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forest management

Multicriteria Forest Decisionmaking under Risk with Goal-Programming Markov Decision Process Models

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As multiple risks pervade forest decisionmaking, Markov decision process (MDP) models offer an analytically tractable approach to seek optimal policies that are straightforward for implementation in practice. By incorporating goal programming (GP), this study extended MDP models with both average and discounted criteria to deal with multiple, often noncommensurable and conflicting, objectives. This method (GPMDP) was applied to the management of mixed loblolly pine-hardwood forests in the southern United States. The decision criteria were the values of harvests, carbon sequestered by trees, diversity of tree species and sizes, and fraction of old-growth stands in the forested landscape. For the case study, the results showed that given equal weights for normalized criteria, with both average and discounted GPMDPs, minimum deviations from the highest diversity of tree size and species were achieved at the cost of, on average, one-third of the decline of other criteria from their maximum levels.

Keywords: forest management, loblolly pine, multiple objectives, risk, optimization

Forest management in practice rarely has a single objective, especially in nonindustrial forests. It usually involves multiple criteria: economic, ecological, and recreational, often noncommensurable and conflicting. Furthermore, biological, climatic, and economic conditions change constantly, and catastrophes may strike (Martell et al. 1998, Amacher and Brazee 2014). Consequently, forest decisionmaking under risk, i.e., when outcomes are uncertain but their probabilities can be assessed, is an active research area (Yin and Newman 1996, Lohmander 2000, Alvarez 2004, Gong et al. 2005, Amacher and Brazee 2014).

Decisionmaking problems under risk are harder to solve than their deterministic counterparts, and the difficulty increases with the number of stochastic variables. Accordingly, much of the literature has concentrated on single stochastic variables. One line of work deals with natural hazards only and focuses on risk assessment (for a review, see Hanewinkel et al. 2011). For risky timber prices, harvesting rules have been developed with the assumption that they follow geometric Brownian motions (Yin and Newman 1996) or random draw processes (Haight 1991) or that they display large jumps (Saphores et al. 2002) or regime switches (Chen 2010). Another approach has used scenarios for various price levels and based decisions on their respective probabilities (Pukkala and Kangas 1996, Alonso-Ayuso et al. 2011, Veliz et al. 2015). Real options models have also been used to deal with price volatility (Thorsen 1999).

Among the few studies that deal with multiple stochastic variables, Insley and Lei (2007) apply a real options approach, whereas Pukkala and Miina (1997) use scenario optimization and Chang (1998) adopts forward recursive programming. Markov decision processes (MDPs) have standard solutions, including linear programming, capable of dealing with large problems efficiently. They give simple solutions of classic infinite horizon problems that are difficult with other methods (Buongiorno 2001, Insley and Rollins 2005). MDPs are also readily combined with simulation (Bertsekas and Tsitsiklis 1995, p. 94). Simulators of complex systems "bring the real world into the laboratory," where they can be optimized in a simpler form with MDP methods (Holling et al. 1986, p. 453–473). Given numerous stochastic factors, analytical solutions do not exist for most methods; thus, MDPs and heuristic approaches such as scenario optimization (Dembo 1991) may be the only computationally tractable ones. Although MDPs are subject to the curse of dimensionality, compactly representing large problems

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as factored MDPs and approximately solving them have been active research areas (Guestrin et al. 2003).

MDP models have proved effective in dealing with multiple sources of risk in forest growth, timber prices, and weather (Lembersky and Johnson 1975, Lin and Buongiorno 1999a, 1999b, Rollin et al. 2005, Buongiorno and Zhou 2015). There has been controversy on whether forest dynamics can be represented with Markov chains (Roberts and Hruska 1986, Johnson et al. 1991, Acevedo et al. 1995, Logofet and Lesnaya 2000). However, the issue has less to do with the appropriateness of the Markov model, which simply says that future can be predicted (with indeterminacy) with current information, than with the definition of the system state in itself. In the case of a stand of trees, the description of the current state must be such that it contains all the information needed to correctly predict the probability of moving to different states (Buongiorno and Gilless 2003). Similarly, Markov models for prices are very general, embracing random walk, rational expectations, autoregressive, and "any stochastic model in which the price is conditional on previous prices" (Taylor 1984, p. 351). Recent work has extended the models to include stochastic interest rates (Buongiorno and Zhou 2011, Zhou and Buongiorno 2011) and the regime switch of climate policies (Zhou 2015).

Despite the existence of multiple solution methods for MDPs including dynamic programming, policy iteration, and reinforcement learning, the majority of previous forestry applications use the linear programming (LP) formulation. One particular advantage of the LP formulation is the ease of dealing with multiple objectives, by setting a criterion as the objective function and the others as constraints, e.g., maximizing economic returns while maintaining species richness, or, symmetrically, one may seek the highest species richness while keeping discounted returns above a minimum threshold. Solving these constrained problems is also useful for finding the trade-off between ecological and economic objectives.

Lin and Buongiorno (1998, 1999a, 1999b) maximize discounted returns with ecological constraints by limiting the decision space to the decisions that do not result immediately in an undesirable state. This may give a satisfactory approximate solution, but it does not guarantee that the state attained, over a long sequence of decisions, will be in a desirable domain. Solving for a maximum expected criterion subject to a discounted constraint is even more difficult, and there is no obvious similar approximation. Rollin et al. (2005) optimize discounted objectives subject to undiscounted constraints, or the reverse, by exploiting the relation between the decision variables of undiscounted and discounted MDP models. However, the relation consists of many nonlinear constraints, making the model hard to solve.

