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MULTIDIMENSIONAL FIXED POINT RESULTS FOR CONTRACTION MAPPING PRINCIPLE WITH APPLICATION

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Abstract. The main aim of this article is to study the existence and uniqueness of fixed point for isotone mappings of any number of arguments under contraction mapping principle on a complete metric space endowed with a partial order. As an application of our result, we have studied the existence and uniqueness of the solution to an integral equation. The results we have obtaied will generalize, extend and unify several classical and very recent related results in the literature in metric spaces.
Keywords: fixed point; contraction mapping principle; partially ordered metric space; non-decreasing mapping; integral equation.

1. Introduction

The Banach contraction principle is one of the most popular tools in solving the existence in many problems of mathematical analysis. Due to its simplicity and usefulness, there are a lot of generalizations of this principle in the literature. Ran and Reurings [14] extended the Banach contraction principle in partially ordered sets with some applications to linear and nonlinear matrix equations. While Nieto and Rodriguez-Lopez [12] extended the result of Ran and Reurings [14] and applied their main results to obtain a unique solution for a first-order ordinary differential equation with periodic boundary conditions.

The concept of multidimensional fixed point was introduced by Roldan et al. in [16], which is an extension of Berzig and Samet's notion given in [2], which extended and generalized the mentioned fixed point results to higher dimensions. However, they used permutations of variables and distinguished between the first and the last variables. For more details one can consult ([3] [4], [5], [6], [7], [8], [9], [10], [11],[13], [16], [17], [18], [19], [20], [21], [22]).

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In this article, we have studied the existence and uniqueness of fixed point for isotone mappings of any number of arguments under contraction mapping principle on a complete metric space endowed with a partial order. As an application of our result we study the existence and uniqueness of the solution to an integral equation. We improve and generalize the results of Alsulami [1], Razani and Parvaneh [15], Su [20] and many other famous results in the literature.

2. Preliminaries

In order to establish our main results, we will use the following notions. If X is a non-empty set, then we denote $X \times X \times ... \times X$ (n times) by X^n, where $n \in \mathbb{N}$ with $n \geq 2$. If elements x, y of a partially ordered set $(X, \preceq)$ are comparable (that is $x \preceq y$ or $y \preceq x$ holds), then we will write $x \asymp y$. Let $\{A, B\}$ be a partition of the set $\Lambda_n = \{1, 2, ..., n\}$, that is, A and B are non-empty subsets of Λ_n such that $A \cup B = \Lambda_n$ and $A \cap B = \emptyset$. We will denote

$$\begin{aligned} \Omega_{A,\, B} &= \{\sigma : \Lambda_n \to \Lambda_n : \sigma(A) \subseteq A,\ \sigma(B) \subseteq B\}, \\ \text{and } \Omega'_{A,\, B} &= \{\sigma : \Lambda_n \to \Lambda_n : \sigma(A) \subseteq B,\ \sigma(B) \subseteq A\}. \end{aligned}$$

Henceforth, let $\sigma_1, \sigma_2, ..., \sigma_n$ be n mappings from Λ_n into itself and let Υ be the $n-$tuple $(\sigma_1, \sigma_2, ..., \sigma_n)$. Let $F : X^n \to X$ and $g : X \to X$ be two mappings. For brevity, $g(x)$ will be denoted by gx.

A partial order $\preceq$ on X can be extended to a partial order $\sqsubseteq$ on X^n in the following way. If $(X, \preceq)$ be a partially ordered space, $x, y \in X$ and $i \in \Lambda_n$, we will use the following notations:

$$x \preceq_i y \Rightarrow \begin{cases} x \preceq y, & \text{if } i \in A, \\ x \succeq y, & \text{if } i \in B. \end{cases} \tag{2.1}$$

Consider on the product space X^n the following partial order: for $Y = (y_1, y_2, ..., y_i, ..., y_n)$, $V = (v_1, v_2, ..., v_i, ..., v_n) \in X^n$,

$$Y \sqsubseteq V \Leftrightarrow y_i \preceq_i v_i. \tag{2.2}$$

We say that two points Y and V are comparable, if $Y \sqsubseteq V$ or $V \sqsubseteq Y$. Obviously, $(X^n, \sqsubseteq)$ is a partially ordered set.

