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Multidimensional Born velocity inversion : Single wide band point source

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ABSTRACT

The inverse scattering problem for an acoustic medium is considered within the approximate direct inversion framework. As opposed to iterative methods, the direct inversion approach gives an estimate of the medium velocities by operating on the observed scattered field without repeated solutions of the forward problem. Previous solutions to the multidimensional Born velocity inversion problem require either collocated source and receiver arrays or plane wave sources. In both cases an array of point sources is required to collect the proper data for inversion. In this paper, the solution to the single point source problem is derived. It is shown that by extrapolating and imaging the observed scattered field appropriately, the projections of the velocity function at all angles can be obtained. The velocities are, then, reconstructed by the inverse Radon transform method of tomography.

Introduction

The general inverse scattering problem can be stated as follows. The medium of interest is probed by sources located outside the medium and the scattered field is recorded at various locations. The problem is to obtain the properties of the medium such as the propagation velocity or the material density from the observed data. Similar inverse scattering problems arise in areas such as acoustics, electromagnetics and optics although the corresponding physical phenomenon are somewhat different. In this paper, an acoustic medium probed by sound waves is considered. The medium velocity may vary in all directions, but the density is assumed to be constant. We also assume that the velocity does not depend on frequency; i.e., a non-dispersive medium.

The differential equation governing the multidimentional wave propagation in an inhomogeneous medium can be transformed into an integral equation known as the Lippmann-Schwinger equation from which we obtain the following integral representation of the scattered field, (Taylor [1]),

$$P_s(\underline{x},\omega) = k^2 \int_V d\underline{x}' \, \gamma(\underline{x}') \, P(\underline{x}',\omega) \, G_0(\underline{x},\underline{x}',\omega) \ . \tag{1}$$

Here P and P_s are the total and scattered fields and G_0 is the free space Green's function. The velocity function γ is related to the medium velocities $v(\underline{x})$ by

$$\gamma(\underline{x}) = n^2(\underline{x}) - 1 \ , \tag{2}$$

where $n(\underline{x}) = \dfrac{c}{v(\underline{x})}$ and $c = \dfrac{\omega}{k}$ is a reference velocity. Since $P(\underline{x},\omega)$ is also a function of the unknown velocities the scattered field is a nonlinear function of the medium parameters. In the approximate direct inversion approach, the total field in the integrand in equation (1) is replaced by a known incident field. The resulting equation is, then, inverted to obtain the velocity function. Here, the Lippmann-Schwinger equation is considered within the first Born approximation; that is, the total field is replaced by $P_0(\underline{x},\omega)$, the background field computed by assuming that $\gamma(\underline{x}) = 0$. This is a very common linearization in inverse scattering problems; physically it corresponds to assuming that the scattered field inside the medium is small compared to the incident field.

Direct velocity inversion within the Born approximation has been an active research topic in recent years. This problem has been investigated for various dimensions, observation geometries and background velocity models, although most solutions assume an homogeneous background velocity model. The simplest problem, namely, that of a one dimensional medium probed by a broadband plane wave was considered by Cohen and Bleistein [2], who showed that the velocities can be obtained by an inverse Fourier transform over wavenumber. This solution was extended by Gray et. al. [3] to the case of scattering from a layered medium probed by a point source. Here again the Born inversion can be done with one receiver, and the inversion is obtained via Fourier transforms and a change of variables. Most solutions for the multidimensional case assume an observation geometry consisting of coincident sources and receivers (zero-offset). One reason for the interest in the zero-offset problem is that for this geometry the Lippmann-Schwinger integral representation becomes simpler, in that the second and the third terms in the integrand of equation (1) become identical. This leads to a one to one mapping between the observed data and the velocity function in the two (or three) dimensional Fourier transform domain. Cohen and Bleistein [4] describe a time domain method for zero-offset (homogeneous) Born inversion in seismic application, while Norton and Linzer [5] describe similar methods for ultrasonic reflectivity imaging.

A slightly different approach to the study of the direct velocity inversion problem has been developed by extending x-ray tomographic techniques to ultrasonic imaging. In this case, the acoustic medium is probed from various directions by plane waves and the scattered field is recorded for each plane wave separately. At a fixed frequency, plane waves incident on the medium from all directions are required for complete inversion of the velocities. Mueller et al. [6] discuss the diffraction effects of ultrasound tomography and derive the volume integral representation within the Born and Rytov approximations. Greenleaf [7] gives some examples of images reconstructed with this approach. In diffraction tomography, with plane-wave sources, the incident field in equation (1) become a complex exponential, providing a Fourier transform relation. In fact, the scattered field due to each plane wave gives the velocity function along a circular trajectory in the Fourier transform domain as first pointed out by Wolf [8]. Velocities can be reconstructed by interpolating the available data over a rectangular grid and then taking the inverse Fourier transform in rectangular coordinates. Devaney [9] introduced the backpropagation method for reconstruction as an alternative to the interpolation in the frequency domain. Examples of velocity reconstructions with both complete and incomplete sets of plane waves are given by Devaney [10], while Devaney and Beylkin [11] describe the theoretical extension to a point source or "fan beam" geometry.

