optical signal processing. Until now, the latter has been restricted to processing 1-D and 2-D signals only because in order for a signal to be processed optically, it has to be represented on a planar screen.

Although holography allows planar representation of 3-D images, it does not allow us to compute set and morphological operations on them. Specifically, suppose we have two holograms (of two 3-D images, respectively), and wish, by manipulating them, to obtain the hologram of either union, intersection, dilation, or erosion of these images. There is no way of doing so.

On the contrary, suppose two 3-D images are encoded as proposed above. Then, the planar encoding of their union, intersection, dilation, or erosion can be obtained by computing these operations on the above encodings. Thus, the proposed 3-D-to-2-D mapping paves the way for implementation of 3-D mathematical morphology optically (2-D mathematical morphology has been already implemented optically by many researchers [4], [5]).

Finally, in practice, it is, of course, impossible to map all rational points as well as to map a rational point to an irrational point. Instead, only nodes of a cubical mesh should be mapped. The spacing of the mesh should satisfy the Nyquist criteria in order not to loose information. In addition, the spacing should be much higher than the precision of the computer arithmetics. In the case of displaying the mapping on a screen or other planar material, the spacing should also be much higher than the resolution of the screen/material.

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Multichannel Distance Filter

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***Abstract*—A nonlinear multichannel digital filter is presented in this correspondence. The output is a weighted sum of all samples in the filter window, with a single parameter controlling the filter nonlinearity. Although input data ordering is not required, performance can surpass the performance of other ordering-based multichannel filters.**

***Index Terms*—Distance filters, multichannel signals, nonlinear filters.**

I. Introduction

A discrete multichannel signal is defined as a time series vector of components called channels, which are generally correlated and characterized by their joint probability density function (pdf). Two-dimensional (2-D) multichannel signals, e.g., color images and motion fields, are frequently met in practice; rather high correlation is usually exhibited between their components. Multivariate signals can be processed by applying single-channel techniques to each channel separately. However, no correlation between signal components is taken into account in the single-channel approach. In this correspondence, a novel nonlinear multichannel digital filter will be studied.

The nonlinear digital filtering techniques have been extensively used in signal and image processing [1]. Although most of them were related to scalar signal processing, many efforts have been made recently to extend them to the multichannel case. A marginal ordering scheme for ordering multivariate data, as well as various multichannel estimators such as the marginal median, the marginal α-trimmed mean, and the modified trimmed mean filters, have been proposed in [2] and [3]. Multichannel L-filters based on marginal ordering, as well as their theoretical properties and optimal design, is presented in [4]. Vector median filters (VMF's) have been derived from multidimensional exponential pdf's by adopting a maximum likelihood estimate approach [5]. The reduced ordering is another scheme for ordering multivariate data, which has been used to define nonlinear filters [6], [7]. The vector directional filters (VDF's) process separately the direction and magnitude of a signal vector [8], whereas the directional-distance filters (DDF's) combine the characteristics of both VMF's and VDF's [9]. An overview of most of these filtering techniques is presented in [10] and [11].

Most of the methods that were presented in the previous paragraph have a computation step in common: Some sort of ordering is performed on the input data vectors. Certain filters have been researched that avoid this step by producing output that is a weighted sum of the sample vectors inside the filter window with filtering performance similar to the one of order statistic filters. Data-dependent filters have been presented in [12] (also in [13]) for single channel data, with coefficients that are expressed as sums of absolute differences. Their multichannel extensions have been presented in [14]. Another class of multichannel data-dependent filters (called multichannel distance filters) that has been developed independently has been presented in [15]. In these cases, coefficients can also be expressed in terms of Euclidean distances. The data-dependent filters exhibit a locally

Manuscript received November 12, 1997; revised April 16, 1999. The associate editor coordinating the review of this paper and approving it for publication was Prof. P. C. Ching.

