

Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment

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The paper presents the correlation and correlation coefficient of single-valued neutrosophic sets (SVNSs) based on the extension of the correlation of intuitionistic fuzzy sets and demonstrates that the cosine similarity measure is a special case of the correlation coefficient in SVNS. Then a decision-making method is proposed by the use of the weighted correlation coefficient or the weighted cosine similarity measure of SVNSs, in which the evaluation information for alternatives with respect to criteria is carried out by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree under single-valued neutrosophic environment. We utilize the weighted correlation coefficient or the weighted cosine similarity measure between each alternative and the ideal alternative to rank the alternatives and to determine the best one(s). Finally, an illustrative example demonstrates the application of the proposed decision-making method.

Keywords: neutrosophic set; single-valued neutrosophic set; correlation; correlation coefficient; multicriteria decision-making

1. Introduction

Smarandache introduced Neutrosophy in 1995. Neutrosophy is a branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra (Smarandache, 1999). Neutrosophic set is a powerful general formal framework which generalizes the concept of the classic set, fuzzy set (Zadeh, 1965), interval-valued fuzzy set (Turksen, 1986), intuitionistic fuzzy set (Atanassov, 1986), interval-valued intuitionistic fuzzy set (Atanassov & Gargov, 1989), paraconsistent set (Smarandache, 1999), dialetheist set (Smarandache, 1999), paradoxist set (Smarandache, 1999), and tautological set (Smarandache, 1999). A neutrosophic set A is defined on a universe of discourse U . An element x in set A is denoted as $x = x(T, I, F) \in A$, where T is a truth-membership function, I is an indeterminacy-membership function, and F is a falsity-membership function, then T , I , and F are the real standard or non-standard subsets of $]0^-, 1^+[$ (Smarandache, 1999). Neutrosophic sets have many applications such as information fusion in which the data are combined from different sensors. Recently, neutrosophic sets have mainly been applied to image processing (Cheng & Guo, 2008; Guo & Cheng, 2009).

The intuitionistic fuzzy set considers both the truth-membership $t_A(x)$ and the falsity-membership $f_A(x)$ with $t_A(x), f_A(x) \in [0, 1]$ and $0 \leq t_A(x) + f_A(x) \leq 1$ and can only handle incomplete information (set incompletely known), but cannot handle the indeterminate

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information which is the zone of ignorance of a proposition's value between truth and falsehood (inconsistent information). In intuitionistic fuzzy sets, the indeterminacy is $1 - t_A(x) - f_A(x)$ (i.e. hesitancy or unknown degree) by default. In a neutrosophic set, the indeterminacy is quantified explicitly, then the component ' I ', indeterminacy, can be split into more subcomponents in order to better catch the vague information we work with (Smarandache, 1999). However, the truth-membership, the indeterminacy-membership, and the falsity-membership are independent of the neutrosophic set. Its components T , I , and F are not only non-standard subsets included in the unitary non-standard interval $]0^-, 1^+[$ but also standard subsets included in the unitary standard interval $[0, 1]$ as in the intuitionistic fuzzy set (Wang, Smarandache, Zhang, & Sunderraman, 2010). Furthermore, the connectors in the intuitionistic fuzzy set are defined with respect to T and F , i.e. membership and non-membership only (hence the indeterminacy is what is left from 1), while in the neutrosophic set, they can be defined with respect to any of them (no restriction). For example (Wang et al., 2010), when we ask the opinion of an expert about certain statement, he or she may say that the possibility in which the statement is true is 0.5 and the statement is false is 0.6 and the degree in which he or she is not sure is 0.2. For neutrosophic notation, it can be expressed as $x(0.5,0.2,0.6)$. For another example, suppose there are 10 voters during a voting process. Four vote 'aye', three vote 'blackball', and three are undecided. For neutrosophic notation, it can be expressed as $x(0.4,0.3,0.3)$. However, these expressions are beyond the scope of the intuitionistic fuzzy set. So the notion of neutrosophic set is more general and overcomes the aforementioned issues.

