Multigap Discrete Vector Solitons

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We analyze nonlinear collective effects in periodic systems with multigap transmission spectra such as light in waveguide arrays or Bose-Einstein condensates in optical lattices. We reveal that the interband interactions in nonlinear periodic structures can be efficiently managed by controlling their geometry. We predict novel types of discrete vector solitons supported by nonlinear coupling between different band gaps and study their stability.

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Periodic structures are common in nature, with the crystalline lattice being the most familiar example. One of the important common features of such systems is the existence of frequency gaps in the transmission spectra which can dramatically alter both propagation and localization of waves. Moreover, the modern technology allows creating different structures with an artificial periodicity, and the recent examples are photonic crystals, which can control propagation and emission of electromagnetic waves [1], and optical lattices, which are used to trap and manipulate atomic Bose-Einstein condensates (BECs) [2]. An unprecedented level of control over such engineered structures can be realized by tailoring both location and width of multiple band gaps with additional modulation of the structure parameters. For example, it has been shown that atomic BEC can demonstrate a rich variety of phase transitions in optical superlattices [3], and the reduced-symmetry photonic crystals allow self-localization of waves in minigaps [4].

The response of many systems becomes nonlinear at higher energy densities. This phenomenon may have various physical origins, such as the charge recombination in biased photorefractive crystals, excitation of higher energy levels in semiconductors, or atomic interaction in BEC. In periodic media, nonlinearity produces a shift of the band-gap spectrum, and this physical mechanism is responsible for a number of remarkable effects, including the formation of gap solitons [5]. Such nonlinear localized modes can be excited within multiple spectral gaps of a periodic structure, as was first demonstrated experimentally for temporal optical pulses in fiber Bragg gratings [6]. Since the pulses extend over hundreds of grating periods, the gap-soliton dynamics is usually described by averaged coupled-mode equations with constant coefficients [7]. In contrast, spatial optical beams in waveguide arrays and matter waves in BEC can span over a few periods of the structure. Under such conditions the wave profiles are essentially discretized by the underlying periodic structure, strongly affecting the properties of discrete solitons [8,9].

Spatial optical solitons associated with the first (semi-infinite) spectral gap of the multiband transmission spectrum have been extensively studied both theoretically and experimentally in arrays of coupled optical waveguides [8,9]. Very recently, spatial gap solitons localized in higher-order bands have been observed as well [10]. A similar observation of matter-wave gap solitons in BEC loaded into an optical lattice is also expected. However, the interaction properties of localized modes which belong to different gaps are not known. The existence of gap solitons is directly related to the band-gap spectrum, and the latter can be fine-tuned in superlattices. In this Letter, we reveal, for the first time to our knowledge, the fundamental links between periodic modulation of the medium parameters and nonlinear wave coupling between different gaps, and we also predict the existence of multigap discrete vector solitons with nontrivial symmetry and stability properties, which differ significantly from conventional discrete vector solitons [11]. We believe that our results can stimulate and guide the future experiments in optics and matter-wave physics.

Self-action and interaction of optical beams in a one-dimensional periodic structure of coupled optical waveguides with the normalized refractive index profile $V(x) = V(x + h)$ can be described by a set of coupled nonlinear Schrödinger equations,

$$i\frac{\partial \boldsymbol{\psi}}{\partial z} + \frac{\partial^2 \boldsymbol{\psi}}{\partial x^2} + V(x)\boldsymbol{\psi} + \mathbf{G}(\boldsymbol{\psi})\boldsymbol{\psi} = 0, \quad (1)$$

where, in the case of optical waveguides, x and z are the transverse and longitudinal coordinates, respectively, h is the spatial period, and $\boldsymbol{\psi} = (\psi^{(1)}, \psi^{(2)}, \ldots)^T$ are the normalized electric field envelopes of several copropagating beams having different polarizations or detuned frequencies. It is assumed that the beams interact incoherently through the Kerr-type nonlinear change of the refractive index, $G_{mm} = \sum_j \Gamma_{mj} |\psi^{(j)}|^2$, where $\Gamma_{m=j}$ and $\Gamma_{m\neq j}$ are the self- and cross-phase modulation coefficients, respectively.

