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Multidimensional Scaling in Riemann Space

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Multidimensional Scaling in Communication Research

Although the first multidimensional (MDS) scaling algorithms developed were metric algorithms (Torgerson, 1958), the development of nonmetric methods (Shepard, 1966) led to a rapid and nearly complete abandonment of the metric procedures in favor of these newer algorithms. Recently, however, there has been a resurgence of interest in metric algorithms, particularly within the field of human communication research, where the use of metric procedures predominates. In order to understand this reversion to what many psychometricians believe to be an outdated technique it is necessary to understand the difficulties and philosophy which led to the increased interest in metric scaling.

While there is little doubt that the development of nonmetric multidimensional scaling algorithms represents a great advance in the methods available to the contemporary social scientist, there exist some areas of inquiry in which the metric scaling routines may offer certain advantages. This article discusses one such case, familiar to communication researchers in particular. The reader should understand at the outset that this article is not an argument against the use of nonmetric scaling, but rather a suggestion that, for a specific theory dealing with a specific set of research questions, metric multidimensional scaling, modified in certain important ways, provides useful results obtainable either only with great difficulty or not at all by nonmetric means.

A THEORY OF CULTURAL PROCESSES

For several years, many communication researchers have held a theory of intercultural processes which presents several special measurement problems

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and possibilities (Woelfel and Fink, 1980). Briefly, this theory describes the belief system of any culture in terms of the average relationships thought to exist among the "objects" which a culture distinguishes within its environment. An "object" is defined as "anything that can be designed or referred to..." (Blumer, 1966), and the relationships among these "objects" are defined specifically as their perceived dissimilarities.

Multidimensional scaling is the preferred analytic vehicle within this theory, and each object is represented as a point in multidimensional space. Unlike typically psychometric practice, attributes within the space are not equated with the dimensions themselves, but rather each attribute is defined as an object and included as a point in the space. The extent to which any object embodies any attribute is given by the inverse of its distance from the attribute object in the space. On theoretical grounds, the dimensions of the space themselves are therefore of no substantive significance, but rather simply mathematically convenient reference vectors which simplify subsequent analyses (Woelfel and Fink, 1980; Woelfel and Danes, 1980).

CULTURAL PROCESSES

While some investigators maintain an interest simply in the structure of spaces such as those described above, by far the larger part of human communication scientists concern themselves with cultural processes, i.e., with the changes that take place within these spaces when different cultural groups communicate. Typical examples include marketing research studies, in which advertising messages modify the structure of a belief system in order to affect sales; studies of the changes in the structures of the belief systems of immigrants as a result of exposure to their host culture; and abstract experimental studies in which controlled "messages" are introduced into groups of people to determine the effects of such information inputs on processes within the space (Gillham and Woelfel, 1977; Woelfel and Saltiel, 1978; Woelfel et al., 1980a; Barnett et al., 1976; Cody, 1980; Woelfel and Fink, 1980).

The utility of the multidimensional scaling methodology for this theory is immediately apparent when it is realized that processes within the MDS configuration are always representable as motions of points in space. Since the mathematics of point motion is well developed, this constitutes an important advantage. Most recent research, in fact, has been concerned with the extent to which objects (points) in the space obey generalized forms of the Newtonian equation (Woelfel, 1978; Barnett and Kincaid, 1982)

$$m\ddot{x} + C\dot{x} + kx = 0 \tag{1}$$

where m is the inertial mass (or resistance to acceleration) of any point, $\ddot{x}$ is the acceleration of the point, C is the viscosity of the medium through which the point moves, $\dot{x}$ is the velocity of the point, k is the strength of equilibrium forces (restoring forces) which tend to move the point toward its starting position, and x is the position of the point.

The main goals of this research are twofold: first, to determine the extent to which this equation fits cultural processes across a variety of domains and circumstances, and second, to determine the empirical values of the coefficients m, C and k in a variety of circumstances.

Special Methodological Considerations

A careful consideration of the details of this theory or of its utility and fit to data is beyond the scope of this article, and the reader is referred to Woelfel and Fink (1980) for a more thorough introduction. A bibliography of several dozen studies is available (Barnett, 1980). What is to the point here, however, is that the kinds of problem addressed within this theory make special demands on the methods available to the researcher, while making available some new opportunities.

A Metric Measurement Model

Solutions to the problems posed above make serious demands on data. First of all, since the equation in question specifically contains ratios of differential elements, data suitable to the theory must be at least approximately ratio-level at the outset. A second requirement, again resulting from the presence of ratios of differential elements, is that the data be measured quite precisely, first because the differentials themselves must be precise, and second because the propagation of error inherent in taking ratios makes special demands on the data. A final requirement, which is quite unusual in social scientific practice, is that investigators must be able to maintain a standard unit size across measurement sessions and across different studies if the coefficients of the equation are to have any meaning. These special requirements obviously preclude the use of ordinal scaling methods, or even of common categorical scales such as the Likert and semantic differential types.

Investigators in the area of communication research have attempted to meet these stringent requirements by means of a pairwise comparison magnitude estimation task. In general, this method takes any two objects A and B in the domain under study, assigns an arbitrary (but agreed upon) unit size to the distance between them, and asks respondents to estimate the dissimilarities of all pairs of objects in the domain under study as ratios of this standard measure. In practice, this scale usually takes the form: "If A and B are u units apart, how far apart are x and y?", where x varies between 1 and $k-1$, and y varies between $x+1$ and k, where k is the number of objects in the study.