More simply, Buongiorno and Zhou (2015) suggest that multicriteria problems can be approached by discounting or not discounting the criteria in both the objective function and in the constraints, thus maintaining the classic LP formulation. This is in line with the argument of Howarth (2009) that nonmarket values provided by public goods should be discounted at a risk-free interest rate, just like financial returns. Zhou et al. (2012) investigate the consequences of discounting ecological criteria in forestry. We continue to take advantage of the simplicity of LP while extending the formulation to a goal-programming (GP) MDP (GPMDP) addressing numerous criteria simultaneously. GP, initiated by Charnes et al. (1955) and first formally introduced by Charnes and Cooper (1961), is a prevailing technique in the family of multiple criteria decisionmaking (MCDM) (for a comprehensive introduction, see Jones and Tamiz 2010).

As noted by Romero et al. (1998), "GP has the underlying satisficing philosophy which is intuitively more appealing to many decision makers." It also has a general structure so that other MCDM models such as compromise programming (CP) and the reference point method can be restated as GPs. Here we show how GP can be combined with MDP to deal with multiobjective decisionmaking under risk in forestry.

The following section presents the classic LP formulations of discounted and undiscounted MDPs followed by their extensions as weighted GPMDPs. These methods are then applied to the management of all-aged mixed loblolly pine (*Pinus taeda* L.) and hardwood forests in the southern United States. Three sources of risk were considered: forest growth, stumpage prices, and catastrophic storms. The results gave optimum management rules that simultaneously accounted for economic and ecological objectives.

Materials and Methods

The methods extended classic MDP models, consisting of state variables, transition probabilities, decision sets, and expected rewards (Winston and Goldberg 2004). Here, the state variables joined the forest stand state and the market state (price level). Transition probabilities referred to movements between stand-market states. A decision consisted of changing a stand state with a harvest. The decision rewards were biological (expected tree species and size diversity, basal area, CO_2 equivalent [CO_2e] stored by trees, and fraction of area in old growth) and financial (net present value and volume of harvests), over an infinite horizon.

Undiscounted GPMDP

Expected average criteria, such as average annual harvest per unit area, are easy to interpret and compare with current conditions. Moreover, by not discounting, they conform to the philosophy by which "we should treat future generations as we would ourselves, so that the pure rate of pure time preference should be zero" (Arrow 1999, p. 13). However, a zero interest rate implies not only the absence of time preference but also zero productivity of investments (Buongiorno and Gilless 2003, p. 393). The classic LP formulation of this undiscounted MDP model is (Manne 1960)

$$\max_{z_{sd}} H = \sum_{s=1}^{S} \sum_{d \in D_s} H_{sd} z_{sd}$$

Management and Policy Implications

Forest management in practice rarely has a single objective. It usually involves multiple criteria, economic, ecological, and recreational, often noncommensurable and conflicting. Furthermore, biological, climatic, and economic conditions change constantly and catastrophes may strike, which introduce risk in decisionmaking. The method proposed in this article deals with diverse objectives of forest management while taking multiple sources of risk into account. It combines Markov decision process models and goal programming into a general structure covering several classes of multiple criteria decisionmaking models. The resulting decision tables provide straightforward guidelines for practical implementation by public and private nonindustrial forest owners with diverse and often conflicting goals.

subject to:

$$\sum_{d\in D_s} z_{s'd} - \sum_{s=1}^{S}\sum_{d=1}^{D_s} z_{sd}\,p(s'|s,\,d) = 0 \qquad s' = 1,\ldots,S \quad (1)$$

$$\sum_{s=1}^{S}\sum_{d\in D_s} z_{sd} = 1$$

$$z_{sd} \geq 0,\; s = 1,\ldots,S;\; d \in D_s$$

where H_{sd} is the instantaneous return of the criterion of interest with decision d in state s. The decision variable, z_{sd}, is the probability of state s and decision d. There are S possible states, and a decision is a harvest that changes the stand instantaneously to another state. D_s is the set of possible decisions in state s. $p(s'|s, d)$ is the probability of ending in state s' in a year, when in state s and making decision d. After solving 1, the best policy is given by

$$X_{sd} = \frac{z_{sd}}{\sum_{d\in D_s} z_{sd}} \tag{2}$$

where $X_{sd} = 0$ or 1; i.e., the best policy is deterministic, and for each state there is only one best decision. The best policy is also independent of the initial stand state, but in contrast with the discounted problem introduced in the following section, so is the value of the objective function, the expected yearly reward, H.

The GPMDP extension of Model 1 introduced goal variables, i.e., positive (D_i^+) and negative (D_i^-) deviations from the desired level of each criterion, i. With the objective of minimizing the total weighted deviation from all goals, the GPMDP became

$$\min_{z_{sd},D_i^-,\,D_i^+} \sum_{i=1}^{N}(\alpha_i^- D_i^- + \alpha_i^+ D_i^+)$$

subject to:

$$\sum_{d\in D_{s'}} z_{s'd} - \sum_{s=1}^{S}\sum_{d\in D_s} z_{sd}\,p(s'|s,\,d) = 0 \qquad s' = 1,\ldots,S$$

$$\sum_{s=1}^{S}\sum_{d\in D_s} z_{sd} = 1 \tag{3}$$

$$\sum_{s=1}^{S}\sum_{d\in D_s} H_{i,sd}z_{sd} + D_i^- - D_i^+ = C_i^* \qquad i = i,\ldots,N$$

$$z_{sd} \geq 0\; s = 1,\ldots,S;\; d \in D_s$$

$$D_i^-, D_i^+ \geq 0\; i = 1,\ldots,N$$

where with N criteria, (α_i^-) and (α_i^+) are the weights of negative and positive deviations from the target value of criterion i, respectively. Each weight can be viewed as the product of a normalizing constant that reconciled the incommensurability of objectives and of a preferential indicator to reflect the decisionmaker's preference for a negative or positive deviation. The first two constraints were the same as in Model 1. In the third set of constraints, $H_{i,\,sd}$ was the value of criterion i when decision d was made in state s and C_i^* was the target level of criterion i.