Definition 2.1. ([10], [16], [18]). A point $(x_1, x_2, ..., x_n) \in X^n$ is called a $\Upsilon-$fixed point of the mapping $F : X^n \to X$ if

$$F(x_{\sigma_i(1)},\ x_{\sigma_i(2)},\ ...,\ x_{\sigma_i(n)}) = x_i, \text{ for all } i \in \Lambda_n.$$

This definition extends the notions of coupled, tripled, and quadruple fixed points. In fact, if we represent a mapping $\sigma : \Lambda_n \to \Lambda_n$ throughout its ordered image, that is, $\sigma = (\sigma(1), \sigma\ (2), ..., \sigma\ (n))$, then

(i) Gnana-Bhaskar and Lakshmikantham's coupled fixed points occur when $n = 2$, $\sigma_1 = (1, 2)$ and $\sigma_2 = (2, 1)$,

(ii) Berinde and Borcut's tripled fixed points are associated with $n = 3$, $\sigma_1 = (1, 2, 3)$, $\sigma_2 = (2, 1, 2)$ and $\sigma_3 = (3, 2, 1)$,

(iii) Karapinar's quadruple fixed points are considered when $n = 4$, $\sigma_1 = (1, 2, 3, 4)$, $\sigma_2 = (2, 3, 4, 1)$, $\sigma_3 = (3, 4, 1, 2)$ and $\sigma_4 = (4, 1, 2, 3)$.

These cases consider A as the odd numbers in $\{1, 2, ..., n\}$ and B as its even numbers. However, Berzig and Samet [2] use $A = \{1, 2, ..., m\}$, $B = \{m+1, ..., n\}$ and arbitrary mappings."

Definition 2.2. [16]. Let $(X, \preceq)$ be a partially ordered space. We say that F has the mixed monotone property if F is monotone non-decreasing in arguments of A and monotone non-increasing in arguments of B, that is, for all $x_1, x_2, ..., x_n, y, z \in X$ and all i

$$y \preceq z \Rightarrow F(x_1,\ ...,\ x_{i-1},\ y,\ x_{i+1},\ ...,\ x_n) \preceq_i F(x_1,\ ...,\ x_{i-1},\ z,\ x_{i+1},\ ...,\ x_n).$$

Definition 2.3. ([18], [21]). Let (X, d) be a metric space and define $\Delta_n, \rho_n : X^n \times X^n \to [0, +\infty)$, for $Y = (y_1, y_2, ..., y_n)$, $V = (v_1, v_2, ..., v_n) \in X^n$, by

$$\Delta_n(Y,\ V) = \frac{1}{n}\sum_{i=1}^{n} d(y_i,\ v_i) \text{ and } \rho_n(Y,\ V) = \max_{1\le i\le n} d(y_i,\ v_i).$$

Then Δ_n and ρ_n are metric on X^n and (X, d) is complete if and only if (X^n, Δ_n) and (X^n, ρ_n) are complete. It is easy to see that

$$\begin{aligned} \Delta_n(Y^k,\ Y) &\to 0 \Leftrightarrow d(y_i^k,\ y_i) \to 0 \text{ (as } k\to\infty) \\ \text{and } \rho_n(Y^k,\ Y) &\to 0 \Leftrightarrow d(y_i^k,\ y_i) \to 0 \text{ (as } k\to\infty),\ i \in \Lambda_n, \end{aligned}$$

where $Y^k = (y_1^k, y_2^k, ..., y_n^k)$ and $Y = (y_1, y_2, ..., y_n) \in X^n$.

Lemma 2.1. *([18], [21], [22]). Let $(X, d, \preceq)$ be an ordered metric space and let $F : X^n \to X$ and $g : X \to X$ be two mappings. Let $\Upsilon = (\sigma_1, \sigma_2, ..., \sigma_n)$ be an $n-$tuple of mappings from Λ_n into itself verifying $\sigma_i \in \Omega_{A,B}$ if $i \in A$ and $\sigma_i \in \Omega'_{A,B}$ if $i \in B$. Define $F_\Upsilon, G : X^n \to X^n$, for all $y_1, y_2, ..., y_n \in X$, by*

