ORIGINAL RESEARCH

Multiobjective centralized DEA approach to Tokyo 2020 Olympic Games

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Abstract

There exist two types of Data Envelopment Analysis (DEA) approaches to the Olympic Games: conventional and fixed-sum outputs (FSO). The approach proposed in this paper belongs to the latter category as it takes into account the total number de medals of each type awarded. Imposing these constraints requires a centralized DEA perspective that projects all the countries simultaneously. In this paper, a multiobjective FSO approach is proposed, and the Weighted Tchebychef solution method is employed. This approach aims to set all output targets as close as possible to their ideal values. In order to choose between the alternative optima, a secondary goal has been considered that minimizes the sum of absolute changes in the number of medals, which also renders the computed targets to be as close to the observed values as possible. These targets represent the output levels that could be expected if all countries performed at their best level. For certain countries, the targets are higher than the actual number of medals won while, for other countries, these targets may be lower. The proposed approach has been applied to the results of the Tokyo 2020 Olympic Games and compared with both FSO and non-FSO DEA methods.

1 Introduction

Data Envelopment Analysis (DEA) is a non-parametric methodology for the assessment of the relative efficiency of a set of homogeneous operating units for which the term Decision-Making Unit (DMU) is generally used. These DMUs are assumed to consume inputs in order to produce outputs. Data Envelopment Analysis can compute the relative efficiency of a DMU by looking if it is feasible to reduce its input consumption (without reducing its outputs), or whether it is feasible to increase its outputs (without increasing its inputs). If that is the case, then the DMU is said to be inefficient, and an efficiency score that takes into account the excess input consumption and the shortfall in output production is computed.

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Furthermore, a target that reflects the estimated input and output improvements is provided for each DMU. This is the basic DEA methodology, which, depending on the application, may have specific refinements. In particular, in the case of applying DEA to the Olympic Games, the inputs usually considered are the Population and the Gross Domestic Product (GDP) of the participating nations. These inputs are considered non-discretionary and hence cannot be reduced (see Banker & Morey, 1986). Another feature that needs to be taken into account in DEA Olympics applications is the higher value of gold medals with respect to silver medals and of silver medals with respect to bronze medals. In this respect, note that the official ranking of participating National Olympic Committees (NOCs) is of lexicographic type whereby NOCs are ranked based on the number of gold medals won, using silver medals and bronze medals, in that order, to break ties. This lexicographic approach is non-compensatory and implicitly means that a gold medal is worth more than any number of silver medals and that a silver medal is worth more than any number of bronze medals.

Another drawback of the official ranking is that it is based on the absolute results, not on the relative efficiency of the countries, that is, it fails to take into account the size and the wealth of the countries. Data Envelopment Analysis approaches carry out a more objective assessment of NOC performance by bearing these factors in mind, thereby estimating their efficiency in relative terms.

As can be observed in the literature review presented in the next section, a number of DEA approaches have been proposed to analyse the efficiency of NOCs in Olympic Games. They can be divided into two categories: those that take into account that the total number of medals is fixed and those that do not. The latter generally result in DEA projections (i.e., output targets) that greatly exceed the total number of medals available. This problem is not unique to Olympic Games and affects not only DEA. Collier et al., (2011) argue that all deterministic methods that measure technical efficiency in sports using the distance to the efficient frontier of the production technology inferred from the observations suffer from serial correlation, that is, the efficiency estimates are biased downward. This also happens in football and basketball competitions, for example. Assuming that certain teams are inefficient, the removal of their inefficiency would result in increasing their number of wins, but if that happened, then the most efficient teams would be unable to maintain their total number of wins that were partly due to the inefficient performance of their rivals. Collier et al., (2011) proposes a correction that downwardly adjusts the efficient frontier to account for this serial correlation. They argue that additive DEA models are preferable to multiplicative or proportional adjustment approaches. Their model, however, considers that only one of the outputs (e.g., number of wins) is serially correlated while the other outputs are not. In Olympic Games, however, all three outputs generally considered, namely number of gold, silver, and bronze medals won, are fixed-sum outputs (FSO). The imposition of these three FSO constraints leads to centralized DEA models (when an envelopment formulation is used) or to DEA models with a common set of weights (CSW) (in the case of multiplier formulations).

In this paper, a multiobjective FSO DEA approach is proposed. Recall that in multiobjective optimization, the different objective functions are in conflict with each other, that is, improving one generally implies worsening another. By definition, this is what happens in the corresponding Pareto optimal set. Hence, in the case of Olympic Games efficiency analysis, a multiobjective perspective seems appropriate since all countries want to increase their number of medals but, since the total number of these is fixed, if one country wins more medals then another must win fewer. Moreover, we consider each type of medal won by a country as a different objective function. In this regard, note that the number of medals won of each type are also in conflict with each other. Thus, it is assumed that a country aspires to win a medal in a certain sport. If it wins a gold medal, then it cannot win a silver or a bronze

medal and vice versa. Therefore, if a country wins more gold medals, it can win fewer silver and bronze medals and vice versa.

There are many multiobjective DEA approaches in the literature (e.g., Ebrahimnejad & Tavana, 2014; Estellita Lins et al., 2004; Korhonen et al., 2003; Malekmohammadi et al., 2011). The proposed multi-objective FSO DEA approach is of centralized DEA type in the sense that all the DMUs are jointly projected in a single model that imposes the three FSO constraints. In order to solve this multiobjective optimization model, the Weighted Tchebychef Method (WTM), is used, which has been used in several multiobjective DEA approaches (e.g., Arana-Jiménez et al., 2020; Despotis et al., 2016; Gutiérrez & Lozano, 2016; Lozano & Adenso-Díaz, 2018). The WTM is a common solution method in multiobjective optimization. It is based on the concept of the ideal point, that is, the point whose components are the optimal value of each of the objective functions when optimized separately. The idea behind WTM involves finding a solution whose objective function vector is as close as possible to the ideal point, using the Tchebychef metric. The WTM only guarantees a weak Pareto optimal solution but there are several extensions, such as the Augmented WTM, Modified WTM, and Lexicographic WTM (see Marler & Arora, 2004). Aiming to get all outputs as close as possible to their respective ideal values involves a fairness principle that acknowledges that all countries can aspire to reach their maximum potential in every type of medal. However, this has to be carried out taking into account the FSO constraints as well as the integrality of the targets. The Tchebychef metric used by the WTM implies that it focuses on the cases that remain most distant from their ideal values, in an effort to prevent any country from being left behind in any type of medal. Since WTM can generally only guarantee a weak Pareto optimal solution, it is common to use a secondary objective function that maximizes the sum of the different objective functions. However, since in our application that sum is constant, we use another, more appropriate, secondary goal, namely that of minimizing the sum of the absolute value of the changes in the number of medals of the different NOCs. The idea is that, although the targets of some countries will involve more medals and those of other countries will involve fewer, the absolute sizes of those changes are minimized, subject, of course, to the optimal value of the maximum distance to the ideal values computed by the WTM.

The main contribution of the paper is, therefore, the introduction of an innovative central-ized multiobjective perspective that focuses on setting fair FSO output targets from which performance indices and corresponding efficiency ranking are derived *ex post*. Almost all existing DEA approaches to the Olympic Games, regardless of whether they consider FSOs or not, use an output-oriented radial metric and focus on computing efficiency scores, not on output targets. The output targets computed by the proposed approach lie on the FSO efficient frontier, which is below the conventional efficient frontier and represent a fairer assessment of the performance of each NOC since this is what each country could aspire to obtain if all countries were efficient. Obtaining better results than these targets is possible (because in reality not all countries perform efficiently) but that should be interpreted as a sort of super-efficient performance.

As regards the novelty of the proposed approach, note that there is no FSO DEA approach that provides targets for the DMUs in this Olympic Games application. Thus, on one hand, non-FSO DEA approaches (such as those found in Lozano et al., 2002 & Wu et al., 2010) can compute efficiency scores as well as targets but experience the problem that the total number of medals of those targets exceeds the actual number of medals that are awarded in the Games. On the other hand, existing FSO DEA approaches take into account the FSO character of the problem, but they aim at only efficiency estimation. They use variables that represent the changes in the number of medals of the DMUs (they have to, in order to

impose the FSO constraints) but they do not compute targets as such. The proposed approach, instead of focusing on the computation of efficiency scores, focuses on computing sound and fair targets, and determines efficiency scores ex-post based on those targets. Note also that, although a FSO DEA target setting approach based on Nash Bargaining has been recently proposed (Lozano, 2023), it cannot be used to compute integer targets as required in this application.