An alternative objective was the Chebyshev variant (Flavell 1976), which minimized the largest unwanted weighted deviation from the target criteria:

$$\min_{\lambda,z_{sd},D_i^-,D_i^+} \lambda$$

subject to:

$$\sum_{d\in D_{s'}} z_{s'd} - \sum_{s=1}^{S}\sum_{d\in D_s} z_{sd}\,p(s'|s,\,d) = 0 \qquad s' = 1,\ldots,S$$

$$\sum_{s=1}^{S}\sum_{d\in D_s} z_{sd} = 1 \tag{4}$$

$$\sum_{s=1}^{S}\sum_{d\in D_s} H_{i,sd}z_{sd} + D_i^- - D_i^+ = C_i^* \qquad i = i,\ldots,N$$

$$z_{sd} \geq 0 \qquad s = 1,\ldots,S;\; d \in D_s$$

$$D_i^-,\, D_i^+ \geq 0 \qquad i = 1,\ldots,N$$

$$\alpha_i^- D_i^-, \alpha_i^+ D_i^+ \leq \lambda \qquad i = 1,\ldots,N$$

Like Model 1, both Models 3 and 4 were linear and the policy that gave the best compromise decisions was obtained with Equation 2.

Discounted GPMDP

Financial rewards, the income generated by successive harvests, are typically discounted with a specific interest rate, leading to the expected net present value (NPV) criterion. It has also been argued that ecological assets and damages should be discounted as well, although a consensus on the discount rate is lacking (Gollier 2010). We used the LP approach (d'Epenoux 1963) to find the policies that maximized the expected NPV of each criterion over an infinite horizon, given a particular initial stand-market state:

$$\max_{y_{sd},D_i^-,D_i^+} \text{NPV} = \sum_{s=1}^{S}\sum_{d\in D_s} H_{sd}y_{sd}$$

subject to: (5)

$$\sum_{d\in D_{s'}} y_{s'd} - \frac{1}{(1+r)}\sum_{s=1}^{S}\sum_{d\in D_s} y_{sd}\,p(s'|s,\,d) = \pi_{s'} \qquad s' = 1,\ldots,S$$

$$y_{sd} \geq 0, \qquad s = 1,\ldots,S;\; d \in D_s$$

where the variable y_{sd} is the infinite sum of the discounted probability of state s and decision d. Parameter r is the yearly discount rate, and $\pi_{s'}$ is the initial probability of state s'. Given the solution of Problem 5, the best management policy is

$$X_{sd} = \frac{y_{sd}}{\sum_{d\in D_s} y_{sd}} \tag{6}$$

Table 1. Basal area threshold defining low and high basal area level by tree category, and average basal area at each level.

Basal area	Pines			Hardwoods		
	Pulpwood	Small sawtimber	Large sawtimber	Pulpwood	Small sawtimber	Large sawtimber
	($m^2\ ha^{-1}$)					
Threshold	2.5	4.3	3.6	2.9	1.2	1.7
Average						
Low (L)	1.1	2.4	1.7	1.5	0.4	0.5
High (H)	4.9	7.2	6.3	4.8	2.5	2.7

where X_{sd} is the probability of making decision d in stand-market state s. Similar to the average criterion MDP, X_{sd} equals 0 or 1, so a unique decision is optimal for each stand-market state. Furthermore, the decision is independent of the initial distribution of states, $\{\pi_{s'}\}$ (Hillier and Lieberman, 2005). Nevertheless, the maximum expected NPV depends very much on the initial state because potential rewards depend on the state and with discounting, the initial state matters the most.

After introducing goal variables, the discounted GPMDP became

$$\min_{z_{sd},D_i^-,D_i^+} \sum_{i=1}^{N} (\alpha_i^- D_i^- + \alpha_i^+ D_i^+)$$

subject to:

$$\sum_{d \in D_{s'}} y_{s'd} - \frac{1}{(1+r)} \sum_{s=1}^{S} \sum_{d \in D_s} y_{sd} p(s'|s, d) = \pi_{s'} \qquad s' = 1, \ldots, S$$

$$\sum_{s=1}^{S} \sum_{d \in D_s} H_{i,sd} y_{sd} + D_i^- - D_i^+ = C_i^* \qquad i = i, \ldots, N \quad (7)$$

$$y_{sd} \geq 0 \; s = 1, \ldots, S; \; d \in D_s$$

$$D_i^-, D_i^+ \geq 0 \qquad i = 1, \ldots, N$$

Likewise, the GPMDP with a Chebyshev objective became

$$\min_{\lambda, z_{sd}, D_i^-, D_i^+} \lambda$$

subject to:

$$\sum_{d \in D_{s'}} y_{s'd} - \frac{1}{(1+r)} \sum_{s=1}^{S} \sum_{d \in D_s} y_{sd} p(s'|s, d) = \pi_{s'} \qquad s' = 1, \ldots, S$$

$$\sum_{s=1}^{S} \sum_{d \in D_s} H_{i,sd} y_{sd} + D_i^- - D_i^+ = C_i^* \qquad i = i, \ldots, N \quad (8)$$

$$y_{sd} \geq 0 \qquad s = 1, \ldots, S; \; d \in D_s$$

$$D_i^-, D_i^+ \geq 0 \qquad i = 1, \ldots, N$$

$$\alpha_i^- D_i^-, \alpha_i^- D_i^+ \leq \lambda \; i = 1, \ldots, N$$

Table 2. Range of the market index, I_t (\$ t^{-1}, 2010 = 100) defining the market states and mean price level in each market state.