We note that the model (1) is equivalent to a system of coupled Gross-Pitaevskii equations describing the dynamics of multicomponent BEC in 1D optical lattice, where $\psi^{(m)}$ is the mean-field wave function for atoms in the mth quantum state, z stands for time, $V(x)$ is the periodic potential of an optical lattice, and $\mathbf{G}$ is the effective mean-field nonlinearity which appears due to the s-wave atom interaction. Although below we use the terminology from the guided wave optics, our results are equally applicable to the nonlinear dynamics of BEC in optical lattices.

Linear wave propagation through a periodic structure can be entirely described by the Floquet-Bloch spectrum of the eigenmode solutions of Eqs. (1) in the form $\psi = \psi_b \exp(i\beta z + iK_b x/h)$, where K_b is the normalized Bloch-wave number and β is the propagation constant. The Bloch-wave amplitudes decay exponentially when $\mathrm{Im}K_b(\beta) \neq 0$, and this condition defines the location of gaps in the transmission spectrum. At the gap edges, $K_b = 0, \pi$. Nonlinearity manifests itself through an effective change of the optical refractive index which results in a local shift of the bands and gaps. As we demonstrate below, this physical mechanism is responsible for the formation of multigap solitons.

Let us first consider small-amplitude solitons, so that the band shifts are small. Then, we can seek solutions of Eqs. (1) near the gap edges ($\beta = \beta_m$) in the form of modulated Bloch waves [12], $\psi^{(m)} = \varphi^{(m)}\psi_b^{(m)} \times \exp(i\beta_m z + iK_b^{(m)} x/h)$, and derive a system of coupled nonlinear Schrödinger equations for the slowly varying envelopes,

$$i\frac{\partial \varphi^{(m)}}{\partial z} + \frac{D^{(m)}}{2}\frac{\partial^2 \varphi^{(m)}}{\partial x^2} + \sum_j \gamma_{mj}\Gamma_{mj}|\varphi^{(j)}|^2\varphi^{(m)} = 0. \quad (2)$$

Here $D^{(m)} = -h^2\partial^2\beta/\partial K_b^2|_{\beta_m}$ are the effective diffraction coefficients, and $\gamma_{mj} = \int_0^h |\psi_b^{(m)}\psi_b^{(j)}|^2 dx$ characterize nonlinear coupling between different bands, where we assume the normalization $\int_0^h |\psi_b^{(m)}|^2 dx \equiv 1$.

Model (2) is known to possess multicomponent soliton solutions, and this proves the existence of multigap solitons where every component is localized in a different gap. It also follows that a soliton always supports multiple guided modes in other band gaps. We note that the diffraction coefficients $D^{(m)}$ are positive near the lower gap edges, and negative at the upper edges [13], and, therefore, multigap solitons can contain both the bright and dark components in the nonlinear media with either self-focusing or self-defocusing nonlinearities. The simplified model (2) predicts stability of bright vector solitons when all Γ_{mj} have the same sign [14].

We now demonstrate the unique properties of multigap solitons which distinguish them from conventional vector solitons. The soliton components are coupled together through nonlinear interband interactions, and we show below that this coupling can be enhanced or suppressed in engineered superlattices. Moreover, we find that the nonlinear coupling effectively depends on the soliton power and width, and location of the soliton center with respect to the periodic structure.

As an example, we consider a binary superlattice where the effective periodic potential is composed of two types (A and B) of separated individual potential wells, $V(x) = \sum_n [V_A(x + nh) + V_B(x + nh)]$, and $\Gamma_{mj} = \Gamma$. Such superlattices can be produced by etching waveguides on top of a AlGaAs substrate [15], or induced dynamically by two overlapping mutually incoherent interference patterns in a photorefractive medium [16]; see Figs. 1(a) and 1(b). In order to analyze the properties of nonlinear waves of such superlattices, we employ the tight-binding approximation [17,18]. This approach allows us to describe correctly the first two spectral bands, and the "superlattice" gap which appears due to a difference between the A- and B-type lattice sites, whereas the applicability of Eqs. (2) is limited to small-amplitude solitons in the vicinity of the gap edges. We present the total field as a superposition of the guided modes supported by individual potential wells (ψ_A and ψ_B), $\boldsymbol{\psi}(x, z) = \sum_n [\mathbf{a}_n(z)\psi_A(x - nh) + \mathbf{b}_n(z)\psi_B(x - nh)]e^{Ikz}$, where k is the average propagation constant, and derive a normalized system of coupled discrete equations for the mode amplitudes,

