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MULTILEVEL CROSSING RATES FOR AUTOMATED SIGNAL CLASSIFICATION

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Abstract

An investigation was made of multilevel crossing rates as a means of time series analysis of random signals. Pattern recognition techniques based on the Mahalanobis distance were implemented as a means of evaluating the discriminating power of level crossings. Measurement of multilevel crossing rates was found to be an easily implementable means for detection of changes in general frequency content. Level crossing analysis was also shown to be applicable for the study of conductivity measurements of two-phase flow of air and water, where knowledge of the relationship between amplitude and frequency was beneficial in characterizing the process.

I. Introduction

Multilevel crossing analysis represents an approach to interpretation and characterization of time signals by readily relating frequency and amplitude information. In particular, measurement of a signal's level crossing rate, defined to be the number of crossings per unit time of a level, yields information as to the "apparent frequency" associated with that level. Observation of the crossing rates for a number of levels is a procedure that may be easily implemented whereby the crossing behavior may be categorized with respect to the amplitude of the signal. Regardless of the applicability of level crossings, it would not be unreasonable to assume that much of the effort in the literature may be due to the vast number of problems which remain unsolved [1-13] 5

The objective of this investigation was to use multilevel crossing rates for characterizing random processes and to use pattern recognition techniques to distinguish among level crossing observations. In general, a finite number of pattern classes are defined such that a classifier assigns the signal to one of the classes. The ability of the classifier to distinguish among classes is based upon the information obtained through the processing of sample data from the classes during a training period.

Two types of signal classification problems are considered in the following discussion. The first is the classical surveillance problem in which one is interested in classifying an input as either normal or abnormal and the only available sample data for training are from the normal signal. Therefore, the typical approach is to establish bounds of normal behavior for the signal and to classify as abnormal any observation outside these bounds. The second classification problem considered will be referred to as the multiclass case, in which sample data for training are available for the characterization of all defined classes. The task of the classifier is to assign each observation to the class whose features most resemble the observation.

The classification technique employed is the Mahalanobis distance, which is a measure of similarity between an observation and a class characterized by the training data. Thus the task of discriminating multivariate observations is reduced to evaluation of a scalar quantity. Appropriate restrictions are considered in using the Mahalanobis distance in light of the fact that each observation is a set of level crossing rates. The Mahalanobis distance approach has been successfully applied to a variety of problems in the field of surveillance of nuclear reactors [~~14-18~~]. 6-10

II. Level Crossing for Continuous Random Processes

The expected crossing rate of level u by a continuous, stationary, and ergodic random signal, x(t), is given by [3] 2 6

$$\overline{N}_u[x;1] = \int_{-\infty}^{\infty} |v| p_{x,x'}(u, v)dv \qquad (1)$$

where

where $p_{x,x'}(u, v)$ is the joint probability density function of x(t) and the first derivative, $x'(t)$. By the assumption of stationarity, the number of crossings during time T is obtained from

$$\overline{N}_u[x;T] = T\,\overline{N}_u[x;1]. \qquad (2)$$

Observation of the number of crossings of level u during time T by a sample function provides an estimate of the crossing rate, such that [3]

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$$N_u[x; 1] = \frac{1}{T} \sum_{i=1}^{n} Z_u[x; I_{mi}] \qquad (3)$$

where

$$Z_u[x; I_{mi}] = \begin{cases} 1 \text{ for } [x(\frac{i-1}{m})-u][x(\frac{i}{m} T)-u] < 0 \\ 0 \text{ otherwise} \end{cases} \qquad (4)$$

and m subintervals are defined by

$$I_{mi} = [\frac{i-1}{m} T, \frac{i}{m} T], \quad i = 1, 2, \ldots, m. \qquad (5)$$

If an estimate of the crossing rate is obtained for each member of a set of levels, Eq. (1) may be approximated by a set of level crossing rates, called a level crossing profile [10], or LCP.