Parameter	Market state		
	Low	Medium	High
	(\$ t^{-1})		
Range	$I_t < 26.68$	$26.68 < I_t \leq 38.77$	$P_t' > 38.77$
Mean pine price	28.60	39.49	54.03
Mean hardwood price	16.47	20.27	25.70

In both cases the best policy was obtained with Equation 6.

Markov Chain Model of Forest Growth

The forests considered for the case study were uneven-aged mixed loblolly pines and hardwoods in the southern United States that grow naturally, except for the periodic harvest of some trees. Regeneration of new trees occurs by seeding from the old trees, and there is no fertilization. We used the Markov chain model of forest stand growth developed by Zhou and Buongiorno (2006) in this application.

The forest stand states (Table 1) were defined by the basal area of trees in six classes of tree size and species (Schulte et al. 1998). Each state was written as a string of six letters such as (*LHL*,*HHL*), where the first three letters stand for the basal area (*L* for low, *H* for high) of pulpwood, small sawtimber, and large sawtimber of pines, respectively, and the last three for hardwoods. Six tree classes with two levels of basal area in each class made up $2^6 = 64$ possible stand states.

To each stand state corresponded an expected stand basal area, CO_2e stored in trees, a diversity of tree size and tree species, measured with Shannon's entropy index (Pielou 1975), and an expected timber volume by tree size and species group. The yearly harvest changed the stand state instantaneously by reducing the basal area in one or more tree groups. It was further assumed conservatively that a hurricane reduced the stand basal area in the six tree groups to its lowest level (state 1) regardless of the current state. Thus, the transition probability from state s to state 1 was modified to $p(1|s) \times (1 - 0.975) + 0.025$, whereas the other probabilities became $p(s'|s) \times 0.975$ for $s' \neq 1$.

Markov Models of Stumpage Market

Because the main products in the forests under consideration are pine and hardwood sawtimber, we constructed an index based on their stumpage prices to represent the overall price level. The two quarterly price series (\$ t^{-1}) were from 1977 to 2014, averaged over the 21 regions in the US South, reported by *TimberMart-South* (Frank W. Norris Foundation 1977–2014). The quarterly Producer Price Index (PPI) (2010 = 100) of the US Bureau of Labor Statistics (2016) was used to adjust the two series for inflation. The market

Table 3. Probabilities of annual change in the market index (I_t).

I_t	I_{t+1}		
	Low	Medium	High
Low	0.71	0.25	0.04
Medium	0.27	0.52	0.21
High	0.04	0.21	0.75

index was the average of real pine and hardwood sawtimber prices, weighted by their relative volume in the sample plots used to calibrate the forest growth model.

Three market states, low, medium, and high, were defined by setting the threshold between low and medium and that between medium and high at the 33.3% and 66.7% percentiles of the calculated quarterly market index from 1977 to 2014, respectively. Table 2 shows the average sawtimber prices of pine and hardwood in each state over that period. The relative frequency of quarterly transitions between market states, P_{ij}, was then obtained and from it the annual transition probabilities, P_{ij}^4 (Table 3). In each market state, the prices

Table 4. Effects of policies that minimized the total or the maximum weighted deviation from undiscounted criteria.

Criterion	Unit	Target[1]	Minimizing total weighted deviation[2]		Minimizing maximum weighted deviation[2]	
			Achieved value	Relative deviation (%)	Achieved value	Relative deviation (%)
Annual income	$ ha^{-1}	326.0	210.9	35.3	217.8	33.2
Species diversity	Shannon index	1.0	0.9	1.6	0.9	2.4
Size diversity	Shannon index	2.0	1.9	3.2	1.9	3.9
Basal area	m^2 ha^{-1}	21.7	17.7	18.5	16.6	23.6
CO_2e	t ha^{-1}	218.6	158.3	27.6	146.1	33.2
Old growth	Fraction	0.3	0.2	29.1	0.2	33.2
Harvests	m^3 ha^{-1} yr^{-1}	7.5	4.3	42.6	5.0	33.2

[1] Maximum unconstrained, undiscounted value.
[2] Deviation is from the undiscounted maximum; weight is the inverse of undiscounted maximum.

Table 5. Decisions that minimized the total weighted deviation from undiscounted criteria.

