



## Multidimensional deprivation: contrasting social welfare and counting approaches

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**Abstract.** Adoption of a multidimensional approach to deprivation poses the challenge of understanding the interaction between different dimensions. Are we concerned with the union of all those deprived on at least one dimension or with the intersection of those deprived on all dimensions? How does the approach of counting deprivations relate to approaches based on social welfare? The paper brings out key features of different approaches and sets them in a common framework.

**Key words:** deprivation, multidimensional, poverty.

### 1. Introduction: multidimensional deprivation

There is widespread agreement that deprivation is multidimensioned. It is not enough to look only at income poverty; we have also to look at other attributes. As Sen has put it, “the role of income and wealth... has to be integrated into a broader and fuller picture of success and deprivation” ([20], p. 20). But this poses the challenge as to how this integration should take place. How can the different attributes be aggregated? Here there is less agreement, and a number of questions arise. A distinction may be drawn between those who adopt a *union* approach and those who use an *intersection* measure (see Atkinson *et al.* [3], p. 73, and Duclos, Sahn and Younger [10]). Some people are concerned about those who have *either* low income *or* poor access to housing *or* a low level of education? Other people are concerned with those who have low income *and* poor housing access *and* a low level of education. The target adopted, for instance, by the Irish Government in its National Anti-Poverty Strategy considered those who were *both* below a relative income line *and* experiencing deprivation as measured by non-monetary indicators (see Layte, Nolan, and Whelan [15]).

How are the union and intersection approaches related to underlying concepts of social welfare and to multidimensional poverty measures (Tsui [23])? In particular, the social welfare approach adopted in a recent series of papers by Bourguignon and Chakravarty [4–6] has shown how conditions on the union and intersection can be linked to the assumed properties of the social welfare function. This approach is however rather different from the “counting” approach widely used in applied studies, which concentrate on counting the number of dimensions in which people suffer deprivation. In the Belgian reports on poverty by Vranken and colleagues,

for instance, people have scores corresponding to the number of dimensions on which they fall below the threshold. In the example given by Vranken [24], people score  $-3$  if they have low education, are inactive/unemployed, and in arrears on payments. People score  $-2$  if they are deprived on two of the three dimensions,  $-1$  for one deprivation and otherwise  $0$ . Can this counting approach be put in a similar analytical framework? How does it relate to the union/intersection distinction?

The aim of this paper is to bring out some key elements in these two approaches and to describe ways of moving forward based on recent contributions. Identifying directions of progress is particularly important, given that the European Union has recently adopted a common set of social indicators (Social Protection Committee [21], and Atkinson *et al.* [3]). At present, they deal with single dimensions, such as financial poverty, income inequality, long-term unemployment, living in a jobless household, low educational qualifications, and poor health. Each of these is given a separate score, but in time, people are likely to ask about the extent of overlap in deprivation. How far do countries differ in the extent of multiple deprivation? This in turn leads to the questions posed above concerning the definition of an aggregate measure. Section 2 of the paper sets out the social welfare function approach. This section is largely expository, describing the application of the results derived by Bourguignon and Chakravarty, but it also brings out the role played by the assumptions regarding cardinalisation (the degree of concavity of the social welfare function) and the weighting of different attributes. Section 3 departs further from the recent literature in that it seeks to understand how the counting approach can be expressed in the same framework. The final Section 4 summarises the main conclusions and the prospects for future application of these approaches.

In considering the aggregation of deprivation, we should distinguish two different forms of aggregation (see Micklewright [16]). The first – that described above – combines different elements of deprivation at the individual level, which are then summed over individuals to form an aggregate index for the country. The second sums across individuals first, to form a total indicator for all individuals in one dimension, and then combines the total indicators for different attributes. An example of the latter is provided by Anand and Sen [1], who recommend a human poverty index,  $P$ , based on three sub-indices,  $P_1$ ,  $P_2$  and  $P_3$ , where these relate to proportion expected to die before the age of 40, illiteracy, and economic deprivation. Anand and Sen propose using the weighted mean of order  $\beta$ :

$$P \equiv [w_1 P_1^\beta + w_2 P_2^\beta + w_3 P_3^\beta]^{1/\beta}, \quad (1)$$

where the weights sum to unity and  $\beta$  is a parameter. It may appear that the difference between the two forms of aggregation is simply an issue of the order of summation, but there is a substantive difference. If, as here, we are concerned about the way in which multiple deprivation impinges on the individual, the method of aggregation across attributes may well be different from the way in which we would combine sub-indices to arrive at a single ranking of countries. While we

shall later be applying at an individual level a function of the same form as (1), there is no reason why the values of  $\beta$  or the weights should be the same as when it is applied to combine sub-indices.