The neutrosophic set generalizes the above-mentioned sets from philosophical point of view. From scientific or engineering point of view, the neutrosophic set and set-theoretic operators need to be specified. Otherwise, it will be difficult to apply in the real applications. Therefore, Wang et al. (2010) proposed a single-valued neutrosophic set (SVNS), which is an instance of neutrosophic set, and provided the set-theoretic operators and various properties of SVNSs. The SVNS can be used for the scientific and engineering applications of SVNSs because SVNS theory is valuable in modelling uncertain, imprecision, and inconsistent information. Due to its ability to easily reflect the ambiguous nature of subjective judgements, SVNSs are suitable for capturing imprecise, uncertain, and inconsistent information in the multicriteria decision analysis. Therefore, the main purposes of this paper are to present the correlation coefficient of SVNSs based on the extension of the correlation of intuitionistic fuzzy sets (Gerstenkorn & Manko, 1991; Ye, 2010) and to demonstrate that the cosine similarity measure is a special case of the correlation coefficient in SVNS. Then, a decision-making method using the weighted correlation coefficient or the weighted cosine similarity measure of SVNSs is established in which the evaluation information for alternatives with respect to criteria is carried by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree under single-valued neutrosophic environment. The weighted correlation coefficient or the weighted cosine similarity measure between each alternative and the ideal alternative is utilized to rank the alternatives and to determine the best one(s). Finally, an illustrative example demonstrates the application of the proposed decision-making method. However, the existing fuzzy multicriteria decision-making methods cannot deal with the decision-making problem in this paper. The main advantage of the proposed single-valued neutrosophic decision-making method can handle not only incomplete information but also the indeterminate information and inconsistent information which exist commonly in real situations.

2. Some concepts of neutrosophic sets and SVNNS

2.1 Neutrosophic sets

Neutrosophic set is a part of neutrosophy, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra (Smarandache, 1999), and is a powerful general formal framework, which generalizes the above-mentioned sets from philosophical point of view.

Smarandache (1999) gave the following definition of a neutrosophic set.

DEFINITION 1. Let X be a space of points (objects), with a generic element in X denoted by x (Smarandache, 1999). A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is $T_A(x): X \rightarrow]0^-, 1^+[$, $I_A(x): X \rightarrow]0^-, 1^+[$, and $F_A(x): X \rightarrow]0^-, 1^+[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

DEFINITION 2. The complement of a neutrosophic set A is denoted by $c(A)$ and is defined as $T_{c(A)}(x) = \{1^+\} - T_A(x)$, $I_{c(A)}(x) = \{1^+\} - I_A(x)$, and $F_{c(A)}(x) = \{1^+\} - F_A(x)$ for every x in X (Smarandache, 1999).

DEFINITION 3. A neutrosophic set A is contained in the other neutrosophic set B , $A \subseteq B$ if and only if $\inf T_A(x) \leq \inf T_B(x)$, $\sup T_A(x) \leq \sup T_B(x)$, $\inf I_A(x) \geq \inf I_B(x)$, $\sup I_A(x) \geq \sup I_B(x)$, $\inf F_A(x) \geq \inf F_B(x)$, and $\sup F_A(x) \geq \sup F_B(x)$ for every x in X (Smarandache, 1999).

2.2 SVNNS

An SVNNS is an instance of a neutrosophic set, which can be used in real scientific and engineering applications. In the following, we introduce the definition of an SVNNS (Wang et al., 2010).

DEFINITION 4. Let X be a space of points (objects) with generic elements in X denoted by x (Wang et al., 2010). An SVNNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ for each point x in X , $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$.

When X is continuous, an SVNNS A can be written as

$$A = \int_X \frac{\langle T_A(x), I_A(x), F_A(x) \rangle}{x}, \quad x \in X.$$

When X is discrete, an SVNNS A can be written as

$$A = \sum_{i=1}^n \frac{\langle T_A(x_i), I_A(x_i), F_A(x_i) \rangle}{x_i}, \quad x_i \in X.$$

DEFINITION 5. The complement of an SVNNS A is denoted by $c(A)$ and is defined as $T_{c(A)}(x) = F_A(x)$, $I_{c(A)}(x) = 1 - I_A(x)$, $F_{c(A)}(x) = T_A(x)$ for any x in X (Wang et al., 2010).

DEFINITION 6. An SVNS A is contained in the other SVNS B , $A \subseteq B$, if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$ for any x in X (Wang et al., 2010).

DEFINITION 7. Two SVNSs A and B are equal, written as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$ (Wang et al., 2010).

3. Correlation coefficient of SVNSs

An SVNS is a generalization of classic set, fuzzy set, intuitionistic fuzzy set, and paraconsistent set. In this section, based on the extension of the correlation of intuitionistic fuzzy sets (Gerstenkorn & Manko 1991; Ye, 2010), we define the so-called informational energy of SVNS, correlation of two SVNSs, and the correlation coefficient of two SVNSs, which can be used in real scientific and engineering applications.