Stand state		Decision[2] for market state			Stand state		Decision for market state		
No.	Basal area[1]	Low	Medium	High	No.	Basal area	Low	Medium	High
1	*LLL,LLL*	—	—	—	33	*HLL,LLL*	1	1	1
2	*LLL,LLH*	—	—	—	34	*HLL,LLH*	2	2	2
3	*LLL,LHL*	—	—	—	35	*HLL,LHL*	—	—	—
4	*LLL,LHH*	—	—	—	36	*HLL,LHH*	—	—	—
5	*LLL,HLL*	—	—	—	37	*HLL,HLL*	—	—	5
6	*LLL,HLH*	—	—	—	38	*HLL,HLH*	—	—	—
7	*LLL,HHL*	—	—	—	39	*HLL,HHL*	7	7	7
8	*LLL,HHH*	—	—	—	40	*HLL,HHH*	36	36	36
9	*LLH,LLL*	1	1	1	41	*HLH,LLL*	41	1	1
10	*LLH,LLH*	2	2	2	42	*HLH,LLH*	—	2	2
11	*LLH,LHL*	—	—	3	43	*HLH,LHL*	—	35	35
12	*LLH,LHH*	—	—	—	44	*HLH,LHH*	12	12	12
13	*LLH,HLL*	5	5	5	45	*HLH,HLL*	—	37	5
14	*LLH,HLH*	—	6	6	46	*HLH,HLH*	—	—	—
15	*LLH,HHL*	—	—	7	47	*HLH,HHL*	—	—	7
16	*LLH,HHH*	—	—	—	48	*HLH,HHH*	—	—	—
17	*LHL,LLL*	1	1	1	49	*HHL,LLL*	—	—	1
18	*LHL,LLH*	—	—	2	50	*HHL,LLH*	18	18	2
19	*LHL,LHL*	—	3	3	51	*HHL,LHL*	—	—	35
20	*LHL,LHH*	—	—	4	52	*HHL,LHH*	—	—	36
21	*LHL,HLL*	—	—	5	53	*HHL,HLL*	—	—	5
22	*LHL,HLH*	6	6	6	54	*HHL,HLH*	—	—	—
23	*LHL,HHL*	—	7	7	55	*HHL,HHL*	—	—	—
24	*LHL,HHH*	—	—	—	56	*HHL,HHH*	—	—	—
25	*LHH,LLL*	x	x	x	57	*HHH,LLL*	49	49	1
26	*LHH,LLH*	—	18	2	58	*HHH,LLH*	26	18	2
27	*LHH,LHL*	—	11	3	59	*HHH,LHL*	51	51	35
28	*LHH,LHH*	12	12	12	60	*HHH,LHH*	12	12	12
29	*LHH,HLL*	21	21	5	61	*HHH,HLL*	53	53	5
30	*LHH,HLH*	—	—	6	62	*HHH,HLH*	46	46	54
31	*LHH,HLH*	—	—	7	63	*HHH,HHL*	—	—	55
32	*LHH,HHH*	—	16	16	64	*HHH,HHH*	48	48	48
						Harvests	21	30	44
						Total			95

[1] *L*, low; *H*, high; left side before comma, softwoods; right side after comma, hardwoods.
[2] Stand state no. after decision. — denotes doing nothing; x denotes undefined decision as the probability of this stand state was 0.

Table 6. Decisions that minimized the maximum weighted deviation from undiscounted criteria.

Stand state		Decision[2] for market state			Stand state		Decision for market state		
No.	Basal area[1]	Low	Medium	High	No.	Basal area	Low	Medium	High
1	*LLL,LLL*	—	—	—	33	*HLL,LLL*	1	1	1
2	*LLL,LLH*	—	—	—	34	*HLL,LLH*	2	2	2
3	*LLL,LHL*	—	—	—	35	*HLL,LHL*	—	—	—
4	*LLL,LHH*	—	—	—	36	*HLL,LHH*	—	—	—
5	*LLL,HLL*	—	—	—	37	*HLL,HLL*	5	5	5
6	*LLL,HLH*	—	—	—	38	*HLL,HLH*	6	6	6
7	*LLL,HHL*	3	3	3	39	*HLL,HHL*	35	35	35
8	*LLL,HHH*	—	—	—	40	*HLL,HHH*	36	36	36
9	*LLH,LLL*	1	1	1	41	*HLH,LLL*	x	x	x
10	*LLH,LLH*	2	2	2	42	*HLH,LLH*	x	x	x
11	*LLH,LHL*	3	3	3	43	*HLH,LHL*	—	35	35
12	*LLH,LHH*	—	—	—	44	*HLH,LHH*	12	12	36
13	*LLH,HLL*	5	5	5	45	*HLH,HLL*	x	x	x
14	*LLH,HLH*	6	6	6	46	*HLH,HLH*	x	x	x
15	*LLH,HHL*	—	3	3	47	*HLH,HHL*	—	—	35
16	*LLH,HHH*	—	—	—	48	*HLH,HHH*	—	—	—
17	*LHL,LLL*	1	1	1	49	*HHL,LLL*	x	x	x
18	*LHL,LLH*	2	2	2	50	*HHL,LLH*	x	x	x
19	*LHL,LHL*	—	3	3	51	*HHL,LHL*	—	—	35
20	*LHL,LHH*	—	4	4	52	*HHL,LHH*	—	—	36
21	*LHL,HLL*	5	5	5	53	*HHL,HLL*	x	x	x
22	*LHL,HLH*	6	6	6	54	*HHL,HLH*	x	x	x
23	*LHL,HHL*	19	3	3	55	*HHL,HHL*	—	—	—
24	*LHL,HHH*	—	—	—	56	*HHL,HHH*	—	—	36
25	*LHH,LLL*	x	x	x	57	*HHH,LLL*	x	x	x
26	*LHH,LLH*	x	x	x	58	*HHH,LLH*	x	x	x
27	*LHH,LHL*	—	3	3	59	*HHH,LHL*	51	51	35
28	*LHH,LHH*	12	12	12	60	*HHH,LHH*	12	12	12
29	*LHH,HLL*	x	x	x	61	*HHH,HLL*	x	x	x
30	*LHH,HLH*	x	x	x	62	*HHH,HLH*	x	x	x
31	*LHH,HLH*	—	—	3	63	*HHH,HHL*	—	55	35
32	*LHH,HHH*	16	16	16	64	*HHH,HHH*	48	48	48
						Harvests	23	29	34
						Total			86

[1] *L*, low; *H*, high; left side before comma, softwoods; right side after comma, hardwoods.
[2] Stand state no. after decision. — denotes doing nothing; x denotes undefined decision as the probability of this stand state was 0.

of small sawtimber of pine and hardwood were assumed to be half of those of large sawtimber prices (Zhou 2005), whereas pulpwood prices were set constant at 10.2 $\$ t^{-1}$ for pines and 6.1 $\$ t^{-1}$ for hardwoods (2010 = 100). All of the stumpage prices were for fresh weight.