The structure of the paper is as follows. In Sect. 2, a literature review of DEA applications to Olympic Games is presented. Subsequently, in Sect. 3, the proposed approach is formulated and discussed. In Sect. 4, the proposed approach is applied to the recent Tokyo 2020 Olympics, and its results are compared with those of other methods from the literature. Finally, in Sect. 5, conclusions are drawn and further research is outlined.

2 Literature review

The first paper applying DEA to Olympic Games was that by Lozano et al., (2002), which considered the aforementioned basic features (non-discretionary inputs and weight restrictions). It set the default of using a radial output orientation. Certain refinements, such as the imposition of the integrality of the output targets, have also been proposed (e.g., Wu et al., 2010). Intended for ranking the NOCs more than for target setting, Zhang et al., (2009) proposed a lexicographic approach that ranks the NOCs first by their relative efficiency in winning gold medals, while using the relative efficiency in winning silver and bronze medals (in that order) to break ties. Their approach was output-oriented but non-radial in that only one output was increased at a time.

Lei et al., (2015) proposed a shared-input, parallel network DEA (NDEA) approach with two stages corresponding to the Summer and Winter Olympic Games, respectively. This approach used a multiplier formulation, weight restrictions, and an output orientation. Li et al., (2012) also used a multiplier DEA formulation, weight restrictions, and an output orientation but on a two-stage configuration that considered two inputs (Population and GDP per capita), an intermediate product (Number of athletes integrating the NOC team) and the usual three outputs (Number of gold, silver, and bronze medals won). Jablonsky (2018) used a similar approach, except for the inclusion of an additional input (Number of medals won in the previous Olympics). In the same way as Lei et al., (2015), Jablonsky (2018) computes efficiency scores but not output targets.

Although not a strictly DEA approach, Calzada-Infante & Lozano (2016) used a Dominance Network (DN) for the efficiency analysis of Olympic Games. The nodes of a DN are the DMUs, and an arc exists from DMU i to DMU j if the latter dominates the former, that is, if DMU j consumes less input and produces more output than DMU i. Using complex network analysis tools, a number of local and global indicators can be computed to assess and visualize the relative performance of the different countries.

Recently, Sekitani and Zhao (2021) have presented a complex restricted multiplier DEA approach in which the minimum sum of the absolute deviation of an affine function fitting of the total number of medals is imposed as a constraint. Apart from estimating and decomposing the efficiency scores, the method provides a regression function that predicts the total number of medals based on three inputs (Population, GDP, and Total number of medals won at the previous Olympic Games). Furthermore, from the corresponding dual formulation, a production possibility set is implied and can be employed to predict the number of medals won by a certain country. Indeed, the number of medals predicted for Japan in Tokyo 2020

(26 gold, 15 silver, 22 bronze) is very close to the actual number of medals won (27 gold, 14 silver, 17 bronze). Note, however, that this type of prediction differs from the concept of efficient FSO targets considered in this paper, which assumes that all countries perform efficiently.

Other input variables, apart from GDP and Population, have also been considered. Thus, for example, Soares de Mello et al., (2012) uses Population and Number of athletes of each NOC as input variables (as a proxy for the country investments in sports). The latter input is considered discretionary, and an input orientation is used in that paper which means that it does not suffer from the drawback of all the output-oriented approaches mentioned so far, which is that they compute efficiency scores (and corresponding rankings) for each NOC separately which leads to output targets whose sum exceeds the total number of medals of each type to be awarded in the Games. Flegl & Andrade (2018) use three inputs (GDP, Economic Active Population, and Corruption Perception Index) and as outputs, apart from the weighted number of gold, silver, and bronze medals won, a fourth input is considered that represents the aggregated weighted number of 4th to 8th positions. The DEA model used was of a Constant Returns to Scale (CRS) cross-efficiency approach. Wu et al., (2009b) also proposed a CRS cross-efficiency approach, using an aggressive formulation and Assurance Region (AR) constraints to reflect the different valuation of the medals. They utilized a hierarchical clustering algorithm to identify similar groups of countries based on their cross-efficiency scores. Wu et al., (2009a) also computed cross-efficiency scores but under a game theory approach and assumed Variable Returns to Scale (VRS).

Other multiplier DEA formulations include the context-dependent AR approach of Li et al., (2008), the separation vector approach of Chiang et al., (2011), and the interval efficiency approach of Azizi and Wang (2013). Churilov & Flitman (2006) used a standard CRS DEA ratio formulation (without AR constraints), and considered four inputs (Population, GDP per capita, Disability Adjusted Life Expectancy Index, and Index of Equality of Child Survival) and four outputs, corresponding to aggregating the medals won by a given country using four different sets of weights. Soares de Mello et al., (2009) used a pure-output multiplier DEA formulation with AR constraints, and averaged the computed weights of the medal across the different countries that won medals in each cluster of sports. These average weights are employed to compute the efficiency index (and rank) of the participants in each cluster. The efficiency in the different clusters is aggregated to obtain an overall performance index. The idea behind considering these clusters is to distinguish between sports that have numerous different competitions (e.g., athletics, gymnastics, swimming) and those with a fewer number (e.g., baseball, football, volleyball).

Of all the approaches reviewed above, the only one that takes into account the FSO character of the output variables is that of Soares de Mello et al., (2012). They do so, however, in an implicit way, that is, by using an input-oriented approach. Another approach that takes FSO into account in an implicit way (by projecting each DMU assuming a certain output reduction strategy on the part of the other DMUs) is the Zero-Sum Gains (ZSG) model of Lins et al., (2003). Although Lins et al., (2003) considers a single output that is a weighted aggregation of the number of gold, silver, and bronze medals won, the ZSG approach has been extended in Bi et al., (2014) to handle multiple outputs. These ZSG approaches use envelopment DEA formulations and do not impose the FSO constraints explicitly. Indeed, Bouzidis & Karagiannis (2022) have argued that the ZSG efficiency scores of Lins et al., (2003) are not really comparable across DMUs and proposed an alternative performance metric that is consistent with the FSO constraints (which they label as output interdependency). The explicit imposition of FSO constraints that establish that the total number of medals of each type is fixed requires a centralised DEA approach or its equivalent

dual, a CSW DEA model. The Equilibrium Efficient Frontier (EEF) approach used in Yang et al., (2011), despite considering the FSO constraints explicitly does not consider CSW when computing the EEF and hence the efficiency assessment is not carried out using a common platform. This issue was corrected in Yang et al., (2015), whose Generalized Equilibrium Efficient Frontier (GEEF) approach uses FSO constraints as well as CSW. Moreover, GEEF has also been extended by Li et al., (2021) to a two-stage NDEA configuration similar to that in Li et al., (2015), although applied to the Winter Olympic Games. Although there are other GEEF approaches (e.g., Amirteimoori et al., 2017; Zhu et al., 2020) they have not been applied to the Olympic Games. Note, finally, that neither the ZSG nor the GEEF approaches take into account the integrality of the Olympic Games output variables or compute output targets, but instead compute only efficiency scores and rankings. This is the research gap we identified: a DEA approach able to compute fair, FSO-compliant targets for the DMUs.

Table 1 summarizes the main features of the aforementioned DEA applications to the Olympic Games. It can be observed that no FSO-explicit envelopment approach, such as the one proposed in this paper, aimed at computing output targets, has been proposed. Furthermore, although this is, in some respect, a secondary consideration, instead of the usual radial approach, the proposed centralized DEA approach is multiobjective and uses a non-radial solution method that includes a secondary goal.

3 Proposed multiobjective FSO approach

Before formulating the proposed approach, let us introduce the notation to be used.

3.1 Data

$j, r \in \{1, 2, ..., n\}$ indices on the set of DMUs.

$\mathbf{x}_j = (x_{1j}, x_{2j}, ..., x_{mj})$ inputs of DMU j.

$\mathbf{y}_j = (y_{Gj}, y_{Sj}, y_{Bj})$ number of medals won by DMU j (G = Gold, S = Silver, B = Bronze).

$y_k^{total} = \sum_{j=1}^{n} y_{kj}$ total amount of medals of type $k \in \{G, S, B\}$.

$\mathbf{w} = (w_G, w_S, w_B)$ relative weight/value of medals of type k.

3.2 Decision variables

$(\lambda_{1r}, \lambda_{2r}, ..., \lambda_{nr})$ intensity variables utilized to compute the target of DMU r.

$\hat{\mathbf{y}}_r = (\hat{y}_{Gr}, \hat{y}_{Sr}, \hat{y}_{Br})$ target of the number of medals to be won by DMU r.