$$F_\Upsilon(y_1,\ y_2,\ ...,\ y_n) = \begin{pmatrix} F(y_{\sigma_1(1)},\ y_{\sigma_1(2)},\ ...,\ y_{\sigma_1(n)}), \\ F(y_{\sigma_2(1)},\ y_{\sigma_2(2)},\ ...,\ y_{\sigma_2(n)}), \\ ...,\ F(y_{\sigma_n(1)},\ y_{\sigma_n(2)},\ ...,\ y_{\sigma_n(n)}) \end{pmatrix}, \tag{2.3}$$

and

$$G(y_1,\ y_2,\ ...,\ y_n) = (gy_1,\ gy_2,\ ...,\ gy_n). \tag{2.4}$$

(1) *If F has the mixed $(g, \preceq)-$monotone property, then F_Υ is monotone $(G, \sqsubseteq)-$non-decreasing.*

(2) If F is $d-$continuous, then F_Υ is also Δ_n-continuous and ρ_n-continuous.

(3) If g is $d-$continuous, then G is Δ_n-continuous and ρ_n-continuous.

(4) A point $(y_1, y_2, ..., y_n) \in X^n$ is a $\Upsilon-$fixed point of F if and only if $(y_1, y_2, ..., y_n)$ is a fixed point of F_Υ.

(5) A point $(y_1, y_2, ..., y_n) \in X^n$ is a $\Upsilon-$coincidence point of F and g if and only if $(y_1, y_2, ..., y_n)$ is a coincidence point of F_Υ and G.

(6) If $(X, d, \preceq)$ is regular, then $(X^n, \Delta_n, \sqsubseteq)$ and $(X^n, \rho_n, \sqsubseteq)$ are also regular.

Lemma 2.2. *[8]. Let $(X, d, \preceq)$ be a partially ordered metric space and let $F : X^n \to X$ be a mapping. Then*

(a) If there exists $y_0^1, y_0^2, ..., y_0^n \in X$ verifying $y_0^i \preceq_i F(y_0^{\sigma_i(1)}, y_0^{\sigma_i(2)}, ..., y_0^{\sigma_i(n)})$, for $i \in \Lambda_n$, then there exists $Y_0 \in X^n$ such that $Y_0 \sqsubseteq F_\Upsilon(Y_0)$.

(b) If F is a mixed monotone mapping, then F_Υ is an isotone mapping.

(c) If for each $i \in \Lambda_n$ and $y_i, v_i \in X$ there exists $z_i \in X$ which is $\preceq_i -$comparable to y_i and v_i, then there exists $Z \in X^n$ which is $\sqsubseteq -$comparable to Y and V.

Definition 2.4. [20]. A generalized altering distance function is a function $\psi : [0, +\infty) \to [0, +\infty)$ which satisfied the following conditions:

(i_ψ) ψ is non-decreasing,

(ii_ψ) $\psi(t) = 0$ if and only if $t = 0$.

3. Main results

Theorem 3.1. *Let $(X, \preceq)$ be a partially ordered set and suppose that there is a metric d on X such that (X, d) is a complete metric space. Let $T : X \to X$ be a non-decreasing mapping for which there exist a generalized altering distance function ψ and a right upper semi-continuous function $\varphi : [0, +\infty) \to [0, +\infty)$ such that*

$$\psi(d(Tx,\ Ty)) \leq \varphi(d(x,\ y)), \tag{3.1}$$

for all $x, y \in X$ with $x \preceq y$, where $\psi(t) > \varphi(t)$ for all $t > 0$ and $\varphi(0) = 0$. Suppose either

(a) T is continuous or

(b) $(X, d, \preceq)$ is regular.

If there exists $x_0 \in X$ such that $x_0 \asymp Tx_0$, then T has a fixed point. Moreover, if for each $x, y \in X$ there exists $z \in X$ which is $\preceq -$comparable to x and y, then the fixed point is unique.

We omit the proof of the previous result since its proof is similar to the main theorem in [20].

Put $\psi(t) = t$ and $\varphi(t) = kt$ with $k < 1$ in Theorem 3.1, we get the following result:

Corollary 3.1. *Let $(X, \preceq)$ be a partially ordered set and suppose that there is a metric d on X such that (X, d) is a complete metric space. Let $T : X \to X$ be a non-decreasing mapping such that*

$$d(Tx,\ Ty) \leq kd(x,\ y),$$

for all x, $y \in X$ with $x \preceq y$, where $k < 1$. Suppose either

(a) T is continuous or

(b) $(X, d, \preceq)$ is regular.