As was mentioned above, most multidimensional velocity inversion methods assume either zero-offset geometry or plane-wave sources. In this paper, the following problem is considered. The three (or two) dimensional acoustic medium is excited by a point (or line) source located outside the inhomogeneous region and the scattered field is observed at all frequencies on the surface surrounding the region. The single-source problem is interesting for several reasons. First, unlike in the zero-offset case both reflected and transmitted data can be used for inversion. This problem differs also from diffraction tomography in that the information in the frequency content of a single experiment replaces many experiments each involving monochromatic plane waves. The single-source problem, therefore, may help to better understand and relate the other methods of inversion. Second, the problem is obviously important in applications where the zero-offset configuration is either impractical or physically impossible and where the number and the location of the sources may not be sufficient to produce virtual plane sources. Finally, understanding the single source problem will help to develop direct inversion methods for more realistic problems, such as for multiple bandlimited sources.

Data processing efforts for single source experiments have mainly concentrated on reflector imaging or migration. The two main approaches to migration - the finite difference method (Claerbout [12], Claerbout and Doherty [13]) and the Kirchhoff integral method (Schneider [14], Jain and Wren [15]) - are readily applicable to a single-source experiment. A review of wavefield extrapolation methods for migration can be found in Berkhout [16]. In general, the purpose of migration is to map the locations of the sharp velocity changes in the medium rather than, e.g., to obtain quantitative estimates of the velocities. Kirchhoff migration can be viewed as a delay and sum array processing. To image a given point in the medium the receiver array is focused on that point by appropriate phase delays, and the image at the point is obtained by integrating over the receivers. The imaged quantity is obviously a function of the amount of scattering from that point but it is also a function of the relative positions of the source, the receiver array and the image point. Therefore, the imaged quantity is not a direct measure of the velocity changes at the image point. Better images of the discontinuities can be obtained by employing a weighted delay and sum operation for focusing, where a different receiver weight is used for each point in the medium. Miller et. al. [17] describe the focusing weights with examples of the migrated synthetic data for the case of homogeneous background. A formal derivation of the receiver weights for a variable background within the geometrical optics approximation is given by Beylkin [18].

In this paper a different approach to the inversion problem is taken. Instead of operating directly on the observed scattered data, we consider the field extrapolated by the wave equation from the receivers into the medium. The volume integral representation of the extrapolated field was first derived by Porter [19] and then by Bojarski [20] for homogeneous Green's functions in the context of holographic imaging and the inverse source problem. A derivation of the volume integral representation for arbitrary Green's functions is given in the following section. Although extrapolated field does not contain more information than the observed scattered field, there is more flexibility in the type of operations that can be performed with it. For example, for a two dimensional problem the observed scattered field $P_s(\xi,\omega)$ is a function of two variables, namely, receiver location and frequency (or time). On the other hand the extrapolated field $P_e(x,z,t)$ is a function of two space variables and time or frequency. Therefore, there is one more free parameter in this domain. Moreover, the extrapolated field and the unknown velocity function $\gamma(x,z)$ share the same spatial parameters. In fact, in this domain migration is an operation where the time variable of the extrapolated field is simply set equal to the travel time corresponding to each point. In the next section the extrapolated field is defined and its volume integral representation in terms of the medium velocities is derived. In section 3, the exact analytical solution of the velocities is obtained from the extrapolated field within the Born approximation.

1. The Extrapolated Field

Consider the wave propagation operator $\mathbf{D}$ for a constant density acoustic medium with refraction index $n(\underline{x})$,

$$\mathbf{D} = \nabla^2 + k^2 n^2(\underline{x}) \ . \tag{3}$$

A variety of volume integral representations for the scattered field can be obtained by decomposing this operator into a background operator and a residual. Let

$$\begin{aligned} \mathbf{D} &= \mathbf{D}_0 + \gamma(\underline{x},\omega) \\ \mathbf{D}_0 &= \nabla^2 + k^2 n_0^2(\underline{x}) \\ \gamma(\underline{x},\omega) &= k^2 \left[n^2(\underline{x}) - n_0^2(\underline{x}) \right] , \end{aligned} \tag{4}$$

where $n_0(\underline{x})$ represents the background model which incorporates any a priori knowlegde of the medium. Since it does not complicate the following discussion an arbitrary varying background is considered in this section. For the homogeneous background case $n_0(\underline{x}) = 1$. Define the volume V surrounded by the closed surface S to be a domain containing the values of $\underline{x}$ such that

$$n(\underline{x}) \neq n_0(\underline{x}) . \tag{5}$$

Let $P(\underline{x},\omega), \underline{x} \in V$ be the total field (incident plus scattered) and $P_s(\underline{x},\omega), \underline{x} \in S$ be the observed scattered field due to sources located outside V. Then,

$$\mathbf{D}_0 P_s(\underline{x},\omega) = -\gamma(\underline{x},\omega)\ P(\underline{x},\omega) \quad ; \quad \underline{x} \in V , \tag{6}$$

and an integral solution for P_s is

$$P_s(\underline{x},\omega) = \int_V d\underline{x}'\ \gamma(\underline{x}',\omega)\ P(\underline{x}',\omega)\ G_0(\underline{x},\underline{x}',\omega) , \tag{7}$$

where the Green's function is given by

$$\mathbf{D}_0 G_0(\underline{x},\underline{x}',\omega) = -\delta(\underline{x} - \underline{x}') . \tag{8}$$

