Multifrequency Backscattering Tomography: Principles and Reconstruction Methods

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SUMMARY

Single-frequency diffraction tomography, though is well known in the literature, has its inherent problems, such as the limited resolution and image distortion due to the existence of "blind areas" of the object spectrum. The introduction of multi-frequency methods can improve the resolution and partly fill out the "blind areas" of the spectrum. The existing multi-frequency methods, such as the multi-frequency holography (prestack migration) or the wide band Born inversion, are time consuming procedures. The Multi-Frequency Backscattering Tomography (MFBT) is a fast method which uses only the backscattered waves (after plane wave decomposition) but meanwhile maintains the merit of multi-frequency methods. Two reconstruction methods are presented: the backpropagation method and the direct Fourier transform method. In the latter method, the backpropagation of plane waves in the z-direction is implememted by FFT through a change of variable, this increases significantly the computation speed. Compared with the single-frequency diffraction tomography, the MFBT has a better resolution and image quality, and its reconstruction speed is faster by a factor N_z/log_2N_z, where N_z is the number of grid points in z-direction. When N_z is large, the time saving of MFBT is remarkable.

INTRODUCTION

Since the diffraction tomography was introduced to geophysical applications (Devaney 1984, Wu and Toksöz 1987), it has been recognized as a special inversion method (Lo et al. 1988, Pratt and Worthington 1988, 1990, Mora 1989). However, the diffraction tomography was formulated for the case of monochromatic wave field and therefore has its inherent problems, such as the limited resolution and the image distortion due to the existence of "blind areas" in the object spectrum. Since the sources used in seismic exploration or other applications (such as the geo-radar subsurface imaging) are often broad banded, naturally the further development of geophysical diffraction tomography will have one direction toward the use of multi-frequencies, which can improve the resolution and partly fill out those "blind areas" of the object spectrum. Multi-frequency holography (Wu et al. 1977, Wu and Toksöz 1987), which is similar to the process of prestack migration (e.g. Stolt and Benson 1986), has been shown to have improved resolution, especially the vertical resolution, and image quality. For the geometry of nondestructive testing of materials, in which the plane wave source is used to illuminate the object from different directions, the MF diffraction tomography has been used in a straightforward way (Langenberg, 1987). However, in the case of geophysical applications, multi-frequency diffraction tomography has rarely been discussed in the literature.

In this paper, following the proposal of Wu (1991), we formulate a special multi-frequency linear inversion method: Multi-Frequency Backscattering Tomography (MFBT), which has a fast computation speed and meanwhile offers an improved resolution and quality of image compared with the single-frequency diffraction tomography (SFDT). Here we present two reconstruction methods: the backpropagation method and the direct Fourier transform method. In the latter method, the backpropagation of plane waves in the z-direction is implemented by FFT through a change of variable, this increases significantly the computation speed. The method presented in this work offers a feasible fast algorithm of imaging the subsurface 3D heterogeneities by wave tomography using 2D seismic array data. The method is also useful for the image reconstruction of geo-radar using electromagnetic waves.

PRINCIPLE OF THE METHOD

For the geometry of Surface Reflection Profiling (SRP), the double Fourier transform of the scattered field along both the source line and receiver (geophone) line, both on the surface, gives the angular spectrum of the scattered field $\tilde{U}(k_g, k_s)$, which has a simple relation with the object spectrum $\tilde{O}(K_x, K_z)$. Under the assumption of weak scattering (see Wu and Toksöz 1987):

$$\tilde{O}(\vec{K} = \vec{k}_g + \vec{k}_s) = 4\frac{\gamma_g \gamma_s}{k^2}\,\tilde{U}(k_g, k_s, k) \qquad (1)$$

where $\tilde{O}(\vec{K})$ is the 2D-FT of the object function $O(\vec{r})$ defined as

$$O(\vec{r}) = 1 - \frac{c_0{}^2}{c^2(\vec{r})}\,, \qquad (2)$$

where c_0 is the wave propagation velocity in the background medium, and $c(\vec{r})$ is the actual velocity which varies with position $\vec{r} = (x, z)$, and

$$\begin{aligned} \gamma_g &= \sqrt{k^2 - k_g{}^2}\ ,\ \ \gamma_s = \sqrt{k^2 - k_s{}^2}\,, \\ \vec{k}_g &= (k_g, \gamma_g) = k\hat{g}\ ,\ \ \vec{k}_s = (k_s, \gamma_s) = k\hat{s}\,. \end{aligned} \qquad (3)$$