$$\begin{aligned} i\frac{d\mathbf{a}_n}{dz} + \rho\mathbf{a}_n + \kappa^{-1}\mathbf{b}_{n-1} + \kappa\mathbf{b}_n + \chi^{(a)}|\mathbf{a}_n|^2\mathbf{a}_n &= 0, \\ i\frac{d\mathbf{b}_n}{dz} - \rho\mathbf{b}_n + \kappa\mathbf{a}_n + \kappa^{-1}\mathbf{a}_{n+1} + \chi^{(b)}|\mathbf{b}_n|^2\mathbf{b}_n &= 0. \end{aligned} \quad (3)$$

Here, the key characteristics of the binary superlattice are defined by free parameters: ρ is proportional to the detuning between the propagation constants of the A- and B-type guided modes, κ characterizes the relative coupling strength between the neighboring wells on the right- and left-hand sides, and $\chi^{(a,b)} = \chi\Gamma \int_{-\infty}^{+\infty} |\psi_{A,B}|^4 dx$ (where $\chi > 0$) are the nonlinear coefficients.

According to Eqs. (3), the linear Bloch-wave dispersion is defined as $K_b = \cos^{-1}(-\eta/2)$, where $\eta = \kappa^2 + \kappa^{-2} + \rho^2 - \beta^2$. The transmission bands correspond to real K_b, and they appear when $\beta_- \leq |\beta| \leq \beta_+$, where $\beta_\pm = (\kappa^2 + \kappa^{-2} + \rho^2 \pm 2)^{1/2}$. A characteristic dispersion relation and the corresponding band-gap

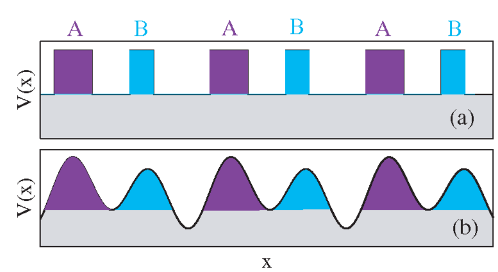

FIG. 1 (color online). Examples of a binary superlattice which can be created by (a) an array of two types of coupled waveguides or (b) an optical superlattice induced by two overlapping mutually incoherent interference patterns.

structure are presented in Fig. 2(a). The upper gap at $\beta > \beta_+$ is due to the effect of total internal reflection (IR), whereas additional gaps appear due to the resonant Bragg reflection (BR).

It is well known that the material dispersion can be completely compensated by the geometrical dispersion in optical fibers. More recently, diffraction management was realized in periodic waveguide arrays [19]. The question is whether it is possible to control nonlinear coupling between the gaps by appropriate design of periodic structures. In order to answer this fundamental question, we study the dependence of the nonlinear coupling coefficients on the superlattice parameter ρ, while preserving exactly the same linear dispersion of Bloch waves. Our results are presented in Fig. 2(b), and they uncover the remarkable feature: nonlinear interband interaction coefficients strongly depend on the symmetry of the periodic structure, and this relation cannot be fully characterized just by the linear Bloch-wave dispersion. Thus, by changing the superlattice parameters it is possible to selectively enhance or suppress interband interaction, and this can be used, in particular, to control the properties of multigap solitons.

To be specific, we now consider the soliton formation in a superlattice with symmetric intersite coupling ($\kappa = 1$) between wide (A) and narrow (B) waveguides in a self-focusing medium. Such a lattice can support two fundamental types of one-component bright solitons centered at either A or B sites, and these solitons exist in both the IR and BR gaps described by model (3). We find that the powers of A- and B-type solitons become significantly different away from the band edges; see Fig. 3(a). The solitons of type B in the IR gap are always unstable with respect to a translational shift (symmetry breaking); however, the stability is reversed for discrete gap solitons in the first BR gap [left part of Fig. 3(a)] where type-A solitons become unstable. Additionally, the discrete gap solitons become oscillatory unstable above a critical power due to (i) internal resonance within the gap, first discovered for the fiber Bragg solitons [20], and (ii) interband resonances, first found for nonlinear defect modes in a periodic medium [21].