Integrating Forest and Market Markov Chains

With 64 stand states and three price levels, there were $64 \times 3 = 192$ stand-market states. The transition probability between each pair of stand-market states was the product of the transition probabilities between the stand states and between the market states. The expected immediate financial reward depended on the volume of harvest due to a decision and the market state. Other criteria, independent of the market state, included harvest volume, tree size, and species diversity measured with Shannon's index, basal area, CO_2e stored in trees, and fraction of the landscape in old-growth state (Zhou and Buongiorno 2006).

Results

Multiple-Objective Optimization with Undiscounted Criteria

The maximum unconstrained value obtained with Model 1 was set as the target level for each criterion. Thus, the positive deviational variables (D_i^+) were unnecessary, leading to one-sided GPMDP models with only the negative deviational variables (D_i^-) penalized. The normalizing constant in Models 3 and 4 was the inverse of its maximum unconstrained value, whereas the preferential indicator was 1. This gave the same importance to a 1% deviation from each goal target.

When the total weighted deviations from all criteria were maximized, as shown in Table 4, the smallest deviation from the maximum value was for tree species diversity, only 1.6% less, followed by tree size diversity, 3.2% less than its maximum. The largest deviation from the target value was for the harvest volume (43% less than the maximum achievable value) and annual income (35% less).

With the Chebyshev (minmax) criterion, tree species and size diversities were also the closest to their targets, both less than 4% smaller than their maximum achievable values. The same largest difference (33.2%) occurred for annual income, stored carbon, old-growth fraction, and volume of harvests. In sum, the Chebyshev objective function led to better financial performance and more harvest, whereas the total weighted objective function achieved higher ecological benefits.

Tables 5 and 6 show the optimal policies derived with undiscounted criteria. Minimizing the total weighted deviation called for harvest in 95 of the 192 possible stand-market states (21 at low price level, 30 at medium, and 44 at high), and in states wanting a harvest, 14 were independent of the price level.(Table 5). With the Chebyshev criterion, 86 stand-market states suggested a harvest (23 at low

Table 7. Effects of policies that minimized the total or the maximum weighted deviation from discounted criteria.

Criterion	Unit	Target[1]	Minimizing total weighted deviation[2]		Minimizing maximum weighted deviation[2]	
			Achieved value	Relative deviation (%)	Achieved value	Relative deviation (%)
Net present value	$ ha^{-1}	13,552.7	9,594.7	29.2	9,497.4	29.9
Species diversity	Shannon index	31.9	31.1	2.6	31.1	2.6
Size diversity	Shannon index	65.2	62.9	3.5	62.6	4.1
Basal area	m^2 ha^{-1}	718.4	566.5	21.2	535.9	25.4
CO_2e	t ha^{-1}	7,242.1	4,984.2	31.18	4,785.5	33.9
Old growth	Fraction	7.2	5.2	27.9	4.7	33.9
Harvests	m^3 ha^{-1} yr^{-1}	350.6	206.2	41.2	231.7	33.9

[1] Maximum unconstrained, discounted value.
[2] Deviation is from the discounted maximum; weight is the inverse of discounted maximum.

Table 8. Decisions that minimized the total weighted deviation from discounted criteria.

Stand state		Decision[2] for market state:			Stand state		Decision for market state:		
No.	Basal area[1]	Low	Medium	High	No.	Basal area	Low	Medium	High
1	*LLL,LLL*	—	—	—	33	*HLL,LLL*	1	1	1
2	*LLL,LLH*	—	—	—	34	*HLL,LLH*	2	2	2
3	*LLL,LHL*	—	—	—	35	*HLL,LHL*	—	—	—
4	*LLL,LHH*	—	—	—	36	*HLL,LHH*	—	—	—
5	*LLL,HLL*	—	—	—	37	*HLL,HLL*	5	5	5
6	*LLL,HLH*	—	—	—	38	*HLL,HLH*	—	—	6
7	*LLL,HHL*	—	—	—	39	*HLL,HHL*	35	35	35
8	*LLL,HHH*	—	—	—	40	*HLL,HHH*	36	36	36
9	*LLH,LLL*	1	1	1	41	*HLH,LLL*	—	1	1
10	*LLH,LLH*	2	2	2	42	*HLH,LLH*	2	2	2
11	*LLH,LHL*	—	—	3	43	*HLH,LHL*	11	11	35
12	*LLH,LHH*	—	—	—	44	*HLH,LHH*	12	12	12
13	*LLH,HLL*	5	5	5	45	*HLH,HLL*	—	5	5
14	*LLH,HLH*	—	—	6	46	*HLH,HLH*	—	—	—
15	*LLH,HHL*	—	—	7	47	*HLH,HHL*	—	—	—
16	*LLH,HHH*	—	—	—	48	*HLH,HHH*	—	—	—
17	*LHL,LLL*	1	1	1	49	*HHL,LLL*	—	—	1
18	*LHL,LLH*	—	2	2	50	*HHL,LLH*	18	2	2
19	*LHL,LHL*	—	3	3	51	*HHL,LHL*	—	35	35
20	*LHL,LHH*	—	—	—	52	*HHL,LHH*	—	—	—
21	*LHL,HLL*	—	5	5	53	*HHL,HLL*	—	—	5
22	*LHL,HLH*	6	6	6	54	*HHL,HLH*	—	—	—
23	*LHL,HHL*	—	7	7	55	*HHL,HHL*	—	—	—
24	*LHL,HHH*	—	—	—	56	*HHL,HHH*	—	—	—
25	*LHH,LLL*	1	1	1	57	*HHH,LLL*	49	49	1
26	*LHH,LLH*	—	2	2	58	*HHH,LLH*	26	2	2
27	*LHH,LHL*	—	11	3	59	*HHH,LHL*	27	11	35
28	*LHH,LHH*	12	12	12	60	*HHH,LHH*	12	12	12
29	*LHH,HLL*	21	5	5	61	*HHH,HLL*	53	53	5
30	*LHH,HLH*	—	—	6	62	*HHH,HLH*	46	46	46
31	*LHH,HLH*	—	—	7	63	*HHH,HHL*	—	—	55
32	*LHH,HHH*	—	16	16	64	*HHH,HHH*	48	48	48
						Harvests	24	34	43
						Total			101