$\mathbf{s}_r = (s_{Gr}, s_{Sr}, s_{Br})$ shortfall (i.e., output slacks) in the number of medals won by DMU r.

σ_r inefficiency score of DMU r.

Assuming VRS, taking into account the non-discretionary character of the input variables and the integrality of the output variables, and considering a weighted additive (WA) metric (e.g., Cooper et al., 2011; Lovell & Pastor, 1995), the following conventional DEA model can be employed to compute an efficient target (and associated inefficiency score) for a certain DMU r

Table 1 Summary of DEA Olympic Games applications

Reference	Envelop	Multiplier	Radial	AR	Integer Targets	VRS	Eff. Scores	Ranking	FSO Targets	NDEA	Remarks
Lozano et al., (2002)	✓		✓	✓		✓	✓	✓			Technical/Scale efficiency
Wu et al., (2010)	✓		✓	✓			✓	✓			CRS
Zhang et al., (2009)	✓				✓	✓	✓	✓			Lexicographic
Soares de Mello et al., (2012)	✓		✓			✓	✓	✓			Input-oriented
Churilov and Flitman (2006)		✓	✓				✓	✓			CRS
Li et al., (2008)		✓	✓	✓		✓	✓	✓			Context-dependent AR
Wu et al., (2009a)		✓	✓	✓		✓	✓	✓			Game XE
Wu et al., (2009b)		✓	✓	✓			✓	✓			XE aggressive, CRS
Soares de Mello et al., (2009)		✓	✓	✓			✓	✓			Pure output, clusters, CRS
Chiang et al., (2011)		✓	✓	✓			✓	✓			CSW, separation vector, CRS
Azizi and Wang (2013)		✓	✓			✓	✓	✓			Interval efficiency
Flegl and Andrade (2018)		✓	✓				✓	✓			XE, Original inputs/outputs
Calzada-Infante and Lozano (2016)	✓		✓	✓		✓	✓	✓			ZSG single output
Lei et al., (2015)		✓	✓	✓		✓	✓			✓	Two parallel subsystems
Li et al., (2015)		✓	✓	✓		✓	✓			✓	Two-stage series system

Table 1 (continued)

Reference	Envelop	Multiplier	Radial	AR	Integer Targets	VRS	Eff. Scores	Ranking	FSO Targets	NDEA	Remarks
Jablonsky (2018)		✓		✓		✓	✓	✓		✓	Two-stage series, SBM
Sekitani and Zhao (2021)		✓	✓	✓	✓	✓	✓				Eff. decomp., prediction
Lins et al., (2003)	✓		✓	✓		✓	✓	✓			ZSG single output
Bi et al., (2014)	✓		✓	✓		✓	✓	✓			ZSG multiple outputs
Yang et al., (2011)		✓	✓			✓	✓	✓			EEF, single output
Yang et al., (2015)		✓	✓	✓		✓	✓	✓			GEEF
Li et al., (2021)		✓	✓	✓		✓	✓	✓		✓	GEEF, Two-stage series
Bouzidis and Karagiannis (2022)	✓		✓			✓	✓	✓			ZSG single output
Proposed approach	✓				✓	✓	✓	✓	✓		Multiobjective, WTM, secondary goal

$$\sigma_r = Max \quad \sum_k w_k s_{kr}$$

$$s.t.$$

$$\sum_j \lambda_{jr} x_{ij} \leq x_{ir} \quad \forall i$$

$$\sum_j \lambda_{jr} y_{kj} \geq \hat{y}_{kr} = y_{kr} + s_{kr} \quad \forall k \tag{1}$$

$$\sum_j \lambda_{jr} = 1$$

$$\lambda_{jr} \geq 0 \ \ \forall j \ \ s_{kr} \geq 0 \ \ \forall k$$

$$\hat{y}_{kr} \text{ integer } \forall k \quad \Leftrightarrow \quad s_{kr} \text{ integer } \forall k$$

This VRS DEA model maximizes the weighted sum of the output shortfalls. The corresponding input constraints take into account their non-discretionary character. The output targets are forced to be integers. Indeed, the integrality of the output variables can be imposed either on the output targets $\hat{y}_{kr}$ themselves or, alternatively, on the associated slack variables s_{kr}. Moreover, note that the latter are assumed to be non-negative. This is a key feature of conventional DEA models of technical efficiency: the DMUs are always projected onto efficient targets that dominate them. The problem with this approach is that, in the case that the sum of the outputs is fixed, the targets computed in this way violate the corresponding FSO constraints, that is, $\sum_{r=1}^{n} \hat{y}_{kr} > y_k^{\text{total}}$, rendering them invalid.

As regards the inefficiency scores σ_r, they are non-negative, whereby a zero value indicates relative efficiency. Let $E = \{r : \sigma_r = 0\}$ be the set of efficient DMUs. For the inefficient DMUs, $\sigma_r > 0$ indicates technical inefficiency and represents the potential improvement in the number of medals of DMU r, and values the corresponding medals according to the given weights $\mathbf{w} = (w_G, w_S, w_B)$.

The aim of this paper is to develop an alternative approach that, by taking into account the FSO constraints, would lead to valid targets. Since these constraints involve the targets of all the DMUs, this means that the proposed approach must project all the DMUs at the same time and, hence, it must use a centralized DEA perspective. The approach must also enable the number of medals to be won by a country to be increased or decreased. This is clear. Since the total number of medals is fixed, the only possibility for some countries to win more medals involves assuming that other countries win fewer medals. Hence, in what follows, the non-negative slack variables $\mathbf{s}_r = (s_{Gr}, s_{Sr}, s_{Br})$ are not used. Instead, we will consider output change variables $\boldsymbol{\delta}_r = (\delta_{Gr}, \delta_{Sr}, \delta_{Br})$ that are taken as free variables (i.e., unrestricted in sign).

The main difficulty, however, lies in defining the objective function to be used by this centralized DEA model. To this end, a multiobjective optimization approach is proposed so that all DMUs can aim to maximizing the number of medals of every type, that is,

$$\text{Max} \quad (\delta_{G1}, \delta_{S1}, \delta_{B1}, \delta_{G2}, \delta_{S2}, \delta_{B2}, ..., \delta_{Gn}, \delta_{Sn}, \delta_{Bn})$$

$$s.t.$$

$$\sum_j \lambda_{jr} x_{ij} \leq x_{ir} \quad \forall i \, \forall r$$

$$\sum_j \lambda_{jr} y_{kj} \geq \hat{y}_{kr} = y_{kr} + \delta_{kr} \geq 0 \quad \forall k \, \forall r$$

$$\sum_j \lambda_{jr} = 1 \quad \forall r \tag{2}$$

$$\lambda_{jr} \geq 0 \;\; \forall j \, \forall r \qquad \delta_{kr} \text{ free } \;\; \forall k \, \forall r$$

$$\hat{y}_{kr} \text{ integer } \forall k \, \forall r \quad \Leftrightarrow \quad \delta_{kr} \text{ integer } \forall k \, \forall r$$

$$\sum_r \hat{y}_{kr} = y_k^{\text{total}} \;\; \forall k \quad \Leftrightarrow \quad \sum_r \delta_{kr} = 0 \;\; \forall k$$

This is a multiobjective optimization problem that uses a centralized DEA approach. This can be observed since all DMUs are projected at the same time. This is necessary if the FSO constraints are to be imposed. As in the case of Model (1), VRS, non-discretionary inputs, and integer output targets are considered. It should also be borne in mind that, in the same way that the integrality of the output variables can be imposed on the output targets $\hat{y}_{kr}$ or, alternatively, on the output change variables δ_{kr}, the FSO constraints can also be formulated using the output targets or the output change variables. Apart from projecting all DMUs, there is also a major difference between Models (1) and (2). Thus, while (1) considers non-negative output shortfalls s_{kr}, in (2) the corresponding output change variables δ_{kr} are free, in that they can be positive or negative. Moreover, for each medal type, the sum of these changes is fixed to zero.

Among the many multiobjective optimization solution methods, we propose using the Weighted Tchebychef method, also known as the Weighted min–max method. This is a well-known multiobjective optimization method that computes a solution whose objective function vector is as close as possible, using the Tchebychef metric, to the ideal point (see Marler & Arora, 2004). Since the objective functions are in conflict with one another, the ideal point cannot be reached in general. Getting as close to it as possible, however, is a sensible strategy. This is the idea behind several multiobjective optimization methods, such as Compromise Programming (e.g., Lozano et al., 2020) and the WTM (e.g., Gutiérrez & Lozano, 2016). As mentioned above, the WTM, in particular, uses the weighted Tchebychef distance, which, in this context, means the maximum weighted deviation from the output targets to their corresponding ideal values. This implements a certain fairness principle that strives to leave no objective function behind with a large deviation from its ideal value. In our application, this means that all medal types of all countries simultaneously strive to approach the ideal point, subject to the FSO constraints.

