace technique needs neither matrix inversion nor eigenvalue decomposition (EVD), and it can provide better performance than the EVD-based methods [237, 238].



Persymmetry is another useful structure for the covariance matrix estimation with low sample support. For the persymmetric covariance matrix, it is persymmetric about its cross diagonal and Hermitian about its diagonal. This structure exists when symmetrically spaced linear arrays and/or pulse trains are used. In addition, persymmetry can be found in other situations, e.g., standard rectangular arrays, uniform cylindrical arrays (with an even number of elements), and some standard exagonal arrays [245]. In [246] and [247], the maximum likelihood estimates of persymmetric covariance matrices were provided in the absence and presence of white noise, respectively. It has been proven in [248, 249] that the exploitation of persymmetry is tantamount to doubling the number of training data in adaptive processing. For target detection by exploiting the persymmetry, references [250–254] considered the case of point target with a single observation in HE, while references [255–258] considered the case of distributed target or point target with multiple observations/multi-bands in HE. Persymmetric detectors in HE with improved rejection capabilities were given in [259]. In the PHE, several persymmetric detection algorithms were designed in [260–265]. The above references exploiting persymmetry mainly focus on rank-one signal detection. In contrast, persymmetric detection of subspace signals was considered in [266–271]. Moreover, persymmetry can also be used in non-Gaussian noise [70, 250, 272–275] or multiple-input multiple-output (MIMO) radars [276–279].

Spectral symmetry exists in ground clutter, when it is observed by a stationary monostatic radar system. Precisely, the ground clutter has a symmetric power spectral density (PSD) centred around the zero-Doppler frequency. This special structure is confirmed by real data in [280, 281]. Other situations where spectral symmetry exists were discussed in details in [245]. Exploiting the spectral symmetry, adaptive detectors were proposed for the HE [245, 282] and PHE [283]. Simulation results indicate that utilizing the spectral symmetry is equivalent to doubling the number of the training data.

The above special structures can be combined together to further improve the performance, for instance, Bayesian method plus parametric method [284], parametric method plus persymmetry [285, 286], low-rank structure plus persymmetry [287], persymmetry plus spectral symmetry [288–290].

### 3.5.2 Dimension reduction methods

The method of utilizing *a priori* information may suffer from significantly performance loss, if the prior information greatly departs from the actual one. Another approach to alleviate the requirement of sufficient IID training data is dimension reduction, which is data-independent. To this end, a reduced-dimension transformation is applied to the test and training data before adaptive processing. This has the effect of projecting the noise covariance matrix onto a low-dimension subspace. As a result, the required number of IID training data can be considerably reduced, and the computational complexity is reduced as well. Various reduced-dimension approaches have been proposed, such as auxiliary channel receiver (ACR) [291], extended factor approach (EFA) [292], space-time multiple-beam (STMB) [293], sum-difference STAP ( $\Sigma\Delta$ -STAP) [294], best channel method (BCM) [295], alternating low-rank decomposition (ALRD) [296], among others [297].

The aforementioned approaches were proposed for filtering. In contrast, the joint-domain localized GLR (JDL-GLR) detector was proposed in [298] for airborne radar target detection. The JDL-GLR first transforms the test and training data into a reduced-dimension space, and then uses the KGLRT structure to form the final detector. Another similar reduced-dimension GLRT was proposed in [299]. In [52] two reduced-dimension detectors were proposed, which adopted the AMF structure. In [300], a reduced-dimension detector was proposed by using subarray processing. Recently, a random matrix-based reduced-dimension detector was given in [301]. The detector also uses the KGLRT structure. However, the reduced-dimension matrix is chosen in a different manner. Precisely, one column of the reduced-dimension matrix is aligned with signal steering vector, while the other columns are chosen randomly in the subspace orthogonal to the signal steering vector.

In [302, 303] the test and training data were first projected on the one-dimensional signal subspace, resulting in scalar data. Then using the resultant scalar data, two reduced-dimension detectors were designed. The above reduced-dimension detectors are mainly for rank-one signals. In contrast, a reduced-dimension detector for subspace signal detection was proposed in [304], referred to as subspace transformation-based detector (STBD). It is shown in [304] that the STBD, which can also serve as a filter, can provide improved detection and filtering performance even in some sample-abundant scenarios, besides the case of limited training data.

Besides the *A priori* information-based method and dimension reduction method, there may be some

other technologies to alleviate (or even not need) the requirement of training data. For example, in [305] the authors considered the problem of detecting a multichannel spatial signal in unknown noise without training data. To estimate the unknown noise covariance matrix, a number of echo signals reflected by the test data were utilized.

### 3.6 Adaptive detection for MIMO radar

**Table 6** Related Taxonomy in Subsection 3.6

Taxonomy	Meaning
Distributed MIMO radar	MIMO radar with widely separate antennas.
Colocated MIMO radar	MIMO radar with closely spaced antennas.
Spatial diversity	The transmit antennas are far enough from each other, and hence the target radar cross sections can be taken as independent random variables for different transmit-receive paths. With a spatially diverse set of “looks”, each set of received data carries independent information about the target.

A MIMO radar adopts multiple elements at both transmit and receive antennas. The transmitted waveforms are linearly independent or orthogonal [306]. According to the antenna configuration, there are two basic categories for MIMO radar. One is distributed MIMO radar, whose antennas are far from each other [307], while the other is colocated MIMO radar, whose antennas are closely spaced [308].

Strictly speaking, the review of MIMO radar target detection can also be carried out from above five aspects, or be included in the above five aspects. However, as an emerging research area, MIMO radar has received considerable attention. Hence, we would like to review MIMO radar target detection in an independent subsection from the following three aspects: adaptive detection for distributed MIMO radar, adaptive detection for colocated MIMO radar, and adaptive detection for other types of MIMO radar.

#### 3.6.1 Adaptive detection for distributed MIMO radar

For the distributed MIMO radar detection, it was shown in [309] that the distributed MIMO radar can provide better detection performance than traditional phased-array radar in high SNR regions. This improvement is due to the fact that spatial diversity can alleviate the impact of target scintillation, and spatial diversity gain is higher than the coherent processing gain of phased-array radar. Based on the results in [309], the expressions for the PD of the GLRT was derived in [310] when the target consists of a finite number of small scatterers. Reference [311] considered the problem of joint target detection and parameter estimation, and it was shown that distributed MIMO radars provide significant improvement over phased-array radars for distributed targets. Reference [312] dealt with the MIMO radar detection problem when phase synchronization mismatch arose between the transmit and receive antennas. The phase error was modelled as the von Mises distribution, and the corresponding GLRT was derived. Polarimetric MIMO radar detection in Gaussian noise was investigated in [313], and it was shown that optimal design of the antenna polarizations leads to better detection performance than MIMO radars transmitting fixed polarized waveforms over all antennas.

In the above references, the target’s movement feature was not taken into consideration. When the transmitted waveform was orthogonal and Doppler processing was adopted, the GLRT and 2S-GLRT were derived in [314] for moving target detection in Gaussian background, and the expression for the PFA of the GLRT was given in [315]. It was assumed in [276] that the noise covariance matrix had the persymmetric property, then the GLRT, as well as its statistical property, was derived for distributed MIMO radar which transmitted orthogonal waveforms and adopted Doppler processing. Under the same antenna configuration, as well as adopting the Doppler processing, the 2S-GLRT was given in [316] for compound-Gaussian clutter, while the corresponding 2S-Rao and 2S-Wald tests were derived in [317]. When the distributed MIMO radar transmitted orthogonal waveforms and adopted Doppler processing, reference [318] derived the GLRT for polarimetric moving target detection in the Gaussian noise. The detection problem in [314] was generalized in [319] by assuming that the target velocity was unknown, and it was shown that distributed MIMO radar has better detection performance than the phased-array radar when detecting a target with small radial velocities and environment is homogeneous. The distributed MIMO radar detection in non-homogeneous clutter was considered in [320], where the corresponding GLRT was derived and analytically evaluated. It was also shown that the GLRT in [320] has better detection performance than the detector in [319], as well as the corresponding phased-array detector.

Note that in the above references orthogonal waveforms are adopted. Under the assumption of white Gaussian noise, it is shown in [321] that a detector will suffer from certain detection performance loss if the orthogonality property of the waveforms transmitted by different antennas is not satisfied. However, the above result may not be suitable for colored noise. Reference [322] derived the GLRT for distributed MIMO radar with arbitrary transmitted waveforms and arbitrary time-correlation of the noise, and it was shown that there is an inherent trade-off between diversity and integration, and that no uniformly optimum waveform design strategy exists. In [323], the GLRT was derived for distributed MIMO radar, with arbitrary transmit waveform and adopting Doppler processing. It was assumed that all transmit-receive pairs share the same known covariance matrix, then the expressions for the PD of the GLRT was given [323], according to which the optimum transmit waveform was given. Reference [324] generalized the data model in [323] to the case that different transmit-receive pairs have different but known covariance matrices. Then the statistical performance of the corresponding GLRT was given for Swerling I target.