If there exists $x_0 \in X$ such that $x_0 \asymp Tx_0$, then T has a fixed point. Moreover, if for each x, $y \in X$ there exists $z \in X$ which is $\preceq -$comparable to x and y, then the fixed point is unique.

Next we give an $n-$dimensional fixed point theorem for mixed monotone mappings. For brevity, $(y_1, y_2, ..., y_n)$, $(v_1, v_2, ..., v_n)$ and $(y_0^1, y_0^2, ..., y_0^n)$ will be denoted by Y, V and Y_0 respectively.

Theorem 3.2. *Let $(X, \preceq)$ be a partially ordered set and suppose that there is a metric d on X such that (X, d) is a complete metric space. Let $\Upsilon = (\sigma_1, \sigma_2, ..., \sigma_n)$ be an $n-$tuple of mappings from Λ_n into itself verifying $\sigma_i \in \Omega_{A,B}$ if $i \in A$ and $\sigma_i \in \Omega'_{A,B}$ if $i \in B$. Let $F : X^n \to X$ be a mixed monotone mapping for which there exist a generalized altering distance function ψ and a right upper semi-continuous function $\varphi : [0, +\infty) \to [0, +\infty)$ satisfying*

$$\psi(d(F(y_1,\ y_2,\ ...,\ y_n),\ F(v_1,\ v_2,\ ...,\ v_n))) \leq \varphi\left(\max_{1\leq i\leq n} d(y_i,\ v_i)\right), \tag{3.2}$$

for which y_i, $v_i \in X$ such that $y_i \preceq_i v_i$ for all $i \in \Lambda_n$, where $\psi(t) > \varphi(t)$ for all $t > 0$ and $\varphi(0) = 0$. Also, suppose that either F is continuous or $(X, d, \preceq)$ is regular. If there exists y_0^1, y_0^2, ..., $y_0^n \in X$ such that

$$y_0^i \preceq_i F(y_0^{\sigma_i(1)},\ y_0^{\sigma_i(2)},\ ...,\ y_0^{\sigma_i(n)}),\ \textit{for } i \in \Lambda_n.$$

Then F has a $\Upsilon-$fixed point. Moreover, if for each $i \in \Lambda_n$ and y_i, $v_i \in X$ there exists $z_i \in X$ which is $\preceq_i -$comparable to y_i and v_i. Then F has a unique $\Upsilon-$fixed point.

Proof. For fixed $i \in A$, we have $y_{\sigma_i(t)} \preceq_t v_{\sigma_i(t)}$ for $t \in \Lambda_n$. By using (3.2), we have

$$\begin{aligned} &\psi(d(F(y_{\sigma_i(1)},\ y_{\sigma_i(2)},\ ...,\ y_{\sigma_i(n)}),\ F(v_{\sigma_i(1)},\ v_{\sigma_i(2)},\ ...,\ v_{\sigma_i(n)}))) \\ \leq\ &\varphi\left(\max_{1\leq i\leq n} d(y_i,\ v_i)\right), \end{aligned} \tag{3.3}$$

for all $i \in A$. Similarly, for fixed $i \in B$, we have $y_{\sigma_i(t)} \succeq_t v_{\sigma_i(t)}$ for $t \in \Lambda_n$. It follows from (3.2) that

$$\begin{aligned} & \psi(d(F(y_{\sigma_i(1)},\ y_{\sigma_i(2)},\ ...,\ y_{\sigma_i(n)}),\ F(v_{\sigma_i(1)},\ v_{\sigma_i(2)},\ ...,\ v_{\sigma_i(n)}))) \\ \leq\ & \psi(d(F(v_{\sigma_i(1)},\ v_{\sigma_i(2)},\ ...,\ v_{\sigma_i(n)}),\ F(y_{\sigma_i(1)},\ y_{\sigma_i(2)},\ ...,\ y_{\sigma_i(n)}))) \\ \leq\ & \varphi\left(\max_{1\leq i\leq n} d(y_i,\ v_i)\right), \end{aligned} \tag{3.4}$$

for all $i \in B$. Now by using (2.2), (2.3), (3.3), (3.4) and by the monotonicity of ψ, we have

$$\psi(\rho_n(F_\Upsilon(Y),\ F_\Upsilon(V))) \leq \varphi(\rho_n(Y,\ V)),$$

for all $Y,\ V \in X^n$ with $Y \sqsubseteq V$. It is only required to apply Theorem 3.1 with the help of Lemma 2.1 and Lemma 2.2 for the mapping $T = F_\Upsilon$ in the ordered metric space $(X^n,\ \rho_n,\ \sqsubseteq)$. ∎