Much of the paper is concerned with the *dominance conditions*. These conditions are significant since there is likely to be differing views about the form of the deprivation measure. We may not be able to agree on a family of multivariate poverty measures, only on certain broad properties, or – if agreed on a family of measures – there may still be parameters about whose value there is disagreement (such as  $\beta$  in Equation (1)). Dominance conditions identify the circumstances under which we can make a statement of the form that “multidimensional deprivation in country A is lower than in country B” for all deprivation measures satisfying certain general properties. This approach only yields a partial ordering, just as Lorenz dominance cannot rank income distributions where Lorenz curves cross, and the extent of the ordering depends on the strength of the assumptions made about the form of the deprivation indicator. As in the literature on the measurement of inequality, the heart of the matter is the interplay between the nature of social judgments and the application of statistical criteria or numerical measures to distributional data.

## 2. A social welfare approach

The circumstances of an individual (or family or household) are assumed to be described by two attributes,  $x$  and  $y$ , which take on non-negative values. (For simplicity of exposition, I limit attention to the two-attribute case.) It should be noted that these attributes cannot be traded one for the other. It is for this reason that I referred earlier to “access to housing”, not to “housing”. If the only reason that a family is poorly housed is that they cannot afford better housing, then income (variable  $x$ ) is a sufficient indicator of deprivation. If, on the other hand, a family is prevented by discrimination from living in better housing, then the housing standard (variable  $y$ ) acquires an independent significance. Alternatively, a family may enjoy adequate housing at a subsidised or zero rent, but have an income below the poverty line. In these cases, a family may be deprived on one dimension but not the other.<sup>1</sup>

Where the aggregation is taking place at the level of the individual, it is natural to commence with the social welfare approach. By this I do not mean an adoption of utilitarianism. Obviously if individuals evaluate different attributes according to a utility function, say  $U(x, y)$  in two dimensions, and this encapsulates all our concerns, then the problem of aggregation ceases to exist. This is not the route followed here. Rather, I assume that  $x$  and  $y$  are the arguments in a social welfare function evaluating the position of an individual, just as in the literature on inequality measurement income is the observed variable entering the social welfare function. The social welfare function approach to multidimensional deprivation has been explored in a series of papers by Bourguignon and Chakravarty, who

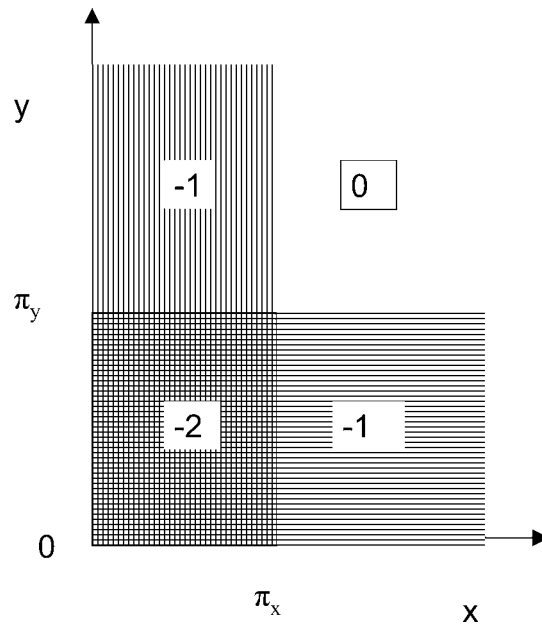


Figure 1. Deprivation in two dimensions.

have both described a family of multivariate poverty measures [5] and derived dominance conditions under which distributions can be ranked [6], and by Duclos, Sahn and Younger [10], who have derived dominance conditions and techniques of statistical inference.

I start with the dominance conditions, since, as these authors have shown, they lead us directly to the union/intersection distinction. The two-dimensional case is illustrated in Figure 1, where the deprivation threshold in dimension  $x$  is labelled as  $\pi_x$  and that in dimension  $y$  as  $\pi_y$ . We denote the cumulative distribution by  $F(x, y)$ , defined on non-negative values of  $x$  and  $y$ , and normalised to integrate to unity over the positive orthant. We denote the marginal distributions by  $F(x)$  and  $F(y)$ , and the density function by  $f(x, y)$ . We know that a proportion  $F(\pi_x)$  are deprived on the  $x$  dimension and a proportion  $F(\pi_y)$  on the  $y$  dimension. If we were to add these, we would double count the proportion  $F(\pi_x, \pi_y)$  who are deprived on both dimensions. The union is given by  $F(\pi_x) + F(\pi_y) - F(\pi_x, \pi_y)$ . But we may want to attach particular significance to the multiply deprived group.