The signal model in [323] was also adopted in [325–329], however, the noise was assumed to be compound-Gaussian. Precisely, the 2S-Rao and 2S-Wald tests for distributed MIMO radar were given in [325], while several Bayesian 2S-GLRTs were derived in [326, 327]. The 2S-Rao and 2S-Wald tests in [325] was generalized to polarimetric distributed MIMO radar detection in [330] for point targets and in [331] for distributed targets. Moreover, reference [328] derived the 2S-GLRT for point target detection with polarimetric distributed MIMO radar in compound-Gaussian clutter, and it was generalized in [329] for the case of distributed target detection.

### 3.6.2 Adaptive detection for colocated MIMO radar

For the colocated MIMO radar detection, reference [332] derived the GLRT and its asymptotic statistical distribution for colocated MIMO radar after beamforming in white Gaussian noise. Reference [333] proposed three 2S-GLRTs for colocated MIMO radar with randomly distributed arrays in compound-Gaussian clutter, and it was shown that the configuration of randomly distributed arrays achieve detection performance improvement at the directions with strong clutter.

Remarkably, it was shown in [334] that colocated MIMO radars make it possible that detecting a target or estimating its parameters does not need training data or even range compression. Without the training data, the problem of parameter estimation for colocated MIMO radar was addressed in [335], where the GLRT was derived to suppress the false peak induced by strong jammer. The corresponding Rao and Wald tests, as well as their statistical properties, were given in [336]. When signal mismatch occurs, a tunable MIMO radar detector was proposed in [135], which includes the Rao and Wald tests in [336] as special cases. The proposed tunable detector in [135] has flexibility in controlling the direction property, selectivity or robustness, for mismatched signals. Two robust detectors were proposed in [337] for mismatched signals by assuming the actual signal lying in certain subspaces. The GLRT in [335] was generalized in [277] when the persymmetry of noise covariance matrix was exploitation. It was shown by simulation and experimental data that by utilization the persymmetry, the proposed detector in [277] can achieve better detection performance. The correspond persymmetric Rao test Wald test were given in [279] and [278], and in [279] a two-stage detector was also given for mismatched signal detection by cascading the above persymmetric Rao and Wald tests.

More recently, for a colocated MIMO radar, a robust Wald-type test were proposed in [338]. Performance analysis showed that there always exists a sufficient number of (virtual) antennas such that the required performance are satisfied, without prior knowledge of the noise statistical property. This type of MIMO radar was referred to as the massive MIMO radar therein. Moreover, in [339] three adaptive GLRTs were proposed for colocated MIMO radar equipped with frequency diverse array (FDA).

### 3.6.3 Adaptive detection for other types of MIMO radar

There are several types of variations of the distributed MIMO radar and colocated MIMO radar, such as phased MIMO (Phased-MIMO) radar [340], hybrid MIMO phased array radar (HMPAR) [341], transmit subaperturing MIMO (TS-MIMO) radar [342], and multi-site radar system MIMO (MSRS-MIMO) [343]. The MSRS-MIMO radar has multiple widely separate sub-arrays, and each sub-arrays has multiple colocated antennas. According to the waveforms, the MSRS-MIMO radar can be classified as two kinds. One is that the waveforms are different or orthogonal in different transmit antennas [344]. The other is that the waveforms transmitted by the antennas are scaled versions of a single waveform [345]. For

convenience, the first type of MSRS-MIMO radar is referred to as the distributed-colocated MIMO radar, while the latter one is referred to as the distributed-phased MIMO radar<sup>17)</sup>.

When the waveform are orthogonal, the GLRT for distributed-colocated MIMO radar was obtained in non-Gaussian environment in [2], and the expression for the PFA was given therein under the constraint that the product of the number of transmit elements and receive elements is the same for each pair of transmit-receive sub-array. The results for the PFA in [2] was generalized in [348] by eliminating the above constraint. For distributed-colocated MIMO radar with non-orthogonal waveform, the GLRT in Gaussian noise was derived in [349], while the two-step Rao and Wald tests were given in [350]. In non-Gaussian background, the 2S-GLRT, Rao test, and Wald test were exploited in [351] for distributed-colocated MIMO radar with non-orthogonal waveform. Moreover, reference [352] considered the problem of detecting a mismatched signal in distributed-phased MIMO radar, and proposed three selective detectors.

Before closing this section, we summarize important progress in Table I.

**Table 7** Important Progress in Multichannel Adaptive Signal Detection

Year	Important Progress	Author(s)	Ref.
1986	first paper on adaptive detection	Kelly	[4]
1989	solid mathematical background for adaptive signal detection	Kelly and Forsythe	[100]
1991	well-known detector for point targets: AMF	Chen, Fobey, et al.	[43]
1992	tunable detector for mismatched signals	Kalson	[127]
1992	persymmetry structure based detector with limited training data	Cai and Wang	[256]
1995/ 1999	well-known detector for point targets: ACE	Conte, Lops, Kraut, Scharf, et al.	[61] [55]
1996	subspace-based signal detection	Raghavan, Pulsone, et al.	[79]
1996	direction detection	Bose and Steinhardt	[96]
1997	distributed target detection	Gerlach, Steiner, et al.	[353]
2000	two-stage detector for mismatched signals	Pulsone and Zatman	[136]
2000	low-rank structure based detector with limited training data	Ayoub and Haimovich	[299]
2000/ 2006	parametric detector with limited training data	Roman, Rangaswamy, Li, Michels, et al.	[202] [205]
2001	selective detector for mismatched signals	Pulsone and Rader	[118]
2003	adaptive detectors based on Rao and Wald tests	Conte and De Maio	[272]
2004	multiple target detection	Gini, Bordonni, et al.	[354]
2005	adaptive detection based on convex optimization	De Maio	[112]
2007	adaptive detection in subspace interference	Bandiera, De Maio, et al.	[153]
2007	Bayesian detector in heterogeneous environment	Besson, Tournet, et al.	[194]
2008	MIMO radar detection in unknown noise	Xu, Li, et al.	[335]
2010	spectral symmetry based detector with limited training data	De Maio, Orlando, et al.	[245]
2014	Rao and Wald tests for complex-valued signals with circularly symmetric random parameters	Liu, Wang, et al.	[29]
2014	double subspace signal detection	Liu, Xie, et al.	[82, 101]
2016	Rao test for complex-valued signals with circularly or non-circularly symmetric random parameters	Kay and Zhu	[30]
2016	adaptive detection based on covariance structure classification	Carotenuto, De Maio, et al.	[355]

<sup>17)</sup> It is worth pointing out that the data model of the distributed-phased MIMO radar is the same as the conventional distributed MIMO radar which adopts coherent pulse processing in each sub-array, such as [320, 346, 347].

## 4 Typical adaptive detectors for different detection problems

Multichannel adaptive signal detection was first investigated for a point target in 1986 by Kelly [4]. Based on Kelly's work, all kinds of problems were dealt with, and numerous detectors were proposed. In this section, we first summarize the statistical properties of many well-known detectors for point targets, since the statistical properties are the primary tool to evaluate the detection performance of the detectors. Then, we generalize the case of point target detection to distributed target detection and signal detection in the presence of interference<sup>18</sup>).

### 4.1 Adaptive detectors for point targets and their statistical distributions

Note that subspace signal model is more general than the rank-one signal model adopted in (20). It is pointed out in [146] that the matched subspace detector is the general building block of signal processing, and it contains the rank-one matched filter or detector as a special case. Hence, in this subsection the detectors for point targets are all based on subspace signal model.

For the detection problem in (1), if the signal  $\mathbf{s}$  lies in a known subspace spanned by an  $N \times p$  full-column-rank matrix  $\mathbf{H}$ , then we have  $\mathbf{s} = \mathbf{H}\boldsymbol{\theta}$ , with  $\boldsymbol{\theta}$  being a  $p \times 1$  unknown coordinate vector. In the HE, the noise covariance matrix in the test data  $\mathbf{x}$  is the same as that in the training data  $\mathbf{x}_{e,l}$ . Then, for the detection problem in (1) with  $\mathbf{s}$  being replaced by  $\mathbf{H}\boldsymbol{\theta}$ , the GLRT [84], Rao test [82], and Wald test [82] are

$$t_{\text{SGLRT}} = \frac{\tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{x}}}{1 + \tilde{\mathbf{x}}^H \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{x}}}, \quad (35)$$

$$t_{\text{SRao}} = \frac{\tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{x}}}{(1 + \tilde{\mathbf{x}}^H \tilde{\mathbf{x}})(1 + \tilde{\mathbf{x}}^H \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{x}})}, \quad (36)$$

and

$$t_{\text{SAMF}} = \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{x}}, \quad (37)$$

respectively, where  $\tilde{\mathbf{H}} = \mathbf{S}^{-\frac{1}{2}} \mathbf{H}$  and  $\mathbf{P}_{\tilde{\mathbf{H}}} = \tilde{\mathbf{H}}(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}})^{-1} \tilde{\mathbf{H}}^H$ . The detectors in (35)-(37) are referred as the SGLRT, SRao, and SAMF, respectively.