Therefore, in order to apply the WTM to the multiobjective optimization problem (2) the first step is to compute the corresponding ideal point. The coordinates of the ideal point are the maximum of each of the objective functions when they are optimized individually. This involves solving the following single-objective optimization model for each value of $r' \in \{1, 2, ..., n\}$ and $k' \in \{G, S, B\}$, that is, for each country and medal type.

$$\delta_{k'r'}^{ideal} = Max \quad \delta_{k'r'}$$
$$s.t.$$
$$\sum_j \lambda_{jr} x_{ij} \leq x_{ir} \quad \forall i \, \forall r$$
$$\sum_j \lambda_{jr} y_{kj} \geq \hat{y}_{kr} = y_{kr} + \delta_{kr} \geq 0 \quad \forall k \, \forall r$$
$$\sum_j \lambda_{jr} = 1 \quad \forall r \tag{3}$$
$$\lambda_{jr} \geq 0 \; \forall j \, \forall r \quad \delta_{kr} \, free \; \forall k \, \forall r$$
$$\hat{y}_{kr} \, integer \; \forall k \, \forall r \quad \Leftrightarrow \quad \delta_{kr} \, integer \; \forall k \, \forall r$$
$$\sum_r \hat{y}_{kr} = y_k^{total} \; \forall k \quad \Leftrightarrow \quad \sum_r \delta_{kr} = 0 \; \forall k$$

This Mixed-Integer Linear Program (MILP) model is employed to separately maximize each of the objective functions of Model (2). In this way, the corresponding components of the ideal point can be computed. Note that the constraints of Model (3) are the same and have the same meaning as those of Model (2).

Proposition 1 *The optimal solution of Model (3) is always non-negative, that is, $\delta_{k'r'}^{ideal} \geq 0 \; \forall k' \, \forall r'$.*

Proof The solution that projects each DMU onto itself, that is,

$$\lambda_{jr} = \begin{cases} 1 & if \; j = r \\ 0 & otherwise \end{cases}$$
$$\delta_{kr} = 0 \quad \forall k \, \forall r$$
$$\hat{y}_{kr} = y_{kr} + \delta_{kr} \quad \forall k \, \forall r$$

is always feasible in Model (3), for any $r' \in \{1, 2, ..., n\}$ and $k' \in \{G, S, B\}$, and its objective function value is zero. This establishes zero as a lower bound on the optimal objective function value of Model (3).

The above result implies the following corollary, which states that the maximum number of medals that a country can obtain of a given type $y_{k'r'}^{ideal} = y_{k'r'} + \delta_{k'r'}^{ideal}$ is never below the observed value.

Corollary $y_{k'r'}^{ideal} \geq y_{k'r'} \; \forall k' \, \forall r'$

Note that, in the WTM, the deviations of the different objective functions with respect to their ideal values are generally weighted, with weights that represent the preferences of the DMU. In the application presented in this paper, the weights used for the different objective functions are.

$$\omega_{kr} = \begin{cases} \frac{w_k}{y_{kr}^{ideal}} & if \; y_{kr}^{ideal} > 0 \\ 0 & if \; y_{kr}^{ideal} = 0 \end{cases} \tag{4}$$

where w_k corresponds, as indicated above, to the relative value of medals of type k. The interpretation of these weights is that, on the one hand, the different types of medals should be weighted in proportion to their relative value, and, on the other hand, the deviation of the number of medals with respect to their optimal values should be measured in relative terms rather than in absolute terms. Another way of interpreting the denominator is as if we consider normalized objective functions using y_{kr}^{ideal} as normalization coefficients. The weight ω_{kr} is assigned a zero value if $y_{kr}^{ideal} = 0$ because if $y_{kr}^{ideal} = 0$ then the observed value must be $y_{kr} = 0$ and therefore it is known *ex ante* that $\hat{y}_{kr} = 0$ and $\delta_{kr} = 0$. Hence, in that case, the value of ω_{kr} plays no role since it is always multiplied by zero.

Furthermore, to help the proposed FSO_WT centralized DEA to choose from among the large number of alternative optima that may exist, it is useful to include certain additional constraints that act as cutting planes, and reduce the feasible region, which, due to the FSO constraints and the integrality of the targets, has a combinatorial structure. Specifically, two sets of additional constraints are considered. The first set states that the net weighted change in the number of medals of the efficient DMUs must be negative. This is imposed since, in an FSO context, these efficient DMUs should reduce their outputs so that the inefficient DMUs can increase theirs. The net weighted change in the number of medals of any DMU r can be expressed as

$$\xi_r = \sum_{k\in\{G,S,B\}} \omega_{kr}\delta_{kr} = \sum_{k\in\{G,S,B\}} \omega_{kr}\hat{y}_{kr} - \sum_{k\in\{G,S,B\}} \omega_{kr}y_{kr} \tag{5}$$

Hence, the first set of additional constraints can be expressed as

$$\xi_r \leq 0 \quad \forall r \in E \tag{6}$$

The second set of additional constraints is that the net weighted change in the number of medals ξ_r of the efficient DMUs must be lower (or at least equal) to that of the inefficient DMUs. The rationale for this is that, as indicated above, efficient DMUs are the first that should reduce their number of medals (to make room for the most inefficient DMUs to increase theirs) but this reduction in the number of medals may also occur to some slightly inefficient DMUs which may therefore also have a negative ξ_r. The idea is that although inefficient DMUs may have negative values of ξ_r these values cannot go below the ξ_r of the efficient DMUs. In other words, the reduction in the number of medals (in terms of a net weighted change) should occur first and mainly to the efficient DMUs. Mathematically, this second set of additional constraints can be expressed as

$$\xi_r \leq \xi_{r'} \quad \forall r \in E, r' \notin E \tag{7}$$

Therefore, the proposed FSO_WT centralized DEA model is the following:

$$\mu^{\mathrm{min\,max}} = Min \quad \mu$$

$$s.t.$$

$$\omega_{kr} \cdot \left(\delta_{kr}^{\mathrm{ideal}} - \delta_{kr}\right) \leq \mu \quad \forall k \, \forall r$$

$$\sum_j \lambda_{jr} x_{ij} \leq x_{ir} \quad \forall i \, \forall r$$

$$\sum_j \lambda_{jr} y_{kj} \geq \hat{y}_{kr} = y_{kr} + \delta_{kr} \geq 0 \quad \forall k \, \forall r$$

$$\sum_j \lambda_{jr} = 1 \quad \forall r \tag{8}$$

$$\lambda_{jr} \geq 0 \;\; \forall j \, \forall r \qquad \delta_{kr} \text{ free } \forall k \, \forall r$$

$$\hat{y}_{kr} \text{ integer } \forall k \, \forall r \quad \Leftrightarrow \quad \delta_{kr} \text{ integer } \forall k \, \forall r$$

$$\sum_r \hat{y}_{kr} = y_k^{\mathrm{total}} \; \forall k \quad \Leftrightarrow \quad \sum_r \delta_{kr} = 0 \;\; \forall k$$

$$\sum_{k \in \{G,S,B\}} \omega_{kr} \delta_{kr} \leq 0 \;\; \forall r \in E$$

$$\sum_{k \in \{G,S,B\}} \omega_{kr} \delta_{kr} \leq \sum_{k \in \{G,S,B\}} \omega_{kr'} \delta_{kr'} \;\; \forall r \in E, r' \notin E$$

This MILP model implements the Minmax criterion characteristic of the WTM (see Marler & Arora, 2004). In our case, the aim is to minimize the maximum weighted deviation of each output change variable with respect to its ideal value. As in the previous models, VRS, non-discretionary inputs, integer output targets and FSO constraints are considered. As mentioned above, two additional sets of constraints have been included to reduce the feasibility region and discard solutions with unwanted features. In particular, these constraints forbid positive net weighted output changes for efficient DMUs (because these should reduce their outputs in order to let inefficient DMUs increase theirs). They also prevent inefficient DMUs from obtaining lower net weighted output changes than efficient DMUs.