Let any SVNS be $A = \sum_{i=1}^n \langle T_A(x_i), I_A(x_i), F_A(x_i) \rangle / x_i$, $x_i \in X$ in the universe of discourse $X = \{x_1, x_2, \dots, x_n\}$, where $T_A(x_i), I_A(x_i), F_A(x_i) \in [0, 1]$ for every $x_i \in X$. we define

$$T(A) = \sum_{i=1}^n [T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)]. \tag{1}$$

Then the above formula expresses the so-called informational energy of the SVNS A .

Assume that two SVNSs A and B in the universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ are denoted by $A = \sum_{i=1}^n \langle T_A(x_i), I_A(x_i), F_A(x_i) \rangle / x_i$, $x_i \in X$ and $B = \sum_{i=1}^n \langle T_B(x_i), I_B(x_i), F_B(x_i) \rangle / x_i$, $x_i \in X$, where $T_A(x_i), I_A(x_i), F_A(x_i), T_B(x_i), I_B(x_i), F_B(x_i) \in [0, 1]$ for every $x_i \in X$. Then we define the following so-called correlation of the SVNSs A and B :

$$C(A, B) = \sum_{i=1}^n [T_A(x_i) \cdot T_B(x_i) + I_A(x_i) \cdot I_B(x_i) + F_A(x_i) \cdot F_B(x_i)]. \tag{2}$$

It is obvious that the correlation of SVNSs satisfies the following properties:

- (1) $C(A, A) = T(A)$,
- (2) $C(A, B) = C(B, A)$.

Therefore, we now define the correlation coefficient of the SVNSs A and B by the formula:

$$K(A, B) = \frac{C(A, B)}{[T(A) \cdot T(B)]^{1/2}} = \frac{\sum_{i=1}^n [T_A(x_i) \cdot T_B(x_i) + I_A(x_i) \cdot I_B(x_i) + F_A(x_i) \cdot F_B(x_i)]}{\sqrt{\sum_{i=1}^n [T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)]} \sqrt{\sum_{i=1}^n [T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)]}}. \tag{3}$$

Then, it is obvious that the correlation coefficient of SVNSs satisfies the following properties:

- (1) $A = B \Rightarrow K(A, B) = 1$,
- (2) $K(A, B) = K(B, A)$.

THEOREM 1. For two SVNNS A and B in the universe of discourse $X = \{x_1, x_2, \dots, x_n\}$, the correlation coefficient of SVNNS satisfies the following property:

$$0 \leq K(A, B) \leq 1.$$

Proof. Considering $n = 1$, from Equation (3), we yield the following correlation coefficient:

$$K(A, B) = \frac{T_A(x_i) \cdot T_B(x_i) + I_A(x_i) \cdot I_B(x_i) + F_A(x_i) \cdot F_B(x_i)}{\sqrt{T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)} \sqrt{T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)}}. \quad (4)$$

Assume that three parameters $T_A(x_i)$, $I_A(x_i)$, and $F_A(x_i)$ in the SVNNS A or $T_B(x_i)$, $I_B(x_i)$, and $F_B(x_i)$ in the SVNNS B can be considered as a vector representation with the three elements. Based on the cosine similarity measure (Ye, 2011), the cosine similarity measure between SVNNS A and B is defined as follows:

$$S(A, B) = \frac{T_A(x_i) \cdot T_B(x_i) + I_A(x_i) \cdot I_B(x_i) + F_A(x_i) \cdot F_B(x_i)}{\sqrt{T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)} \sqrt{T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)}}. \quad (5)$$

According to the value range of the cosine function, we can obtain the following property:

$$0 \leq S(A, B) \leq 1.$$

Hence, there is $0 \leq K(A, B) \leq 1$ for $i = 1$ because $S(A, B) = K(A, B)$ for $i = 1$.

When $i = n$, we can deduce the property of $0 \leq K(A, B) \leq 1$ from Equations (3)–(5) according to the cosine property.

Thus the proof is finished. \square

Obviously, the cosine similarity measure Equation (5) is a special case of the correlation coefficient Equation (3) for $i = 1$.

However, the differences of importance are considered in the elements in the universe. Therefore, we need to take the weights of the elements x_i ($i = 1, 2, \dots, n$) into account. In the following, we develop a weighted correlation coefficient between SVNNSs.