Most psychometricians will be aware of the large random component to be expected in such a scaling task, but the theory in question defines its main variable of interest as the average of all such dissimilarities. As is well known, this averaging process has two important consequences. First, the random component can be reduced as a function of the square root of the sample size by averaging increasingly large number of observations into the means. Within limits of economy, there is no theoretical limit to the precision that can be obtained by these methods, so random variation is in this context only an economic problem.

The second consequence of averaging, of course, is that individual variation is obscured, a fact of which researchers in the area are well aware. For certain problems, this obscuring of individual attributes and perceptions is unacceptable, as is well known, and individual differences models exist and have been discussed extensively elsewhere. For some purposes, however, the obscuring of individual differences is not only acceptable but desirable. One such purpose for which this consequence is particularly desirable is the investigation of the central tendencies of cultural belief systems (precisely the area the present theory was designed to approach, as suggested by Durkheim:

> Currents of opinion, with an intensity varying according to the time and place, impel certain groups either to more marriages, for example, or to more suicides, or to a higher or lower birthrate, etc. ... Since each of these figures [i.e., average marriage, suicide and birth rates] contains all the individual cases indiscriminately, the individual circumstances [or, we may here say, attributes] which may have had a share in the production of the phenomenon are neutralized... The average, then, expresses a certain state of the group mind (Durkheim, 1953, p. 10).

It is neither right nor wrong, then, to average individual dissimilarities matrices, but rather each procedure describes a different level of the phenomena considered. In an analysis of motivations for behavior at an individual psychological level, averaging of individual scores is a mistake. On the other hand, in investigations of the sociological behavior of aggregates of people or cultures, the dissimilarities matrix averaged over respondents is an appropriate basis.

MULTIDIMENSIONAL SCALING IN RIEMANN SPACE

Although multidimensional scaling in Riemann space is unfamiliar to many outside the field of communication, communication researchers have administered scales of this type to tens of thousands of respondents in dozens of cultures on every continent (Barnett, 1980). Two characteristics of the data which usually result from these procedures are of interest here. First, the dissimilarities often violate the "triangle inequality" rule; that is, frequently two sides of a triangle formed of three points will not sum to at least the length of the third side. When this occurs, the latent roots (eigenvalues) of the scalar products matrix formed from these dissimilarities are both positive and negative. Since the eigenroots are the sums of the squares of the coordinates, negative eigenroots are associated with imaginary eigenvectors in the space. Forms which have both positive and negative eigenroots are called indefinite, and constitute a general Riemann space (Sokolnikoff, 1951).

A second characteristic of these data is their precision. Often the average dissimilarities matrices exhibit uncertainties of less than 5 or 10%. In special cases, uncertainties of less than 1% have been observed (Brandt, 1980).

The combination of these two characteristics in the same data sets means that violations of the triangle inequality are in fact statistically significant, and that no transformation of the data within the typical 95 or 99% confidence intervals is sufficient to eliminate the imaginary eigenvectors. This fact is sufficient to rule out a casual application of nonmetric smoothing procedures such as those embodied in the nonmetric multidimensional scaling programs on the assumption that such violations represent simply unreliability of measurement.

RELIABILITY OF THE IMAGINARY EIGENVECTORS

A simple way to determine whether the non-Euclidian components of a scaling solution are the result of random errors of measurement is to examine the correlations among the imaginary eigenvectors across random splits of a data set, or over time in time-ordered data sets. Since the orientation of the eigenvectors is arbitrary within each data set, comparison of multiple data sets requires some Procrustean rotation prior to such calculations. While the logic of Procrustean rotations has been examined carefully (Cliff, 1966; Schonemann, 1966; Lissitz et al., 1979), most such examinations rest on the tacit assumption that all the eigenvectors to be rotated are real. When indefinite forms including both real and imaginary eigenvectors are considered, complete pairwise rotations of the typical kind are not appropriate, since vector lengths do not remain invariant under such transformations. Consider the Riemannian vector with coordinates 2, $2i$. The length of this vector is given by

$$|A| = [2 \cdot \cdot 2 + 2i \cdot \cdot 2] \cdot \cdot (1/2) = [4 + (-4)] \cdot \cdot (1/2) = 0$$

The reader can easily verify that any rotation will change the length of the vector. Two strategies are possible. First, the normal trigonometric functions in the rotation equations can be replaced by hyperbolic trigonometric functions. Alternatively, the spaces may be partitioned into their real and imaginary parts and rotations performed separately within each subspace. This operation leaves the lengths of the vectors invariant, since each subspace is orthogonal to the other by definition.

Cody (1980) scaled 13 descriptive attributes (competent, experienced, just, reliable, intelligent, ideal credible source, attractive, repulsive, unintelligent, unreliable, unjust, incompetent, inexperienced) by means of the procedures described above, and rotated the dimensions of each of the three treatment groups to give a least-squares best fit on a control group by means of the rotation technique just described. Even though the treatments (not described here) changed the configurations as expected, nonetheless the four largest imaginary eigenvectors correlated with their counterparts in the control group quite highly, as Table I shows.

In another experiment, M. Woelfel (1978) measured the pairwise dissimilarities among a set of 11 concepts which were most frequently mentioned by 43 undergraduate students as being associated with the women's movement, using the procedures described above. Several weeks later these students each engaged in a five minute discussion about the women's movement with two confederates, then filled out the original questionnaire a second time. Even though considerable time had elapsed and the students' measured stress levels during the discussion averaged ~25% above normal, the two largest imaginary eigenvectors of this solution correlated 0.87 and 0.88 with their counterparts in the original results.