As Bourguignon and Chakravarty [6] show, the relation between union and intersection measures can profitably be seen in terms of a correlation-increasing perturbation in the density function. An example of such a switch is given in Figure 2, where the density is either increased or reduced by a small amount  $\delta$  in such a way as to leave the marginal distributions unchanged, while increasing the correlation between  $x$  and  $y$ . The example in Figure 2 is chosen to show a case where this switch strictly *raises* the proportion falling in the intersection by  $\delta$ . But the proportion falling in the union is *reduced* by  $\delta$ . The parallel with multidimen-

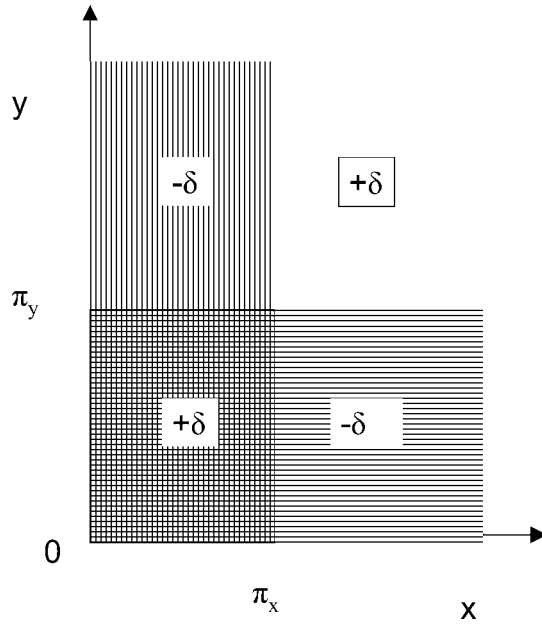


Figure 2. A correlation-increasing perturbation to the density function.

sional inequality measurement (Atkinson and Bourguignon [2]) suggests that the impact of such a correlation increasing perturbation in the density function depends on the form of the social welfare function. Suppose that we consider the class of deprivation measures,  $D$ , formed by integrating over the distribution a function  $p(x, y)$ , where this is zero when  $x$  and  $y$  are both above the poverty thresholds:

$$D \equiv \int_0^{\pi_x} \int_0^{\pi_y} p(x, y) f(x, y) dy dx. \tag{2}$$

From the standpoint of social welfare, deprivation is to be minimised. For this reason, when later graphing deprivation functions, I plot  $(-D)$  rather than  $D$ . The  $p$ -function is assumed to be non-increasing in  $x$  and  $y$ . (Placing a minus sign in front of  $p$  would give the non-decreasing relation with  $x$  and  $y$  associated with social welfare functions: an increase in income raises social welfare.) Bourguignon and Chakravarty show that a deprivation measure is increased, or remains the same, as a result of a correlation increasing perturbation if the cross-derivative of  $p$  with respect to  $x$  and  $y$  is positive (non-negative). They refer to this as the case of the attributes being *substitutes*, and draw a contrast with the case where the derivative is negative, referred to as *complements*. They are using these terms in the Auspitz–Lieben–Edgeworth–Pareto (ALEP) sense (parallel with the properties of the utility function in consumer theory) rather than the Hicks *Value and Capital* sense (of the properties of the indifference contours) – see Kannai [13]. In the present application, whether  $x$  and  $y$  are substitutes or complements is a matter of social judgment and may depend on the attributes in question.

The first-degree dominance conditions delineate the situations in which we can rank two distributions. In one dimension, first-degree dominance requires that the headcount in country A be lower (or no higher) than in country B for all possible deprivation thresholds up to and including that actually applied. In the two dimension case, we require (Bourguignon and Chakravarty [6], Propositions 1 and 2) *both* the same statement about the marginal distributions ( $F(x)$  be lower for all  $x$  up to  $\pi_x$  and  $F(y)$  be lower for all  $y$  up to  $\pi_y$ ) *and* a condition on the joint distribution, where this condition depends on the nature of the deprivation measure. For poverty measures that are *substitutes*, the dominance condition for the joint distribution involves the *intersection*: the value of  $F(x,y)$  must be lower in country A than in country B for  $(0 \leq x \leq \pi_x$  and  $0 \leq y \leq \pi_y)$ . For measures that are *complements*, the dominance condition for the joint distribution involves the *union*: the value of  $[F(x) + F(y) - F(x, y)]$  must be lower in country A than in country B for  $(0 \leq x \leq \pi_x$  and  $0 \leq y \leq \pi_y)$ . (This includes the conditions on the marginal distributions.) It should be noted that, in neither case, is it sufficient that the condition holds at the deprivation threshold: it must hold for all values of  $(x, y)$  below the threshold.