The mutual trapping of the modes localized in different gaps and the physics of multigap vector solitons can be understood in terms of the soliton-induced waveguides. Therefore, the effect of discreteness on the intergap coupling can be captured by studying the guided modes supported by a scalar soliton in other gaps: the larger is the eigenvalue shift from the band edge; the stronger is the interaction. In Figs. 3(b) and 3(c), we plot the eigenvalues of the guided modes supported by the BR (gap) and IR solitons and observe two remarkable features which cannot be captured by the simplified envelope approximation (2). First, the strength of the intergap coupling depends strongly on the soliton symmetry. Indeed, the type-B soliton always creates a stronger effective waveguide, despite the fact that the soliton power in the BR gap is smaller compared to the type-A solitons. Second, the nonlinear intergap coupling decreases for strongly localized discrete solitons in the IR regime,

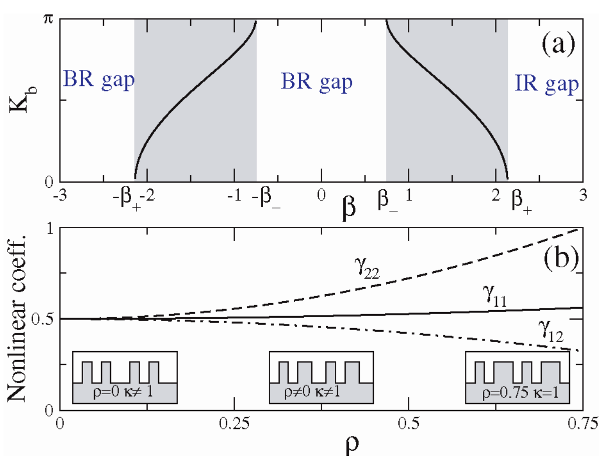

FIG. 2 (color online). (a) Characteristic dependence of the Bloch wave number (K_b) on the propagation constant β. Gray shadings mark the transmission bands. (b) Dependence of the normalized nonlinear coupling coefficients [$\gamma_{mj} = |a^{(m)}a^{(j)}|^2 + |b^{(m)}b^{(j)}|^2$, where $\{a, b\}^{(m,j)}$ are the Bloch-wave solutions of Eq. (3), and it is assumed that $\chi^{(a)} \simeq \chi^{(b)}$] between the gap edges $\beta_1 = \beta_+$ and $\beta_2 = -\beta_-$ vs the parameter ρ. The values of $\beta_\pm$ correspond to the plot (a) by a proper choice of κ. The insets show possible symmetries of superlattices corresponding to different parameter values, but the same linear dispersion.

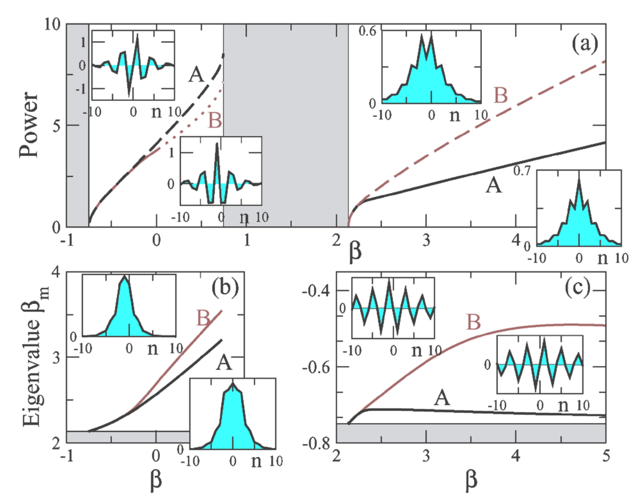

FIG. 3 (color online). (a) Power vs propagation constant for discrete solitons centered at A (black) and B (gray) lattice sites. Solid line: stable, dashed line: unstable, and dotted line: oscillatory unstable modes. (b),(c) Eigenvalues of the guided modes supported by the discrete solitons localized in the complimentary gap. The insets show characteristic profiles of solitons and their guided modes defined as $u_{2n} = a_n$ and $u_{2n+1} = b_n$. The array parameters are $\rho = 0.75$, $\kappa = 1$, and $\chi^{(a,b)} = 1$