TABLE I

Correlations of Four Largest Imaginary Eigenvectors in Three Treatment Groups with their Counterparts in a Control Condition

Dimension	Group 1	Group 2	Group 3	Variance (%)
13	0.26	0.97	0.90	−3.3
14	0.63	0.96	0.95	−4.6
15	0.86	0.93	0.93	−5.3
16	0.83	0.99	0.98	−6.0

Barnett (1982) gathered longitudinal data from 20 subjects over 12 points in time. The objects scaled represented the candidates, major issues and political parties, together with a series of attributes and a self-point, during the 1976 Presidential election campaign in the U.S.A. There was good reason to expect considerable true change among the various variables as the subjects' political attitudes changed during the campaign. We show here, however, that the largest imaginary dimension remained relatively stable during the experiment.

An initial question of interest is whether this dimension accounts for a stable proportion of the variance. The answer appears to be that it does. The mean variance accounted for by the largest imaginary dimension was 25%, with a range of 11%. The standard deviation about the mean was only 4.01, indicating a high degree of stability. Also, the largest imaginary dimension was the second largest dimension (in absolute terms) for the first seven measurement sessions, and the third largest for the last five sessions. If a scree test were performed on the absolute values of the eigenvalues, this dimension would always be retained by any standard interpretation (Barnett and Woelfel, 1979).

The reliability of the loadings on this dimension was estimated by correlating the scale values at each point in time with the corresponding values at every other point in time.

The lags among the measures were then examined. The correlations among the dimensions would be expected to decrease as the lag interval increased if the changes were due to true change rather than unreliability. This occurred. The mean correlation for each lag was regressed on the length of the lag. This resulted in a negative slope ($b = -0.038$, $r = -0.278$). The intercept ($a = 0.603$) provides perhaps the best available estimate of the "true" reliability of the loadings on the largest imaginary eigenvector, since, by interpolation, it represents the correlation of the eigenvector with itself at lag 0. Once again, careful analysis rather clearly rules out the likelihood at least that the largest imaginary dimension is the result of random errors of measurement.

These three examples are in no way atypical, but rather represent the common pattern of findings using MDS methods. Virtually all studies show the same pattern, and seem to demonstrate unambiguously that imaginary eigenvectors are not the result of random errors of measurement.

TRANSFORMATIONS OF THE DATA

Given that the Riemannian character of the resulting spaces is not always attributable to unreliability of measurement, there still remains the possibility that this outcome is the result of a systematic error in the measurement of the dissimilarities. As Woelfel and Fink (1980) have argued, such an assertion could be evaluated only if some knowledge of the "true" form of the dissimilarities could be obtained independent of measurements. Nonetheless, for the sake of demonstration it may temporarily be assumed that there is a strong reason to suspect that all cultural spaces are Euclidean, or that Euclidian outcomes are strongly preferred on some grounds. It is therefore, of interest to investigate the class of transformations which would render the data Euclidian prior to analysis.

There are several mathematical procedures by which Riemannian data may be fitted into Euclidian space. One such method is to transform the data in such a way that the triangle inequalities are eliminated. The first such set of transformations are the monotonic transformations which form the basis of modern nonmetric multidimensional scaling algorithms. Used naively, such transformations seem inappropriate, since they are based on the assumption that the metric properties of the raw data are untrustworthy, and that only the ordinal properties of the data may be relied upon. As we have shown, the metric properties of data obtained in the ways discussed above are quite reliable, and may frequently be more robust than the ordinal properties of the same data sets. Consider the "face" in Fig. 1. This Figure is the first principal plane of a metric multidimensional scaling analysis (Galileo version 1.0) of the interpoint distances among the principal features of a human face. A face is a symmetric structure such that the distance, for example, between the left eye and the tip of the nose is the same as the that

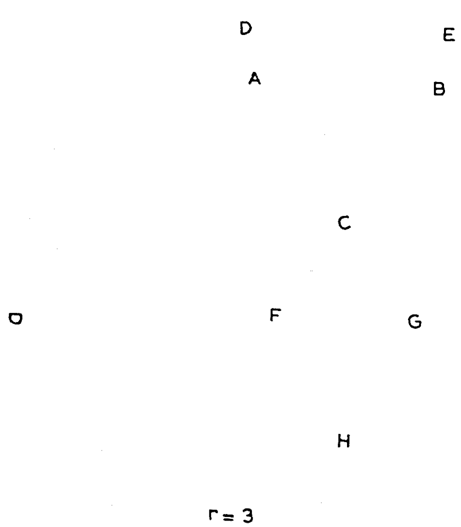

Fig. 1. Galileo Face.

Fig. 2. TORSCA face.

between the right eye and the tip of the nose. A normal face contains many such symmetries.

Whenever these conditions are met in a data structure, the ordinal properties of the data will be less robust than their metric properties, since vanishingly small changes in the magnitudes of a few of the distances can make major changes in the rank orders of those same distances. These changes are often sufficient to render deceptive the most competent nonmetric scaling programs. Figure 2, for example, illustrates the first principal plane of the TORSCA nonmetric solution for the same data set after including only a small (less than 5%) random component in the data.