To be more concrete, suppose that  $x$  denotes income relative to the median, and the poverty line is set at 60%, and that  $y$  denotes rooms per person and the deprivation threshold is set at 1. If the multidimensional deprivation measure ( $p$ -function) satisfies the substitutes condition, but we know nothing more about its properties, then for country A to have less multiple deprivation, it must be the case that there is a smaller proportion who both have income below 60% and less than 1 room per person. We also require that there be a lower proportion with less than 60% of median income and less than 0.9 rooms per person, less than 0.8 rooms per person and so on. And we require that there be a lower proportion with less than 50% of the median, and less than 1 room per person, 0.9 rooms per person, and so on. In contrast, if the multidimensional deprivation measure ( $p$ -function) satisfies the complementarity condition, the requirements are different. We require, for example, taking the same point as before, that the proportion is lower of people with income less than 60% of the median added to people with income greater than or equal to 60% but less than 1 room per person. It is a different condition. The empirical importance of the distinction between the two conditions depends on the degree of correlation. If everyone is located along the diagonal passing through the thresholds, then moving to a multidimensional measure adds nothing to the comparisons based just on financial poverty.

The dominance conditions described above are first-degree conditions. They are relatively demanding, and we may seek to weaken them by moving to second, or higher, degree conditions.<sup>2</sup> These however involve third and fourth (or higher) derivatives of the function  $p(x, y)$ . As shown by Duclos, Sahn and Younger [10], Theorem 2 moving to the second-degree in  $x$  involves the cross-derivative being decreasing in  $x$ . It is not easy to form an intuition about such higher derivatives, and for this reason it is natural to turn to specific functional forms.

## FAMILY OF MEASURES

To illustrate these results, Bourguignon and Chakravarty take the weighted mean of order  $\beta$ , the analogue at the individual level of the Anand–Sen [1] combination of aggregate sub-indices. If we write the relative shortfalls as

$$g_x \equiv \max[0, (1 - x/\pi_x)] \quad \text{and} \quad g_y \equiv \max[0, (1 - y/\pi_y)] \quad (3)$$

then we can define the deprivation indicator

$$p(x, y) \equiv [g_x^\beta + b g_y^\beta]^{\alpha/\beta}. \quad (4)$$

In (4) there are three non-negative constants  $\alpha$ ,  $\beta$  and  $b$  ( $b^{\beta/\alpha}$  in Bourguignon and Chakravarty [5]). The parameter  $\alpha$  is a measure of the concavity of the function  $-p(x, y)$ , again viewed from a social welfare standpoint, and  $\alpha = 1$  is the least concave representation. (The indicator embodies homothetic preferences over shortfalls, so that the least concave representation is linear homogeneous – see Kihlstrom and Mirman [14], p. 272.)

The parameter  $\beta$  alone governs the shape of the contours in  $(x, y)$  space, as illustrated in Figure 3. It is important to note, as is stressed by Bourguignon and Chakravarty [5], p. 12, that the function exhibits constant elasticity *with respect to the shortfalls*, not  $x$  and  $y$ . The contours are centred (for  $b = 1$ ) on  $(\pi_x, \pi_y)$ , not on the origin, and are convex with respect to that centre. This explains why the elasticity of substitution is defined with the opposite of the sign usual for isoquants, as  $1/(\beta - 1)$ . Where  $\beta = 1$  we have a straight line, with infinite elasticity; with  $\beta = 2$  we have quarter circles, and as  $\beta$  tends to infinity the contours approach a

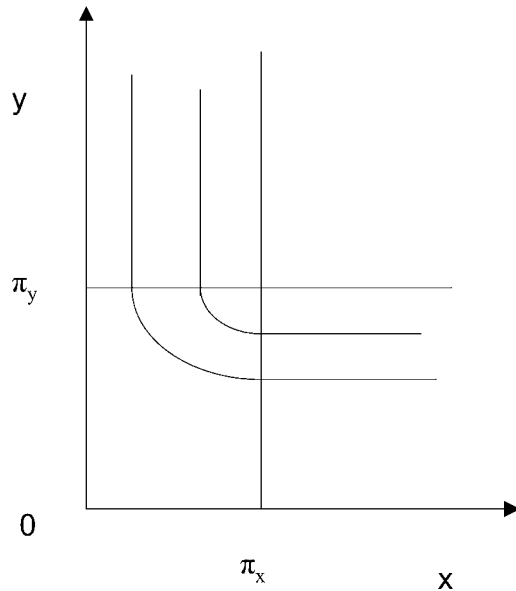


Figure 3. Iso-deprivation contours.