Knowing the spatial spectum of the object, we can reconstruct the object function by an inverse 2D-FT:

$$O(\vec{r}) = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} d^2\vec{K}\,\tilde{O}(\vec{K})\,e^{i\vec{K}\cdot\vec{r}}\,. \qquad (4)$$

However, the problem is how much information can be obtained from the scattering measurements and how much will be used in the reconstruction process. For SFDT the spectral coverage is shown in Fig. 1a, where k_0 is the central frequency (assuming $50Hz$). We can see that the object spectrum is limited to the range of $2\,k_0$ and has two big holes non reachable, the so called "blind areas". The introduction of multi-frequency methods can improve the resolution and reduce those "blind areas" of the spectrum. The existing multi-frequency imaging procedures are time consuming. The proposed method of MFBT uses only the backscattered waves (after plane wave decomposition),saving computation time but still maintaining the image quality of the multi-frequency methods. We can see from Fig. 1b and 1c that the spectral coverage for the case $k_s = k_g$, in MFBT, is quite uniform. In Fig. 1b the frequency band is assumed to be $f = 2 - 100Hz$, and in Fig. 1c, $f = 30 - 70Hz$ with $\Delta f = 2Hz$. In drawing the spectral coverages, we assume that the data are collected along a line with length of $800m$ and

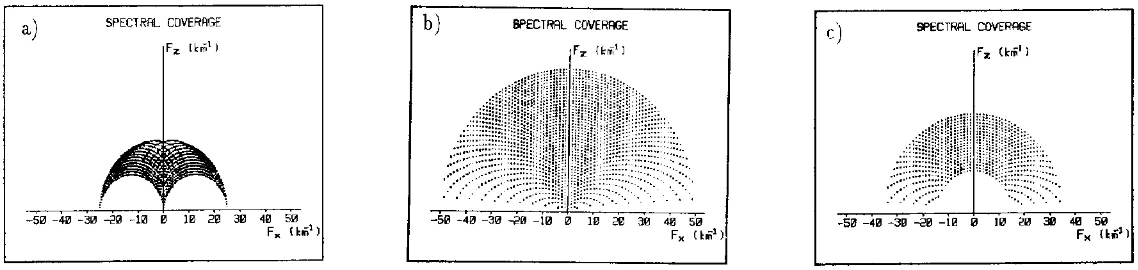

Fig. 1 Comparison of the spectral coverages: a) single frequency diffraction tomography with $f_0 = 50Hz$; b) multi-frequency backscattering tomography with $f = 2 - 100Hz$; c) multi-frequency backscattering tomography with $f = 30 - 70Hz$.

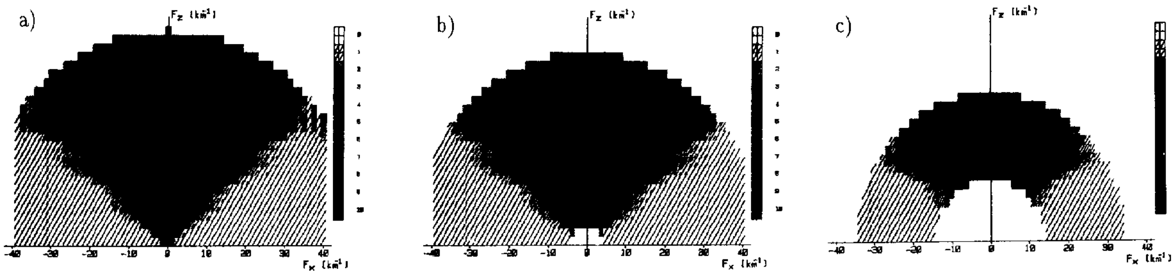

Fig. 2 The object spectra derived by MFBT for a point scatterer: a) $f = 2 - 100Hz$; b) $f = 10 - 90Hz$; c) $f = 30 - 70Hz$.

that the background velocity is $4000m/s$. Therefore, if the frequency band of the signal is broad enough, the spectral coverage of MFBT is expected to be much better than SFDT, resulting in a better image quality.