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Publisher Item Identifier S 1053-587X(99)09199-0.

adaptive characteristic, which is a highly desirable feature in signal processing. Additionally, techniques that utilize ordering exhibit hard nonlinearity characteristics, making them rather unsuitable for filtering multichannel signals corrupted by short- or medium-tailed noise. Consequently, the ability to control the filter linearity is very important.

The *multichannel distance filter (MDF)* presented in this correspondence attempts to address these two goals. As will be shown later on, no data ordering is performed, yet performance similar to the one presented above can be attained. Finally, a single scalar parameter controls the filter linearity, which makes these methods natural candidates for adaptation.

A review of multivariate data ordering is presented in Section II. The MDF is defined in Section III. In Section IV, experimental results of the MDF filtering performance are obtained. For comparison purposes, the performance of other well-known multivariate filters, on the same test signals, has also been reported. Finally, certain conclusions are drawn in Section V.

II. Overview of Multivariate Data Ordering

Ordering of multivariate data is not unambiguous, as it is in the case of single-channel data. Several ways of ordering multivariate data are discussed in the literature [3]. Three classes of ordering methods (marginal, reduced, and distance ordering) will be used to define nonlinear filters. These methods will be shortly presented here. Let $\mathbf{X}$ denote a p-dimensional random vector variable, i.e., a p-dimensional vector of random variables $\mathbf{X} = [X_1, X_2, \cdots, X_p]^T$. Let $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N$ denote N samples of $\mathbf{X}$ with $\mathbf{x}_i = [x_{i_1} \; x_{i_2} \; \cdots \; x_{i_p}]^T$. Finally, let $\mathbf{x}_{(i)}$ denote the ith sample of the ordered set.

A. Fundamentals

In marginal ordering (M-ordering), the samples are ordered independently, along each one of the p channels. The marginal median is the vector

$$\mathbf{x}_{\text{med}} = [\text{med}\{x_{1_1}, \cdots, x_{N_1}\} \; \cdots \; \text{med}\{x_{1_p}, \cdots, x_{N_p}\}]^T.$$

The statistical analysis of marginal order statistics is given in [3].

In reduced ordering (R-ordering) sample vectors, $\mathbf{x}_i$ are ordered according to their distance from a reference point $\mathbf{a}$. The ordered vectors satisfy the relationship

$$\|\mathbf{x}_{(1)} - \mathbf{a}\| \le \cdots \le \|\mathbf{x}_{(N)} - \mathbf{a}\|. \tag{1}$$

The arithmetic mean or the marginal median of a subset of the input data set may be utilized as the reference vector $\mathbf{a}$. Obviously, the number of samples used to determine a reference point does not have to be equal to the number of ordered samples. The operator $\|\cdot\|$ in (1) may represent either the Euclidean or any other (e.g., Mahalanobis) distance for that purpose.

Finally, an ordering technique, which is proposed in this correspondence, is based on ordering input vectors on the sum of the distances between a certain sample and all the other ordered ones (distance ordering)

$$\sum_{j=1}^{N} \|\mathbf{x}_{(1)} - \mathbf{x}_j\| \le \cdots \le \sum_{j=1}^{N} \|\mathbf{x}_{(N)} - \mathbf{x}_j\|.$$

B. L-Filters for Multivariate Data

The methods of Section II-A for ordering multivariate data can be used to define nonlinear multichannel filters. These filters can be classified as L-filters because the output of each one is a linear combination of the ordered input samples. Consequently, they can be thought of as generalizations of single-channel L-filters [16], [17].

The output $\mathbf{y} = [y_1 \; y_2 \; \cdots \; y_p]^T$ of the M-ordering L-filter is equal to

$$\mathbf{y} = \sum_{i_1=1}^{N} \cdots \sum_{i_p=1}^{N} \mathbf{A}_{i_1,\cdots,i_p} \mathbf{x}_{(i_1,\cdots,i_p)} \tag{2}$$

where $\mathbf{A}_{i_1,\cdots,i_p}$ is a coefficient matrix. The ordered vector $\mathbf{x}_{(i_1,\cdots,i_p)}$ has its jth component equal to the i_jth largest statistic of the vectors' jth component. A theoretical analysis and an optimization method for the design of this filter is given in [4].