Since the number of crossings of a level must be a non-negative integer (finite for any physically realizable process), the observed crossing rate of any level within the bounds of the process x will be a non-negative, discrete random variable. The restriction of the level to the bounds insures that the observed crossing rate will have a non-zero variance. Since N_u is discrete, its corresponding probability density will be a sequence of impulses. Fig. 1 is an example of an experimentally determined histogram approximation of $p_u(N_u)$ for some level u. The probability of observing a particular crossing rate of that level is equal to the magnitude of the impulse at that crossing rate. The smaller impulses which appear alternately represent those observations in which there occurred an odd number of crossings. In terms of the sample record, the endpoints were on opposite sides of the level, so that the number of upcrossings did not equal the number of downcrossings. As the level approaches the mean, it can be expected that, for sufficiently long T, the probability of observing an odd number of crossings approaches the probability of observing an even number of crossings. Thus an envelope of the probability density impulses would tend toward a smooth curve.

III. Generation of the LCP

Figure 2 shows $\hat{x}_\ell(t, T_s)$, a linearly interpolated approximation to a signal $x(t)$, sampled every T_s seconds, and digitized (denoted by "ˆ") at each sample point to an integer value in the range 0 to $2^M - 1$, M representing the number of bits used in the discrete representation. This approximation to x(t) is the waveform for which the LCP will actually be calculated. Once sampling and digitization have been performed, the level crossing point for each level which has a value between the amplitudes of consecutive sample points is incremented by one. The total number of counts for each level is divided by the time of each record to give the level crossing count per unit time, or estimated crossing rate.

If a digitized sample point at time t_i equals the value of an assigned level L_j, further analysis of the signal may be required. If the sample points at times t_{i-1} and t_{i+1} are such that $[\hat{x}_\ell(t_{i-1}, T_s) - L_j] \cdot [\hat{x}(t_{i+1}, T_s) - L_j] < 0$, then L_j has been crossed and the count should be incremented. Otherwise, a minimum or a maximum occurred in the interval of resolution about the level and the decision to increment the crossing count will depend upon the round-off or truncation characteristics of the digitizer.

Unlike the histogram approach to approximating the probability density function (PDF) for which only the amplitude bin in which a sample point occurs is incremented, the LCP algorithm linearly interpolates between sample points. The difference in philosophy is due to the fact that the probability density function $p_x(u)$ is a descriptor of the relative amount of time x spends in the interval u, u + du, whereas the level crossing rate, $\overline{N}_u$ is a measure of the number of times x crosses level u. Obviously, if a continuous x(t) lies between u_j, u_{j+1} at time t_i and between u_k, u_{k+1} at time t_{i+1}, with $u_j < u_{j+1} < u_k < u_{k+1}$, then x(t) must have crossed levels u_{j+1} through u_k at least once between times t_i, t_{i+1}. The probability that each of these levels was crossed only once approaches one as the rate of sampling is increased. However, in approximating $p_x(u)$, the relative amount of time x(t) spends in all bins between times t_i, t_{i+1} is regarded as insignificant, so that only bins with sample points are considered. This results in a relatively smooth LCP as opposed to the often jagged histogram approximation of the PDF, as illustrated in Fig. 3 for a Gaussian process.

The manner with which levels were chosen for the experiments of this investigation was heuristic. Some general guidelines which were found to be useful are as follows:

1) Spacing between levels is uniform.
2) The bounds on the amplitude of the levels are chosen such that all levels are crossed at least once during the training period, as a necessary condition for implementation of the Mahalanobis distance.
3) The number of levels is kept moderate, typically 15 to 25, to hold computations involving the pattern recognition techniques to a reasonable amount of time.

As illustrated in Fig. 4 for a pure sine wave, level crossing counts will be missed when the signal is sampled near the Nyquist rate. If the amplitude of the sine wave is random, then regardless of the sampling rate, the expected crossing rate of the sampled version will always be less than of the

count

continuous signal. There will be a missed count whenever a local maximum of the continous signal occurs above a level while the preceding and following sample points are below the level. An anologous statement may be made for a local minimum.