In the PHE, the noise covariance matrices in the test and training data can be modified as  $\mathbf{R}_e = \sigma^2 \mathbf{R}$  and  $\mathbf{R}$ , respectively, with  $\sigma^2$  being an unknown positive scaling factor, standing for the power mismatch between the test and training data. In the PHE, the GLRT, Rao test, and Wald test coincide with each other, and are found to be [101]

$$t_{\text{ASD}} = \frac{\tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{x}}}{\tilde{\mathbf{x}}^H \tilde{\mathbf{x}}}, \quad (38)$$

which is named as ASD in [83]. Note that the SGLRT, SRao, SAMF, and ASD are the subspace generalizations of the KGLRT, DMRao, AMF, and ACE, respectively.

The above four detectors are designed without taking into account the possibility of signal mismatch. On the one hand, signal mismatch may be caused by antenna error, mutual coupling, or target maneuvering. On the other hand, signal mismatch can also be caused by a strong target or a jamming signal located in the radar sidelobe, generated by the ECM. For different sources of signal mismatch, different directivity properties (the capability of selectivity or robustness to signal mismatch) of the detector are preferred. For the first case, a robust detector is needed, which maintains good detection performance in the presence of signal mismatch. In contrast, for the second case, a selective detector is preferred, whose detection performance decreases rapidly with the increase of signal mismatch.

To design selective detectors in the case of signal mismatch, an effective approach is adding artificially determinant factitious jammer under hypothesis  $H_0$  [118]. Then, the detection problem in (1) can be modified to be

$$\begin{cases} H_0 : \mathbf{x} = \mathbf{n} + \mathbf{q}, \mathbf{x}_{e,l} = \mathbf{n}_{e,l}, l = 1, 2, \dots, L, \\ H_1 : \mathbf{x} = \mathbf{H}\boldsymbol{\theta} + \mathbf{n}, \mathbf{x}_{e,l} = \mathbf{n}_{e,l}, l = 1, 2, \dots, L, \end{cases} \quad (39)$$

where the  $N \times 1$  unknown vector  $\mathbf{q}$  denotes the artificially injected determinant factitious jammer. Remarkably, the injection of the factitious jammer  $\mathbf{q}$  makes the resulting detector tend to choose hypothesis

<sup>18</sup>) We choose the above three cases because they are representative in the field of adaptive detection and extensively studied in the literature.



$H_0$  if signal mismatch occurs. When  $\mathbf{q}$  is constrained to be orthogonal to the signal subspace in the quasi-whitened space, i.e.,

$$\mathbf{H}^H \mathbf{S}^{-1} \mathbf{q} = \mathbf{0}_{p \times 1}, \quad (40)$$

the GLRT for the detection problem in (39) is

$$t_{\text{SABORT}} = \frac{1 + \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{x}}}{1 + \tilde{\mathbf{x}}^H \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{x}}}, \quad (41)$$

which is a special case of the adaptive direction detector with mismatched signal rejection of type 2 (ADD-MSR2) in [123], and a subspace generalization of the ABORT, proposed in [118]. Hence, for convenience, the detector in (41) is referred to as the subspace-based ABORT (SABORT).

If the determinant factitious jammer in (40) is modified as

$$\mathbf{H}^H \mathbf{R}^{-1} \mathbf{q} = \mathbf{0}_{p \times 1}. \quad (42)$$

That is to say, the factitious jammer is orthogonal to the signal subspace in the truly whitened space. Then the GLRT for the detection problem in (39) becomes

$$t_{\text{W-SABORT}} = \frac{1 + \tilde{\mathbf{x}}^H \tilde{\mathbf{x}}}{(1 + \tilde{\mathbf{x}}^H \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{x}})^2}, \quad (43)$$

which is a special case of the adaptive direction detector with mismatched signal rejection of type 1 (ADD-MSR1) in [123], and a subspace generalization of the W-ABORT in [119]. The detector in (43) is denoted as the whitened SABORT (W-SABORT) for convenience.

Moreover, another approach to devise a selective detector is injecting an unknown, rank-one, noise-like, fictitious jammer  $\mathbf{v}$  under both hypotheses. As a results, the noise covariance matrix in the test data becomes  $\mathbf{R} = \mathbf{R}_e + \mathbf{v}\mathbf{v}^H$ . Then, the selective detector derived according to the Rao test is

$$t_{\text{DN-SAMF}} = \frac{\tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{x}}}{\tilde{\mathbf{x}}^H \tilde{\mathbf{x}} (1 + \tilde{\mathbf{x}}^H \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{x}})}, \quad (44)$$

which is a special case of the Rao test in [179] and a subspace generalization of the DN-AMF in [126]. For convenience, the detector in (44) is denoted as the doubly normalized SAMF (DN-SAMF).

Different from the above devised selective detectors, a robust detector to the signal mismatch may be preferred in many applications. A robust detector can be designed by assuming the desired signal to be detected is completely unknown. In other words, the signal  $\mathbf{s}$  in (1) is unknown, or equivalently, the dimension of the signal matrix  $\mathbf{H}$  is  $N \times N$ . Then the corresponding GLRT is given by [131]

$$t_{\text{AED}} = \tilde{\mathbf{x}}^H \tilde{\mathbf{x}}, \quad (45)$$

which can be denoted as AED. It is shown in [82] that the Rao test and Wald test are both equivalent to the GLRT, i.e., the AED in (45).

Since the case of signal match can be taken as a special case of signal mismatch (i.e., the mismatched angle is zero), we only summarize the statistical properties of the above detectors in the presence of signal mismatch. As mentioned above, the statical performance of the detectors in the presence of signal mismatch was first dealt with by Kelly in [105] for the KGLRT in the case of rank-one signal. Based on this result, the statistical performance of the SGLRT, SAMF, and ASD was given in [107]. In the following, we summarize the statical properties of the above eight detectors, some of which were not found in the open literature.

To obtain the statical distributions of the detectors, it is convenient to introduce the following quantity

$$\beta = \frac{1}{1 + \tilde{\mathbf{x}}^H \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{x}}}, \quad (46)$$

which can be taken as a loss factor.

If signal mismatch happens, the actual signal, denoted as  $\mathbf{s}_0$ , may not completely lie in the signal subspace spanned by the columns of  $\mathbf{H}$ . Then, it is shown in [107] that the statistical distribution of

the SGLRT in (35), with  $\beta$  given, under hypothesis  $H_1$  is complex noncentral F-distribution, with  $p$  and  $L - N + 1$  DOF and a noncentrality parameter  $\beta\rho \cos^2\phi$ , written symbolically as

$$t_{\text{SGLRT}}|[\beta, H_1] \sim \mathcal{CF}_{p, L-N+1}(\beta\rho_{\text{pnt}} \cos^2\phi), \quad (47)$$

where  $\rho_{\text{pnt}}$  is the output SNR, defined as

$$\rho_{\text{pnt}} = \mathbf{s}_0^H \mathbf{R}^{-1} \mathbf{s}_0, \quad (48)$$

$$\cos^2\phi = \frac{\mathbf{s}_0^H \mathbf{R}^{-1} \mathbf{H} (\mathbf{H}^H \mathbf{R}^{-1} \mathbf{H})^{-1} \mathbf{H}^H \mathbf{R}^{-1} \mathbf{s}_0}{\mathbf{s}_0^H \mathbf{R}^{-1} \mathbf{s}_0}, \quad (49)$$

and the notation “[ $\beta, H_1$ ]” denotes the fact that the above statistical distribution holds under hypothesis  $H_1$  on the condition that  $\beta$  is given. Equation (49) can be rewritten as

$$\cos^2\phi = \frac{\bar{\mathbf{s}}_0^H \mathbf{P}_{\bar{\mathbf{H}}} \bar{\mathbf{s}}_0}{\bar{\mathbf{s}}_0^H \bar{\mathbf{s}}_0}, \quad (50)$$

where  $\mathbf{s}_0 = \mathbf{R}^{-\frac{1}{2}} \mathbf{s}_0$ ,  $\bar{\mathbf{H}} = \mathbf{R}^{-\frac{1}{2}} \mathbf{H}$ ,  $\mathbf{P}_{\bar{\mathbf{H}}} = \bar{\mathbf{H}} (\bar{\mathbf{H}}^H \bar{\mathbf{H}})^{-1} \bar{\mathbf{H}}^H$ . It follows from (50) that the quantity  $\cos^2\phi$  measures cosine-squared of the angle between the whitened actual signal  $\bar{\mathbf{s}}_0$  and the whitened nominal signal subspace spanned by the columns of  $\bar{\mathbf{H}}$ .  $\cos^2\phi$  plays a key role in controlling the detection performance of a detector in the presence of signal mismatch. This is numerically shown in the next section.

Moreover, it is shown in [107] that the statistical distribution of the loss factor  $\beta$  in (46) under hypothesis  $H_1$  is a complex noncentral Beta distribution, with  $L - N + p + 1$  and  $N - p$  DOFs and a noncentrality parameter  $\delta^2$ , written symbolically as

$$\beta|H_1 \sim \mathcal{CB}_{L-N+p+1, N-p}(\delta^2), \quad (51)$$

where

$$\delta^2 = \rho_{\text{pnt}} \sin^2\phi, \quad (52)$$

and  $\sin^2\phi = 1 - \cos^2\phi$ .