The output $\mathbf{y}$ of the R-ordering L-filter is given by

$$\mathbf{y} = \sum_{i=1}^{N} a_i \mathbf{x}_{(i)} \tag{3}$$

where a_i, with $i = 1, \cdots, N$, denote filter coefficients. The R_1 filter [7], the ranked-order estimators R_E and R_M [6], and the double window modified trimmed mean (DW MTM) filter [16] represent special cases of this filter class.

Finally, the distance ordering L-filter output is also given by (3). The vector median filter is the most known representative of this category [5].

III. The Multichannel Distance Filter

The MDF is a data-dependent nonlinear filter with output $\mathbf{y}$ given by

$$\mathbf{y} = \frac{\sum_{i=1}^{N} a_i \mathbf{x}_i}{\sum_{i=1}^{N} a_i}. \tag{4}$$

The coefficients a_i are given by

$$a_i = \left(\sum_{k=1}^{N} \|\mathbf{x}_i - \mathbf{x}_k\|^2 \right)^r \tag{5}$$

where r is a filter parameter. Equation (4) can be written in a more compact notation if normalized coefficients are utilized:

$$\mathbf{y} = \sum_{i=1}^{N} a_i' \mathbf{x}_i \tag{6}$$

where

$$a_i' = \frac{a_i}{\sum_{j=1}^{N} a_j}. \tag{7}$$

A small number must be added to the sum of the distances in order to avoid division by zero if the input signal $\mathbf{x}$ is constant, and $r < 0$.

The MDF coefficients depend only on the distances between samples. All samples within the filter window are used in the calculation of the filter output. It should be obvious that no input data ordering is performed. The parameter r controls the amount of applied nonlinearity. For small values of r, the filter behaves in an almost linear fashion, whereas for larger values of r, linearity diminishes. Thus, filter behavior can be easily altered. Indeed, this filter becomes equivalent to the vector median filter if the linearity parameter r tends to infinity $(r \to -\infty)$, as can be easily shown.

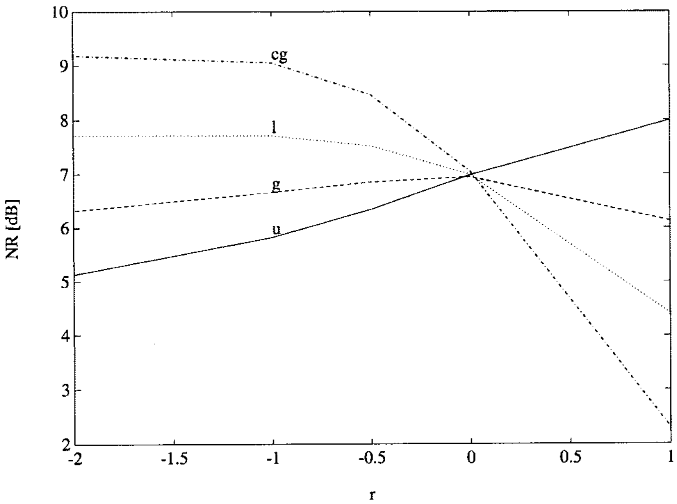

Fig. 1. MDF Noise reduction for different noise distributions (u: uniform, g: Gaussian, l: Laplacian, cg: cont. Gaussian) for a 1-D, two-channel constant signal and filter length $N = 5$.

IV. Experimental Results

Simulations were carried out in order to estimate MDF performance on the following multivariate signals:

- one-dimensional, two-channel sequences;
- velocity fields (2-D, 2-channel sequences);
- color images (2-D, 3-channel sequences).