[1] *L*, low; *H*, high; left side before comma, softwoods; right side after comma, hardwoods.
[2] Stand state no. after decision.—denotes doing nothing.

price level, 29 at medium, and 34 at high), and in states requiring a harvest, 20 were price independent. Maximizing the total weighted deviation led to zero steady-state probability for one single-stand state (*LHH,LLL*), suggesting that it would be absent from the forest landscape under this policy, in the long run. In contrast, with the Chebyshev formulation, 16 stand states were absent in the steady state (Table 6). Thus, the long-term landscape generated by the policy maximizing total weighted deviations was more diverse than the policy obtained with the other variant.

Multiple-Objective Optimization with Discounted Criteria

The maximal value of each discounted criterion was obtained by solving Model 5 with a discount rate of 3.08% per year, roughly the upper bound of the social rate for intra- and intergenerational discounting used by the US Environmental Protection Agency (2010). The initial frequency of stand states ($\pi_{s'}$) was that observed in the study region (Zhou and Buongiorno 2006). The initial market states (low, medium, or high) were given equal probabilities. The results of Model 5 were used as the targets in the discounted GPMDPs 7 and 8. The weights were the inverse of each target, thus giving equal importance to a 1% deviation from each goal target.

As shown in Table 7, minimizing the total weighted deviation from the discounted criteria, or the maximum weighted deviation, gave similar results for tree species and tree size diversity, at most 4%

Table 9. Decisions that minimized the maximum weighted deviation from discounted criteria.

Stand state		Decision[2] for market state			Stand state		Decision for market state:		
No.	Basal area[1]	Low	Medium	High	No.	Basal area	Low	Medium	High
1	*LLL,LLL*	—	—	—	33	*HLL,LLL*	1	1	1
2	*LLL,LLH*	—	—	—	34	*HLL,LLH*	2	2	2
3	*LLL,LHL*	—	—	—	35	*HLL,LHL*	3	3	3
4	*LLL,LHH*	—	—	—	36	*HLL,LHH*	—	—	—
5	*LLL,HLL*	—	—	—	37	*HLL,HLL*	5	5	5
6	*LLL,HLH*	—	—	—	38	*HLL,HLH*	6	6	6
7	*LLL,HHL*	3	3	3	39	*HLL,HHL*	3	3	3
8	*LLL,HHH*	—	—	—	40	*HLL,HHH*	36	36	36
9	*LLH,LLL*	1	1	1	41	*HLH,LLL*	1	1	1
10	*LLH,LLH*	2	2	2	42	*HLH,LLH*	x	x	x
11	*LLH,LHL*	3	3	3	43	*HLH,LHL*	—	—	—
12	*LLH,LHH*	—	—	—	44	*HLH,LHH*	12	12	12
13	*LLH,HLL*	5	5	5	45	*HLH,HLL*	5	5	5
14	*LLH,HLH*	6	6	6	46	*HLH,HLH*	—	—	—
15	*LLH,HHL*	—	—	—	47	*HLH,HHL*	—	—	—
16	*LLH,HHH*	—	—	—	48	*HLH,HHH*	—	—	—
17	*LHL,LLL*	1	1	1	49	*HHL,LLL*	1	1	1
18	*LHL,LLH*	2	2	2	50	*HHL,LLH*	2	2	2
19	*LHL,LHL*	3	3	3	51	*HHL,LHL*	—	—	—
20	*LHL,LHH*	4	4	4	52	*HHL,LHH*	—	—	—
21	*LHL,HLL*	5	5	5	53	*HHL,HLL*	—	—	—
22	*LHL,HLH*	6	6	6	54	*HHL,HLH*	—	—	—
23	*LHL,HHL*	3	3	3	55	*HHL,HHL*	—	—	—
24	*LHL,HHH*	—	—	—	56	*HHL,HHH*	52	52	52
25	*LHH,LLL*	1	1	1	57	*HHH,LLL*	1	1	1
26	*LHH,LLH*	2	2	2	58	*HHH,LLH*	x	x	x
27	*LHH,LHL*	—	—	—	59	*HHH,LHL*	27	27	27
28	*LHH,LHH*	12	12	12	60	*HHH,LHH*	12	12	12
29	*LHH,HLL*	5	5	5	61	*HHH,HLL*	53	53	5
30	*LHH,HLH*	—	—	—	62	*HHH,HLH*	46	46	46
31	*LHH,HLH*	—	—	—	63	*HHH,HHL*	63	63	63
32	*LHH,HHH*	16	16	16	64	*HHH,HHH*	48	48	48
						Harvests	38	38	38
						Total			114

[1] *L*, low; *H*, high; left side before comma, softwoods; right side after comma, hardwoods.
[2] Stand state no. after decision. — denotes doing nothing; x denotes undefined decision as the steady-state probability of this stand state was 0.

less than the highest unconstrained values. For the other goals, the relative deviations were much larger (21–41%). Overall, the total weighted objective function gave results that were inferior to those of the Chebyshev objective function for harvested volume only, whereas they were similar or superior for all other criteria.