A second, less naive use of the nonmetric procedure involves producing a nonmetric solution and then examining the resulting "Shephard diagram" (see Fig. 3) to help estimate the form of the transforming function needed to eliminate the non-Euclidian characteristics from the data set.

Figure 3 represents the "Shepard diagram" or scatter of the relation between the original dissimilarities matrices and the final distances obtained by using the TORSCA monotonic smoothing function to fit a set of ten concepts into three-dimensional Euclidian space. The ten concepts (walking, sitting, strolling, running, sleeping, fighting, revolution, marrying, singing, practicing medicine) were assessed by 45 undergraduate sociology students at a large Midwestern university. An examination of Fig. 3 reveals that the TORSCA transformation is typically many-to-one; that is, it assigns the

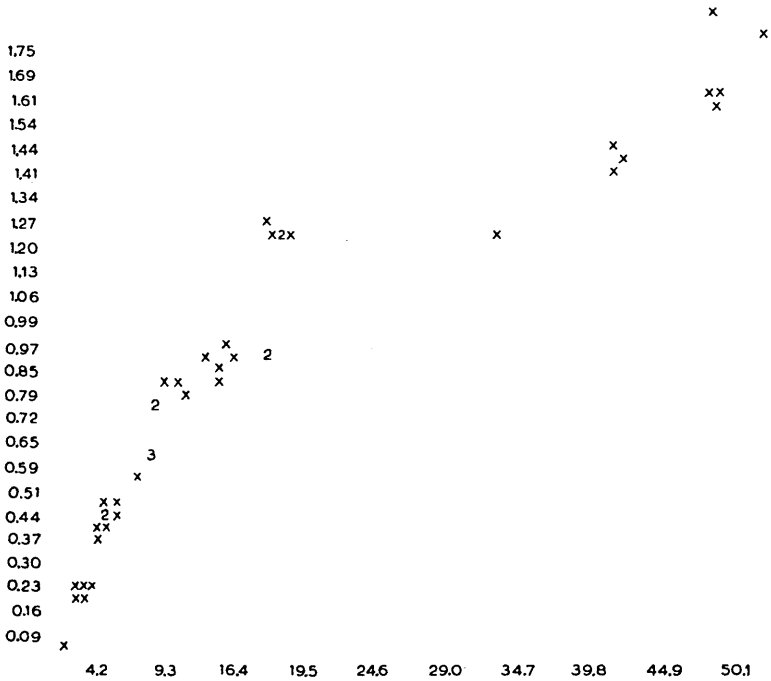

Fig. 3. Distances versus Original Data for Ten Behaviors—initial results (see text) (TORSCA solution).

same final value to several different initial values after transformation. Such transformations, while monotonic, have no inverses, and thus it is never possible to re-generate the original dissimilarities from the solution by any function whatever. While this may be acceptable for noisy data sets characterized by large random components, it does not seem appropriate for precise data sets, and it makes it impossible to maintain a standard metric across measurement sessions and across studies.

Figure 4 represents the relations among the same set of concepts as in Fig. 3 but for data obtained from the same subjects two weeks later. In the interval, classes were suspended owing to unrest during the Cambodia invasions and the killings of the Kent State students. Two of the concepts in the study (fighting and revolution) were relevant to the events on campus during the two-week period. Of greatest importance is that events on campus seem to have affected the overall structure of the space in such a way that a different transformation is required to fit the concepts into a Euclidian configuration. When data are arrayed in a time series, as are these, it seems clearly inappropriate to choose a transformation based on the configuration at only one point in time, or to choose different transformations for each time period. If the departures from the Euclidian configuration are the

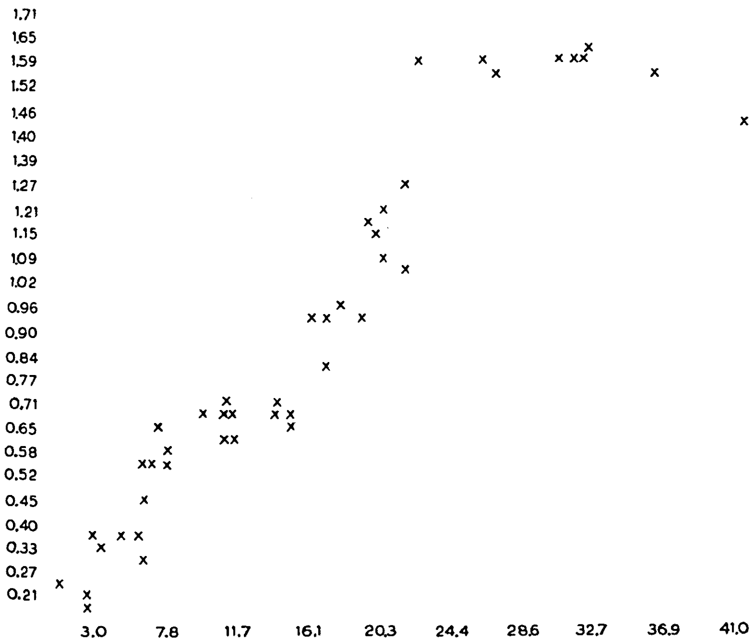

Fig. 4. Distances versus Original Data for Ten Behaviors— subsequent results (see text) (TORSCA solution).

consequence of simple unreliability, choosing a different transformation for each data set may not require much justification, but if this is not so (as it is not for this or the other data sets described in this paper), then the advocate of transformations to the Euclidian definite form faces the obligation of explaining why the "scaling bias" should change from one measurement session to another.