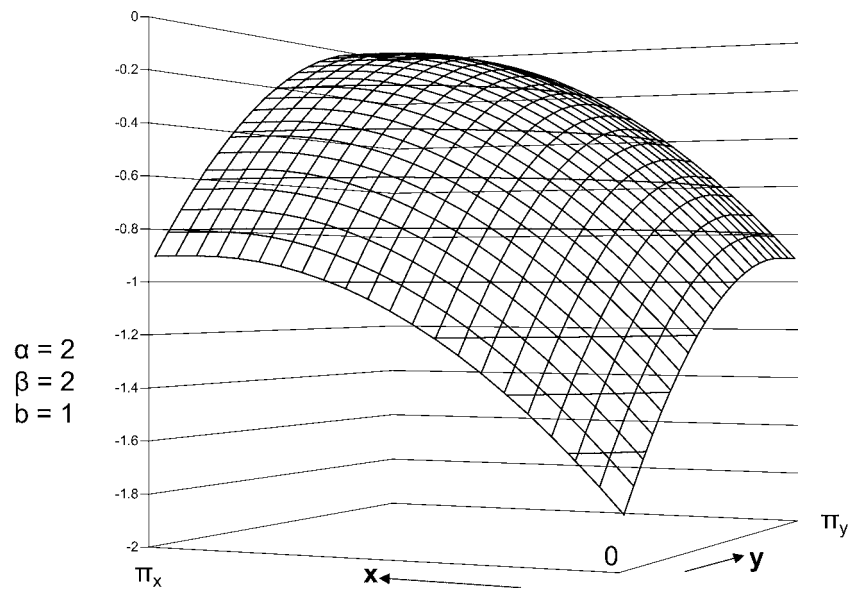


Figure 4. Shape of social welfare surface.

right angle. It should be re-emphasised that changes in  $\alpha$  do not alter the shape of the social indifference curves between  $x$  and  $y$ ; changes in  $\alpha$  affect only the social welfare numbering of these contours. The combined effect of  $\alpha$  and  $\beta$  is illustrated in Figure 4, which shows the shape of the social welfare surface within the box defined by the deprivation thresholds. Here, and in the subsequent graphs, I place a minus sign in front of the poverty measure, so that moving upwards represents an improvement, as with a social welfare function.

In the construction of Figure 4, the parameter  $b$  is taken as equal to 1, but we may wish to weight the attributes differentially. In our report on social indicators, Atkinson *et al.* [3], p. 25, we were concerned that the weight of single attributes should be *proportionate*. We felt that it would be difficult to make sense of an aggregate indicator that combined measures of central importance, such as national poverty rates, with indicators of a more specialised nature or which refer to subsets of the population (such as those in a particular age group). The parameter  $b$  is in a sense answer: a measure only affecting half the population could be given, for example, half the weight of a national measure. In their discussion of aggregation, Desai and Shah [9] distinguish between objective and subjective weights. The latter correspond to individual feelings of deprivation, and they suggest that these feelings depend, negatively, on the proportion of the community who are deprived. I do not consider this approach here, but note that it requires an extension of the results described, since the weights would then be functions of the distribution. The choice of weights for different attributes has been much discussed: see, for example, Brandolini and D'Alessio [7]. As they observe, there is considerable room for disagreement, even where they are fixed across samples. They cite the



proposal of Sen ([19], p. 30) that a range of weights be considered. The dominance results cited earlier in essence allow  $b$  to range from 0 to infinity. The marginal conditions effectively allow each attribute a veto.

The cross-derivative of  $p$  is positive where  $\alpha > \beta$ . The interpretation in terms of  $x$  and  $y$  then being substitutes is illuminating, but has to be fully understood. As the condition makes clear, it is not simply a property of the contours (which depend only on  $\beta$ ). It depends on the cardinalisation. (This is why it is described as being of the ALEP rather than of the Hicksian variety.) Where  $\beta$  is greater than 1, the least concave representation ( $\alpha = 1$ ) cannot satisfy the substitutability condition, so that  $x$  and  $y$  are complements, in the sense defined above, and the relevant dominance condition is of the union form. But by taking a sufficiently concave representation of the poverty indicator ( $\alpha$  sufficiently large that it exceeds  $\beta$ ), we can ensure that the relevant dominance condition is of the intersection form. This is geometrically apparent from considering the perturbation shown in Figure 2. While the two  $(-\delta)$  elements may lie on a contour, and the top right-hand element  $(+\delta)$  has no impact, the weight attached to the bottom left  $(+\delta)$  depends on the numbering of the contours. If a sufficiently large negative weight is given to the bottom left element, its effect on social welfare will dominate.