The extrapolated field is obtained by solving the homogeneous wave equation for the background velocity model with Dirichlet boundary conditions given by the complex conjugate of the scattered field on the observation surface, i.e.

$$\begin{aligned} \mathbf{D}_0 P_e(\underline{x},\omega) &= 0 \quad ; \quad \underline{x} \in V , \\ P_e(\underline{x},\omega) &= P_s^*(\underline{x},\omega) \quad ; \quad \underline{x} \in S . \end{aligned} \tag{9}$$

In the time-domain, this can be interpreted as running the homogeneous wave equation "backwards" in time with boundary conditions given by the time-reversed scattered field. The solution of this boundary value problem is (Morse and Feshbach [21])

$$P_e^*(\mathbf{r},\omega) = -\int_S d\boldsymbol{\xi}' \; [\; P_s(\boldsymbol{\xi}',\omega)\; \nabla_{\xi'} G_0^*(\mathbf{r},\boldsymbol{\xi}',\omega) - G_0^*(\mathbf{r},\boldsymbol{\xi}',\omega)\; \nabla_{\xi'} P_s(\boldsymbol{\xi}',\omega)\;] \cdot \hat{\mathbf{n}}(\boldsymbol{\xi}') \;, \tag{10}$$

where $\hat{\mathbf{n}}$ is the unit vector outward normal to the surface S. For inversion, we want to represent the extrapolated field in terms of the velocity function. From equations (7) and (10), it follows that

$$P_e^*(\mathbf{r},\omega) = -\int_V d\mathbf{r}'\; \gamma(\mathbf{r}',\omega)\; P(\mathbf{r}',\omega) \int_S d\boldsymbol{\xi}' \; [\; G_0(\boldsymbol{\xi}',\mathbf{r}',\omega)\; \nabla_{\xi'} G_0^*(\mathbf{r},\boldsymbol{\xi}',\omega) - G_0^*(\mathbf{r},\boldsymbol{\xi}',\omega)\; \nabla_{\xi'} G_0(\boldsymbol{\xi}',\mathbf{r}',\omega)\;] \cdot \hat{\mathbf{n}}(\boldsymbol{\xi}') \;. \tag{11}$$

By applying Green's theorem (Morse and Feshbach [21]) to the surface integral in equation (11), we obtain

$$\int_V d\boldsymbol{\xi}' \; [\; G_0(\boldsymbol{\xi}',\mathbf{r}',\omega)\; \mathbf{D}_0 G_0^*(\mathbf{r},\boldsymbol{\xi}',\omega) - G_0^*(\mathbf{r},\boldsymbol{\xi}',\omega)\; \mathbf{D}_0 G_0(\boldsymbol{\xi}',\mathbf{r}',\omega)\;] = -G_0(\mathbf{r},\mathbf{r}',\omega) + G_0^*(\mathbf{r},\mathbf{r}',\omega) \quad ; \quad \mathbf{r} \in V \;. \tag{12}$$

Therefore, the volume integral representation of the extrapolated field in terms of the velocity function γ is given by

$$P_e^*(\mathbf{r},\omega) = \int_V d\mathbf{r}'\; \gamma(\mathbf{r}',\omega)\; P(\mathbf{r}',\omega)\; 2i\; Im[\; G_0(\mathbf{r},\mathbf{r}',\omega)\;] \;. \tag{13}$$

Comparing equations (7) and (13), the extrapolated field has the same form as the scattered field except that the Green's function is replaced by its imaginary part. Since the imaginary part of the Green's function is odd symmetric in time, the extrapolated field consists of scattered waves propagating in positive and negative time directions. This property will be used in the next section to invert equation (13) in the case of homogeneous background.

2. Inversion for the Velocities

Consider an experiment where the two (three) dimensional medium V is probed by a line (point) source and the scattered field is observed on the surface S as shown in figure 1. From equation (13), for a source with flat frequency spectrum, the volume integral for the extrapolated field within the Born approximation can be written as

$$P_e^*(\underline{x},\omega) = \int_V d\underline{x}' \; \gamma(\underline{x}',\omega) \; G_0(\underline{x}',\underline{x}_s,\omega) \; 2i \; Im[\; G_0(\underline{x},\underline{x}',\omega) \;] \;, \qquad (14)$$

where $\underline{x}_s$ is the location of the source. A geometric interpretation of the extrapolated field is as follows. Let the travel time from the source to a point $\underline{x}_0$ be τ_0. When the incident field reaches this point at time τ_0 the waves scattered from the point propagate forward and backward in time. At times $2\tau_0$ and zero, the extrapolated field due to scattering from $\underline{x}_0$ lies on a curve passing through the source location as shown in figure 2. The shape of the curve is determined by the travel times of the background velocity model. For example with a homogeneous background the extrapolated field maps onto circles in two dimensions and onto spherical surfaces in three dimensions.