Even if such transformations can be justified, they still create problems. When coefficients for eqn. (1) are estimated after such many-to-one transformations, they are of no value to the communication researcher, since each coefficient is actually expressed in arbitrary units whose functional relation to the arbitrary units of any other study is completely unknown.

A third possibility is to transform the data by a noniterative function. It is easy to show (although no demonstration is offered here) that violations of the triangle inequality rule can be transformed away by a monotonic function which foreshortens long distances, and similarly that such violations are exaggerated by any transformation which increases large distances relative to small. A transformation of the form

$$s' = s^c$$

where s denotes the measured value, s' the "true" value, and c is a constant, can be shown to increase the number of violations of the triangle inequality and hence the number of imaginary eignevectors when c is greater than one, and to reduce these numbers when c is less than one. When $c=0$, any configuration will converge on a unit hypersphere in Euclidian space.

This approach too, however, has problems. First, large bodies of sound evidence, primarily from psychophysics, show unambiguously that human respondents already foreshorten long distances, particularly when their expression requires the use of fairly large numbers (Shinn, 1977). Gordon (1976) used the exact scaling method described here for nine groups each of ~ 100 students, changing only the arbitrary standard and/or its modulus (numerical value assigned to the standard), and showed a slight logarithmic rolling-off of high scores when the scale used a small unit and required large numerical estimates (these data are discussed in some detail in Woelfel and Fink, 1980, Chap. 5).

Application of the logarithmic transformation to the raw data, of course, is based on the assumption that the raw scores are systematically rather than randomly in error. But, as the evidence shows, since such systematic errors are almost certain to be in the form of logarithmic foreshortening of long distances, which in fact attenuates rather than creates triangle inequality violations, still further foreshortening seems highly undesirable.

Several other such transformations are known to the writers. These include additive models, such as Attneave's additive constant, in which the smallest constant which can be added to every measured dissimilarity to eliminate the non-Euclidian components of the space is found by an iterative method (Torgerson, 1958), and Lingoes' "lambda min" solution, in which the largest (absolute) latent root is subtracted from each root, and the eigenvectors renormalized to the new eigenvectors (Lingoes, personal correspondence).

The former method is based on the assumption that the scales of measurement are interval-level scales, with apparent zero points differing from the true zero points by the value of the additive constant. All non-Euclidian characteristics of the data are seen as consequences of the arbitrary zero point on the scale. It is difficult to see the application of this reasoning to the scales discussed in the present article, since the meaning of the zero point—that there is no difference at all between A and B—on the direct magnitude estimation scales is fairly unambiguous. Moreover, both of these procedures suffer from the same problem as do the logarithmic and nonmetric monotonic transformations when applied to longitudinal data sets. In the case of the additive constant method, it is necessary to inquire as to why the limen or zero point should move systematically between measurement sessions, and in the latter case (Lingoes' lambda min method), to establish why the transforming function needed to eliminate the non-Euclidian characteristics should shift between measurement sessions.

The last transformation discussed here is one considered by the authors for several years in the early 1970's. This method consists of relaxing the assumption that each stimulus item be considered a point in space, and assuming rather that each may in fact be considered a hypersphere of radius $r(i)$, where the $r(i)$'s are fitted to the stimuli so as to be the smallest possible which render the data set completely Euclidian (operationally, so that the largest negative eigenroot vanishes). While this remains an intriguing possibility to the writers, a completely satisfactory mathematical solution to the problem is not known to us, and the method also suffers from the same problem as do the other transformations when applied to time-ordered data sets, although in this case the problem takes the form of finding a meaningful theory about why different radii need to be assigned to each object scaled in each data set.

In general, the major problem lies with theory. Many transformations which can project Riemannian surfaces onto Euclidian surfaces are known, but in general there is no theory which can lend a substantive interpretation to any of them.

Should theoretical grounds for such a transformation be found in the future (at present we are aware of none), it is important to note that not only the form of the transformation but the actual coefficients of the transforming equation have to be the same for every measurement session if the empirical coefficients of eqn. (1) are to be meaningful across studies and across investigators.

This last objection is far more serious than it first appears. Typically, when dealing with a data set which exhibits non-Euclidian characteristics, an initial impulse is to find some empirical transformation which will eliminate them. But what of the case when an investigator is faced with several data sets of which each exhibits some non-Euclidian elements, particularly when a different transformation is required to make each set Euclidian? If each data set is transformed by a different function, how can they be compared meaningfully afterward? And even if the investigator is sufficiently careful (and lucky!) to find a single transformation which eliminates the non-Euclidian characteristics from all data sets at once, how will comparisons be made with data sets analyzed by different investigators? (It is simple enough to show that each of the data sets described earlier in this article requires a unique transformation function to eliminate its non-Euclidian components.

