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Multicomponent Strongly-Interacting Few-Fermion Systems in One Dimension

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Jonathan Lindgren et al. (2013) arXiv:1304.2992
Artem G. Volosniev et al. (2013) arXiv:1306.4610

Outline

1. System
2. Aims
3. Motivation
4. Approach
5. Results
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System

N particles of equal mass in one spatial dimension:

- the particles are divided into classes of identical spinless fermions;
- the system is trapped by the external potential;
- the interparticle interaction is assumed to be of zero range $V(x_i - x_j) = g\delta(x_i - x_j)$.

Aims

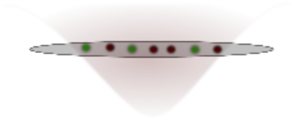
$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + g\sum_{i>j}\delta(x_i - x_j)$$

The aim is to obtain analytically the eigenvalues and the corresponding eigenstates for such Hamiltonian for large repulsive interaction, i.e. in the vicinity of $1/g \to 0$.

Motivation to study such systems

1. The system is experimentally realizable[1] and needs a thorough theoretical description.

- classes of spinless fermions: ^6Li hyperfine states
- quasi-one-dimensional geometry: optical lattices with different aspect ratios
- interparticle interaction can be tuned using external magnetic field

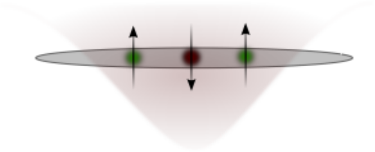


2. The analytical solution can be used as a reference point for different numerical simulations.

[1] See for example G. Zürn (2012) PhD thesis *Few-fermion systems in one dimension*

Approach to obtain solutions

The approach can be shown by using the simple system of *three particles trapped in the harmonic oscillator*: *two particles* out of three from one class that will be referred to as *spin up* and *one particle* will be called *spin down*.

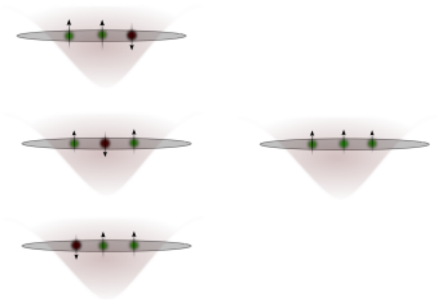


$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_{\uparrow(1)}^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_{\uparrow(2)}^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_{\downarrow}^2} + g\delta(x_{\uparrow(1)} - x_{\downarrow}) + g\delta(x_{\uparrow(2)} - x_{\downarrow})$$

Approach to obtain solutions

Note that for $1/g = 0$

- impenetrable regime (two particles cannot exchange their position)
- fermionization: $\Psi(x_{\uparrow(1)} < x_{\downarrow} < x_{\uparrow(2)}) \sim \Psi_{spinlessfermions}(x_{\uparrow(1)}, x_{\uparrow(2)}, x_{\downarrow})$

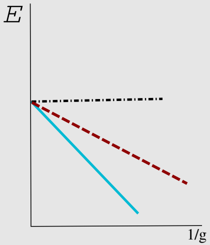


Approach to obtain solutions

It follows

- the energy spectrum is the same as for three spinless fermions.
- the system can be found in the following independent configurations $\uparrow\uparrow\downarrow, \uparrow\downarrow\uparrow, \downarrow\uparrow\uparrow$, which means that the energy spectrum is three times degenerate.

- find $E \simeq E(1/g = 0) - K/g$
- find the corresponding wave functions



Approach to obtain solutions

$$K = \lim_{g\to\infty} g^2 \frac{\partial E}{\partial g} = \lim_{g\to\infty} \frac{g^2 \int |\Psi|^2 \left(\delta(x_{\uparrow(1)} - x_\downarrow) + \delta(x_{\uparrow(2)} - x_\downarrow)\right) \mathrm{d}x_{\uparrow(1)} \mathrm{d}x_{\uparrow(2)} \mathrm{d}x_\downarrow}{\langle\Psi|\Psi\rangle}$$