An analytical solution of equation (14) is possible when the background model is homogeneous. The procedure is the same for the two dimensional medium with a line source and for the three dimensional medium with a point source. The Green's function of a two dimensional homogeneous medium (i.e., line source) is given by

$$G_0(\underline{x},\underline{x}',\omega) = \frac{i}{4} H_0^{(1)}(k\,|\underline{x}-\underline{x}'|), \tag{15}$$

and its imaginary part is

$$Im[G_0(\underline{x},\underline{x}',\omega)] = \frac{1}{4} sgn(k)\; J_0(k\,|\underline{x}-\underline{x}'|), \tag{16}$$

where $H_0^{(1)}(\cdot)$ and $J_0(\cdot)$ are the Hankel and Bessel functions of the first kind and $sgn(k) = \pm 1$ denotes the sign of k. Also, for homogeneous background the scattering potential is

$$\gamma(\underline{x},\omega) = k^2\,\gamma(\underline{x}) \quad ; \quad \gamma(\underline{x}) = n^2(\underline{x}) - 1. \tag{17}$$

Then, from equation (14) the extrapolated field can be written as

$$P_e^*(\underline{x},\omega) = -\frac{k^2}{8} sgn(k) \int_V d\underline{x}'\,\gamma(\underline{x}')\,H_0^{(1)}(k\,|\underline{x}'-\underline{x}_s|)\,J_0(k\,|\underline{x}-\underline{x}'|). \tag{18}$$

By taking the real parts of both sides, we have

$$P_e^R(\underline{x},\omega) = -\frac{k^2}{8} \int_V d\underline{x}'\,\gamma(\underline{x}')\,J_0(k\,|\underline{x}'-\underline{x}_s|)\,J_0(k\,|\underline{x}-\underline{x}'|). \tag{19}$$

Now consider the following function obtained from the extrapolated field imaged at time zero

$$\hat{\gamma}(\underline{x}) = -4\,|\underline{x}-\underline{x}_s| \int_{-\infty}^{\infty} d\omega\,\frac{P_e(\underline{x},\omega)}{|\omega|}. \tag{20}$$

The function $\hat{\gamma}(\underline{x})$ can be expressed as

$$\hat{\gamma}(\underline{r}) = -8\ |\underline{r}-\underline{r}_s|\ \int_0^\infty dk\ \frac{P_s^R(\underline{r},\omega)}{k}$$

$$= |\underline{r}-\underline{r}_s|\ \int_V d\underline{r}'\ \gamma(\underline{r}')\ \int_0^\infty dk\ k\ J_0(k\,|\underline{r}'-\underline{r}_s|)\ J_0(k\,|\underline{r}-\underline{r}'|)$$

$$= |\underline{r}-\underline{r}_s|\ \int_V d\underline{r}'\ \frac{\gamma(\underline{r}')}{|\underline{r}'-\underline{r}_s|}\ \delta(\ |\underline{r}'-\underline{r}_s| - |\underline{r}-\underline{r}'|\ ). \tag{21}$$

The roots of the argument of the delta function above clearly lie along the line perpendicular to the $\underline{r}-\underline{r}_s$ vector at its midpoint. To obtain the appropriate weights for integrating along this line the variables in the integral (21) can be changed from (x',z') to (ρ,ξ) by shifting the origin to the source location $\underline{r}_s$ and by rotating as shown in figure 3. It follows that

$$\hat{\gamma}(\hat{\underline{r}},r) = r \int_{\xi_1(\underline{r})}^{\xi_2(\underline{r})} d\xi \int_{\rho_1(\underline{r})}^{\rho_2(\underline{r})} d\rho\ \frac{\gamma(\rho,\xi;\hat{\underline{r}})}{(\rho^2+\xi^2)^{1/2}}\ \delta\left\{ (\rho^2+\xi^2)^{1/2} - [(\rho-r)^2+\xi^2]^{1/2} \right\}, \tag{22}$$

where $r = |\underline{r}|$ and $(\rho,\xi;\hat{\underline{r}})$ denotes the coordinates (ρ,ξ) for a given direction $\hat{\underline{r}}$ as shown in figure 3. Noting that $\hat{\gamma}(\hat{\underline{r}},0) = 0$ for all directions $\hat{\underline{r}}$, for $r \neq 0$ this integral is evaluated as follows. Viewing ξ as a parameter, the second integral is of the form

$$\int_{\rho_1(\underline{r})}^{\rho_2(\underline{r})} d\rho\ b(\rho,\xi)\ \delta[a(\rho,\xi)] = \left\{ b(\rho,\xi) \left| \frac{\partial a(\rho,\xi)}{\partial\rho} \right|^{-1} \right\}_{\rho=\rho_0} \quad ; \quad \rho_1<\rho_0<\rho_2$$

$$= 0 \quad ; \quad \text{otherwise,} \tag{23}$$

where ρ_0 is the only root of $a(\rho,\xi)$ for all ξ. In equation (22) the only root of the argument is $\rho_0 = r/2$ and

$$\left\{ \frac{\partial a(\rho,\xi)}{\partial\rho} \right\}_{\rho=\rho_0} = \frac{\rho}{(\rho^2+\xi^2)^{1/2}} - \frac{\rho-r}{[(\rho-r)^2+\xi^2]^{1/2}} = \frac{r}{[(\frac{r}{2})^2+\xi^2]^{1/2}}. \tag{24}$$

Therefore, for $r \neq 0$

$$\hat{\gamma}(\hat{\underline{r}},r) = \int_{\xi_1(\underline{r})}^{\xi_2(\underline{r})} d\xi \; \gamma(\frac{r}{2},\xi;\hat{\underline{r}})$$

$$= \int_{(\underline{r}'-\underline{r}_s)\cdot\hat{\underline{r}}=\frac{r}{2}} dl \; \gamma(\underline{r}') \; . \tag{25}$$