Yet another strategy for avoiding the Riemannian configuration involves embedding the Riemann surface in a larger (i.e., higher-dimensional) Euclidian space. Following Sokolnikoff (1951), an estimate of the dimensionality of the required Euclidian space can be obtained as follows. Consider the Riemann space R_n generated by

$$ds^2 = g_{ij}\, dx^i\, dx^j \quad (i, j = 1, \ldots, n) \tag{2}$$

An estimate of the dimensionality m of the Euclidian manifold in which R_n is embedded can be obtained by considering the transformation T, which maps the points of the Riemann space into Euclidian space:

$$T\colon y^\alpha = y^\alpha(x^1, \ldots, x^n) \quad (\alpha = 1, \ldots, n)$$

Suppose that

$$dy^\alpha\, dy^\alpha = g_{ij}\, dx^i\, dx^j \quad (i, j = 1, \ldots, n)$$

Now,

$$dy^\alpha = (\partial y^\alpha / \partial x^i)\, dx^i$$

Inserting this into eqn. (2) gives $[n(n+1)]/2$ differential equations of the form

$$\partial y^\alpha\, \partial y^\alpha / \partial x^i\, \partial x^j = g_{ij}$$

Since this set of differential equations will, in general, always be solvable if $m > [n + (n+1)]/2$ differential equations exist, a Riemannian space of n dimensions may always be mapped into a Euclidian space of m dimensions.

This procedure can be made free of the problem of handling multiple time-ordered data sets if m is chosen sufficiently large to accommodate the largest Riemannian space ever encountered. All data sets ever scaled, then, can always be fitted into this "superspace" without the need for a different transformation for each data set.

The problem with this solution, however, is that the dimensionality of the Euclidian space required will commonly be much greater than the dimensionality of the Riemann space itself—often sufficiently large to strain the computational capacity of even the largest of computing facilities. Although it may afford psychological comfort to the investigator, this solution offers no advantages to the practical scientist.

AD HOC ADJUSTMENTS

Each of the procedures described above could be considered "automatic" in one sense, in that the functions could be found by some constrained generator of monotonic functions operating with respect to a specified criterion (e.g., that the largest negative eigenroot should be zero). Once encoded into software, such an algorithm could be set to transform the data automatically, although such a procedure would still be subject to the difficulties discussed above. One last alternative might be to examine each data set, and the metric solutions generated from it, to determine whether some clustering of the concepts (or of the subjects) might lead to a new Euclidian solution in a reasonably small and computable space. Or, perhaps, substantive interpretations for each of the eigenvectors might be sought; those imaginary dimensions in which meaningful clustering did not occur, or for which no interpretation could be found, might be eliminated. New distance matrices might be generated from these "tailored" spaces and further analyses undertaken. While such procedures might work in some cases, there is good reason to consider them both inappropriate and infeasible for the type of problems typically studied by the communication worker.

First, as most researchers (including the present writers) are well aware, many researchers attach substantive significance to the dimensions of multidimensional scaling solutions. Some MDS algorithms, for example INDSCAL, depend on such an interpretation for their viability (although even INDSCAL allows dimensions to be nonorthogonal to deal with correlated attributes). Not all researchers accept this type of interpretation, however. Rosenberg and Sedlak (1972), for example, argued persuasively that attributes are frequently mutually dependent (correlated), which both rules out the possibility that they might correspond to the orthogonal dimensions of an MDS solution, and reduces the likelihood that every dimension of the solution will be interpretable. Furthermore, they presented sound empirical evidence that the number of attributes which may lie in any n-dimensional subspace may greatly exceed n, and that they may lie at oblique angles to the n dimensions. Many scaling algorithms themselves (such as PROFIT in the Bell Labs package) rely on this assumption, an assumption that is overwhelmingly supported by data (Rosenberg and Sedlak, 1972).

Not only may attributes fail to correspond to the dimensions in an MDS solution (thus ruling out the necessity that all "genuine" dimensions be interpretable), but several researchers (Cody et al., 1976; Barnett and Woelfel, 1979; Woelfel and Danes, 1980; Woelfel and Fink, 1980) have presented both theoretical and data-based evidence that attributes ought not be thought of as "lines" or dimensions at all. The most common theory held by communication researchers recommends that each of the end points of what were formerly considered linear attributes (e.g., "good–bad") be scaled as points or "monopoles" in the space, so that the extent to which any object in the space exhibits an attribute is given by the inverse of its distance from the point which represents the end point of that attribute. An object is "good", therefore, insofar as it is close to the point "good" in the space, and "bad" insofar as it is close to the point "bad". Such a solution seems inherently superior to the idea that attributes should be represented as line segments, since it leaves open the possibility that an object might increase or decrease the extent to which it manifests any quality without simultaneously increasing or decreasing the extent to which it manifests the "opposite" quality. Changing the chemical composition of a substance, for example, might increase the extent to which it is "sweet" without decreasing the extent to which it is "sour". If such a view is true, then the dimensions of an MDS space need not be substantively interpretable.

Woelfel and Fink (1980) have shown, for example, that for two samples (undergraduate students and professional network analysts) for which data were scaled, the concepts good, bad and evil form not a line segment but a triangle. This can be interpreted either as meaning that "good" constitutes an end point for two attribute dimensions (good–evil and good–bad), or as support for the view that each of the three attribute adjectives (good, evil, bad) ought to be scaled as a monopole.

Should the older theory that attributes must be representable as lines in the space be true, then this newer model would empirically yield that outcome in any case. Empirical evidence available from the works cited above confirms our own experience, and leads us to believe that the notion that the dimensions of an MDS space ought to be interpretable as attributes in their own right is unsupportable.

