Multidimensional Quantum Dynamics

MCTDH Theory and Applications

Edited by
Hans-Dieter Meyer, Fabien Gatti,
and Graham A. Worth

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