Test (reference) signals were either existing or artificially generated ones. Subsequently, reference signals were corrupted by artificially generated noise. The modified noise reduction index NRI [4]

$$\mathrm{NRI} = -10 \log \frac{\sum_k (\mathbf{y}(k) - \mathbf{s}(k))^T (\mathbf{y}(k) - \mathbf{s}(k))}{\sum_k (\mathbf{x}(k) - \mathbf{s}(k))^T (\mathbf{x}(k) - \mathbf{s}(k))} \tag{8}$$

was utilized as a quantitative criterion of filter performance, where $\mathbf{x}(k)$, $\mathbf{y}(k)$, and $\mathbf{s}(k)$ denote the values of the corrupted (noisy), filtered, and reference signals respectively, at sample index k.

Data from uniform (denoted by U), Gaussian (G), Laplacian (L), and contaminated Gaussian (CG) distributions were generated in order to quantify filter performance in dissimilar noise environments. The contaminated Gaussian distribution $CN(\mathbf{m}_1, \mathbf{C}_1, \mathbf{m}_2, \mathbf{C}_2, \varepsilon)$ is a linear combination of two normal distributions

$$\begin{aligned} &CN(\mathbf{m}_1, \mathbf{C}_1, \mathbf{m}_2, \mathbf{C}_2, \varepsilon) \\ &\quad = (1 - \varepsilon) N(\mathbf{m}_1, \mathbf{C}_1) + \varepsilon N(\mathbf{m}_2, \mathbf{C}_2) \end{aligned} \tag{9}$$

where $0 \leq \varepsilon < 1$ denotes the contamination factor.

A. 1-D Sequence Filtering

In simulations on 1-D sequence filtering, the corrupted signal $\mathbf{x} = \mathbf{s} + \mathbf{n}$ was the sum of a constant two-channel signal $\mathbf{s} = (1, 2)^T$ and white, zero-mean noise $\mathbf{n}$. Four noise distributions were examined: uniform, Gaussian, Laplacian, and contaminated Gaussian. The first three distributions had variances equal to $\sqrt{2}$ and correlation coefficient equal to 0.25. The variances and the correlation coefficient of the first distribution of the contaminated Gaussian were equal to 1 and 0.2, respectively. For the second distribution, these parameters were set to 1 and 0.2, respectively. The contamination factor was equal to $\varepsilon = 0.1$.

TABLE I
Optimal Values of the Exponent r and Maximal Noise Reduction for 1-D, Two-Channel Constant Signal and Window Length $N = 5$

Filter type	Noise distribution			
	U	G	L	CG
r	1.092	-0.014	-1.462	-1.637
NRI [dB]	8.002	6.945	7.747	9.209

TABLE II
Comparison of the Noise Reduction (in Decibels) of the MDF and Other Reported Filters for 1-D, Two-Channel Constant Signal and Window Length $N = 5$

Filter type	Noise distribution			
	U	G	L	CG
arithmetic mean	6.98	6.94	6.99	7.05
Marginal median	3.72	5.50	7.62	8.49
Vector median [5]	2.40	4.13	6.24	7.29
R_E $j = 1$ [6]	3.02	4.42	5.82	6.61
R_M $j = 1$ [6]	2.94	4.36	5.78	6.56
R_1 [7]	2.20	3.68	5.36	6.55
R_2 [7]	2.33	3.67	5.18	6.41
adaptive R_2 [7]	2.93	3.97	4.70	5.16
MDF	**7.99** 1)	**6.94** 2)	**7.72** 3)	**9.18** 3)

1) $r = 1$, 2) $r = 0$, 3) $r = -1$

Fig. 1 shows how the performance depends on the value of the exponent r and on the type of the noise distributions. The optimal values of r are shown in Table I. The greatest noise reduction is achieved for $r \simeq 1$ if noise is uniformly distributed. The filter coefficients are proportional to the sum of distances between samples in this case. Consequently, the remotest samples have the greatest weights (as expected). Thus, the MDF behaves like a midrange estimator, which is optimal for this type of noise [16]. In the case of Gaussian noise, the optimal value of r is approximately equal to zero. Thus, all coefficients are equal, and behavior equivalent to the one of an averaging filter (which is an optimal estimator for additive Gaussian noise) is obtained. The optimal value of r lies between -2 and -1 in the case of long-tailed distributions: Laplacian and contaminated Gaussian. In essence, centrally located samples are heavily weighted, and remote samples (outliers) are practically discarded. Thus, filter behavior can be altered at will by adapting the value of the parameter r, depending on the signal local statistics.