Theorem 3.3. *Let $(X,\ \preceq)$ be a partially ordered set and suppose that there is a metric d on X such that $(X,\ d)$ is a complete metric space. Let $\Upsilon = (\sigma_1,\ \sigma_2,\ ...,\ \sigma_n)$ be an $n-$tuple of mappings from Λ_n into itself verifying $\sigma_i \in \Omega_{A,B}$ if $i \in A$ and $\sigma_i \in \Omega^{'}_{A,B}$ if $i \in B$. Let $F : X^n \to X$ be a mixed monotone mapping for which there exist a generalized altering distance function ψ and a right upper semi-continuous function $\varphi : [0,\ +\infty) \to [0,\ +\infty)$ such that*

$$\begin{aligned} & \psi\left(\frac{1}{n}\sum_{i=1}^{n} d(F(y_{\sigma_i(1)},\ y_{\sigma_i(2)},\ ...,\ y_{\sigma_i(n)}),\ F(v_{\sigma_i(1)},\ v_{\sigma_i(2)},\ ...,\ v_{\sigma_i(n)}))\right) \\ \leq\ & \varphi\left(\frac{1}{n}\sum_{i=1}^{n} d(y_i,\ v_i)\right), \end{aligned} \tag{3.5}$$

for all $y_1,\ y_2,\ ...,\ y_n,\ v_1,\ v_2,\ ...,\ v_n \in X$ with $y_i \preceq_i v_i$, for $i \in \Lambda_n$, where $\psi(t) > \varphi(t)$ for all $t > 0$ and $\varphi(0) = 0$. Also, suppose that either F is continuous or $(X,\ d,\ \preceq)$ is regular. If there exists $y_0^1,\ y_0^2,\ ...,\ y_0^n \in X$ verifying $y_0^i \preceq_i F(y_0^{\sigma_i(1)},\ y_0^{\sigma_i(2)},\ ...,\ y_0^{\sigma_i(n)})$, for $i \in \Lambda_n$, then F has a $\Upsilon-$fixed point. Moreover, if for each $i \in \Lambda_n$ and $y_i,\ v_i \in X$ there exists $z_i \in X$ which is $\preceq_i -$comparable to y_i and v_i. Then F has a unique $\Upsilon-$fixed point.

Proof. Note that the contractive condition (3.5) means that

$$\psi(\Delta_n(F_\Upsilon(Y),\ F_\Upsilon(V))) \leq \varphi(\Delta_n(Y,\ V)),$$

for all $Y,\ V \in X^n$ with $Y \sqsubseteq V$. Therefore, it is only necessary to use Theorem 3.1 with the help of Lemma 2.1 and Lemma 2.2 for the mapping $T = F_\Upsilon$ in the ordered metric space $(X^n,\ \Delta_n,\ \sqsubseteq)$. ∎

In a similar way, we may state the results analogue to Corollary 3.1, for Theorem 3.2 and Theorem 3.3.

4. Applications

In this section we give an application to our results. Consider the integral equation

$$u(t) = \int_0^T K(t,\ s,\ u(s))ds + g(t),\ t \in [0,\ T], \tag{4.1}$$

where $T > 0$. Consider the space:

$$C[0,\ T] = \{u : [0,\ T] \to \mathbb{R} : u \text{ is continuous on } [0,\ T]\},$$

equipped with the metric

$$d(x,\ y) = \sup_{t\in[0,\ T]} |x(t) - y(t)|\,,\ \text{for each } x,\ y \in C[0,\ T].$$

It is obvious that $(C[0, T], d)$ is a complete metric space. Furthermore, $C[0, T]$ can be equipped with the following partial order $\preceq$

$$x \preceq y \Longleftrightarrow x(t) \le y(t),\ \text{for each } x,\ y \in C[0,\ T] \text{ and } t \in [0,\ T].$$

It is clear that $(C[0, T], d, \preceq)$ is regular.