RECONSTRUCTION METHODS

a). Reconstruction by Backpropagation

In MFBT, we choose $k_g = k_s$ for each frequency (or wavenumber k), and change the integration variables from (K_x, K_z) to (k_s, k). Since in this case

$$K_x = 2k_s \ , \quad K_z = -2\gamma_s = -2\sqrt{k^2 - {k_s}^2} \ , \tag{5}$$

we have the Jacobian

$$J(K_x, K_z \mid k_s, k) = \frac{\partial(K_x, K_z)}{\partial(k_s, k)} = \frac{-4k}{\gamma_s} \ . \tag{6}$$

Substituting (1), (5) and (6) into (4), we obtain

$$O(x,z) = -\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\tilde{U}(k_s,k)\,\frac{16\gamma_s}{k}\,e^{-i2\gamma_s z)}\right] \cdot e^{i2k_s x}\,dk\,dk_s \ . \tag{7}$$

If we interpolate and stretch the data into $\tilde{U}\left(\frac{k_x}{2}, k\right)$ with

$$k_s = \frac{k_x}{2} \ , \quad \gamma_x = \sqrt{k^2 - \left(\frac{k_x}{2}\right)^2} = \gamma_s \tag{8}$$

then

$$O(x,z) = -\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[8\,\frac{\gamma_x}{k}\,\tilde{U}\left(\frac{k_x}{2}, k\right)\,e^{-i2\gamma_x z}\right] \cdot e^{ik_x x}\,dk\,dk_x \ . \tag{9}$$

We see that the internal integration is in a form of inverse spatial FT in x-direction. Therefore we can use the filtered backpropagation algorithm for the image reconstruction (Devaney 1982, Wu and Toksöz 1987). First we filter the data $\tilde{U}\left(\frac{k_x}{2}, k\right)$ by a transfer function $8\,\gamma_x/k$, backpropagate to depth z using the backpropagator $e^{-i2\gamma_x z}$, and then inverse FT back to the space domain. Coherent superposition of the results from all frequencies forms the final image of MFBT.

b).Reconstruction by Direct FT

The reconstruction method of backpropagation in principle can be used for the case of vertically inhomogeneous background media. If the background medium is homogeneous, we can change the reconstruction formula (9) into a 2D-FT, similar to the case of Stolt F-K migration (see Stolt and Benson 1986). For a given $k_x = 2\,k_s$, γ_x is a function of k. So, doing the coordinate transform from k to γ_x using

$$\frac{dk}{d\gamma_x} = \frac{\gamma_x}{k} \tag{10}$$

and $\gamma_x = k_z/2$, (9) becomes

$$O(x,z) = -\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{k_z}{k_i}\right)^2\,\tilde{U}\left(\frac{k_x}{2}, k_i\right) \cdot e^{-ik_z z + ik_x x}\,dk_x\,dk_z \ , \tag{11}$$

where

$$k_i = \frac{1}{2}\sqrt{{k_x}^2 + {k_z}^2} \ . \tag{12}$$

The reconstruction formula (11) is a double FT, which can be implemented by the Fast Fourier Transform (FFT) algorithm.

Now let us compare the computation speeds of different reconstruction methods. Assume we have the same numbers of sources and receivers, i.e. $N_s = N_g$. The reconstruction method of backpropagation for MFBT will have the same order of computation speed as that for the single-frequency diffraction tomography as can be seen from (9), if $N_s \approx N_f$, where N_f is the number of frequencies used. We see that in the backpropagation method, for each k_x the computation time is mainly taken by the $N_z \times N_f$ complex multiplications. Meanwhile in the algorithm of direct FT, as seen from (11), for each k_x the numbers of complex multiplications needed are only $N_z \times log_2 N_z$ if FFT (Fast Fourier Transform) is used. Of course, there will be an interpolation time needed for each k_x. However, when N_z and N_f are large, the time saving is remarkable. For a case of $N_f = 100$, $N_z = 128$, the factor is $N_f/(log_2 N_z) \approx 14$. Compared to the single-frequency diffraction tomography, the speed factor of the direct FT reconstruction of MFBT is of about $N_z/(log_2 N_z)$. Therefore, when N_z is large the MFBT will be much faster than the SFDT. For MF holography (prestack migration) the factor becomes $N_f\, N_z/(log_2 N_z)$.

NUMERICAL TESTS

First we test the point scatterer response (spread funtion) of MFBT and compose with those of the single frequency method SFDT and other more time-consuming multi-frequency methods, such as the MF holography (prestack migration).