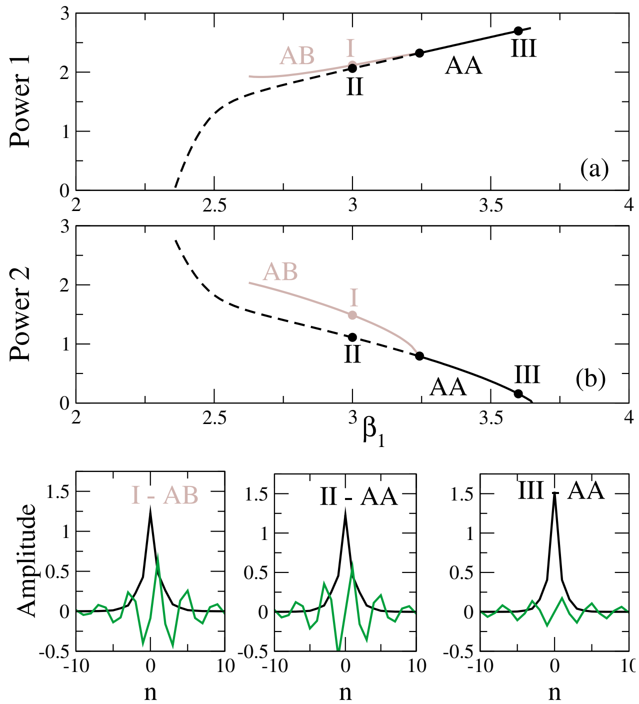

FIG. 4 (color online). (a),(b) Powers in the IR (a) and BR (b) components of the multigap discrete vector solitons vs propagation constant β_1 with $\beta_2 = 0.5 - \beta_1/3$ for the families with symmetric (*AA*) and asymmetric (*AB*) profiles. Dashed lines mark solutions exhibiting symmetry-breaking instability. Bottom: Characteristic profiles of the vector solitons composed of the components localized in two different gaps: IR: unstaggered; BR: staggered. Propagation constants correspond to the points marked as I, II, and III in the upper plots. The array parameters match Fig. 3.

as follows from the nonmonotonic dependence of the gap-mode eigenvalues shown in Fig. 3(c).

The eigenvalues of the linear guided modes define the point where a multigap vector soliton bifurcates from their scalar counterparts. Initially the amplitude of the guided mode is very small, but it increases away from the bifurcation point, and the mode interacts with the soliton waveguide creating a coupled intergap state; see Fig. 4. In the vicinity of the bifurcation point, the soliton symmetry and stability are defined by the large-amplitude soliton component. For example, the *AA*-type discrete vector soliton shown in Fig. 4(III) is stable because the powerful *A*-type mode in the IR gap suppresses instability of the second component. However, as the power in the second component grows, the soliton properties change dramatically: (i) the *AA* state becomes unstable, and at the same time (ii) a stable *AB*-type asymmetric vector soliton emerges; see modes II and I in Fig. 4 (bottom), respectively. These complex existence and stability properties underline a nontrivial nature of nonlinear intergap coupling between the localized components with different symmetries.

We note that such a simple way to engineer nonlinear coupling in the superlattices can lead to novel effects in the soliton collisions. Both coherent interaction of solitons from different gaps and vector solitons with the components from several gaps can be controlled by engineering the superlattice parameters, thus leading to novel features in the soliton switching and steering.

In conclusion, we have studied nonlinear coupling and localization in periodic systems with multigap transmission spectra. We have predicted the existence of novel types of multigap vector solitons and studied their stability. Using the example of a binary waveguide array, we have demonstrated the basic concepts of the engineering of nonlinear interband interactions in such structures, which in turn determine the key soliton properties. We note that multigap solitons were also predicted independently [22].

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