What this highlights is that the key ingredients are both the shape of the contours (new in the multidimensional case) and the degree of concavity. The latter was already present in the one dimension case, and there is not necessarily any reason to change our views about the value of  $\alpha$  simply because we have moved to a higher dimensionality. Figure 5 illustrate different degrees of concavity in the one dimension case, where, as noted earlier, we place a minus sign in front of the poverty measure, so that moving upwards represents an improvement. The case  $\alpha = 1$  corresponds to the poverty gap, and higher values to other measures in the Foster–Greer–Thorbecke [12] class. (These are the solid lines; the dashed curves are discussed in the next section.) We may note that these higher values are not

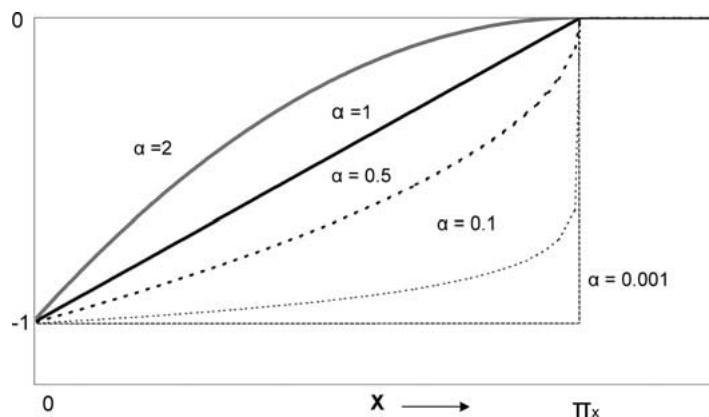


Figure 5. Shortfall in one dimension to power of alpha.

often used in policy analysis, and even the poverty gap is less common than the poverty headcount, to which I turn in the next section.

### 3. Counting deprivations

Empirical studies of multiple deprivation to date have not typically adopted a social welfare function approach. Rather they have tended to concentrate on counting the number of dimensions in which people suffer deprivation. Just as a headcount measure treats poverty as a zero/one condition, so deprivation in two dimensions may represent a discretely different level. In terms of the low income and housing example, we are concerned with the number of people with income below 60% of the median, not distinguishing the extent to which they fall below, the number with less than 1 room per person, again not distinguishing the extent of the shortfall, and the number deprived on both attributes. This is illustrated in Figure 1, where  $-1$  indicates deprivation in one dimension, and deprivation in both dimensions is indicated as  $-2$ . I have again adopted the convention that deprivation is negative, so that an increase in the score denotes an improvement.

Such a counting approach is widely used. In his pioneering study of poverty in the United Kingdom, Townsend [22] constructed a deprivation index that was the number of characteristics (12 in total) on which a person was deprived. In studies based on the Swedish Level of Living Surveys, a count was made of the number of components on which an individual had problematic conditions (Erikson [11], p. 70). In the ESRI studies of poverty in Ireland (for example, Callan *et al.* [8]), scores are calculated for the enforced lack of 23 items. In the last case, the variables are binary, but this is not true of the underlying variables in all studies.

How can this counting approach be related to the welfare approach described in the previous section? (I continue to assume that  $x$  and  $y$  are continuous variables; as noted by Brandolini and D'Alessio [7], where the attributes are binary in form, the measures become simpler.) The headcount provides a useful point of entry. In the one dimension case, we can write the indicator of individual status as

$$i(x) \equiv [\max(0, (1 - x/\pi_x))]^\alpha. \quad (5)$$

For positive  $\alpha$  and  $x$ , the value is less than 1, but by taking  $\alpha$  closer and closer to zero, we can approximate the headcount:  $i$  taking the value 1 for all  $x$  below the poverty line. Dashed lines in Figure 5 (which plots  $-i(x)$ ) show values of  $\alpha$  less than 1. As this brings out, the headcount is ill behaved. By this, I do not mean that it is a discontinuous function; this is not important, since we can regard it as the limit of a continuous function. The problem is that measures with  $\alpha$  less than 1 are not concave. As is well known (Sen [18]), the principle of transfers is not satisfied. A transfer from a person just above the poverty line to a person well below the poverty line is equalising but could *raise* the headcount if it took the donor below the poverty line. The headcount does not therefore fit well with the welfare economic approach widely adopted in the measurement of inequality.