The WTM generally uses a second-phase model. The reason for this is twofold. One is that, in general, the WTM only guarantees weak Pareto optimality, and the other is that there may be alternative optima. In our application, the first reason does not apply since, due to the FSO constraints, the optimal solution of Model (4) (in fact any feasible solution) is Pareto optimal. There remains, however, the possibility of alternative optima. In other words, in this application, the use of a lexicographic approach (i.e., a secondary goal) is not to guarantee Pareto optimality but to choose from among the alternative solutions of Model (4). The standard secondary objective function used in the Augmented WTM and Lexicographic WTM variants (see, for example, Marler & Arora, 2004) corresponds to maximizing the sum of the objective functions considered, that is, $\sum_{r=1}^{n} \sum_{k \in \{G,S,B\}} \delta_{kr}$. However, in our application, this objective function value is zero (not only for the alternative optima but also for all feasible solutions) since $\sum_{r=1}^{n} \sum_{k \in \{G,S,B\}} \delta_{kr} = \sum_{k \in \{G,S,B\}} \sum_{r=1}^{n} \delta_{kr}$ and $\sum_{r=1}^{n} \delta_{kr} = 0 \quad \forall k$. Therefore, a different secondary goal needs to be employed. The secondary goal proposed in this paper involves minimizing the absolute change from the observed values. The rationale for choosing this criterion to set the final FSO_WT efficient targets is that, regardless of whether the DMUs need to increase their outputs or to reduce them, the closer they are to the observed values, the more acceptable and reasonable they become. Mathematically, the

second-phase model is, therefore

$$
\text{Min} \quad \sum_{r=1} \sum_{k\in\{G,S,B\}} |\delta_{kr}|
$$

$s.t.$

$$
\omega_{kr} \cdot \left(\delta_{kr}^{\text{ideal}} - \delta_{kr}\right) \leq \mu^{\text{min max}} \quad \forall k \, \forall r
$$

$$
\sum_j \lambda_{jr} x_{ij} \leq x_{ir} \quad \forall i \, \forall r
$$

$$
\sum_j \lambda_{jr} y_{kj} \geq \hat{y}_{kr} = y_{kr} + \delta_{kr} \geq 0 \quad \forall k \, \forall r
$$

$$
\sum_j \lambda_{jr} = 1 \quad \forall r \tag{9}
$$

$$
\lambda_{jr} \geq 0 \;\; \forall j \, \forall r \qquad \delta_{kr} \text{ free } \forall k \, \forall r
$$

$$
\hat{y}_{kr} \text{ integer } \forall k \, \forall r \quad \Leftrightarrow \quad \delta_{kr} \text{ integer } \forall k \, \forall r
$$

$$
\sum_r \hat{y}_{kr} = y_k^{\text{total}} \;\; \forall k \quad \Leftrightarrow \quad \sum_r \delta_{kr} = 0 \;\; \forall k
$$

$$
\sum_{k\in\{G,S,B\}} \omega_{kr} \delta_{kr} \leq 0 \quad \forall r \in E
$$

$$
\sum_{k\in\{G,S,B\}} \omega_{kr} \delta_{kr} \leq \sum_{k\in\{G,S,B\}} \omega_{kr'} \delta_{kr'} \quad \forall r \in E, r' \notin E
$$

This MILP model selects, from among the alternative optima of Model (8), that in which the targets of the DMUs are closet to their observed values. The idea is to facilitate and reduce the effort for each DMU to reach its target. In other words, this increases the likelihood of the DMUs achieving their targets. Apart from the constraint that guarantees the optimal objective function value of Model (8), the other constraints in Model (9) are the same and have the same meaning as those of Model (8).

The absolute value function of the objective function of Model (9) can be linearized through the usual change of variables, that is,

$$
\delta_{kr} = \delta_{kr}^+ - \delta_{kr}^-
$$
$$
\delta_{kr}^+, \; \delta_{kr}^- \geq 0 \tag{10}
$$
$$
|\delta_{kr}| = \delta_{kr}^+ + \delta_{kr}^-
$$

The final targets computed by the proposed approach, after solving Models (6)-(7), are thus

$$
\left(\hat{y}_{kr}\right)^* = y_{kr} + (\delta_{kr})^* = y_{kr} + \left(\delta_{kr}^+\right)^* - \left(\delta_{kr}^-\right)^* \quad \forall k \, \forall r \tag{11}
$$

It is clear that these targets respect the FSO constraints. In certain cases, that is, for certain DMUs and certain medal types, the target exceeds the observed value while in others the target remains below the observed value. The final net weighted change computed for each DMU r

$$
\xi_r = \sum_{k\in\{G,S,B\}} \omega_{kr} \cdot (\delta_{kr})^* = \sum_{k\in\{G,S,B\}} \omega_{kr} \cdot \left[\left(\delta_{kr}^+\right)^* - \left(\delta_{kr}^-\right)^*\right] \tag{12}
$$

can be used as the performance index of DMU r and can be positive, negative, or zero. A positive value indicates underperformance, that is, there is a clear potential increase in the weighted value of the number of medals won. A negative value, in contrast, indicates super-efficient performance, that is, the DMU has obtained a greater number of medals than those that could be expected/reasonable if all DMUs performed at their best level. A zero value indicates efficiency, that is, the number of medals won are in line with what could be expected if all DMUs performed at their best level. This classification resembles, but is not equivalent to, the classification of high performers, average performers, and low performers in Bouzidis and Karagiannis (2022).

One clarification is in order: in this paper, we distinguish between efficient targets and FSO_WT efficient targets. The former are those computed by the non-centralized DEA model (1), which lie on the efficient frontier of the conventional VRS DEA technology (in this case, of the corresponding integer VRS DEA technology). Conversely, the targets computed by the proposed centralized DEA approach, which considers the FSO character of the problem, are jointly determined and are efficient in a Pareto optimality sense. For these, the term FSO_WT efficient targets is applied. These targets do not generally lie on the efficient frontier of the conventional VRS DEA technology because that would violate the FSO constraints. Instead, certain efficient DMUs will have to reduce their number of medals (and therefore their target would be below the conventional VRS DEA technology) while certain inefficient DMUs would need to increase their number of medals (but not by so many as to reach the conventional VRS DEA technology).

In Fig. 1, we aim to convey an abstract, simplified representation of the proposed approach, by assuming constant inputs and only two outputs, while ignoring the integrality of the variables and considering only a few DMUs (to prevent clutter). Panel a) represents the calculation of the ideal point δ_{kr}^{ideal} (shown as an individual ideal point for each DMU). Panel b) represents Model (4), in which the ideal point is used as a reference so that the weighted Tchebychef distance of all output targets of all DMUs to their corresponding ideal values is minimized. This is represented by the rectangles centred in each of the individual ideal points. The size of the maximum of these rectangles is minimized in Model (4). Since Model (4) explicitly imposes the FSO constraints, the targets computed by that model belong to the FSO efficient frontier. However, there are alternative optima, some of which may be far from the observed values. That is why, following the idea that inspires the approaches for the closest target and the smallest improvement (e.g., Aparicio et al., 2007; Lozano & Khezri, 2021), a secondary goal that involves minimizing the rectangular distance from the observed DMUs to the corresponding targets is considered in Model (9). This is represented by the rhombi centred in the observed DMUs in Panel c). The sum of the sizes of those rhombi is minimized in Model (9). The final changes δ_{kr} associated to the computed targets are shown in Panel d).

Note that one may be tempted to draw lines connecting the FSO efficient targets in panel c) or d) as if defining a type of "interior frontier". However, apart from that oxymoron being a confusing concept, such lines (which probably do not define a convex region) do not mean anything; they most certainly do not represent the FSO efficient frontier. In this regard, note that a point in the FSO efficient frontier corresponds to n operating points in this type of plot. In other words, the whole set of targets computed by the proposed approach for the n DMUs is just one point in the FSO efficient frontier. Therefore, the FSO efficient frontier cannot be drawn in this type of plot. The only certainty is that the n operating points of any FSO efficient frontier must lie within the conventional VRS technology and that their coordinates must respect the FSO constraints.