We put a point scatterer at $x = 400m$, $z = 300m$. The image space is limited to $800 \times 500m^2$ with $\Delta x = 25m$, $\Delta z = 20m$ and a background velocity $c_0 = 4000m/s$. The scattered field is generated by Born approximation either in (x_s, x_g, t) domain or in (x_s, x_g, f) domain with 32 sources and 32 receivers ($N_s = N_g = 32$). We reconstruct the images of MFBT using data with frequencies ranging $2 - 100Hz$, $10 - 90Hz$, and $30 - 70Hz$ respectively at interval $\Delta f = 2Hz$. Fig. 2a, b, and c show the spectra obtained from the data for these three frequency ranges respectively. It is known that the theoretical spectrum of this point object should be a uniform one. Therefore, from these figures we can see how much information has been recovered from the data for each case. The reconstructed images by MFBT for these three frequency ranges are shown in Fig. 3a, b, and c respectively. The image for the ideal case of infinite frequency range should be a perfect point. However, due to the incomplete coverage of the spectrum, the reconstructed images have not only finite sizes, but also some oscillations. We see that the resolution and the image quality are deteriorated when the frequency range is shrinked. Fig. 4 and 5 give the corresponding results by SFDT and by MF holography respectively for comparison. It is evident that the image by MFBT has a much improved resolution and image quality, together with a faster computation speed, over the single frequency method SFDT. Also we can see that MFBT keeps essentially the same merit of multi-frequency methods as compared with MF holography or prestack migration, but with much faster computation speed.

To test the resolution of MFBT for more complex objects we choose, as an example, a letter "P" composed of 12 points separated by one wavelength of the central frequency. In Fig. 6a and b are the results of MFBT reconstruction using the frequency ranges of $10 - 90Hz$ and $30 - 70Hz$ respectively. Even with the reduced frequency range of $30 - 70Hz$, the quality of image is still acceptable.

CONCLUSIONS

The use of multi-frequencies in diffraction tomography can increase the information coverage of the object spectrum and partly fill out the "blind areas" in the spectral domain for single-frequency reconstruction, and therefore improve the resolution and image quality.

Multi-Frequency Backscattering Tomography (MFBT) is a fast algorithm, especially with the direct FT reconstruction method, even faster than the single-frequency method with a factor about $N_z/log_2 N_z$. Meanwhile, the method keeps almost the same quality of image as the other multi-frequency methods, as can be seen from the comparison by numerical tests. Therefore the method is suitable for 3D image reconstruction. The image obtained can serve as an initial model for a more sophisticated nonlinear inversion.

The fast direct-FT reconstruction algorithm is preferable for the case of homogeneous background media, while the backpropagation method may be adapted to the case of vertically inhomogeneous media.

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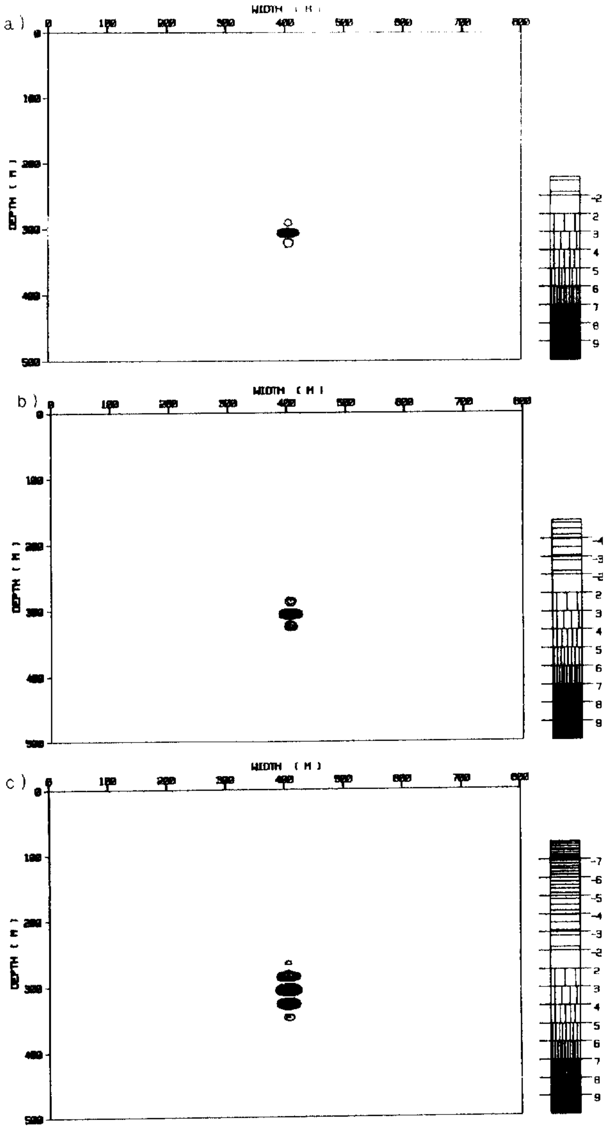