In contrast, under hypothesis  $H_0$ , the statistical distributions of the SGLRT in (35) and the loss factor  $\beta$  in (46) become

$$t_{\text{SGLRT}}|[\beta, H_0] \sim \mathcal{CF}_{p, L-N+1}, \quad (53)$$

and

$$\beta|H_0 \sim \mathcal{CB}_{L-N+p+1, N-p}, \quad (54)$$

respectively.

The analytical expressions for the PDF and cumulative distribution function (CDF) of the complex noncentral F-distribution and complex noncentral Beta distribution were exploited in detail in Kelly and Forsythe’s classic report [100], also summarized in [106, 141]. One can use these CDFs and PDFs to derive the expressions for the PDs and PFAs of the above detectors.

It is straightforward to verify that the following seven equations hold

$$t_{\text{SAMF}} = \frac{t_{\text{SGLRT}}}{\beta}, \quad (55)$$

$$t_{\text{ASD}} = \frac{t_{\text{SGLRT}}}{1 - \beta}, \quad (56)$$

$$t_{\text{SRao}} = \frac{\beta t_{\text{SGLRT}}}{1 + t_{\text{SGLRT}}}, \quad (57)$$

$$t_{\text{SABORT}} = \beta + t_{\text{SGLRT}}, \quad (58)$$

$$t_{\text{W-SABORT}} = (1 + t_{\text{SGLRT}})\beta, \quad (59)$$

$$t_{\text{DN-SAMF}} = \frac{\beta t_{\text{SGLRT}}}{(1 - \beta)(1 - \beta + t_{\text{SGLRT}})}, \quad (60)$$



$$t_{\text{AED}} = \frac{1 - \beta + t_{\text{SGLRT}}}{\beta}. \quad (61)$$

Based on the conditional distribution of the SGLRT in (47) and the statistical distribution of the loss factor  $\beta$  in (51), along with the statistical dependences in (55)-(61), one can readily obtain analytical expressions for the PDs and PFAs of the detectors. Interested readers can refer to [107] for examples. Note that one can obtain the expressions for the PD and PFA of the AED in a more direct manner by deriving the statistical distribution of the AED [356]. Precisely, according to Theorem 3.2.13 in [357, p.98] or Theorem 5.2.2 in [358, p.176], the statistical distribution of the AED in (45) under hypotheses  $H_1$  and  $H_0$  are

$$t_{\text{AED}}|H_1 \sim \mathcal{CF}_{N,L-N+1}(\rho_{\text{pnt}}) \quad (62)$$

and

$$t_{\text{AED}}|H_0 \sim \mathcal{CF}_{N,L-N+1}, \quad (63)$$

respectively.

In order to evaluate the detection performance of the detectors under different numbers of training data, we consider the detector with known noise covariance matrix. Precisely, when  $\mathbf{R}$  is known, the GLRT for the detection problem in (1) with  $\mathbf{s}$  being replaced by  $\mathbf{H}\boldsymbol{\theta}$  is

$$t_{\text{SMF}} = \mathbf{x}^H \mathbf{R}^{-1} \mathbf{H} (\mathbf{H}^H \mathbf{R}^{-1} \mathbf{H})^{-1} \mathbf{H}^H \mathbf{R}^{-1} \mathbf{x}, \quad (64)$$

which is referred to as the subspace-based matched filter (SMF). It can also be obtained by the criteria of GLRT, Rao and Wald tests. The statistical distribution of the SMF in (64) under hypothesis  $H_1$  is a complex noncentral Chi-square distribution with  $p$  DOFs and a noncentrality parameter  $\rho$  [87], written symbolically as

$$t_{\text{SMF}}|H_1 \sim \mathcal{C}\chi_p^2(\rho_{\text{pnt}}). \quad (65)$$

Under hypothesis  $H_0$ , the above distribution becomes central, i.e.,

$$t_{\text{SMF}}|H_0 \sim \mathcal{C}\chi_p^2 \quad (66)$$

## 4.2 Numerical examples

In this subsection we compare the detection performance of the detectors with numerical examples, and only focus on the case of HE. Two cases are considered, namely, the case of no signal mismatch and the case of signal mismatch. The PD curves of all detectors are obtained by using the theoretical results, and confirmed by Monte Carlo simulations, which are not shown for a clear display.

Figure 3 compares the detection performance of the adaptive detectors under different SNRs in the absence of signal mismatch. For comparison purpose, the result for the SMF is also reported. The results indicate that, for the chosen parameters, the SGLRT, among the eight adaptive detectors, has the highest PD and slightly better than the SAMF and SABORT, the DN-SAMF has the lowest PD, and the PDs of the ASD, W-SABORT, SRao, and AED are in between. Moreover, the detection performance loss of the SGLRT in terms of SNR is roughly 4 dB when  $\text{PD} = 0.9$ , compared with the SMF. This is quite different from adaptive filtering, since it is well-known from the RMB rule [42] that  $2N$  independent identically distributed (IID) training data can maintain 3 dB SNR loss, compared with the optimum filter. The above detection loss is owing to two factors [4]. One is the effective SNR loss factor (similar to adaptive filtering), and the other is the CFAR loss of the adaptive detectors. The effective SNR loss factor depends roughly on the ratio of  $L$  to  $N$ , while the CFAR loss depends solely on  $L$ , whose increase results in the decrease of the CFAR loss.

Figure 4 shows the detection performance of the adaptive detectors under different amount of signal mismatch. As expected, the AED is the most robust and its PD does not vary with the change of  $\cos^2 \phi$ . However, its PD cannot attain unity for the chosen parameters. The robustness of the SAMF, SGLRT, SABORT, ASD, W-SABORT, SRao, and DN-SAMF reduces in sequence.

Another method to illustrate the detection performance for mismatched signals is showing the contours of PDs as functions of SNR and  $\cos^2 \phi$ , first introduced in [118] and named as mesa plot. This is displayed in Figure 5 for the above detectors. The directivities of the detectors are the same as those in Figure 4. However, more information can be inferred from Figure 5. Taking the SAMF for example, it is very robust to signal mismatch. It can provide a PD as high as 0.9 as long as the SNR is high enough, even the

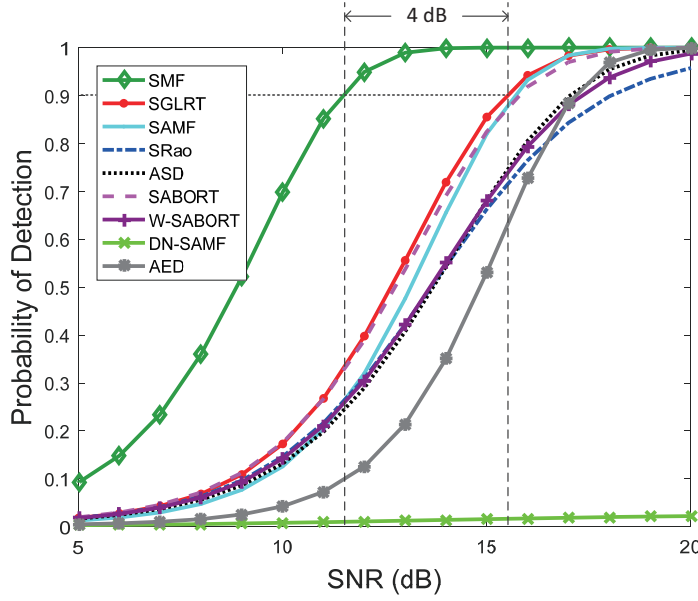


Figure 3 PD versus SNR.  $N = 12$ ,  $p = 2$ ,  $L = 2N$ , and  $\text{PFA} = 10^{-3}$ .

whitened actual signal is orthogonal to the whitened nominal signal subspace, i.e., the case of  $\cos^2 \phi = 0$ . In contrast, for a selective detector, such as the SABORT, it does not achieve a PD higher than 0.5 when  $\cos^2 \phi < 0.55$ , no matter how high the SNR is. It is worth pointing out for the chosen parameters, the SAMF and SABORT have comparable PDs for matched signals as shown in Figure 3. Hence, if a selective detector is needed, the SABORT is a better candidate than the SAMF.

Before closing this section, we would like to give the following three remarks. First, it is known from (53), (54), and (55)-(61) that all the adaptive detectors exploited above have the CFAR property with respect to the noise covariance matrix  $\mathbf{R}$ . Second, only the ASD, among the above eight adaptive detectors, possesses the CFAR property in PHE, although the ASD has lower PD than some other detectors in HE. Third, the DN-SAMF can behave quite well when the number of system dimension  $N$  is large enough, as shown in [126].

### 4.3 Generations of point-target-based adaptive detectors

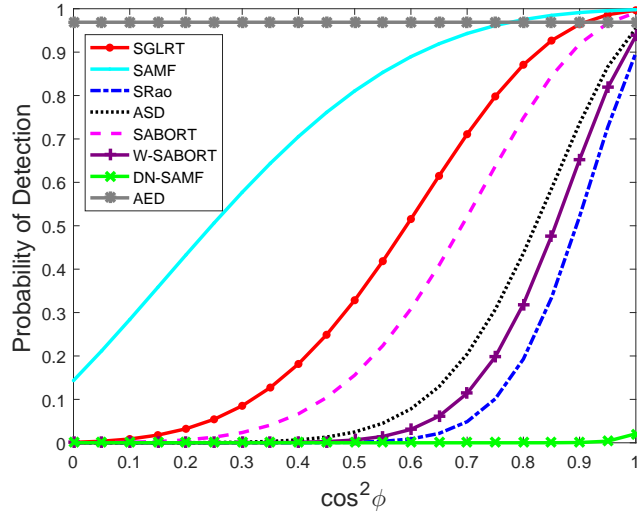
The detection problem in (1) has been generalized in many aspects, and hence, many other adaptive detectors have been proposed besides the ones shown in the above subsection. Distributed target detection (without interference) and signal detection in interference are two important generations, which will be shown below.