(a)

(b)

(c)

(d)
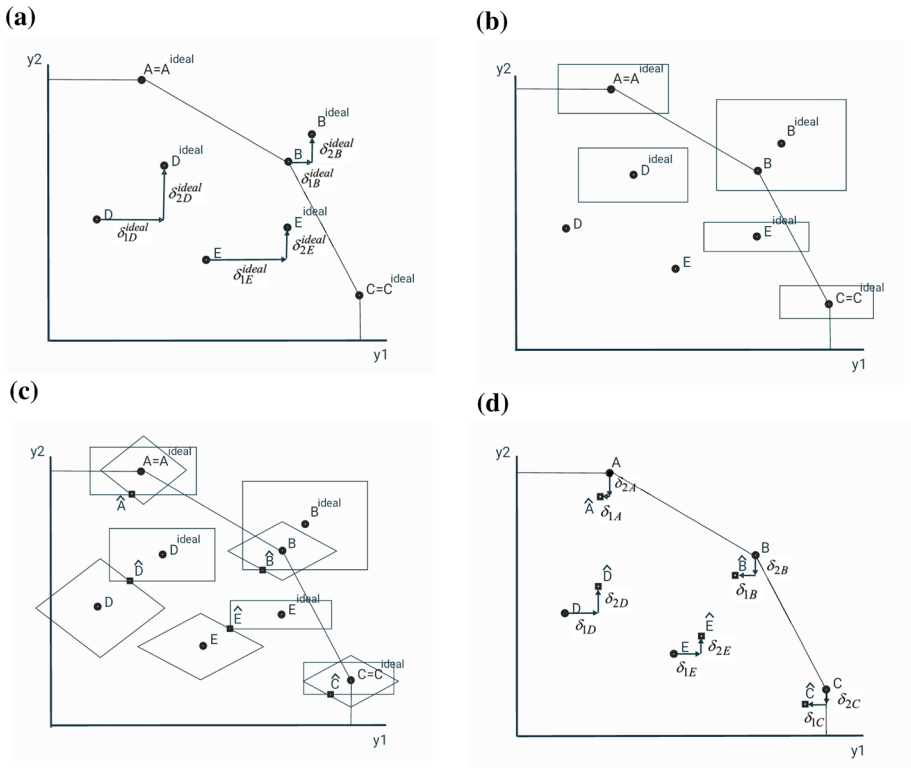
Fig. 1 Abstract representation of the proposed FSO_WT approach

Therefore, summarizing, a two-step lexicographic approach is proposed so that the first step aims at getting all outputs of all DMUs as close to their ideal values as possible (using a Minmax criterion) thereby implicitly defining a subset of the FSO efficient frontier (which we call the FSO_WT efficient frontier). The second step computes specific FSO_WT efficient targets by minimizing the magnitude of the total weighted effort required to reach them from the set of observed DMUs. Note that the two steps of the proposed approach use a centralized DEA perspective which, although not shown in Fig. 1, always maintains the FSO and the integrality constraints.

4 Application to Tokyo 2020

In this section, the application of the proposed approach to the results of the Tokyo 2020 Olympics is presented. The 93 NOCs that won at least one medal according to the Tokyo 2020 official website (https://olympics.com/en/olympic-games/tokyo-2020/medals) have been considered. Table 2 shows the number of medals won by each NOC as well as their corresponding Population and GDP inputs. The sources of the input data include the 2019 Revision of World Population Prospects of the UN Population Division (https://population.un.org/wpp/) and The World Factbook (https://www.cia.gov/the-world-factbook/field/real-gdp-purchasing-power-parity/country-comparison), respectively.

Table 2 Inputs and outputs for Tokyo 2020

NOC	Population	GDP (10^3 US$)	Gold (#)	Silver (#)	Bronze (#)
Argentina (ARG)	45,195,777	991,523	0	1	2
Armenia (ARM)	2,963,234	40,384	0	2	2
Australia (AUS)	25,499,881	1,264,514	17	7	22
Austria (AUT)	9,006,400	498,780	1	1	5
Azerbaijan (AZE)	10,139,175	144,374	0	3	4
Bahamas (BAH)	393,248	14,450	2	0	0
Bahrain (BRN)	1,701,583	73,870	0	1	0
Belarus (BLR)	9,449,321	181,286	1	3	3
Belgium (BEL)	11,589,616	596,414	3	1	3
Bermuda (BER)	62,273	5,228	1	0	0
Botswana (BOT)	2,351,625	40,928	0	0	1
Brazil (BRA)	212,559,409	3,092,216	7	6	8
Bulgaria (BUL)	6,948,445	161,654	3	1	2
Burkina Faso (BUR)	20,903,278	44,266	0	0	1
Canada (CAN)	37,742,157	1,843,053	7	6	11
China (CHN)	1,439,323,774	22,526,502	38	32	18
Chinese Taipei (TPE)	23,816,775	1,143,277	2	4	6
Colombia (COL)	50,882,884	741,099	0	4	1
Côte d'Ivoire (CIV)	26,378,275	134,048	0	0	1
Croatia (CRO)	4,105,268	126,625	3	3	2
Cuba (CUB)	11,326,616	137,000	7	3	5
Czech Republic (CZE)	10,708,982	435,987	4	4	3
Denmark (DEN)	5,792,203	336,335	3	4	4
Dominican Republic (DOM)	10,847,904	197,735	0	3	2
Ecuador (ECU)	17,643,060	197,631	2	1	0
Egypt (EGY)	102,334,403	1,180,890	1	1	4
Estonia (EST)	1,326,539	48,987	1	0	1
Ethiopia (ETH)	114,963,583	248,972	1	1	2
Fiji (FIJ)	896,444	12,178	1	0	1
Finland (FIN)	5,540,718	268,662	0	0	2
France (FRA)	65,273,512	3,097,061	10	12	11
Georgia (GEO)	3,989,175	55,776	2	5	1
Germany (GER)	83,783,945	4,482,448	10	11	16
Ghana (GHA)	31,072,945	164,640	0	0	1

Table 2 (continued)

NOC	Population	GDP (10^3 US$)	Gold (#)	Silver (#)	Bronze (#)
Great Britain (GBR)	67,886,004	3,118,396	22	21	22
Greece (GRE)	10,423,056	319,334	2	1	1
Grenada (GRN)	112,519	1,908	0	0	1
Hong Kong (HKG)	7,497,000	449,299	1	2	3
Hungary (HUN)	9,660,350	321,869	6	7	7
India (IND)	1,380,004,385	9,155,083	1	2	4
Indonesia (INA)	273,523,621	3,196,682	1	1	3
Iran (IRI)	83,992,953	1,027,238	3	2	2
Ireland (IRL)	4,937,796	428,825	2	0	2
Israel (ISR)	8,655,541	363,448	2	0	2
Italy (ITA)	60,461,828	2,562,135	10	10	20
Jamaica (JAM)	2,961,161	28,779	4	1	4
Japan (JPN)	126,476,458	5,231,066	27	14	17
Jordan (JOR)	10,203,140	101,738	0	1	1
Kazakhstan (KAZ)	18,776,707	487,868	0	0	8
Kenya (KEN)	53,771,300	227,638	4	4	2
Kosovo (KOS)	1,767,881	20,396	2	0	0
Kuwait (KUW)	4,270,563	209,738	0	0	1
Kyrgyzstan (KGZ)	6,524,191	33,918	0	2	1
Latvia (LAT)	1,886,202	59,102	1	0	1
Lithuania (LTU)	2,722,291	103,756	0	1	0
Malaysia (MAS)	32,365,998	906,239	0	1	1
Mexico (MEX)	128,932,753	2,525,481	0	0	4
Mongolia (MGL)	3,278,292	39,723	0	1	3
Morocco (MAR)	36,910,558	279,295	1	0	0
Namibia (NAM)	2,540,916	24,040	0	1	0
Netherlands (NED)	17,134,873	986,847	10	12	14
New Zealand (NZL)	4,822,233	210,877	7	6	7
Nigeria (NGR)	206,139,587	1,032,048	0	1	1
North Macedonia (MKD)	2,083,380	34,333	0	1	0
Norway (NOR)	5,421,242	340,303	4	2	2
Philippines (PHI)	109,581,085	963,121	1	2	1
Poland (POL)	37,846,605	1,261,433	4	5	5
Portugal (POR)	10,196,707	358,344	1	1	2
Puerto Rico (PUR)	2,860,840	110,238	1	0	0

Table 2 (continued)

NOC	Population	GDP (10^3 US$)	Gold (#)	Silver (#)	Bronze (#)
Qatar (QAT)	2,881,060	255,010	2	0	1
Republic of Moldova (MDA)	51,269,183	34,680	0	0	1
Romania (ROU)	19,237,682	579,549	1	3	0
Russia (ROC)	145,934,460	3,968,180	20	28	23
San Marino (SMR)	33,938,000	2,008	0	1	2
Saudi Arabia (KSA)	34,813,867	1,609,323	0	1	0
Serbia (SRB)	8,737,370	116,339	3	1	5
Slovakia (SVK)	5,459,643	178,513	1	2	1
Slovenia (SLO)	2,078,932	81,614	3	1	1
South Africa (RSA)	59,308,690	730,913	1	2	0
South Korea (KOR)	51,269,183	2,211,315	6	4	10
Spain (ESP)	46,754,783	1,925,576	3	8	6
Sweden (SWE)	10,099,270	547,595	3	6	0
Switzerland (SUI)	8,654,618	588,472	3	4	6
Syria (SYR)	17,500,657	50,280	0	0	1
Thailand (THA)	69,799,978	1,285,287	1	0	1
Tunisia (TUN)	11,818,618	125,783	1	1	0
Turkey (TUR)	84,339,067	2,371,374	2	2	9
Turkmenistan (TKM)	6,031,187	86,858	0	1	0
Uganda (UGA)	45,741,000	96,838	2	1	1
Ukraine (UKR)	43,733,759	538,388	1	6	12
United States (USA)	331,002,647	20,524,945	39	41	33
Uzbekistan (UZB)	33,469,199	235,021	3	0	2
Venezuela (VEN)	28,435,943	269,068	1	3	0
Total	-	-	340	338	402