As a general guide to sampling, one approach is to represent a band-limited signal, $x(t)$, by its Fourier series. Consider the highest frequency component, f_h, to give the cosine term,

$$x_h(t) = a \cos 2\pi f_h t. \qquad (6)$$

In reference to Fig. 5, as a necessary condition for sampling, no more than one level may be missed by the linearly interpolated, sampled version, $x_{h,\ell}(t, T_s)$. In other words, if $x_h(t)$ crosses L_n, then $x_{h,\ell}(t, T_s)$ must cross L_{n-1}. It follows from Fig. 5 that the shortest sampling time is required when $a_h = L_n$. If n represents the number of uniformly spaced levels of positive amplitude (starting with n = 0 at the zero level), then the maximum sampling time may be given in terms of the number of levels, such that

$$T_s \leq t_o = \frac{1}{\pi f_h} \arccos \left(\frac{n-1}{n}\right). \qquad (7)$$

This choice of T_s may be smaller than necessary to ensure that $x_\ell(t, T_s)$ meets the same condition for sampling as $x_{h,\ell}(t, T_s)$. However, the analysis was made under worst case assumptions to provide some general criterion for sampling of any signal.

IV. Classification Approach

The level crossing profile may be represented as a vector

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad (8)$$

where x_j is the crossing rate [Eq. (3)] of the jth level by the input signal.

A measure of similarity between a vector x and the mean vector of the ith class, denoted by m_i, is given by [6]

italic

$$D_i(x) = (x - m_i)^T C_i^{-1} (x - m_i) \qquad (9)$$

where superscript "T" denotes transposition, and C_i is the covariance matrix of the ith class. Both m_i and C_i may be estimated from the vectors of the training set [6].

For the two class case of normal and abnormal, m and C are estimated for the normal population only and the class subscripts may be dropped. Following training, however, whenever an observation yields a value of D(x) greater than some non-negative T, the input is classified as abnormal; otherwise a decision of normal is given. For $D(x) = T$, a multidimensional ellipse, or hyperellipsoid, is generated about m, with the axes oriented to the directions of maximum variance. Consequently, only those vectors with coordinates lying within the hyperellipsoid are designated as normal. For the multiclass case, m_i and C_i must be estimated for each class. During classification, a separate Mahalanobis distance between the mean of each class, m_i, and the observation x is to be computed, after which x is assigned to the class yielding the smallest distance.

Implementation of the Mahalanobis distance is optimum when a class possesses a multivariate Gaussian distribution. As illustrated in Fig. 1, the observed crossing rate may not be Gaussian; however, the Mahalanobis distance may still be useful when the mean and the variance are regarded as significant descriptors of the probability density.

The threshold, T, may be selected using one of several criteria such as a chi-square distribution [9], a maximum-distance calculation using the training set [9], or receiver operating characteristic (ROC) curves [12].

V. Experimental Results

The following experiments were conducted to evaluate the performance of the level crossing/Mahalanobis distance approach under a variety of practical conditions.

Reactor Experiment

A signal from a neutron detector taken at the Oak Ridge National Laboratory High-Flux Isotope Reactor (HFIR) was analyzed to determine whether level crossings could be used to detect the addition of a small amplitude noise signal (bandwidth 3.5-4.5 Hz.) applied as a demand to the control rod servo system. Previous investigations of this situation have been carried out elsewhere [~~16~~7-10] by frequency analysis.

The signal was lowpass filtered at 10 Hz. and sampled every 0.008 seconds with 2048 sample points per observation. Twenty-one levels were selected for the LCP, spaced throughout the full range of the signal so that levels were also placed at extreme amplitudes. There were 1200 LCPs for the training set and 59 LCPs for the abnormality.

The system classified all the observations from the training set as normal, and 57 of the 59 abnormal signals were properly flagged as being beyond the limits of normal behavior. The classification threshold was selected as the maximum Mahalanobis distance found in the training set.