#### 4.3.1 Adaptive detectors for distributed targets

A large target usually occupies multiple range bins, especially for high-resolution radar system [359]. In this case, the detection problem in (1) should be modified as

$$\begin{cases} H_0 : \mathbf{X} = \mathbf{N}, \mathbf{x}_{e,l} = \mathbf{n}_{e,l}, l = 1, 2, \dots, L, \\ H_1 : \mathbf{X} = \mathbf{s}\mathbf{a}^H + \mathbf{N}, \mathbf{x}_{e,l} = \mathbf{n}_{e,l}, l = 1, 2, \dots, L, \end{cases} \quad (67)$$

where  $\mathbf{X}$  is an  $N \times K$  matrix denoting the test data, with  $N$  being the number of system channels and  $K$  being the number of range bins occupied by the distributed target,  $\mathbf{N}$  is the noise in the test data,  $\mathbf{s}$  is the signal steering vector,  $\mathbf{a}$  is the coordinate vector of the signal,  $\mathbf{x}_{e,l}$  is the  $l$ th training data vector, and  $\mathbf{n}_{e,l}$  is the noise in  $\mathbf{x}_{e,l}$ . The columns of  $\mathbf{N}$  are IID, having the noise covariance matrix  $\mathbf{R}_t$ . Denote the noise covariance matrix of  $\mathbf{n}_{e,l}$  as  $\mathbf{R}$ . Then, in HE,  $\mathbf{R}_t = \mathbf{R}$ , while in PHE  $\mathbf{R}_t = \sigma^2 \mathbf{R}$ , with  $\sigma^2$  being the unknown power mismatch between the test data and training data.



**Figure 4** PD versus  $\cos^2 \phi$ .  $N = 12$ ,  $p = 2$ ,  $L = 2N$ , SNR = 18 dB, and PFA =  $10^{-3}$ .

For the detection problem in (67), the GLRT and its two-step variation for the HE and PHE were all proposed in [53]. Precisely, for the HE, the GLRT and 2S-GLRT are

$$t_{\text{GKGLRT}} = \frac{\tilde{\mathbf{s}}^H \tilde{\mathbf{X}} (\mathbf{I}_K + \tilde{\mathbf{X}}^H \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}^H \tilde{\mathbf{s}}}{\tilde{\mathbf{s}}^H \tilde{\mathbf{s}} - \tilde{\mathbf{s}}^H \tilde{\mathbf{X}} (\mathbf{I}_K + \tilde{\mathbf{X}}^H \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}^H \tilde{\mathbf{s}}} \quad (68)$$

and

$$t_{\text{GAMF}} = \frac{\tilde{\mathbf{s}}^H \tilde{\mathbf{X}} \tilde{\mathbf{X}}^H \tilde{\mathbf{s}}}{\tilde{\mathbf{s}}^H \tilde{\mathbf{s}}}, \quad (69)$$

respectively. Moreover, for the PHE, the GLRT and 2S-GLRT are

$$t_{\text{GLRT-PHE}} = \frac{(\hat{\sigma}_0^2)^{\frac{NK}{L+K}} \left| \mathbf{I}_K + \frac{1}{\hat{\sigma}_0^2} \tilde{\mathbf{X}}^H \tilde{\mathbf{X}} \right|}{(\hat{\sigma}_1^2)^{\frac{NK}{L+K}} \left| \mathbf{I}_K + \frac{1}{\hat{\sigma}_1^2} \tilde{\mathbf{X}}^H \mathbf{P}_{\tilde{\mathbf{s}}}^\perp \tilde{\mathbf{X}} \right|} \quad (70)$$

and

$$t_{\text{GASD}} = \frac{\tilde{\mathbf{s}}^H \tilde{\mathbf{X}} \tilde{\mathbf{X}}^H \tilde{\mathbf{s}}}{\tilde{\mathbf{s}}^H \tilde{\mathbf{s}} \operatorname{tr}(\tilde{\mathbf{X}}^H \tilde{\mathbf{X}})}, \quad (71)$$

respectively. In (70),  $\hat{\sigma}_0^2$  and  $\hat{\sigma}_1^2$  are the sole solutions of

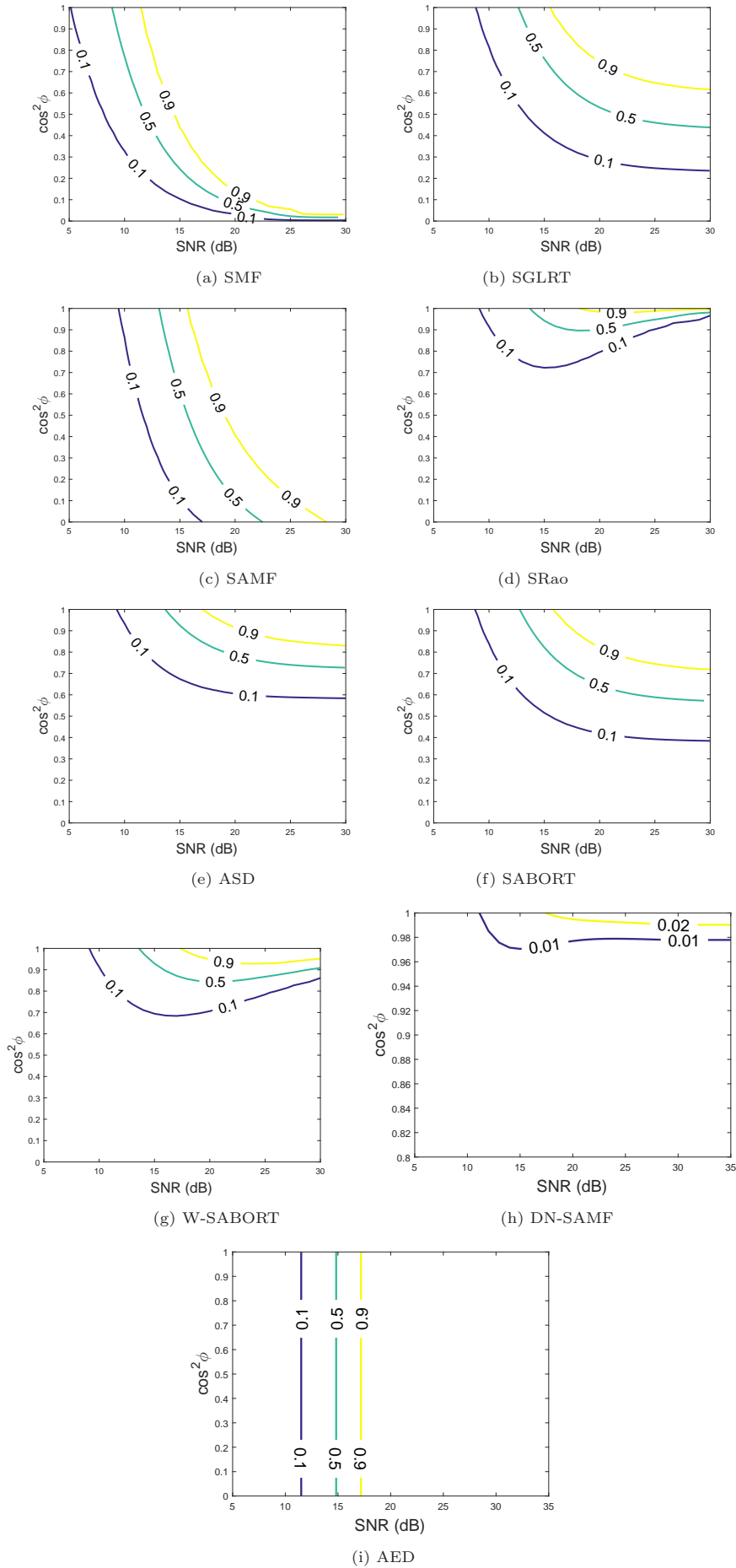
$$\sum_{k_0=1}^{r_0} \frac{\lambda_{k_0}}{\lambda_{k_0} + \sigma^2} = \frac{NK}{L+K} \quad (72)$$

and

$$\sum_{k_1=1}^{r_1} \frac{\xi_{k_1}}{\xi_{k_1} + \sigma^2} = \frac{NK}{L+K}, \quad (73)$$

respectively, where  $r_0 = \min(N, K)$ ,  $r_1 = \min(N-1, K)$ ,  $\lambda_{k_0}$  is the  $k_0$ th non-zero eigenvalue of  $\tilde{\mathbf{X}}^H \tilde{\mathbf{X}}$ ,  $k_0 = 1, 2, \dots, r_0$ , and  $\xi_{k_1}$  is the  $k_1$ th non-zero eigenvalue of  $\tilde{\mathbf{X}}^H \mathbf{P}_{\tilde{\mathbf{s}}}^\perp \tilde{\mathbf{X}}$ ,  $k_1 = 1, 2, \dots, r_1$ .