Fig. 3 The reconstructed images of a point scatterer by MFBT: a) $f = 2 - 100Hz$; b) $f = 10 - 90Hz$; c) $f = 30 - 70Hz$.

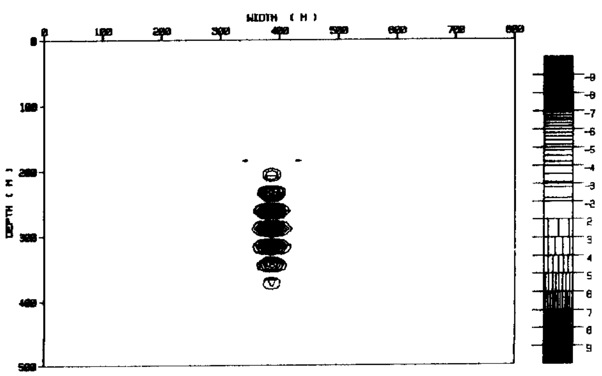

Fig. 4 The reconstructed image of a point scatterer by SFDT with $f_0 = 50Hz$.

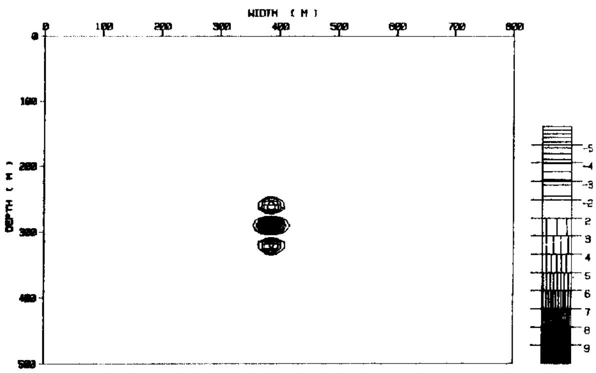

Fig. 5 The reconstructed image of a point scatterer by MF holography (prestack migration) with $f = 30 - 70Hz$.

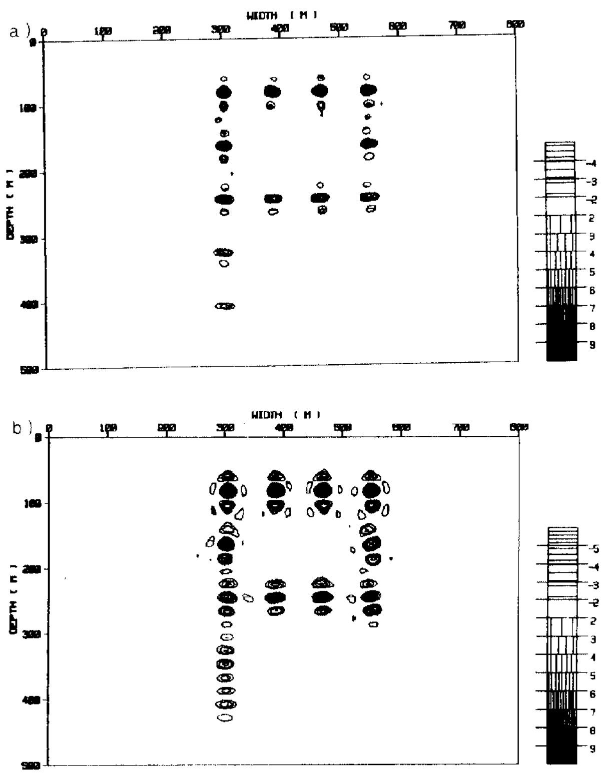

Fig.6 The reconstructed images of "P" composed by 12 discrete point scatterers separated by one wavelength (for $f_0 = 50Hz$) by MFBT: a) $f = 10 - 90Hz$; b) $f = 30 - 70Hz$.