Let $\mathbf{w} = \{w_1, w_2, \dots, w_n\}$ be the weight vector of the elements x_i ($i = 1, 2, \dots, n$), then we have the following weighted correlation coefficient:

$$W(A, B) = \frac{\sum_{i=1}^n w_i [T_A(x_i) \cdot T_B(x_i) + I_A(x_i) \cdot I_B(x_i) + F_A(x_i) \cdot F_B(x_i)]}{\sqrt{\sum_{i=1}^n w_i [T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)]} \sqrt{\sum_{i=1}^n w_i [T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)]}}. \quad (6)$$

If $w = \{1/n, 1/n, \dots, 1/n\}$, then Equation (6) is reduced to the correlation coefficient Equation (3).

It is easy to check that the weighted correlation coefficient $W(A, B)$ between SVNNS A and B also satisfies the property of $0 \leq W(A, B) \leq 1$.

4. Single-valued neutrosophic multicriteria decision-making method

An SVN is a generalization of classic set, fuzzy set, intuitionistic fuzzy set, and paraconsistent set. It is more general and can handle not only incomplete information but also the indeterminate information and inconsistent information which exist commonly in real situations. Therefore, the single-valued neutrosophic decision-making is more suitable for real scientific and engineering applications.

In this section, we present a handling method for the multicriteria decision-making problem under single-valued neutrosophic environment (or called a single-valued neutrosophic multicriteria decision-making method) by means of the weighted correlation coefficient or weighted cosine similarity measure between SVN s.

Let $A = \{A_1, A_2, \dots, A_m\}$ be a set of alternatives and $C = \{C_1, C_2, \dots, C_n\}$ be a set of criteria. Assume that the weight of the criterion C_j ($j = 1, 2, \dots, n$), entered by the decision-maker, is w_j , $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$. In this case, the characteristic of the alternative A_i ($i = 1, 2, \dots, m$) is represented by the following SVN:

$$A_i = \sum_{j=1}^n \frac{\langle T_{A_i}(C_j), I_{A_i}(C_j), F_{A_i}(C_j) \rangle}{C_j}, C_j \in C,$$

where $T_{A_i}(C_j), I_{A_i}(C_j), F_{A_i}(C_j) \in [0, 1], j = 1, 2, \dots, n$, and $i = 1, 2, \dots, m$. An SVN is denoted by $\alpha_{ij} = \langle a_{ij}, b_{ij}, c_{ij} \rangle$ for convenience. Here, an SVN is usually derived from the evaluation of an alternative A_i with respect to a criterion C_j by means of a score law and data processing in practice. Therefore, we can elicit a single-valued neutrosophic decision matrix $D = (\alpha_{ij})_{m \times n}$.

In multicriteria decision-making environments, the concept of ideal point has been used to help identify the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct against which to evaluate alternatives (Ye, 2010). Hence, we define the ideal alternative A^* as the SVN $\alpha_j^* = \langle a_j^*, b_j^*, c_j^* \rangle = \langle 1, 0, 0 \rangle$ for $j = 1, 2, \dots, n$.

Thus, by applying Equation (6), the weighted correlation coefficient between an alternative A_i and the ideal alternative A^* represented by the SVN s is defined by

$$W_i(A_i, A^*) = \frac{\sum_{j=1}^n w_j [a_{ij} \cdot a_j^* + b_{ij} \cdot b_j^* + c_{ij} \cdot c_j^*]}{\sqrt{\sum_{j=1}^n w_j [a_{ij}^2 + b_{ij}^2 + c_{ij}^2]} \sqrt{\sum_{j=1}^n w_j [(a_j^*)^2 + (b_j^*)^2 + (c_j^*)^2]}}. \tag{7}$$

Then, the bigger the value of the weighted correlation coefficient $W_i(A_i, A^*)$ is, the better the alternative A_i is. Through the weighted correlation coefficient $W_i(A_i, A^*)$ ($i = 1, 2, \dots, m$) between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best one can be easily identified as well.

Because the cosine similarity measure Equation (5) is a special case of the correlation coefficient Equation (3) for $i = 1$, we can also define the weighted cosine similarity measure between an alternative A_i and the ideal alternative A^* represented by the SVN s

are defined as

$$M_i(A_i, A^*) = \sum_{j=1}^n w_j \frac{a_{ij} \cdot a_j^* + b_{ij} \cdot b_j^* + c_{ij} \cdot c_j^*}{\sqrt{a_{ij}^2 + b_{ij}^2 + c_{ij}^2} \sqrt{(a_j^*)^2 + (b_j^*)^2 + (c_j^*)^2}}. \quad (8)$$

The measure values of Equation (8) can yield the ranking order of all alternatives and obtain the best alternative.