The detectors in (69) and (71) were referred to as generalized AMF (GAMF) and generalized adaptive subspace detector (GASD), respectively in [53]. For convenience, the detector in (68) is denoted as GKGLRT in this paper.



**Figure 5** Contours of the PDs versus SNR and  $\cos^2 \phi$ .  $N = 12$ ,  $p = 2$ ,  $L = 2N$ , and  $\text{PFA} = 10^{-3}$ .

Moreover, for the detection problem in (67) in HE, the Wald test is the same as the GAMF, while the Rao test was proposed in [89], described as<sup>19)</sup>

$$t_{\text{Rao-HE}} = \frac{\mathbf{s}^H (\mathbf{S} + \mathbf{X}\mathbf{X}^H)^{-1} \mathbf{X}\mathbf{X}^H (\mathbf{S} + \mathbf{X}\mathbf{X}^H)^{-1} \mathbf{s}}{\mathbf{s}^H (\mathbf{S} + \mathbf{X}\mathbf{X}^H)^{-1} \mathbf{s}}. \quad (74)$$

For the detection problem in (67) in PHE, the Rao test and Wald test were proposed in [90], given by

$$t_{\text{Rao-PHE}} = \frac{1}{\hat{\sigma}_0^2} \text{tr} \left[ \mathbf{X}^H \hat{\mathbf{R}}_0^{-1} \mathbf{H} (\mathbf{H}^H \hat{\mathbf{R}}_0^{-1} \mathbf{H})^{-1} \mathbf{H}^H \hat{\mathbf{R}}_0^{-1} \mathbf{X} \right] \quad (75)$$

and

$$t_{\text{Wald-PHE}} = \frac{1}{\hat{\sigma}_1^2} \text{tr} \left[ \mathbf{X}^H \hat{\mathbf{R}}_1^{-1} \mathbf{H} (\mathbf{H}^H \hat{\mathbf{R}}_1^{-1} \mathbf{H})^{-1} \mathbf{H}^H \hat{\mathbf{R}}_1^{-1} \mathbf{X} \right], \quad (76)$$

respectively, where  $\hat{\sigma}_0^2$  and  $\hat{\sigma}_1^2$  are the sole solutions of (72) and (73), respectively,

$$\hat{\mathbf{R}}_0 = \frac{1}{L+K} \left( \mathbf{S} + \frac{1}{\hat{\sigma}^2} \mathbf{X}\mathbf{X}^H \right) \quad (77)$$

and

$$\hat{\mathbf{R}}_1 = \frac{1}{L+K} \mathbf{S}^{\frac{1}{2}} \left( \mathbf{I}_N + \frac{1}{\hat{\sigma}_1^2} \mathbf{P}_{\bar{\mathbf{s}}}^{\perp} \tilde{\mathbf{X}} \tilde{\mathbf{X}}^H \mathbf{P}_{\bar{\mathbf{s}}}^{\perp} \right) \mathbf{S}^{\frac{1}{2}}. \quad (78)$$

Different from the case of point target, it is more difficult to derive the statistical performance for the distributed-target-based detectors. At present, only the statistical performance of the GKGLRT and GAMF is known. The statistical distribution of the GKGLRT was first proposed in [91] for the case of no signal mismatch, and then generalized to the case of signal mismatch in [103]. The statistical distribution of the GAMF was given in [102]. Precisely, under hypothesis  $\mathbf{H}_1$ , the conditional distribution of the GKGLRT in (68) is

$$t_{\text{GKGLRT}} | \mathbf{H}_1 \sim \mathcal{CF}_{K,L-N+1} (\rho_{\text{dstr}} \cos^2 \phi_{\text{rk1}} \beta_{\text{GKGLRT}}), \quad (79)$$

where

$$\rho_{\text{dstr}} = \mathbf{a}^H \mathbf{a} \cdot \mathbf{s}_0^H \mathbf{R}^{-1} \mathbf{s}_0, \quad (80)$$

can be taken as the output SNR, with  $\mathbf{s}_0$  being the actual signal steering vector,

$$\cos^2 \phi_{\text{rk1}} = \frac{|\mathbf{s}_0^H \mathbf{R}^{-1} \mathbf{s}|^2}{\mathbf{s}_0^H \mathbf{R}^{-1} \mathbf{s}_0 \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s}} \quad (81)$$

is generalized cosine-squared between the actual signal  $\mathbf{s}_0$  and the nominal signal  $\mathbf{s}$  in the whitened space, and  $\beta_{\text{GKGLRT}}$  is a loss factor for the GKGLRT, having the statistical distribution

$$\beta_{\text{GKGLRT}} | \mathbf{H}_1 \sim \mathcal{CB}_{L+K-N+1, N-1} (\rho \sin^2 \phi_{\text{rk1}}), \quad (82)$$

with  $\sin^2 \phi_{\text{rk1}} = 1 - \cos^2 \phi_{\text{rk1}}$ . Under hypothesis  $\mathbf{H}_0$ , equations (79) and (82) become

$$t_{\text{GKGLRT}} | \mathbf{H}_0 \sim \mathcal{CF}_{K,L-N+1} \quad (83)$$

and

$$\beta_{\text{GKGLRT}} | \mathbf{H}_0 \sim \mathcal{CB}_{L+K-N+1, N-1}, \quad (84)$$

respectively. Moreover, under hypothesis  $\mathbf{H}_1$  the conditional distribution of the GAMF in (69) is

$$\beta_{\text{GAMF}} t_{\text{GAMF}} | \mathbf{H}_1 \sim \mathcal{CF}_{K,L-N+1} (\beta_{\text{GAMF}} \rho_{\text{dstr}}), \quad (85)$$

where  $\beta_{\text{GAMF}}$  is a loss factor for the GAMF, with the statistical distribution

$$\beta_{\text{GAMF}} | [\mathbf{H}_1 \text{ and } \mathbf{H}_0] \sim \mathcal{CB}_{L-N+2, N-1}. \quad (86)$$

<sup>19)</sup> Using matrix inversion lemma, it is easy to show that equation (74) can be recast as  $t_{\text{Rao-HE}} = \frac{\bar{\mathbf{s}}^H \bar{\mathbf{X}} (\mathbf{I}_K + \bar{\mathbf{X}}^H \mathbf{P}_{\bar{\mathbf{s}}}^{\perp} \bar{\mathbf{X}})^{-1} (\mathbf{I}_K + \bar{\mathbf{X}}^H \bar{\mathbf{X}})^{-1} \bar{\mathbf{X}}^H \bar{\mathbf{s}}}{\bar{\mathbf{s}}^H \bar{\mathbf{s}}}$ .

Under hypothesis  $H_0$ , equation (85) turns to be

$$\beta_{\text{GAMF}} t_{\text{GAMF}} | H_0 \sim \mathcal{CF}_{K, L-N+1}. \quad (87)$$

There are two kinds of further generalizations of the detection problem in (67). One is that the signal steering vector  $\mathbf{s}$  lies in a given subspace spanned by an  $N \times p$  full-column matrix  $\mathbf{H}$ . Hence,  $\mathbf{s}$  can be expressed as  $\mathbf{s} = \mathbf{H}\boldsymbol{\theta}$ , with  $\boldsymbol{\theta}$  being  $p \times 1$  unknown coordinates. It follows that (67) becomes

$$\begin{cases} H_0 : \mathbf{X} = \mathbf{N}, \mathbf{x}_{e,l} = \mathbf{n}_{e,l}, l = 1, 2, \dots, L, \\ H_1 : \mathbf{X} = \mathbf{H}\boldsymbol{\theta}\mathbf{a}^H + \mathbf{N}, \mathbf{x}_{e,l} = \mathbf{n}_{e,l}, l = 1, 2, \dots, L, \end{cases} \quad (88)$$

The GLRT and 2S-GLRT in HE were proposed in [96], described as

$$t_{\text{GLRDD}} = \lambda_{\max} \left[ \tilde{\mathbf{X}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{X}} (\mathbf{I}_K + \tilde{\mathbf{X}}^H \tilde{\mathbf{X}})^{-1} \right] \quad (89)$$

and

$$t_{\text{AMDD}} = \lambda_{\max} \left( \tilde{\mathbf{X}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{X}} \right), \quad (90)$$

respectively, where  $\lambda_{\max}(\cdot)$  denotes the maximum eigenvalue of the matrix argument. It was shown in [97] that there is no reasonable Rao test for the detection problem in (88), the 2S-Wald test is the same as the detector in (90), and the Wald test is given by

$$t_{\text{SNRDD}} = \frac{\boldsymbol{\theta}_{\max}^H \tilde{\mathbf{H}}^H \tilde{\mathbf{X}} \tilde{\mathbf{X}}^H \tilde{\mathbf{H}} \boldsymbol{\theta}_{\max}}{\boldsymbol{\theta}_{\max}^H \tilde{\mathbf{H}}^H \tilde{\mathbf{H}} \boldsymbol{\theta}_{\max}}, \quad (91)$$

where  $\boldsymbol{\theta}_{\max}$  is a principal eigenvector (the eigenvector corresponding to the maximum eigenvalue) of the matrix  $(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}})^{-1} \tilde{\mathbf{H}}^H \tilde{\mathbf{X}} (\mathbf{I}_K + \tilde{\mathbf{X}}^H \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}^H \tilde{\mathbf{H}}$ . The detectors in (89), (90), and (91) are referred to as GLR-based direction detector (GLRDD), adaptive matched direction detector (AMDD), SNR-based direction detector (SNRDD) in [97]. The 2S-GLRT for the detection problem in (88) in PHE was proposed in [95], given by<sup>20)</sup>

$$t_{\text{GADD}} = \frac{\lambda_{\max} \left( \tilde{\mathbf{X}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{X}} \right)}{\text{tr} \left( \tilde{\mathbf{X}}^H \tilde{\mathbf{X}} \right)}, \quad (92)$$

which was denoted as GADD therein.