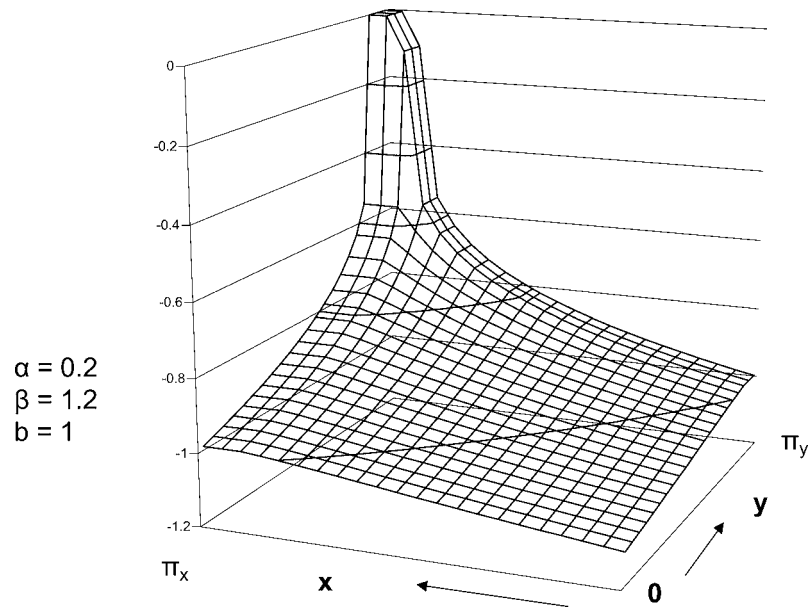


Figure 6. Weighting deprivation in two dimensions.

The same kind of consideration carries over to the two-dimensional case, but it does not simply involve the parameter  $\alpha$ . Suppose that we try to approximate by a continuous function, of the constant elasticity type (4) with  $b = 1$ , the stepped values of the counting measure. Figure 6 shows that a low value of  $\alpha$  gives the right shape in one dimension, but that, even with a value of  $\beta$  close to 1, the normal shaped contours are inappropriate. What is needed, as illustrated by Figure 7, is not only  $\alpha$  but also  $\beta$  approaching zero, with contours in  $(x, y)$  space that are convex to the deprivation threshold. In the two-dimensioned case,  $\beta$  less than 1 means that the contours are concave to the origin: the reverse of those drawn by Bourguignon and Chakravarty [4]. A linear combination of  $x$  and  $y$  generates a lower value of the index.

People may agree on the counting approach but differ with regard to the scaling. Suppose that we consider the limit as  $\alpha$  and  $\beta$  tend to zero with  $\alpha/\beta = k$ , and continue to assume  $b = 1$ . Then we obtain the generalised counting measure equal to 1 where *either*  $x$  is below the threshold but  $y$  is above, *or* where  $y$  is below the threshold and  $x$  is above, and is equal to  $2^k$  where both are below. People may disagree about the value of  $k$ . The scale could be linear ( $k = 1$ ): i.e. that 2 people with one deprivation count the same as 1 person with 2 deprivations. This case of counting deprivations is a useful benchmark, corresponding to the case where the cross-derivative is zero, but  $k$  could vary from zero (giving no weight to multiple deprivation, counting each deprived person once) to infinity (giving all the weight to multiple deprivation).

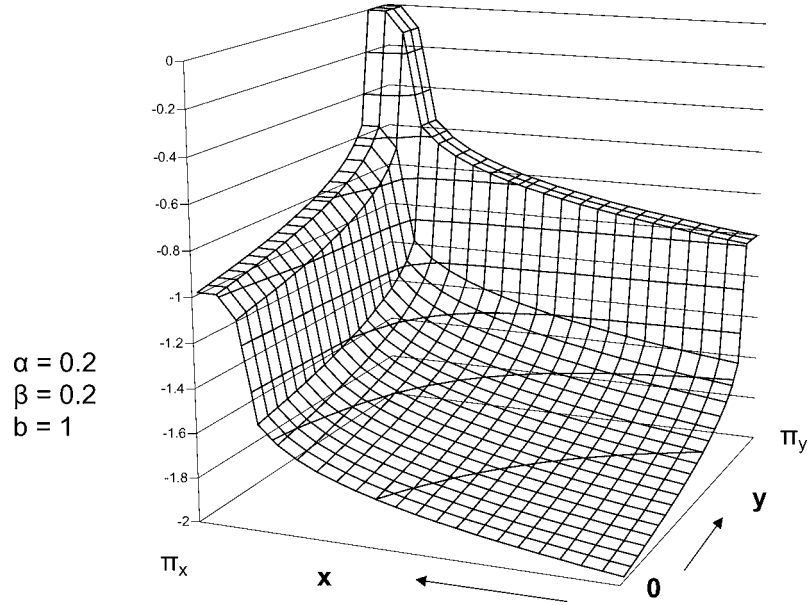


Figure 7. Weighting deprivation in two dimensions with  $\beta$  less than 1.