This simply means that $\hat{\gamma}(\hat{\underline{r}},2r)$ for $\hat{\underline{r}}$ fixed is the projection of $\gamma(\underline{r}')$ onto the line defined by the unit vector $\hat{\underline{r}}$ as shown in figure 4. Since the function $\hat{\gamma}(\underline{r})$ is known everywhere in the medium, projections at all angles $\hat{\underline{r}} = \frac{\underline{r}}{r}$ are obtained by equation (25). Recovering $\gamma(\underline{r})$ from its projections $\hat{\gamma}(\hat{\underline{r}},2r)$ is exactly the problem encountered in tomography and is accomplished by the inverse Radon transform (Ludwig [22]). From the projection-slice theorem

$$\tilde{\gamma}(k\hat{\underline{k}}) = \int_{-\infty}^{\infty} d\rho \; \hat{\gamma}(\hat{\underline{k}},2\rho) \; e^{-ik\rho}, \tag{26}$$

where $\tilde{\gamma}(\underline{k})$; $\underline{k} = k\hat{\underline{k}}$, is the Fourier transform of $\gamma(\underline{r})$. From the Fourier representation of $\gamma(\underline{r})$ and using equation (26) it can be shown that

$$\gamma(\underline{r}) = \frac{1}{4\pi^2} \int_0^{2\pi} d\hat{\underline{k}} \int_0^{\infty} dk \; k \; e^{ik\hat{\underline{k}}\cdot\underline{r}} \int_{-\infty}^{\infty} d\rho \; \hat{\gamma}(\hat{\underline{k}},2\rho) \; e^{-ik\rho}. \tag{27}$$

This equation gives the reconstruction of $\gamma(\underline{r})$ from its projections and it can be implemented by mapping the polar coordinates into cartesian coordinates. An alternative implementation can be obtained by noting that $\hat{\gamma}(-\hat{\underline{k}},-2\rho) = \hat{\gamma}(\hat{\underline{k}},2\rho)$. Using this property equation (27) can be written as

$$\gamma(\underline{x}) = \frac{1}{8\pi^2} \int_0^{2\pi} d\hat{\underline{k}} \int_{-\infty}^{\infty} dk \; e^{ik\hat{\underline{k}}\cdot\underline{x}} \; |k| \int_{-\infty}^{\infty} d\rho \; \hat{\gamma}(\hat{\underline{k}},2\rho) \; e^{-ik\rho}. \qquad (28)$$

This shows that for reconstruction the projections are first filtered with the filter $|k|$, then they are backprojected into the medium. Equation (28) can also be written in compact form

$$\gamma(\underline{x}) = \frac{1}{4\pi} \int_0^{2\pi} d\hat{\underline{k}} \; \mathrm{H} \left\{ \frac{\partial}{\partial s} \hat{\gamma}(\hat{\underline{k}},s) \right\}_{s=2\hat{\underline{k}}\cdot\underline{x}}, \qquad (29)$$

where H denotes the Hilbert transform.

As was pointed out previously, the projections of $\gamma(\underline{x})$ along the lines that go through the origin are not obtained, instead we have the constraint that $\hat{\gamma}(\hat{\underline{x}},0) = 0$. Nevertheless, this has no effect on the reconstructed velocities within the volume V. From equation (25)

$$\hat{\gamma}(\hat{\underline{x}},2r) = \int_{-\infty}^{\infty} d\xi \; \gamma(r,\xi;\hat{\underline{x}}) \quad ; \quad r \neq 0$$

$$= 0 \quad ; \quad r = 0 \, . \qquad (30)$$

The integration limits are extended to infinity since $\gamma(r,\xi;\hat{\underline{x}})$ is zero outside $[\ \xi_1(\underline{x})\ ;\ \xi_2(\underline{x})\ ]$. Let the reconstructed velocity function $\gamma_r(\underline{x})$ be the sum of the true velocity function and an error term

$$\gamma_r(\underline{x}) = \gamma(\underline{x}) + \gamma_e(\underline{x})$$

$$\hat{\gamma}(\hat{\underline{x}},2r) = \int_{-\infty}^{\infty} d\xi \; \gamma_r(r,\xi;\hat{\underline{x}}) \, . \qquad (31)$$

It follows from equations (30) and (31) that the projections of the error term are given by

$$\int_{-\infty}^{\infty} d\xi\, \gamma_e(r,\xi;\hat{x}) = -\int_{-\infty}^{\infty} d\xi\, \gamma(r,\xi;\hat{x}) \quad ; \quad r = 0$$

$$= 0 \quad ; \quad \text{otherwise.} \tag{32}$$

Therefore, the only region where the error term is nonzero is an infinitely close neighbourhood of the source location. Since the source is located outside the medium this does not effect the reconstruction and

$$\gamma_r(\underline{x}) = \gamma(\underline{x}) \quad ; \quad \underline{x} \in V\,. \tag{33}$$

In summary, the velocity function $\gamma(\underline{x})$ is obtained from the observed scattered field in three steps ;

1) Extrapolate the observed data to obtain $P_e(\underline{x},\omega)$. The extrapolation can be done directly with equation (9) using a finite difference scheme. Alternatively, the Kirchhoff integral form in equation (10) can be used.