Even if this ad hoc approach might be made to work, there are strong reasons why it would be neither feasible nor desirable in a typical communication research study. Often the MDS literature leads one to suspect that the attainment of a final MDS configuration is a terminal goal of a research project. This is decidedly untrue of communication research, in which the MDS solution is simply an intermediate step toward the analysis of longitudinal communication or cultural processes. Typical is the study of the 1980 U.S. Presidential election currently in progress. The goal of the study is to determine convergences and divergences among candidates, issues and the position of the "average voter" during the election campaign in response to events within the campaign. To achieve this, a complex multistaged research process was required. First, in-depth interviews were conducted with randomly chosen voters in New York and Hawaii. The issues and terms most frequently mentioned in these interviews were included on a magnitude estimation questionnaire involving pairwise comparison of 14 items. This questionnaire was then administered to a fresh random sample of ~ 25 voters in Hawaii and New York every day for six weeks prior to the election (and for two weeks past the election for the New York voters).

Next, a metric MDS solution for each separate day at each site (New York and Hawaii) was constructed; each of these was rotated to a least-squares best fit on the preceding day's configuration. It is at this point, and only at this point, that the main analysis begins. This main analysis consists of a mathematical description of the trajectories of the candidates, issues and other terms through the space. These trajectories are then plotted against media and other events during the campaign.

Since we are here concerned with a time series of up to 42 distinct MDS configurations, clearly it is not possible to deal with each of these configurations as if it were a separate entity. If each space is treated differently from each other space, the resulting analysis of the entire time series is rendered meaningless. Nor is it possible to treat the data for each separate site differently, and then make sense of the subsequent comparisons. On pragmatic grounds alone, ad hoc adjustment of each separate space would make a project like this economically unmanageable.

INTERPRETING THE RIEMANN SPACE

While the investigations reported here have shown that it is possible to transform away the Riemannian characteristics of spaces generated by averaged magnitude estimation scaling methods based on pairwise comparison, each of the methods considered has both practical and theoretical disadvantages. A clear alternative is simply to deal with the Riemannian configuration on its own terms, and to apply appropriate non-Euclidian mathematical operations to the resulting indefinite mathematical form. One advantage of so doing, as already mentioned, is the saving in computer core required for analysis; another is the ability to save the original metric of the scaling method, an essential feature for addressing the fundamental theoretical questions important to communication scientists. A third advantage is the fact that plausible theoretical interpretations for the Riemannian character of the data sets may be found.

Two such interpretations are presented here. First, there is reason to suspect that non-Euclidian outcomes ought to be expected when objects or stimuli from diverse domains of meaning are scaled in the same measurement session. Second, there are theoretical reasons why similar non-Euclidian manifestations ought to be expected when objects or stimuli are ambiguous or uncertain to the subjects, particularly when subjects may be thought to hold "dissonant" cognitions, following Heider (1958), Festinger (1957), and others.

Cross-Domain Scaling

As Fillenbaum and Rappaport (1971, p. 3) suggested, the "meaning of a lexical item is a function of the set of meaning relations which hold between that item and other items in the same domain". A domain may be defined as a semantic field or a structurally related coherent set sharing some meaning properties or having some common class referents (Fillenbaum and Rappaport, 1971). But how does one define the boundaries of the domain to be scaled? In certain cases, the scaled items may be limited to a "well specified" domain, such as kinship names (Fillenbaum and Rappaport, 1971), or animal names (Henley, 1969). This limitation of the domain follows the approach advocated by Osgood (1968, p. 132), to "...restrict the semantic domain under study to a pure type of system...". This is not always possible or desirable, however:

> ...[This] solution would appear undesirable since, insofar as many domains are not pure systems, such a maxim would restrict our investigations to a subset of "neat" cases, and rule out of consideration many important structured domains" (Fillenbaum and Rappaport, 1971, pp. 239–240).

Osgood's advice, moreover, is often at odds with well-established and plausible methods for choosing the stimuli that are to be included in scaling tasks. Woelfel et al. (1980b) suggest, for example, that MDS studies of applied campaigns, such as election campaigns or marketing studies, should conduct preliminary interviews with samples drawn from the population of interest, asking the respondents to describe the topic freely and at length, and include the most frequently mentioned attributes in the scaling instrument for the main study. Often the resulting list includes elements from different domains, such as person descriptors, political parties, issues, and so forth. The conscientious investigator cannot abide simultaneously by both this advice and Osgood's, since using the most frequently mentioned attributes will often require the scaling of objects from multiple domains, yet attempting to maintain a "pure" scaling task will frequently require leaving out some of the most frequently mentioned attributes. It would seem, then, that it is often necessary and even desirable to scale elements drawn from multiple domains on the same instrument.

Not only do the authors cited above show an awareness of the non-Euclidian results obtained by scaling items from multiple domains, but there exist both theoretical reasons and empirical evidence suggesting that such results are to be expected. Concerning the theoretical grounds, we are aware of no theory current in the social sciences which asserts seriously and without qualification that human subjects have anything approaching a full awareness and understanding of their own cognitive structures and mental processes. That subjects are frequently unaware of the exact (or even general) meanings of scaling stimuli is commonplace knowledge to the experienced researcher. Moreover, it is eminently plausible to suspect that subjects are most imprecise in their understanding of the meaning relations among objects drawn from widely different domains of meaning. That such a situation is likely to result in Riemannian outcomes is not only plausible, but can be shown empirically.

It is possible to examine data gathered from a single domain and data taken from more than one domain in order to see whether sets of the latter type have greater variance on the imaginary dimensions. Accordingly, data were gathered on the dissimilarities among ten political stimuli (a coherent set) at three points in time (Barnett et al., 1976). The ratio of the imaginary variance to the variance explained by the real dimensions was respectively 0.38, 0.24 and 0.27 for the three measurement sessions. Similarly, data were collected on the dissimilarities among ten mass-media items (a coherent set) from four different groups. The same ratio for these four groups was respectively 0.27, 0.27, 0.19 and 0.21 (Barnett, 1977). Harkins (1978) had subjects scale eight items from a well-defined domain of mass-media terms. He found a ratio of 0.22. Barnett (1982), on the other hand, gathered data from more than one domain. The mean ratio of imaginary to real variance was 0.50.