Under what conditions can we say that the distribution in country A dominates that in country B for different values of  $k$ ? The answer may be seen from the value of the deprivation indicator:

$$D = F(\pi_x) + F(\pi_y) + 2(2^{k-1} - 1)F(\pi_x, \pi_y). \quad (6)$$

At one extreme, for  $k$  sufficiently large, the final term dominates any comparison, and we require the intersection condition evaluated at the thresholds:  $F(\pi_x, \pi_y)$  be no larger in country A. This applies where all the weight is attached to multiple deprivation. At the other extreme, as  $k$  approaches zero, we compare the union, again evaluated at the thresholds. In this case, no additional weight is attached to multiple deprivation. In between, a weight of  $k = 1$  means that we count deprivations and the dominance condition concerns the sum  $F(\pi_x) + F(\pi_y)$ .

If therefore we allow  $k$  to vary from 0 to infinity, necessary conditions involve both the union and intersection conditions outlined in Section 2. With the counting approach, we cannot necessarily conclude that a correlation-increasing switch increases (or reduces) multiple deprivation. (In the case where  $k = 1$  and we look at the sum of the margins, then the switch has no effect.) The reason can be understood from the fact that it is possible to approximate the counting measure with  $\alpha/\beta$  either greater or less than 1. At the same time, it should be noted that we only require the conditions to hold *at* the deprivation threshold, whereas the previous conditions have to hold at all  $(x, y)$  up to the threshold. Moreover, if we can agree that  $k$  is at least 1, then it is sufficient that (a) the sum  $F(\pi_x) + F(\pi_y)$  be lower, and (b) that the intersection be lower. Taking our earlier example, with

the poverty line set at 60%, and the threshold for rooms per person set at 1, for country A to have less multiple deprivation, for dominance with  $k$  greater than or equal to 1 we require (a) that there be smaller (no higher) sum of the proportion with income below 60% and the proportion with less than 1 room and (b) that there be a smaller (no higher) proportion who both have income below 60% and less than 1 room per person. How the households are distributed below 60% of the median or below 1 room per person is not germane.

As in the welfare approach, we may seek to weaken the dominance conditions. However, not only does this require us to consider higher derivatives, but we know from the one-dimensional case that the ill-behaved nature of this kind of counting measure is a barrier to deriving second-degree dominance results. (The principle of transfers is the key to moving from first-degree to second-degree dominance in the one dimensional case.)

#### 4. Conclusion

There is wide agreement that we need a multidimensional approach to deprivation, but implementation of this approach poses a number of conceptual problems (in addition to the empirical issues not addressed here). Can these be resolved through the line of attack described here?

A pessimist might conclude that the dominance method offers little to the analysis of multiple deprivation. On a social welfare approach the analogue of first-degree dominance conditions depends on whether the attributes are complements or substitutes, and this depends in turn on the cardinalisation (i.e. not just on the shape of the indifference curves). Even if we could agree on either the union or intersection conditions, they remain strong requirements and may well not yield definite conclusions in many cases. On a counting approach, both union and intersection conditions may be necessary if people adopt very different approaches to scaling. Generalising the conditions to second-degree involves assumptions about third and higher derivatives. On the social welfare approach, these are hard to interpret, and on the counting approach they are unlikely to be satisfied.

An optimist, on the other hand, might conclude that we have learned about the relation between dominance conditions and the underlying assumptions about the social judgments. If, for example, we can agree that the constant elasticity measure is flexible enough for our purposes, then those who feel that the poverty gap is sufficiently concave can deduce that the relevant dominance condition is that on the union of deprivations. Those, on the other hand, who prefer a more concave transformation may find the intersection condition relevant. Those who favour a counting approach may be able to agree that the weight given to multiple deprivation should involve at least counting deprivations, in which case the conditions are weaker. The dominance conditions even when not satisfied allow us to locate the disagreements that are crucial. The analysis of the paper has allowed us to place

in a common framework two approaches apparently at variance, and this serves to identify the key differences in underlying judgments.

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### Notes

<sup>1</sup> For a valuable discussion of the role of “non-income” indicators, see Ravallion [17], who identifies three sets of indicators apart from poverty measures: access to non-market goods, indicators of distribution within households, and certain limiting personal characteristics.

<sup>2</sup> Higher order dominance results for multi-dimensional distributions are discussed by Atkinson and Bourguignon [2], including the relation between the conditions for second-degree dominance and the incomplete covariance.

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