The first thing one may do is to compute the conventional WA DEA targets $(\hat{y}_{Gr}, \hat{y}_{Sr}, \hat{y}_{Br})$ and inefficiency scores σ_r as per Model (1). These are shown in Table 3, where it is apparent that, for each type of medal, the sum of the targets greatly exceeds the total number of medals awarded. As regards the medal weights utilized to obtain these results, $(w_G, w_S, w_B) = (0.5714286, 0.2857143, 0.1428571)$, they correspond to the assumption that $w_G = \beta w_S = \beta^2 w_B$, where $\beta = 2$, that is, one gold medal is worth two silver medals and one silver medal is worth two bronze medals. Note that, according to the conventional DEA approach, there are 16 efficient NOCs, namely AUS, BAH, BER, CUB, GEO, GBR, GRN, HUN, JAM, JPN, NED, NZL, ROC, SMR, UKR, and USA. The most inefficient NOCs are those of IND, INA, MEX, TUR, KSA, BRA, and THA. Interestingly, in spite of the high number of medals won by CHN, it is still labelled as inefficient according to the conventional DEA approach.

Table 3 Targets and efficiency scores computed by WA DEA model (1)

NOC r	$\hat{y}_{G,r}$	$\hat{y}_{S,r}$	$\hat{y}_{B,r}$	σ_r	Rank
Argentina (ARG)	14	7	17	11.857	82
Armenia (ARM)	3	2	3	1.857	22
Australia (AUS)	17	7	22	0.000	1
Austria (AUT)	9	6	10	6.714	68
Azerbaijan (AZE)	6	4	5	3.857	43
Bahamas (BAH)	2	0	0	0.000	1
Bahrain (BRN)	3	2	2	2.286	28
Belarus (BLR)	7	4	6	4.143	44
Belgium (BEL)	10	6	11	6.571	67
Bermuda (BER)	1	0	0	0.000	1
Botswana (BOT)	3	1	3	2.286	28
Brazil (BRA)	21	21	22	14.286	88
Bulgaria (BUL)	6	5	5	3.286	38
Burkina Faso (BUR)	4	1	4	3.000	33
Canada (CAN)	18	11	22	9.286	72
China (CHN)	39	41	33	5.286	55
Chinese Taipei (TPE)	15	7	20	10.286	78
Colombia (COL)	12	6	14	9.286	72
Côte d'Ivoire (CIV)	6	3	5	4.857	51
Croatia (CRO)	5	4	5	1.857	22
Cuba (CUB)	7	3	5	0.000	1
Czech Republic (CZE)	9	6	10	4.429	48
Denmark (DEN)	7	6	7	3.286	38
Dominican Republic (DOM)	7	5	6	5.143	53
Ecuador (ECU)	7	5	6	4.857	51
Egypt (EGY)	16	7	20	12.571	86
Estonia (EST)	2	1	3	1.143	20
Ethiopia (ETH)	7	6	7	5.571	58
Fiji (FIJ)	1	1	2	0.429	17
Finland (FIN)	7	6	7	6.429	65
France (FRA)	21	20	22	10.143	77
Georgia (GEO)	2	5	1	0.000	1
Germany (GER)	23	22	22	11.429	81
Ghana (GHA)	7	4	5	5.714	60
Great Britain (GBR)	22	21	22	0.000	1
Greece (GRE)	8	6	8	5.857	61
Grenada (GRN)	0	0	1	0.000	1
Hong Kong (HKG)	8	6	8	5.857	61
Hungary (HUN)	6	7	7	0.000	1

Table 3 (continued)

NOC r	$\hat{y}_{G,r}$	$\hat{y}_{S,r}$	$\hat{y}_{B,r}$	σ_r	Rank
India (IND)	26	31	26	25.714	93
Indonesia (INA)	22	21	22	20.429	92
Iran (IRI)	14	7	18	10.000	76
Ireland (IRL)	7	6	7	5.286	55
Israel (ISR)	8	6	9	6.143	63
Italy (ITA)	20	17	21	7.857	71
Jamaica (JAM)	4	1	4	0.000	1
Japan (JPN)	27	14	17	0.000	1
Jordan (JOR)	6	2	4	4.143	44
Kazakhstan (KAZ)	9	7	10	7.429	69
Kenya (KEN)	7	6	7	3.000	33
Kosovo (KOS)	2	1	3	0.714	19
Kuwait (KUW)	6	5	6	5.571	58
Kyrgyzstan (KGZ)	3	2	3	2.000	24
Latvia (LAT)	3	1	3	1.714	21
Lithuania (LTU)	4	3	4	3.429	40
Malaysia (MAS)	13	7	16	11.286	80
Mexico (MEX)	20	16	22	18.571	91
Mongolia (MGL)	4	1	4	2.429	30
Morocco (MAR)	7	6	7	6.143	63
Namibia (NAM)	3	1	3	2.143	26
Netherlands (NED)	10	12	14	0.000	1
New Zealand (NZL)	7	6	7	0.000	1
Nigeria (NGR)	14	7	18	12.143	85
North Macedonia (MKD)	3	1	3	2.143	26
Norway (NOR)	7	6	7	3.571	41
Philippines (PHI)	14	6	17	10.857	79
Poland (POL)	16	8	20	9.857	75
Portugal (POR)	8	6	9	6.429	65
Puerto Rico (PUR)	4	3	4	3.143	36
Qatar (QAT)	4	3	4	2.429	30
Republic of Moldova (MDA)	4	1	4	3.000	33
Romania (ROU)	10	6	12	7.714	70
Russia (ROC)	20	28	23	0.000	1
San Marino (SMR)	0	1	2	0.000	1
Saudi Arabia (KSA)	17	10	21	15.286	89
Serbia (SRB)	6	2	5	2.000	24
Slovakia (SVK)	6	5	6	4.429	48
Slovenia (SLO)	3	2	3	0.571	18
South Africa (RSA)	12	6	14	9.429	74

Table 3 (continued)

NOC r	$\hat{y}_{G,r}$	$\hat{y}_{S,r}$	$\hat{y}_{B,r}$	σ_r	Rank
South Korea (KOR)	19	14	22	12.000	83
Spain (ESP)	18	12	22	12.000	83
Sweden (SWE)	9	7	10	5.143	53
Switzerland (SUI)	8	7	9	4.143	44
Syria (SYR)	4	2	3	3.143	36
Thailand (THA)	17	7	22	14.143	87
Tunisia (TUN)	6	3	5	4.143	44
Turkey (TUR)	19	16	21	15.429	90
Turkmenistan (TKM)	5	2	4	3.714	42
Uganda (UGA)	5	3	4	2.714	32
Ukraine (UKR)	1	6	12	0.000	1
United States (USA)	39	41	33	0.000	1
Uzbekistan (UZB)	7	6	7	4.714	50
Venezuela (VEN)	7	6	7	5.286	55
Total	912	687	968	–	–

In order to obtain valid FSO targets and associated performance indices of the DMUs, the proposed FSO_WT approach is used. This implies:

1. Solving Model (3) for each NOC r' and each medal type k', thereby obtaining the corresponding ideal values $\delta_{k'r'}^{\text{ideal}}$ and $y_{k'r'}^{\text{ideal}} = y_{k'r'} + \delta_{k'r'}^{\text{ideal}}$.
2. Solving FSO_WT model (8) using weights as per (4), thereby obtaining the Minmax distance to the ideal point $\mu^{\text{min max}}$.
3. Solving the secondary-goal model (9)-(10), thereby obtaining the FSO_WT efficient targets (11) and the corresponding performance indices (12).

As regards the solution method of all the models in this paper, including Models (1), (3), (8), and (9), these have been solved using Gurobi Optimizer (version 9.1.2), which uses Branch and Cut to solve such MILP problems. Non-centralized DEA models, such as Model (1), have very few (in fact, just three) integer variables and hence are solved very easily. Centralized DEA models, such as Models (3), (8), and (9), have more integer variables (namely three times the number of DMUs) and hence can take somewhat longer to compute. In any case, the computation times in a conventional laptop (with 8 Gb RAM and an i7 processor) are negligible (less than 2 s).