Two-Phase Flow Experiment

The term two-phase flow refers to the flow of two immiscible fluids, which in this experiment happen to have been air and water. Three major types of flow are often considered: 1) bubbly, 2) slug, or large bubble, and 3) annular, where a ring of water surrounds the air. The percentage of air within a volume is known as the void fraction. It is of interest in the nuclear industry to be able to determine the type of flow, the void fraction, or any other relevant information concerning the flow. A review of experimental methods in this area is provided by Hewitt and Lovegrove [13].

One of these methods is the measurement of the relative conductivity across the flow. For a particular type of flow, the conductivity decreases as the void fraction increases. Shown in Fig. 6 are some theoretical curves of the relative conductivity versus void fraction for the three types of flow mentioned above [14]. If the conductivity and the type of flow are known, then it should be possible to determine the corresponding void fraction from the curves.

For this experiment, measurements of the conductivity between a pair of electrodes were obtained [15] for 3%, 25%, and 50% void fractions. Figure 7 illustrates sample voltage waveforms representative of the time measurements of conductivity for the three void fractions. Each descending spike indicates the immediate presence of an air slug or bubble and the depth of the spike is indicative of the size of the slug. The smaller spikes result from bubbly flow. Each signal illustrated in Fig. 7 is the result of the original conductivity measurement having been AC coupled to remove the DC component first and then amplified to increase the dynamic range of recording. The DC value removed and the gain employed were different for each case. As the voide fraction increased, the DC value decreased, due to the low overall conductivity, and the gain of each signal increased. Thus, in terms of the curves of Fig. 6, each waveform reflects the activity about a mean conductivity level. The magnitude of the activity, that is, the size of any spike, is relative to the individual signal only since the gains were different. The crossing rate at any level by the descending spikes indicates the number of air slugs of a particular width or greater which passed between the electrodes in a unit amount of time.

A training period was established whereby 70 profiles were generated and processed for each class of void fraction. The signals were lowpass filtered at 100 Hz. and seventeen levels were chosen, ranging from -4 to 0. This range was set to ensure crossings of all levels by all three signals as a necessary condition for the existence of an inverse covariance matrix for each class. The estimated mean LCP of each class is shown in Fig. 8. Following training, pairwise classification was performed on each LCP of the training set, whereby a Mahalanobis distance was computed from the mean of each class to the LCP. The signal was assigned to the class yielding the smallest distance. Figure 9 shows the three Mahalanobis distances computed for each LCP of the 3% void fraction. Class separation is clearly evident since the distances from the classes of the 25% and 50% void fractions are one to two orders of magnitude greater than the distances from the class of the 3% void fraction. Similar results were found to be valid for the classification of the LCPs of the 25% and 50% void fractions.

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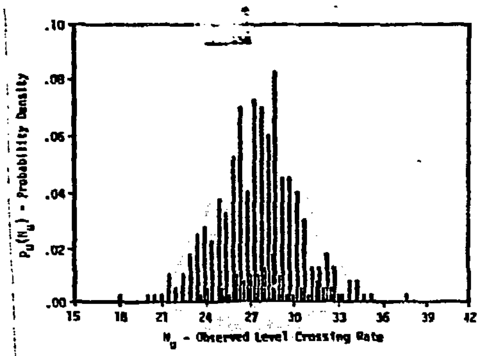


Figure 1. Probability density of observed level crossing rate.