The best decisions obtained by minimizing the sum of the weighted deviations from the discounted criteria (Table 8) were different from those by minimizing the maximum weighted deviation (Table 9) in 55 of the 192 stand-market states. Minimizing the sum of the weighted deviations called for a harvest in 101 of the 192 possible stand-market states (24 at low price level, 34 at medium, and 43 at high). With the Chebyshev objective, 114 stand-market states wanted a harvest. In this case, regardless of price level, 38 stand states needed a basal area reduction, and only in one state (*HHH*,*HLL*) was the reduction price dependent (Table 9). When the sum of the weighted deviations was minimized, there was a well-defined decision for each stand state, whereas with the Chebyshev objective function states (*HLH*,*LLH*) and (*HHH*,*LLH*) had no attendant decision as the probability of observing them was zero under the optimum policy described in Table 9.

Discussion

MDPs make it possible to obtain solutions for systems with multiple objectives and subject to multiple sources of risk. Classic MDPs formulated as LP models are capable of handling numerous management goals by setting one as the objective and the rest as constraints, so long as both objective and constraints are discounted or undiscounted. However, defining specific bounds for some criteria, especially ecological ones, that reflect accurately the decisionmaker's preferences presents considerable challenges (Tamiz et al. 1998). In addition, infeasibility often occurs if conflicting objectives are all set too aggressively.

Here, to aid multiobjective decisionmaking with MDPs, we adopted techniques that take advantage of the "satisfying and sufficing" (Simon 1957) nature of goal programming and its ease of use. To this end, deviational variables were introduced in LP formulations of MDPs with average or discounted criteria. We proposed two formulations for each case: weighted total and Chebyshev; the former allowed straightforward trade-offs between goals and the latter strived to "achieve a good balance between a set of goals" (Jones and Tamiz 2010). There is no strong reason to prefer one formulation over the other, and, in practice, both could be used for more insight, with only slight changes in the model and with the same software. Within each method, the setting of goal targets and weights is also at the discretion of the decisionmaker. Diaz-Balteiro et al. (2013) offer guidance to help analysts choose the methods and parameters that best reflect the preferences of decisionmakers.

The essence of the approach was to minimize the undesired deviations from multiple criteria without imposing fixed levels to be achieved. There was a great amount of flexibility in assigning importance to each objective by utilizing weights as preferential indicators or normalizing metrics, and the resulting models were always feasible. Another advantage of the GPMDP solutions was that best policies were always deterministic; i.e., there was one single decision for each state. In contrast, constrained MDPs are well known to generate nondeterministic policies (Altman 1999), rendering them difficult to implement, unless only a few decisions are stochastic. One limitation was the separation of undiscounted and discounted criteria, but it allowed LP formulations and thus efficient handling of large practical problems with multiple objectives.

In this particular application with mixed pine-hardwood forests in the southern United States, we simultaneously dealt with ecological and economic objectives. Because these objectives were not directly commensurate, the weights were the inverse of their maximal achievable levels, implying that they were of equal importance to the decisionmaker. Moreover, by setting the target level optimistically, i.e., at its maximal unconstrained level, "the dominant underlying philosophy was changed from satisficing to optimizing" (Jones and Tamiz 2010, p. 7), which guaranteed Pareto efficiency (Pareto 1964): no other feasible solution existed that could improve at least one of the criteria while keeping the others as good as in the optimal GPMDP solution. This approach was also in essence equivalent to CP (Tamiz et al. 1998), an ideal-point method in which "to be as close as possible to the perceived ideal is assumed to be the rationale of human choice" (Zeleny and Cochrane 1973).

The empirical results suggested that for the study area, maintaining tree species and size diversity within 4% of their maximum value required falling short of the maximum of other criteria (financial, production, and ecological) by 19 to 43%. Also, in this application, the choice of objective function, weighted total or Chebyshev, had little effect on system performance, but the optimal policies were quite different, especially for discounted criteria. The Chebyshev formulation yielded undefined decisions for several stand states because such states were not present in the steady state.

The limitations of the GPMDPs emerged in part from the MDP component. We must be concerned about the errors due to the discretization of states (Tavella and Randall 2000). Defining a state space sufficiently large for accurate representation, but small enough to be practical, presents considerable challenge. One additional limitation is the stationarity requirement of the system. Yousefpour et al. (2012) provides an overview of some methods dealing with nonstationary processes such as growth under climate change. MDPs with enlarged state space can in principle reflect climate change and related changing disturbance regimes, but again the "curse of dimensionality" poses practical limits to this approach.

As for the GP component, the risk of Pareto inefficient solutions should not be a problem as long as pessimistic target levels are avoided. Setting targets at their maximum unconstrained levels is good practice in that respect if optimizing instead of satisficing is the decisionmaker's goal. If levels other than the ideal ones are specified for the targets, techniques outlined in Tamiz and Jones (1996) could be used to restore Pareto efficiency. More difficult to resolve is the specification of the relative importance/preferences of multiple objectives and therefore of their weights. Several lines of research are needed in that respect on how to set weights for accurate representation of preferences (Martel and Aouni 1990, Linares and Romero 2002), to reflect ambiguous preferences with fuzzy objectives and constraints (Biswas and Pal 2005), or to use the lexicographic variant of GP to rank the objectives (Jones and Tamiz 2010) and thus customize models for decisionmaker's preferences (Diaz-Balteiro et al. 2013). The proposed methods also assumed risk neutrality of decisionmakers. Although approaches of dynamic programming and policy iteration exist for single-objective risk-averse MDPs (Ruszczyński 2010), how to handle risk aversion in MDP models with multiple objectives with LP such as those considered in this article remains a challenging field for further research.

Still, as it stands, the GPMDP method presented in this article offers a pragmatic way to handle forestry decisionmaking with competing objectives under risk. The methods are theoretically rigorous and the numerical solutions based on linear programming handle very large problems efficiently. The adaptive policies that emerge are simple decision tables readily applicable in the field. Used wisely, they should help both public and private nonindustrial forest owners meet their diverse and often conflicting goals.

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