These results seem to confirm what theorists have suggested previously, namely that human subjects seem to place multidomain stimuli within generalized Riemann spaces rather than Euclidian configurations. Both relations within domains and those across domains may be examined by calculating the numbers of triangle inequality violations for samples of stimuli from within a single domain and for samples of stimuli taken from more than one domain. This may be done easily with the aid of the generalized equation of Pythagoras.

In cases where $\cos\theta < 1.0$, the relations may be considered Euclidian. When $\cos\theta > 1.0$, the triangle inequality relation is violated and generalized Riemannian structures result.

Randomly selected samples (both across time and from within or between domains) of three stimuli were taken from the Barnett (1982) data set described above ($N = 56$). In those cases where the three stimuli were members of the same set, 84% of the relations were Euclidian. In those cases where the three stimuli came from more than one domain, only 14% of the relations were Euclidian. These results differ significantly from chance, with $\chi^2 = 32.45$, d.f. = 1, $P < 0.001$.

Taken collectively, the above results present a very strong prima facie case that Riemannian manifolds ought to be expected when scaling stimuli across multiple meaning-domains. This means that cultural spaces may well be globally Riemannian even though locally Euclidian. Since the scaling of items from multiple domains is often either unavoidable or desirable, there are clearly circumstances under which Riemannian configurations are both theoretically and empirically the expected outcome.

SCALING DISSONANT COGNITIONS

A second, equally interesting, set of circumstances exist in which Riemannian relations are to be expected on theoretical grounds. Many theorists, in particular Heider (1958), Newcomb (1953), Festinger (1957) and Osgood et al. (1957), have suggested that human subjects ought frequently to find themselves embedded in dissonant relations with the objects of their experience. Consider the inconsistent freindship relation, where A likes B, A does not like C, and B likes C. Translating into dissimilarities relations, it might be said that A is close to B, B is close to C, but A and C are distant from each other. This can easily be translated into a violation of the triangle inequality relation for this triad. Similarly, for communication data, consider the case in which A and B communicate frequently (i.e., are "close to" one another), B and C communicate frequently, and A and C never or infrequently communicate (Barnett, 1979). If frequencies of communication are taken (as they commonly are) as the inverses of the "communication distances" or "network distances" among these three, then again triangle inequality violations and Riemannian structures are to be expected on theoretical grounds.

Such cases occur frequently for straightforward dissimilarities-data as well. Fink et al. (1975), for example, reported that a random sample of university students saw large distances between themselves and the rich, but small dissimilarities between the rich and big business and between themselves and big business. Each of these relations, considered pairwise, is interpretable and conforms with qualitative impressions taken from interview data, but taken together they result in triangle inequality violations and Riemannian structure.

Such inconsistencies are meaningful both theoretically and pragmatically. Scaling algorithms which eliminate them eliminate meaningful and useful information about the cognitive structures of the sample. Some theorists (Woelfel and Fink, 1980; Barnett and Kincaid, 1982) hypothesize in fact that communication among members of populations having dissonant (i.e., Riemannian) cognitive structures ought to lead these structures to move toward consistency as a consequence of communication. Any scaling method which in the act of scaling eliminated the inconsistencies would make empirical investigation of this hypothesis impossible. Even granting that a comparison of the transforming functions needed to eliminate the non-Euclidian characteristics from each data set might be attempted, such a comparison would itself be a very difficult undertaking whose mechanics are themselves not well understood. By contrast, the simple comparisons illustrated above are very easy to make, and render empirical investigation of such hypotheses relatively easy.

Summary and Conclusions

Although many discussions of multidimensional scaling are mathematical and abstract, most have been guided by a general set of goals whose roots lie in psychology. These goals typically focus on the representation of the cognitive structures of individual persons. The extension of MDS procedures beyond the boundaries of psychology, however, has led to goals quite different from those psychological concerns. Communication researchers in particular often make use of MDS procedures for the analysis of cultural processes. These new goals pose special problems and offer new opportunities to the MDS user. In particular, concentration on aggregates has led to an increase in the use of ratio-level scaling procedures and metric as opposed to nonmetric algorithms. This combination of measurement and analysis procedures tends to yield indefinite mathematical forms or Riemann spaces.

The present paper has shown that the Riemannian character of the spatial configurations resulting from these methods cannot be attributed solely to unreliability of measurement. Although several transformations which could eliminate the Riemannian characteristics from these solutions have been discussed, none of them is completely free of problems of its own. While the writers by no means advocate abandoning the investigation of these types of transformations, on the other hand none of them is so compelling as to rule out the simple expedient of dealing with the Riemannian configurations as they are. In fact, such early research as exists seems to indicate that this expedient may focus needed attention on inconsistencies in cultural belief patterns which might go unnoticed by routine nonmetric MDS analyses. While alternative methods might well exist (including some of the procedures discussed in the present article), the application of Riemannian mathematical analyses to the Riemann spaces which result from metric MDS of ratio-scaled aggregated data seems sufficiently fruitful to warrant increased mathematical and experimental investigation.

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