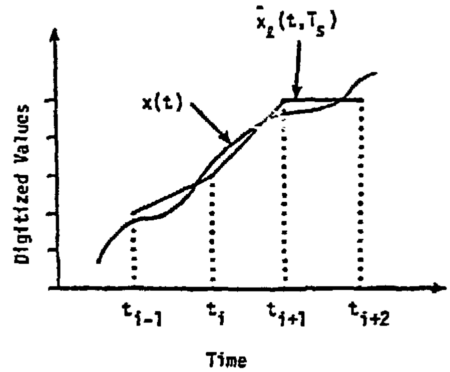
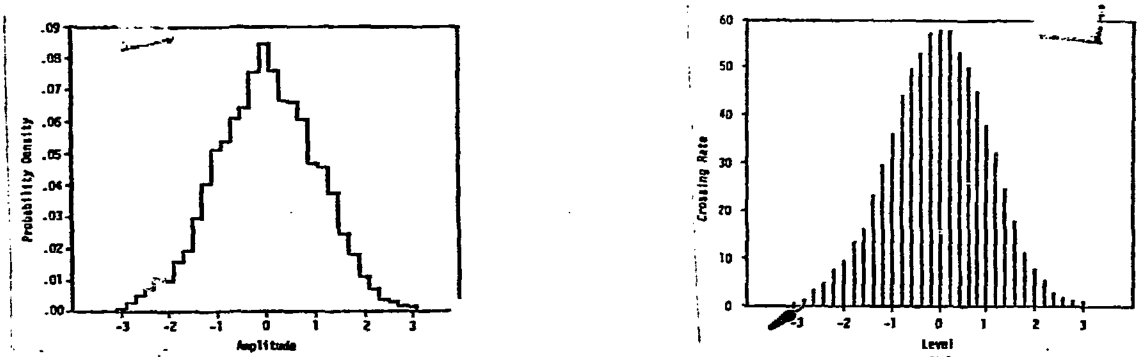


Figure 2. Digitized, linearly interpolated, sampled signal.



(a) (b)

Figure 3. Examples of (a) probability density histogram and (b) level crossing profile.

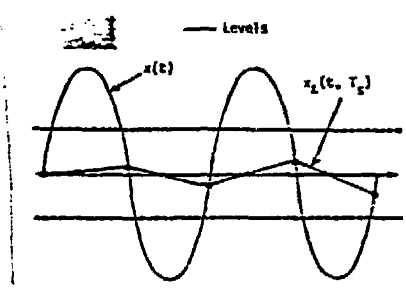


Figure 4. Missed crossings by sampling near the Nyquist rate.

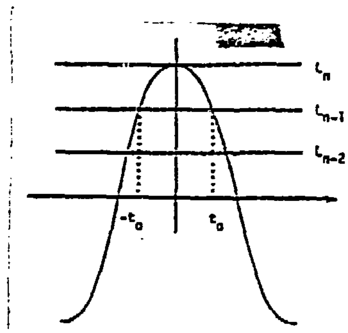


Figure 5. Level crossings by highest frequency component of a random signal.