2) Obtain the zero time image field $\hat{\gamma}(\underline{x})$ by equation (20). In the time domain equation (20) can be implemented by integrating and Hilbert transforming the observed traces before extrapolation. Then, the extrapolated field is imaged at time zero and scaled at all image points by $4\,|\underline{x}-\underline{x}_s\,|$.

3) Reconstruct $\gamma(\underline{x})$ from its projections $\hat{\gamma}(\hat{x},r)$. From equation (28) the reconstruction can be done in two steps:

a) Filter the projections

$$F(k,\hat{k}) = |\,k\,| \int_{-\infty}^{\infty} d\rho\, \hat{\gamma}(\hat{k},2\rho)\, e^{-ik\rho},$$

$$f(\underline{x},\hat{k}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk\;\; F(k,\hat{k})\; {}^{ik\hat{k}\cdot\underline{x}}. \tag{34}$$

b) Backproject the filtered projections

$$\gamma(\underline{x}) = \frac{1}{4\pi} \int_0^{2\pi} d\underline{\hat{\xi}}\, f(\underline{x},\underline{\hat{\xi}}). \tag{35}$$

3. Conclusion

In this paper a new approach to the velocity inversion problem has been presented. The problem was formulated in terms of the field extrapolated from the receivers into the medium by the wave equation. The volume integral representation of the extrapolated field was derived for an arbitrary Green's function. It was shown that a complete set of projections of the velocity function can be obtained by imaging the extrapolated field at zero time. In the case of a homogeneous Born background the projection trajectories become straight lines. Therefore, the single point source problem can be transformed into the classical tomography problem. The imaged field is then inverted for the velocities by the inverse Radon transform.

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Figure 1.

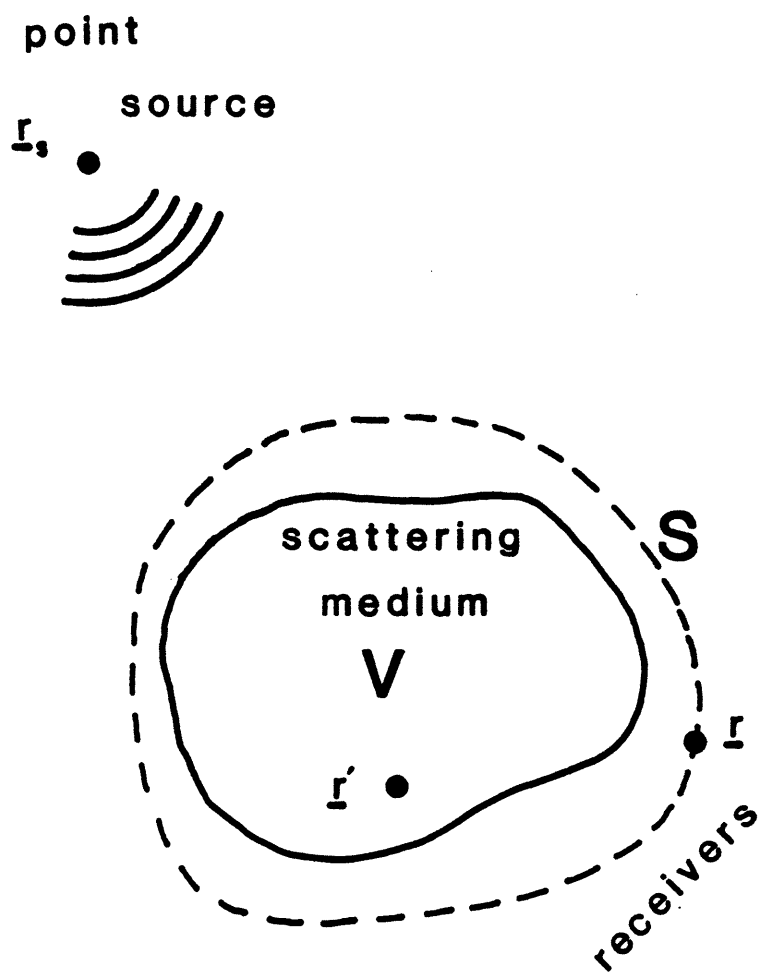

Figure 2.