Performance indices are tabulated in Table II. MDF performance is compared with the performance of other well-known filters:

- arithmetic mean;
- marginal and vector median [5];
- R_E and R_M [6];
- R_1, R_2 and adaptive R_2 filters [7].

From Fig. 1, the highest performance index for the MDF was selected for this table. It is evident that better performance than both marginal and vector median filters was attained in all noise distributions. The performance difference was quite high in the case of uniform noise (more that 4 dB). MDF performance is also better than the one of the arithmetic mean filter in the case of Laplacian and contaminated Gaussian distributions (almost 2 dB and more than 3 dB, respectively).

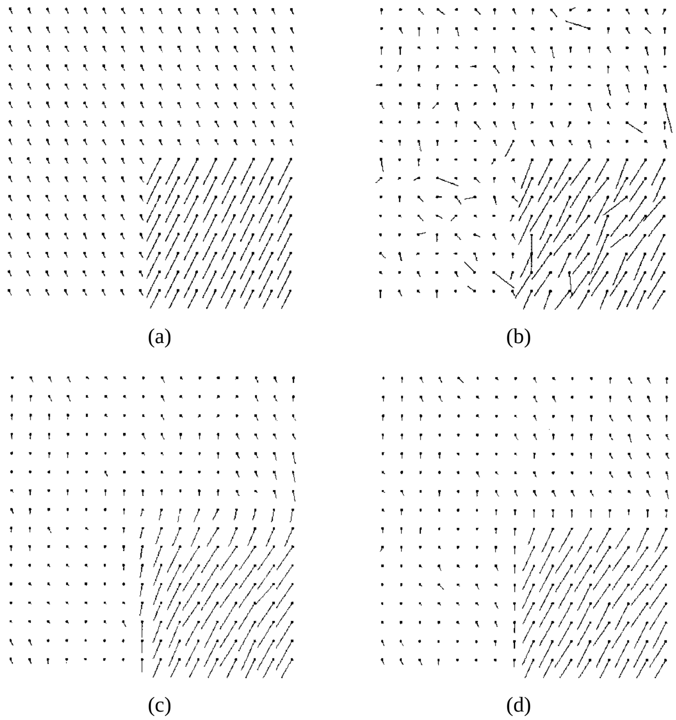

Fig. 2. Velocity field filtering. (a) Original velocity field. (b) Velocity field corrupted by Laplacian noise. (c) Output of a 5 × 5 moving average filter. (d) Output of a 5 × 5 MDF filter.

B. Velocity Field Filtering

An artificially generated 2-D velocity field of size 64 × 64 vectors, composed of two constant velocity regions, was utilized as the reference signal in the experiments on velocity field filtering. It is shown in Fig. 2. Velocity vectors have values $(1, 2)^T$ in the first region and $(-5, 10)^T$ in the second one. The same noise distributions as in the case of 1-D sequence filtering were used to corrupt the reference field.

Fig. 2 shows a graphical representation of a part of the original velocity field (upper-left picture) and the same part corrupted by Laplacian noise (upper-right). This noisy vector field was subsequently filtered by an averaging (bottom-left) and an MDF (bottom-right) filter, with a window of 5 × 5 points. For the MDF, a value of $r = -2$ was used.