It follows from (89)-(92) that the detectors choose a direction among the subspace spanned by the columns of  $\mathbf{H}$ . In other words, the detection problem in (88) tantamount to finding a direction with the largest possibility in a given subspace, and hence it is called direction detection in [95].

The problem of direction detection can be further generalized when both the column component and row component of the signal to be detected lie in given subspaces. To be precise, the test data under hypothesis  $H_1$  becomes  $\mathbf{X} = \mathbf{H}\boldsymbol{\theta}\boldsymbol{\alpha}^H \mathbf{C} + \mathbf{N}$ , with  $\mathbf{C}$  a given  $M \times K$  full-row-rank matrix and  $\boldsymbol{\alpha}$  an  $M \times 1$  vector. This kind of problem is denoted as generalized direction detection in [98], where the GLRT and 2S-GLRT in HE were proposed therein. Moreover, the Wald test in HE was given in [360], and the 2S-GLRT in PHE was derived in [99].

Different from (88), another generalization of (67) is the case that each column of the test data  $\mathbf{X}$  has a slightly different steering vector in the sense that these steering vectors are different but all come from the same subspace. Hence, the detection model in (67) can be modified as

$$\begin{cases} H_0 : \mathbf{X} = \mathbf{N}, \mathbf{x}_{e,l} = \mathbf{n}_{e,l}, l = 1, 2, \dots, L, \\ H_1 : \mathbf{X} = \mathbf{H}\boldsymbol{\Phi} + \mathbf{N}, \mathbf{x}_{e,l} = \mathbf{n}_{e,l}, l = 1, 2, \dots, L, \end{cases} \quad (93)$$

where  $\mathbf{H}$  is an  $N \times p$  full-column-rank matrix, and  $\boldsymbol{\Phi}$  is a  $p \times K$  matrix standing for the coordinates. The GLRT in HE was proposed in Kelly and Forsythe's classic report [100], while the Rao test and Wald test in HE can be obtained according to the results in [82]. Precisely, the GLRT, Rao test, and Wald test are given by

$$t_{\text{GLRT}} = \frac{|\mathbf{I}_K + \mathbf{X}^H \mathbf{S}^{-1} \mathbf{X}|}{\left| \mathbf{I}_K + \mathbf{X}^H \mathbf{S}^{-1} \mathbf{X} - \mathbf{X}^H \mathbf{S}^{-1} \mathbf{H} (\mathbf{H}^H \mathbf{S}^{-1} \mathbf{H})^{-1} \mathbf{H}^H \mathbf{S}^{-1} \mathbf{X} \right|}, \quad (94)$$

<sup>20)</sup> The GADD in (92) can be also derived according to 2S-Wald test.

$$t_{\text{Rao}} = \text{tr} \left\{ \mathbf{X}^H (\mathbf{S} + \mathbf{X}\mathbf{X}^H)^{-1} \mathbf{H} \left[ \mathbf{H}^H (\mathbf{S} + \mathbf{X}\mathbf{X}^H)^{-1} \mathbf{H} \right]^{-1} \mathbf{H}^H (\mathbf{S} + \mathbf{X}\mathbf{X}^H)^{-1} \mathbf{X} \right\}, \quad (95)$$

and

$$t_{\text{Wald}} = \text{tr} \left[ \mathbf{X}^H \mathbf{S}^{-1} \mathbf{H} (\mathbf{H}^H \mathbf{S}^{-1} \mathbf{H})^{-1} \mathbf{H}^H \mathbf{S}^{-1} \mathbf{X} \right], \quad (96)$$

respectively.

Note that when  $p = N$  in the detection problem in (93), the signal steering vectors lie in the whole observation space. Or, equivalently, the steering vectors are completely unknown. The correspond GLRT, 2S-GLRT, and a modified 2S-GLRT (M2S-GLRT) in HE were proposed in [92]. It was also shown in [93] that the M2S-GLRT is essentially the corresponding Rao test, while the 2S-GLRT can also be derived according to the Wald test. Moreover, in [82, 101] the test data in (93) were generalized to the case  $\mathbf{X} = \mathbf{H}\mathbf{\Phi}\mathbf{C} + \mathbf{N}$ , with  $\mathbf{\Phi}$  being a  $p \times M$  unknown matrix,  $\mathbf{C}$  being an  $M \times K$  known full-row-rank matrix. The corresponding signal model was called double subspace (DOS) model<sup>21)</sup> in [82, 101], where many adaptive detectors were proposed.

#### 4.3.2 Adaptive detectors in the presence of interference

Most of the above detectors are designed without taking into account the possibility of interference, which usually exists in practice. Interference can be caused intentionally (jamming due to the ECM) or unintentionally (communication signals or radar signals transmitted by other radar systems). In this case, the detection problem in (1) can be modified as

$$\begin{cases} \text{H}_0 : \mathbf{x} = \mathbf{n}, \mathbf{x}_{e,l} = \mathbf{n}_{e,l}, l = 1, 2, \dots, L, \\ \text{H}_1 : \mathbf{x} = \mathbf{s} + \mathbf{j} + \mathbf{n}, \mathbf{x}_{e,l} = \mathbf{n}_{e,l}, l = 1, 2, \dots, L, \end{cases} \quad (97)$$

where  $\mathbf{j}$  stands for the interference. Roughly speaking, there are two main kinds of interference. One is coherent interference, while the other is noise interference. For the former, it works like a real target, which usually lies in a certain spatially direction and/or occupies a Doppler bin. Hence, the coherent interference can be modelled by a subspace model. For the latter, it works like thermal noise or clutter. As a result, the noise interference changes the noise covariance matrix of the test data.

Based on the above analysis, the coherent interference can be modelled as  $\mathbf{j} = \mathbf{J}\boldsymbol{\phi}$ , where the  $N \times q$  full-column-rank matrix spans the subspace where the interference lies, and the  $q \times 1$  vector  $\boldsymbol{\phi}$  denotes the unknown coordinates. For coherent interference and subspace signals (i.e., the signal in (97) can be expressed as  $\mathbf{s} = \mathbf{H}\boldsymbol{\theta}$ ), the GLRT and 2S-GLRT in HE and PHE for the detection problem in (97) were all proposed in [153], and the GLRT and 2S-GLRT in PHE coincide with each other. Precisely, the GLRT and 2S-GLRT in HE are

$$t_{\text{GLRT-HE-I}} = \frac{\tilde{\mathbf{x}}^H \mathbf{P}_{\mathbf{J}^\perp \tilde{\mathbf{H}}} \tilde{\mathbf{x}}}{1 + \tilde{\mathbf{x}}^H \mathbf{P}_{\mathbf{J}^\perp \tilde{\mathbf{H}}} \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^H \mathbf{P}_{\mathbf{J}^\perp \tilde{\mathbf{H}}} \tilde{\mathbf{x}}} \quad (98)$$

and

$$t_{\text{2S-GLRT-HE-I}} = \tilde{\mathbf{x}}^H \mathbf{P}_{\mathbf{J}^\perp \tilde{\mathbf{H}}} \tilde{\mathbf{x}}, \quad (99)$$

respectively, while the GLRT in PHE is

$$t_{\text{GLRT-PHE-I}} = \frac{\tilde{\mathbf{x}}^H \mathbf{P}_{\mathbf{J}^\perp \tilde{\mathbf{H}}} \tilde{\mathbf{x}}}{\tilde{\mathbf{x}}^H \mathbf{P}_{\mathbf{J}^\perp \tilde{\mathbf{H}}} \tilde{\mathbf{x}}}, \quad (100)$$

where  $\mathbf{P}_{\mathbf{J}^\perp \tilde{\mathbf{H}}} = \mathbf{P}_{\mathbf{J}^\perp} \tilde{\mathbf{H}} (\tilde{\mathbf{H}}^H \mathbf{P}_{\mathbf{J}^\perp} \tilde{\mathbf{H}})^{-1} \tilde{\mathbf{H}}^H \mathbf{P}_{\mathbf{J}^\perp}$ ,  $\mathbf{P}_{\mathbf{J}^\perp} = \mathbf{I}_N - \mathbf{P}_{\mathbf{J}}$ , and  $\mathbf{P}_{\mathbf{J}} = \tilde{\mathbf{J}} (\tilde{\mathbf{J}}^H \tilde{\mathbf{J}})^{-1} \tilde{\mathbf{J}}^H$ . For convenience, the detectors in (98), (99), and (100) are referred to the GLRT in HE with interference rejection (GLRT-HE-I), 2S-GLRT in HE with interference rejection (2S-GLRT-HE-I), and GLRT in PHE with interference rejection (GLRT-PHE-I), respectively.