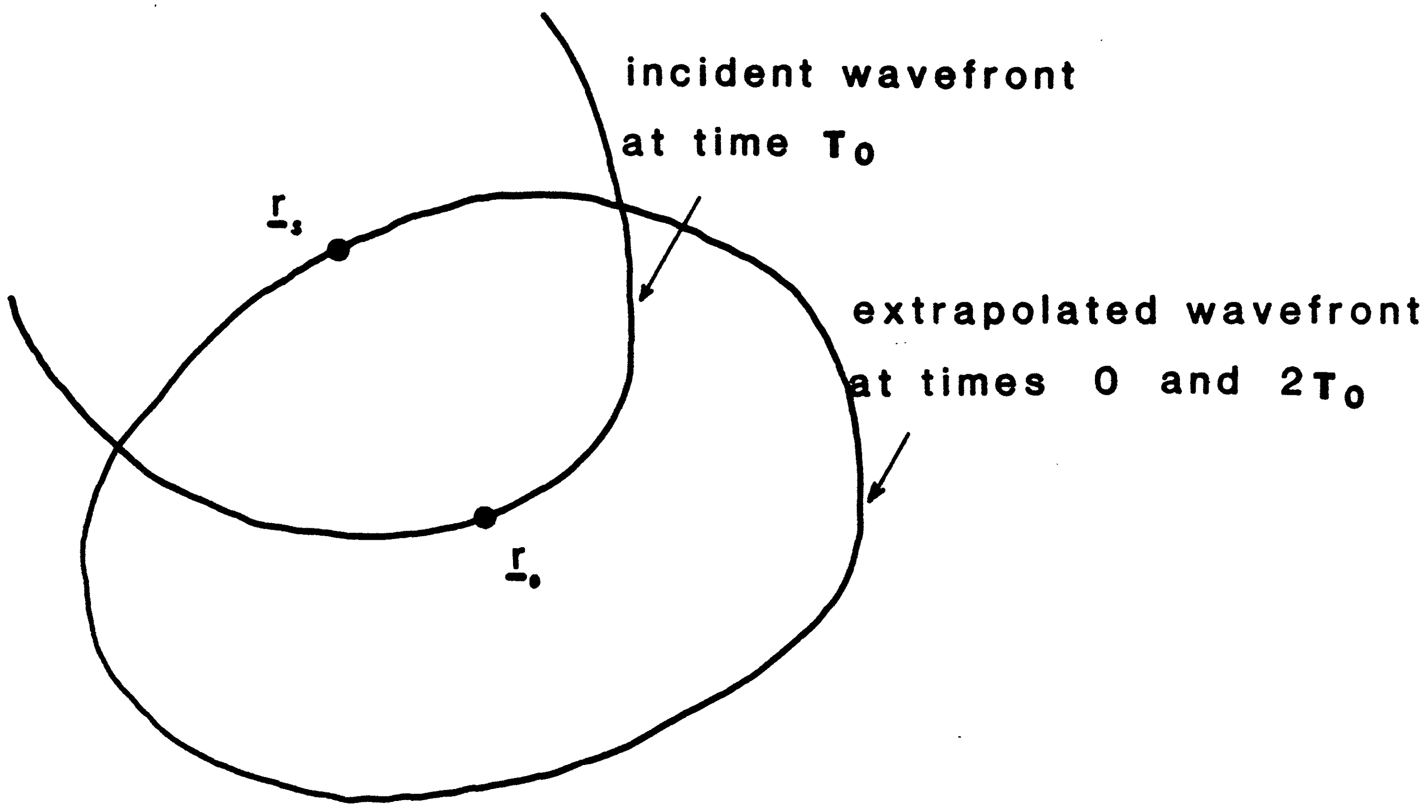
incident wavefront
at time T_0
$\underline{r}_s$
$\underline{r}_0$
extrapolated wavefront
at times 0 and $2T_0$

Figure 3.

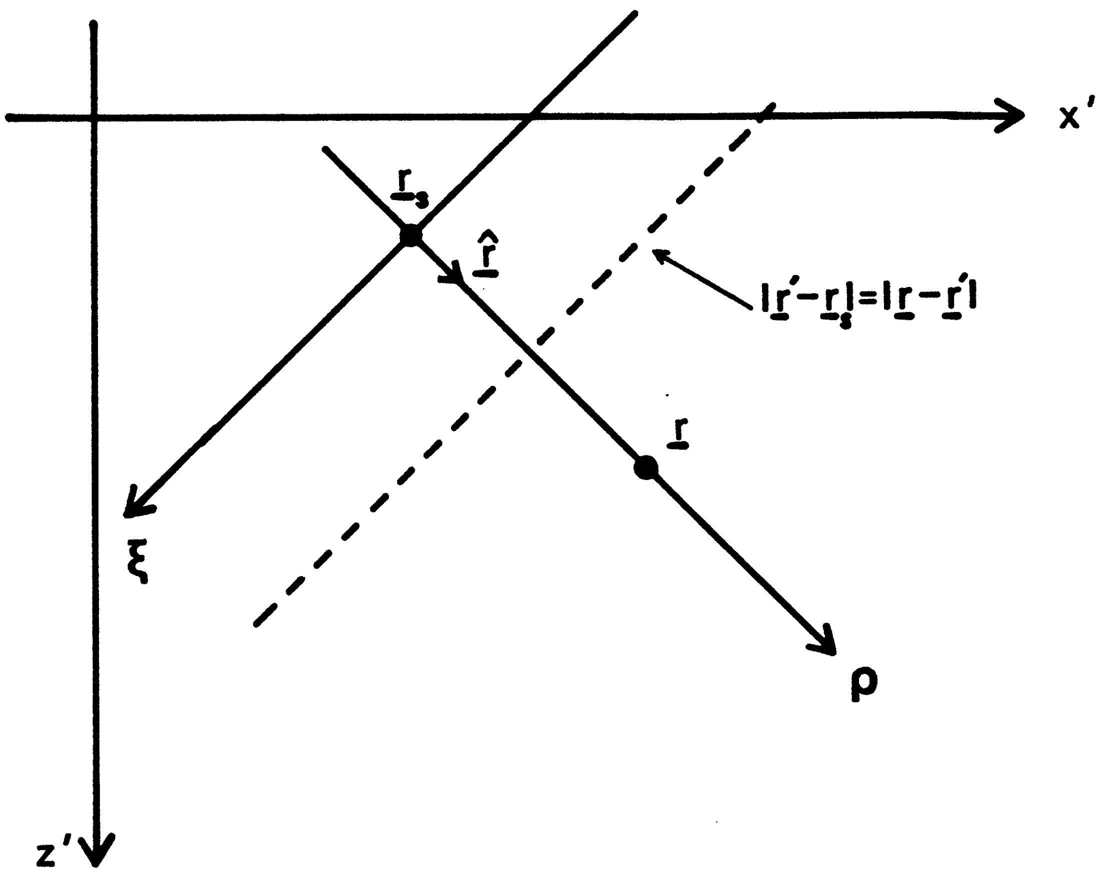
x'
$\underline{r}_s$
$\hat{\underline{r}}$
$|\underline{r}'-\underline{r}_s|=|\underline{r}-\underline{r}'|$
$\underline{r}$
ξ
ρ
z'