Approach to obtain solutions

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$$\left(\frac{\partial\Psi}{\partial x_{\uparrow}} - \frac{\partial\Psi}{\partial x_{\downarrow}}\right)\Bigg|_{x_{\uparrow}-x_{\downarrow}=+0} - \left(\frac{\partial\Psi}{\partial x_{\uparrow}} - \frac{\partial\Psi}{\partial x_{\downarrow}}\right)\Bigg|_{x_{\uparrow}-x_{\downarrow}=-0} = 2g\Psi\Big|_{x_{\uparrow}=x_{\downarrow}}$$

$$K = K[\Psi(1/g = 0)]$$

Approach to obtain solutions

$$K = \lim_{g\to\infty} g^2 \frac{\partial E}{\partial g} = \lim_{g\to\infty} \frac{g^2 \int |\Psi|^2 \left(\delta(x_{\uparrow(1)} - x_\downarrow) + \delta(x_{\uparrow(2)} - x_\downarrow)\right) \mathrm{d}x_{\uparrow(1)} \mathrm{d}x_{\uparrow(2)} \mathrm{d}x_\downarrow}{\langle \Psi | \Psi \rangle}$$

$$\left(\frac{\partial \Psi}{\partial x_\uparrow} - \frac{\partial \Psi}{\partial x_\downarrow}\right)\Bigg|_{x_\uparrow - x_\downarrow = +0} - \left(\frac{\partial \Psi}{\partial x_\uparrow} - \frac{\partial \Psi}{\partial x_\downarrow}\right)\Bigg|_{x_\uparrow - x_\downarrow = -0} = 2g\Psi\Big|_{x_\uparrow = x_\downarrow}$$

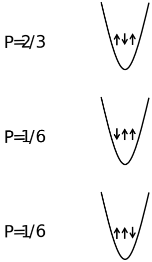
$$K = K[\Psi(1/g = 0)]$$

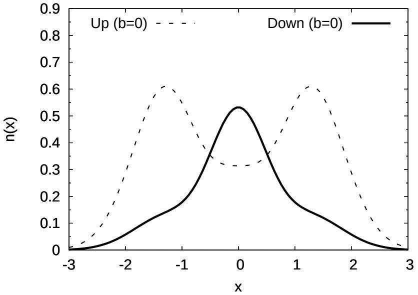
$$\Psi(1/g = 0) = \begin{cases} a_1 \Psi_{spinlessfermions} & : \uparrow\uparrow\downarrow \\ a_2 \Psi_{spinlessfermions} & : \uparrow\downarrow\uparrow \\ a_3 \Psi_{spinlessfermions} & : \downarrow\uparrow\uparrow \end{cases}$$

$$K = K(a_1, a_2, a_3)$$

Results

For the ground state: $a_3 = a_1, a_2 = -2a_1$



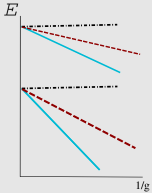


Conclusions

For

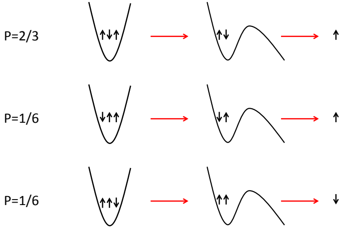
- N particles of equal mass in one spatial dimension;
- the particles are divided into classes of identical spinless fermions;
- the system is trapped by the external potential;
- the interparticle interaction is assumed to be of zero range $V(x_i - x_j) = g\delta(x_i - x_j)$.

It is possible to find analytically the energy spectrum near $1/g \to 0$ up to linear in $1/g$ order and the corresponding wave functions at $1/g = 0$



Conclusions

Experimental probe



This means that the probability to see the particle with spin down is five times smaller than the probability to detect a particle with spin up.

A few relevant publications

Experiment (Heidelberg)

F. Serwane PhD thesis *Deterministic preparation of a tunable few-fermion system* (2011)

G. Zürn PhD thesis *Few-fermion systems in one dimension* (2012)

Andre Wenz et al. arxiv:1307.3443

Theory

J. Lindgren et al. (2013) arXiv:1304.2992

A. Volosniev et al. (2013) arXiv:1306.4610

S. Gharashi and D. Blume (2013) *PRL* **111**, 045302

T. Sowiński et al. (2013) arXiv:1304.8099

X. Cui and T.-L. Ho (2013) arXiv:1305.6361