Theorem 4.1. *Suppose that the following hypotheses hold:*

(*i*) $K : [0, T] \times [0, T] \times \mathbb{R} \to \mathbb{R}$ *and* $g : \mathbb{R} \to \mathbb{R}$ *are continuous.*

(*ii*) *For all* $s, t, x, y \in C[0, T]$ *with* $y \preceq x$, *we have*

$$K(t,\ s,\ y(s)) \le K(t,\ s,\ x(s)).$$

(*iii*) *There exists a continuous function* $G : [0, T] \times [0, T] \to [0, +\infty)$ *such that*

$$|K(t,\ s,\ x) - K(t,\ s,\ y)| \le G(t,\ s) \cdot \frac{|x - y|}{2},$$

for all $s, t \in C[0, T]$ *and* $x, y \in \mathbb{R}$ *with* $y \le x$,

(*iv*) $\sup_{t\in[0,\ T]} \int_0^T G(t, s)^2 ds \le \frac{1}{T}$.

Then the integral (4.1) has a solution $x^* \in C[0, T]$.

Proof. We, first, define $F : C[0, T] \to C[0, T]$ by

$$Fx(t) = \int_0^T K(t,\ s,\ x(s))ds + g(t),\ \text{for all } t \in [0,\ T] \text{ and } x \in C[0,\ T].$$

Suppose $y \preceq x$, then from (ii), for all s, $t \in [0,\ T]$, we have $K(t,\ s,\ y(s)) \leq K(t,\ s,\ x(s))$. Thus, we get

$$Fy(t) = \int_0^T K(t,\ s,\ y(s))ds + g(t) \leq \int_0^T K(t,\ s,\ x(s))ds + g(t) = Tx(t).$$

Now, for all u, $v \in C[0,\ T]$ with $y \preceq x$, due to (iii) and by using Cauchy-Schwarz inequality, we get

$$\begin{aligned} & |Fx(t) - Fy(t)| \\ \leq\ & \int_0^T |K(t,\ s,\ x(s)) - K(t,\ s,\ y(s))|\, ds \\ \leq\ & \int_0^T G(t,\ s) \cdot \frac{|x(s) - y(s)|}{2} ds \\ \leq\ & \left(\int_0^T G(t,\ s)^2 ds \right)^{\frac{1}{2}} \left(\int_0^T \left(\frac{|x(s) - y(s)|}{2} \right)^2 ds \right)^{\frac{1}{2}}. \end{aligned}$$

Thus

$$|Fx(t) - Fy(t)| \leq \left(\int_0^T G(t,\ s)^2 ds \right)^{\frac{1}{2}} \left(\int_0^T \left(\frac{|x(s) - y(s)|}{2} \right)^2 ds \right)^{\frac{1}{2}}. \tag{4.2}$$

Taking (iv) into account, we estimate the first integral in (4.2) as follows:

$$\left(\int_0^T G(t,\ s)^2 ds \right)^{\frac{1}{2}} \leq \frac{1}{\sqrt{T}}. \tag{4.3}$$

For the second integral in (4.2) we proceed in the following way:

$$\left(\int_0^T \left(\frac{|x(s) - y(s)|}{3} \right)^2 ds \right)^{\frac{1}{2}} \leq \sqrt{T} \cdot \frac{d(x,\ y)}{2}. \tag{4.4}$$

Combining (4.2), (4.3) and (4.4), we conclude that

$$|Fx(t) - Fy(t)| \leq \frac{1}{2} d(x,\ y).$$

Taking supremum for each $t \in [0,\ T]$, we get

$$d(Fx,\ Fy) \leq \frac{1}{2} d(x,\ y),$$

for all x, $y \in C[0, T]$ with $y \preceq x$. Thus, the contractive condition of Corollary 3.1 is satisfied with $k = 1/2 < 1$. Hence, all the hypotheses of Corollary 3.1 are satisfied. Thus, F has a fixed point $x^* \in C[0, T]$ which is a solution of (4.1). ∎

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