Fig. 3 shows the dependency of the MDF on the value of the parameter r. This dependency is different from the one depicted in Fig. 1. Higher performance is attained for all noise distributions for negative values of r for all noise distributions. This happens because there are both constant regions and edges in the input velocity field. Edges are preserved better for negative values of r because filter behavior is similar to the one of the vector median filer. Performance in constant regions is better for negative (positive) values of r in the case of long-tailed (short-tailed) distributions. Thus, increasing r from negative values to positive ones decreases the MDF filter performance in both edges and constant regions (if the noise has a long-tailed distribution). On the contrary, if the noise has a short-tailed distribution, the same change of r decreases the filter performance on edges, but increases it in constant regions (compensating, partially at least, for the decrease on edges). Therefore, filter performance dependency on the value of r is greater in the case of long-tailed distributions than in the case of short-tailed ones, as can be seen in Fig. 3.

For performance comparisons, the arithmetic mean, marginal, and vector median R_E, R_1, and R_2 filter performances have also been reported, with respect to the same velocity field. The results of this comparison are depicted in Table III. The MDF filter exhibited the highest NR indices in all noise distributions, although in the case of the Laplacian noise, the performance gain was rather small. However, it should be mentioned that these results were obtained with the same value of the parameter $r = -2$ in all types of noise.

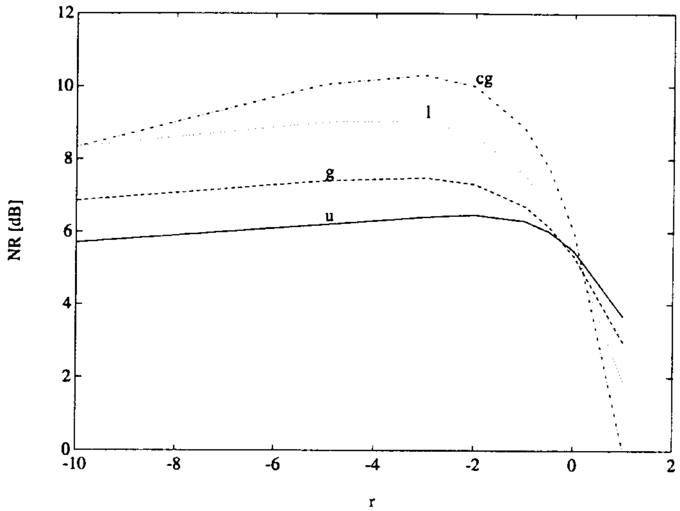

Fig. 3. Noise reduction of the MDF filter, having window size 3 × 3 for a velocity field corrupted by different noise distributions (u: uniform, g: Gaussian, l: Laplacian, cg: cont. Gaussian).

TABLE III
NRI PERFORMANCE COMPARISON OF THE MDF AND OTHER REPORTED FILTERS FOR WINDOW SIZE 3 × 3 ON A VELOCITY FIELD OF SIZE 64 × 64 HAVING TWO REGIONS

Filter Type	Noise Distribution			
	U	G	L	CG
arithmetic mean	5.484	5.335	5.443	6.007
marginal median	4.865	6.467	8.517	9.603
vector median (L_1 norm)	3.638	5.401	7.564	8.669
vector median (L_2 norm)	4.090	5.693	7.714	8.901
R_E filter	4.458	5.596	6.698	7.313
R_1 filter	3.481	5.166	6.869	7.916
R_2 filter	3.547	4.880	6.291	7.330
MDF $r = -2$	**6.473**	**7.317**	**8.661**	**10.004**

C. Color Image Filtering

The color image "Lenna" of size 512 × 512 pixels and 8 bits per color channel was selected for the experiments on color image filtering. Impulsive noise (I) was also utilized, in addition to the four noise distributions previously utilized. Noise distributions had the following parameters:

- uniform and Laplacian: $\sigma_R = \sigma_G = \sigma_B = 20$, $r_{RG} = r_{RB} = r_{GB} = 0$;
- Gaussian: $\sigma_R = \sigma_G = \sigma_B = 20$, $r_{RG} = r_{RB} = r_{GB} = 0.5$;
- contaminated Gaussian:

  a) first distribution: $\sigma_R = \sigma_G = \sigma_B = 10$, $r_{RG} = r_{RB} = r_{GB} = 0.25$;
  b) second distribution: $\sigma_R = \sigma_G = \sigma_B = 50$, $r_{RG} = r_{RB} = r_{GB} = 0.75$;
  c) contamination factor: $\varepsilon = 0.2$;

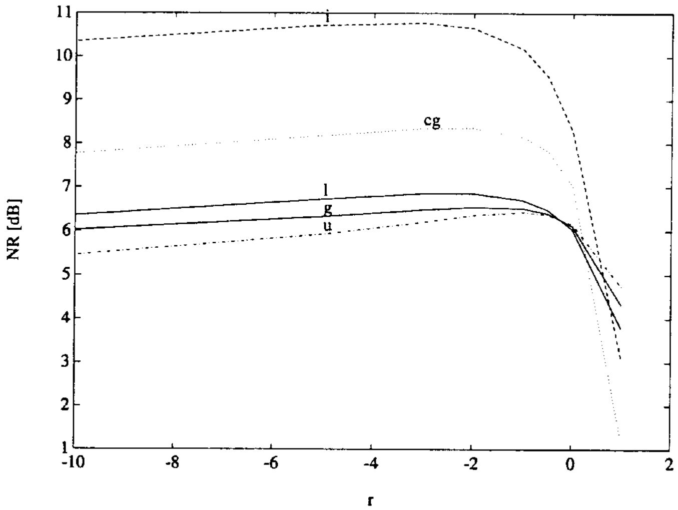

Fig. 4. Noise reduction of the MDF filter on image Lenna corrupted by different noise distributions (u: uniform, g: Gaussian, l: Laplacian, cg: cont. Gaussian, i: impulsive). A 5 × 5 point window was utilized.

TABLE IV
NRI PERFORMANCE COMPARISON OF THE MDF AND OTHER REPORTED FILTERS, ON IMAGE LENNA OF SIZE 512 × 512 PIXELS

Filter Type	Noise Distribution				
	U	G	L	CG	I
arithmetic mean	6.155	6.116	6.043	7.040	8.304
marginal median	5.631	6.328	5.716	**8.390**	**11.038**
vector median (L_1 norm)	3.800	5.116	5.716	7.651	10.895
vector median (L_2 norm)	3.913	4.850	5.071	6.542	8.986
MDF $r = -2$	**6.355**	**6.543**	**6.856**	8.357	10.655

- impulsive: uncorrelated impulsive noise with probability 0.1 in each channel.

Fig. 4 shows the dependency of the MDF performance on the value of the parameter r. This dependency is similar as in the case of velocity field filtering (Fig. 3). The best performance is achieved with $-3 < r < -1$. Noise reduction rapidly decreases for $r > -1$, mainly due to the distortions on the image edges.

Performance results are presented in Table IV. The performance indices of other multichannel filters were also included for comparison purposes. Similarly, as in the case of 1-D sequences and velocity field filtering, the MDF filter was exhibited in three out of five noise types. However, in the case of contaminated Gaussian noise, the MDF was second best, being slightly inferior to the marginal median filter.

V. CONCLUSIONS

The multichannel distance filter presented in this correspondence represents a novel nonlinear filter, which is suitable for multivariate data processing. Experimental results show that MDF performance may be adapted to perform adequately for the noise distributions examined (short-tailed, long-tailed). An important characteristic is that performance can be adapted to the noise pdf by choosing the parameter r appropriately. Simulation results also show that for a wide class of input signals and noise distributions, good results are obtained with small negative values of the parameter r and that filter performance is not very sensitive to the value of this parameter. Thus, the MDF could be utilized when both the desired signal and the noise parameters are not known. Additionally, the filter linearity can be easily controlled. This fact, in turn, suggests that adaptation of the parameter r may yield good performance under any type of the noise environments examined.

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