21) The DOS signal model was first introduced in [100]. However, it was assumed in [100] that no training data were available. Instead, it was assumed  $K \geq M + N$ . This constraint ensures the existence of a set of virtual training data, generated by a certain unitary matrix to the test data.



For coherent interference, the Rao test and 2S-Rao test were proposed in [155], while the Wald test and 2S-Wald test were derived in [156]. Precisely, in HE the Rao test and 2S-Rao test are

$$t_{\text{Rao-HE-I}} = \frac{\tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \mathbf{P}_{\tilde{\mathbf{H}}} \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \tilde{\mathbf{x}}}{(1 + \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \tilde{\mathbf{x}})(1 + \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \mathbf{P}_{\tilde{\mathbf{H}}} \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \tilde{\mathbf{x}})} \quad (101)$$

and

$$t_{2\text{S-Rao-HE-I}} = \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \mathbf{P}_{\tilde{\mathbf{H}}} \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \tilde{\mathbf{x}}, \quad (102)$$

respectively, while in PHE the Rao test is the same as the 2S-Rao test, given by

$$t_{\text{Rao-PHE-I}} = \frac{\tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \mathbf{P}_{\tilde{\mathbf{H}}} \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \tilde{\mathbf{x}}}{\tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \tilde{\mathbf{x}}}. \quad (103)$$

The Wald test is the same as the 2S-Wald test both in HE and PHE, given by

$$t_{\text{Wald-HE-I}} = \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}|\tilde{\mathbf{J}}}^H \mathbf{P}_{\tilde{\mathbf{H}}|\tilde{\mathbf{J}}} \tilde{\mathbf{x}} \quad (104)$$

and

$$t_{\text{Wald-PHE-I}} = \frac{\tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{H}}|\tilde{\mathbf{J}}}^H \mathbf{P}_{\tilde{\mathbf{H}}|\tilde{\mathbf{J}}} \tilde{\mathbf{x}}}{\tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{B}}}^\perp \tilde{\mathbf{x}}}, \quad (105)$$

respectively, where  $\mathbf{P}_{\tilde{\mathbf{H}}|\tilde{\mathbf{J}}} = \tilde{\mathbf{H}}(\tilde{\mathbf{H}}^H \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \tilde{\mathbf{H}})^{-1} \tilde{\mathbf{H}}^H \mathbf{P}_{\tilde{\mathbf{J}}}^\perp$  is the oblique projection matrix onto the subspace spanned by  $\tilde{\mathbf{H}}$  along the subspace spanned by  $\tilde{\mathbf{J}}$ . Detailed analysis and comparison of the above detectors can be found in [156].

At present, only the GLRT-HE-I, 2S-GLRT-HE-I, and GLRT-PHE-I have known statistical properties, given in [159]. Precisely, the conditional distribution of the GLRT-HE-I in (98) with a fixed  $\beta_{\text{I}}$  under hypothesis  $\text{H}_1$ , is

$$t_{\text{GLRT-HE-I}} | [\beta_{\text{I}}, \text{H}_1] \sim \mathcal{CF}_{p, L-N+q+1}(\rho_{\text{eff}} \beta_{\text{I}}), \quad (106)$$

where

$$\rho_{\text{eff}} = \bar{\mathbf{s}}_0^H \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \mathbf{P}_{\tilde{\mathbf{P}}_{\tilde{\mathbf{H}}}} \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \bar{\mathbf{s}}_0 \quad (107)$$

is defined as the effective SNR (eSNR), and  $\beta_{\text{I}}$  is loss factor defined as

$$\beta_{\text{I}} = \frac{1}{1 + \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \tilde{\mathbf{x}} - \tilde{\mathbf{x}}^H \mathbf{P}_{\tilde{\mathbf{P}}_{\tilde{\mathbf{H}}}} \tilde{\mathbf{x}}}. \quad (108)$$

The statistical distribution of  $\beta_{\text{I}}$  under hypothesis  $\text{H}_1$  is

$$\beta_{\text{I}} | \text{H}_1 \sim \mathcal{CB}_{L-N+p+q+1, N-p-q}(\delta_{\text{I}}^2), \quad (109)$$

where

$$\delta_{\text{I}}^2 = \bar{\mathbf{s}}_0^H \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \mathbf{P}_{\tilde{\mathbf{P}}_{\tilde{\mathbf{H}}}} \mathbf{P}_{\tilde{\mathbf{J}}}^\perp \bar{\mathbf{s}}_0, \quad (110)$$

with  $\mathbf{P}_{\tilde{\mathbf{P}}_{\tilde{\mathbf{H}}}}^\perp = \mathbf{I}_N - \mathbf{P}_{\tilde{\mathbf{P}}_{\tilde{\mathbf{H}}}}$ . Under hypothesis  $\text{H}_0$ , (106) and (109) reduce to

$$t_{\text{GLRT-HE-I}} | [\beta_{\text{I}}, \text{H}_0] \sim \mathcal{CF}_{p, L-N+q+1} \quad (111)$$

and

$$\beta_{\text{I}} | \text{H}_0 \sim \mathcal{CB}_{L-N+p+q+1, N-p-q}, \quad (112)$$

respectively.

More geometric interpretation about the eSNR in (107) can be found in [159]. Moreover, the following two equations can be easily verified

$$t_{2\text{S-GLRT-HE-I}} = \frac{t_{\text{GLRT-HE-I}}}{\beta_{\text{I}}}, \quad (113)$$

$$t_{\text{GLRT-PHE-I}} = \frac{t_{\text{GLRT-HE-I}}}{1 - \beta_{\text{I}}}. \quad (114)$$

Using (113) and (114), along with (106), (109), (111) and (112), we can obtain the analytical expressions for the PDs and PFAs of the 2S-GLRT-HE-I and GLRT-PHE-I.

For completely unknown noise interference, it was shown in [178] that the GLRT for rank-one signals is equivalent to the ACE. In The corresponding Rao test was derived in [126], i.e., the DN-AMF, originally adopted for mismatched signal detection. The results in [126, 178] were generalized in [179] when additional coherent interference existed. In [180] the noise interference was assumed to be orthogonal to the signal of interest in the whitened space, and it was shown that the GLRT coincides with the KGLRT. Moreover, it was shown in [181] that the corresponding Rao and Wald tests are the same as the DMRao and AMF, respectively. The results in [180, 181] were generalized in [182] for the case of subspace signals. Some other generalizations for noise interference can be found in [183, 185–189].

## 5 Conclusions

In this paper, we investigated the detector design criteria for adaptive detection, analysed the relationship between adaptive detection and the filtering-then-CFAR detection approach, as well as the relationship between adaptive detectors and adaptive filters, gave a comprehensive review, summarized and compared typical adaptive detectors. Adaptive detection jointly uses the test and training data to form an adaptive detector. Compared with the filtering-then-CFAR detection approach (adaptive or non-adaptive), adaptive detection has many distinct features. It achieves the function of filtering and CFAR processing simultaneously, and hence, it has simple detection procedure. Moreover, it can provide better detection performance.

We hope that this paper will stimulate new researches on adaptive detection. Some possible further research tracks are listed below. 1) The statistical performance of many adaptive detectors are needed to be studies, such as the Rao and Wald tests in subspace interference [155, 156], the 2S-GLRT in HE in the presence of signal mismatch [53]. Obtaining these results can reveal how the signal mismatch and/or interference affect the detection performance. 2) Nowadays, multichannel signal detection has been combined with compressive sensing or sparse representation, which is an emerging signal processing technique for efficiently acquiring and reconstructing a compressible signal, by using much fewer samples. Several compressive sensing-based detectors were proposed, such as [361–367] and the references therein. However, most proposed detectors based on compressive sensing are for known noise or white Gaussian noise with unknown variance. Much more challenging task is for colored noise with unknown covariance matrix. 3) Most existing adaptive detectors were designed under specific assumptions on the noise, either homogeneous, partially homogeneous, compound-Gaussian, or structure nonhomogeneity. However, the actual noise may be different from the assumed one, due to system and environment uncertainties. As a consequence, the designed detectors may suffer from significant performance loss. Therefore, it is necessary to devise fully adaptive detection approaches which can adjust the detection strategy to accommodate the changing environments. Recently, some preliminary analysis on classification of noise covariance structure in Gaussian background was proposed in [290, 355, 368, 369]. 4) Recently, some preliminary results of machine learning were utilized in adaptive detection [370–372]. However, it was not fully addressed the fundamental problem that why and how the detection performance can be improved by using machine learning technologies.

In this paper, we mainly focused on the Gaussian background. In practice, the environment may exhibit non-Gaussian character [60, 61, 373–375]. Interesting readers can refer to a recently overview paper [7] on compound-Gaussian clutter, for the case that the relevant properties of the clutter are assumed to be known in advance.

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