5. Illustrative example

In this section, an example for the multicriteria decision-making problem of alternatives is used as the demonstration of the application of the proposed decision-making method, as well as the effectiveness of the proposed method.

Let us consider the decision-making problem adapted from Ye (2010). There is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money: (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; and (4) A_4 is an arms company. The investment company must take a decision according to the following three criteria: (1) C_1 is the risk analysis; (2) C_2 is the growth analysis; and (3) C_3 is the environmental impact analysis. Then, the weight vector of the criteria is given by $\mathbf{w} = (0.35, 0.25, 0.40)$.

For the evaluation of an alternative A_i with respect to a criterion C_j ($i = 1, 2, 3, 4$; $j = 1, 2, 3$), it is obtained from the questionnaire of a domain expert. For example, when we ask the opinion of an expert about an alternative A_1 with respect to a criterion C_1 , he or she may say that the possibility in which the statement is good is 0.4 and the statement is poor is 0.3 and the degree in which he or she is not sure is 0.2. For the neutrosophic notation, it can be expressed as $\alpha_{11} = \langle 0.4, 0.2, 0.3 \rangle$. Thus, when the four possible alternatives with respect to the above three criteria are evaluated by the expert, we can obtain the following single-valued neutrosophic decision matrix D :

$$D = \begin{bmatrix} \langle 0.4, 0.2, 0.3 \rangle & \langle 0.4, 0.2, 0.3 \rangle & \langle 0.2, 0.2, 0.5 \rangle \\ \langle 0.6, 0.1, 0.2 \rangle & \langle 0.6, 0.1, 0.2 \rangle & \langle 0.5, 0.2, 0.2 \rangle \\ \langle 0.3, 0.2, 0.3 \rangle & \langle 0.5, 0.2, 0.3 \rangle & \langle 0.5, 0.3, 0.2 \rangle \\ \langle 0.7, 0.0, 0.1 \rangle & \langle 0.6, 0.1, 0.2 \rangle & \langle 0.4, 0.3, 0.2 \rangle \end{bmatrix}.$$

Then, we utilize the developed approach to obtain the most desirable alternative(s).

By using Equation (7), we can obtain the following values of weighted correlation coefficient $W_i(A_i, A^*)$ ($i = 1, 2, 3, 4$):

$$W_1(A_1, A^*) = 0.5785, \quad W_2(A_2, A^*) = 0.9108, \quad W_3(A_3, A^*) = 0.7554, \quad \text{and}$$

$$W_4(A_4, A^*) = 0.8848.$$

Therefore, the ranking order of the four alternatives is A_2, A_4, A_3 , and A_1 . Obviously, amongst them A_2 is the best alternative.

Or by applying Equation (8), we can also give the following values of weighted cosine similarity measure $M_i(A_i, A^*)$ ($i = 1, 2, 3, 4$):

$$M_1(A_1, A^*) = 0.5849, \quad M_2(A_2, A^*) = 0.9104, \quad M_3(A_3, A^*) = 0.7511, \quad \text{and} \\ M_4(A_4, A^*) = 0.8779.$$

Thus, the ranking order of the four alternatives is A_2, A_4, A_3 , and A_1 . Obviously, amongst them A_2 is also the best alternative.

Hence, we can see that the above two kinds of ranking orders and the best alternative are the same.

6. Conclusion

In this paper, we defined the information energy of SVN, correlation of SVN, correlation coefficient of SVN, and weighted correlation coefficient of SVN. Also we demonstrated that the cosine similarity measure is a special case of the correlation coefficient for $i = 1$. Then, the multicriteria decision-making method has been established under single-valued neutrosophic environment by means of the weighted correlation coefficient or the weighted cosine similarity measure. Through the weighted correlation coefficient or the weighted cosine similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily identified as well. Finally, an illustrative example illustrated the application of the developed approach. Therefore, the proposed single-valued neutrosophic multicriteria decision-making method is more suitable for real scientific and engineering applications because it can handle not only incomplete information but also the indeterminate information and inconsistent information which exist commonly in real situations. The technique proposed in this paper extends existing decision-making methods and provides a useful way for decision-makers. In the future, we shall continue working in the applications of complex decision-making problems such as group decision-making problems with unknown weights of criteria and other domains such as expert system, information fusion system, bioinformatics, and medical informatics.

Notes on contributor



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