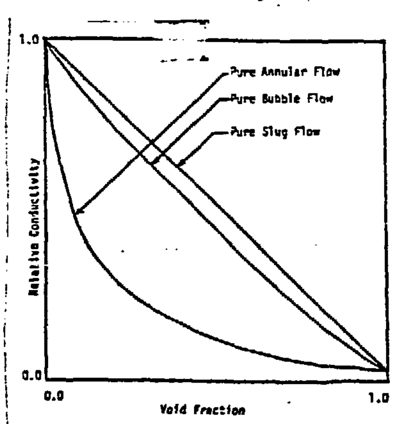
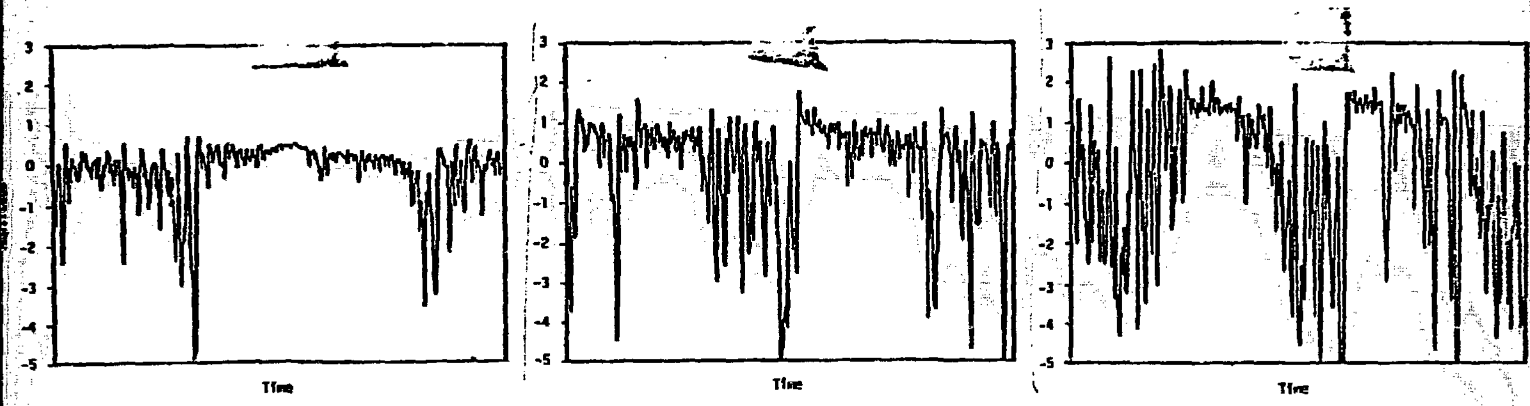
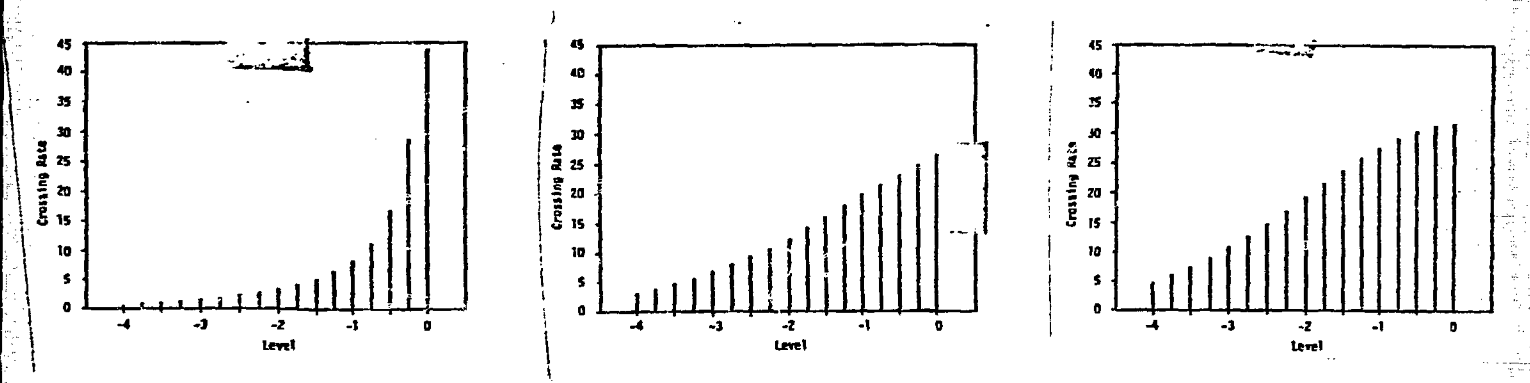


Figure 6. Relative conductivity vs. void fraction curves [14].



(a) (b) (c)

Figure 7. Conductivity measurements of two-phase flow for void fractions of (a) 3% (b) 25% (c) 50%.



(a) (b) (c)

Figure 8. LCP's for conductivity measurements of two-phase flow for void fractions of (a) 3% (b) 25% (c) 50%.

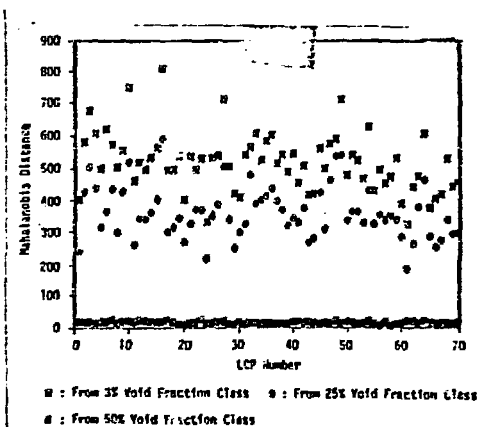


Figure 9. Mahalanobis distances to LCP's of 3% void fraction.