Figure 4.

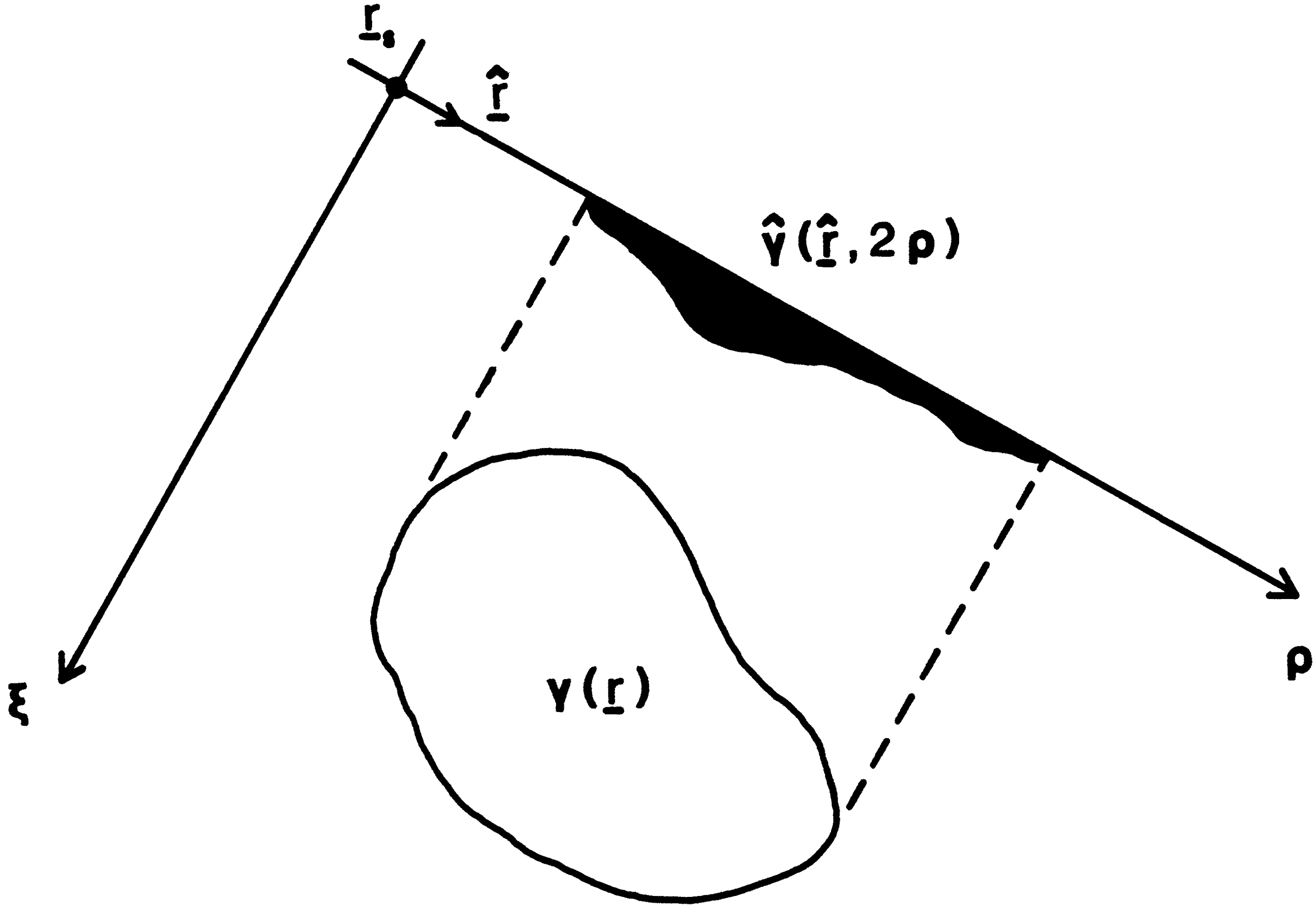
$\underline{r}_s$
$\hat{\underline{r}}$
$\hat{\gamma}(\hat{\underline{r}}, 2\rho)$
ξ
$\gamma(\underline{r})$
ρ

Figure Captions

Figure 1. Single source scattering experiment.

Figure 2. Extrapolated field of a point scatterer imaged at times zero and twice the source travel time.

Figure 3. Change of coordinates for a given source location and a point in the medium.

Figure 4. One projection of the velocity function.