Another important point, raised by one of the reviewers, is the uniqueness of the targets computed. We knew beforehand that the FSO_WT model (8) can have alternative optima, which is the reason why a secondary goal or phase-II model was proposed. However, although unlikely, it is theoretically possible that the secondary-goal model (9)-(10) might also have multiple optimal solutions. Fortunately, this has not been the case, and it could therefore be verified (*ex post*) that the optimal solution of Models (9)-(10) reported below was unique.

Table 4 shows the ideal values y_{kr}^{ideal} for all types of medals of all NOCs, the FSO_WT targets $(\hat{y}_{kr})^*$, performance indices ξ_r, and the corresponding ranking. The optimum of the FSO_WT model (8) is $\mu^{\text{min max}} = 0.4155844$. Note that, as indicated in Corollary 1, the ideal values never fall below the observed values. Indeed, for most countries, the ideal value

Table 4 Ideal point, FSO_WT targets, and performance indices computed by proposed approach

NOC r	$\hat{y}_{Gr}^{ideal}$	$\hat{y}_{Sr}^{ideal}$	$\hat{y}_{Br}^{ideal}$	$(\hat{y}_{G,r})^*$	$(\hat{y}_{S,r})^*$	$(\hat{y}_{B,r})^*$	ξ_r	Rank
Argentina (ARG)	14	12	18	4	0	2	0.139	67
Armenia (ARM)	4	3	4	2	2	2	0.286	89
Australia (AUS)	17	13	22	5	6	22	− 0.425	2
Austria (AUT)	9	8	10	3	0	5	0.091	58
Azerbaijan (AZE)	6	5	5	2	0	4	0.019	45
Bahamas (BAH)	2	1	2	1	0	0	− 0.286	9
Bahrain (BRN)	3	2	3	1	0	0	0.048	50
Belarus (BLR)	7	5	6	2	3	3	0.082	55
Belgium (BEL)	10	9	11	3	1	3	0.000	13
Bermuda (BER)	1	1	2	1	0	0	0.000	13
Botswana (BOT)	3	3	3	1	0	1	0.190	83
Brazil (BRA)	21	23	23	7	6	8	0.000	13
Bulgaria (BUL)	6	5	6	3	1	2	0.000	13
Burkina Faso (BUR)	4	4	4	2	0	1	0.286	89
Canada (CAN)	18	15	22	6	8	11	0.006	39
China (CHN)	39	41	33	38	32	18	0.000	13
Chinese Taipei (TPE)	15	12	20	5	4	6	0.114	63
Colombia (COL)	12	10	14	4	4	1	0.190	84
Côte d'Ivoire (CIV)	6	5	5	2	0	1	0.190	84
Croatia (CRO)	5	5	5	3	3	2	0.000	13
Cuba (CUB)	7	5	5	2	3	5	− 0.408	3
Czech Republic (CZE)	9	7	10	3	6	3	0.018	44

Table 4 (continued)

NOC r	$\hat{y}_{Gr}^{ideal}$	$\hat{y}_{Sr}^{ideal}$	$\hat{y}_{Br}^{ideal}$	$(\hat{y}_{G,r})^*$	$(\hat{y}_{S,r})^*$	$(\hat{y}_{B,r})^*$	ξ_r	Rank
Denmark (DEN)	7	6	7	3	4	4	0.000	13
Dominican Republic (DOM)	7	6	6	2	3	2	0.163	77
Ecuador (ECU)	7	6	6	2	1	0	0.000	13
Egypt (EGY)	16	13	20	5	1	4	0.143	69
Estonia (EST)	2	2	3	1	0	1	0.000	13
Ethiopia (ETH)	7	6	7	2	0	2	0.034	47
Fiji (FIJ)	1	1	2	1	0	1	0.000	13
Finland (FIN)	7	6	7	2	0	2	0.163	77
France (FRA)	21	20	23	6	20	11	0.005	38
Georgia (GEO)	4	5	4	2	5	1	0.000	13
Germany (GER)	23	22	23	7	17	16	0.003	37
Ghana (GHA)	7	5	6	2	0	1	0.163	77
Great Britain (GBR)	22	21	23	7	21	22	− 0.390	7
Greece (GRE)	8	6	8	3	0	1	0.024	46
Grenada (GRN)	0	0	1	0	0	1	0.000	13
Hong Kong (HKG)	8	7	8	3	0	3	0.061	52
Hungary (HUN)	8	7	8	3	7	7	− 0.214	11
India (IND)	30	32	26	9	2	4	0.152	75
Indonesia (INA)	22	23	23	7	1	3	0.156	76
Iran (IRI)	14	12	18	4	1	2	0.017	43
Ireland (IRL)	7	6	7	2	0	2	0.000	13
Israel (ISR)	8	7	9	3	0	2	0.071	53
Italy (ITA)	20	18	22	6	18	20	0.013	40
Jamaica (JAM)	4	2	4	2	0	4	− 0.429	1
Japan (JPN)	27	26	24	8	14	17	− 0.402	5
Jordan (JOR)	6	5	5	2	0	1	0.133	65
Kazakhstan (KAZ)	10	8	11	3	0	8	0.171	80

Table 4 (continued)

NOC r	$\hat{y}_{Gr}^{ideal}$	$\hat{y}_{Sr}^{ideal}$	$\hat{y}_{Br}^{ideal}$	$(\hat{y}_{G,r})^*$	$(\hat{y}_{S,r})^*$	$(\hat{y}_{B,r})^*$	ξ_r	Rank
Kenya (KEN)	7	6	7	3	6	2	0.014	41
Kosovo (KOS)	2	2	3	2	0	0	0.000	13
Kuwait (KUW)	6	5	6	2	0	1	0.190	84
Kyrgyzstan (KGZ)	4	3	4	2	2	1	0.286	89
Latvia (LAT)	3	2	3	1	0	1	0.000	13
Lithuania (LTU)	4	3	4	2	0	0	0.190	84
Malaysia (MAS)	13	11	17	4	0	1	0.150	74
Mexico (MEX)	20	20	22	6	0	4	0.171	80
Mongolia (MGL)	4	3	4	2	0	3	0.190	84
Morocco (MAR)	8	6	8	3	0	0	0.143	69
Namibia (NAM)	3	2	3	1	0	0	0.048	50
Netherlands (NED)	12	12	15	4	12	14	− 0.286	10
New Zealand (NZL)	7	6	7	2	6	7	− 0.408	3
Nigeria (NGR)	14	12	18	4	0	1	0.139	67
North Macedonia (MKD)	3	3	3	1	0	0	0.095	59
Norway (NOR)	7	6	7	2	5	3	0.000	13
Philippines (PHI)	14	11	17	4	0	1	0.071	53
Poland (POL)	16	13	21	5	5	5	0.036	48
Portugal (POR)	8	7	9	3	0	2	0.102	62
Puerto Rico (PUR)	4	3	4	2	0	0	0.143	71
Qatar (QAT)	4	3	4	2	0	1	0.000	13
Republic of Moldova (MDA)	4	3	4	2	0	1	0.286	89
Romania (ROU)	10	8	12	3	3	0	0.114	63

Table 4 (continued)

NOC r	$\hat{y}_{Gr}^{ideal}$	$\hat{y}_{Sr}^{ideal}$	$\hat{y}_{Br}^{ideal}$	$\left(\hat{y}_{G,r}\right)^*$	$\left(\hat{y}_{S,r}\right)^*$	$\left(\hat{y}_{B,r}\right)^*$	ξ_r	Rank
Russia (ROC)	24	28	23	7	28	23	− 0.310	8
San Marino (SMR)	0	1	2	0	1	2	0.000	13
Saudi Arabia (KSA)	17	14	22	5	0	0	0.148	73
Serbia (SRB)	6	5	5	3	1	5	0.000	13
Slovakia (SVK)	6	5	6	2	2	1	0.095	59
Slovenia (SLO)	3	3	4	3	1	1	0.000	13
South Africa (RSA)	12	10	14	4	0	0	0.086	56
South Korea (KOR)	19	17	22	6	4	10	0.000	13
Spain (ESP)	18	16	22	6	8	6	0.095	59
Sweden (SWE)	9	8	10	3	6	0	0.000	13
Switzerland (SUI)	8	7	9	3	4	6	0.000	13
Syria (SYR)	4	4	4	2	0	1	0.286	89
Thailand (THA)	17	13	22	5	0	1	0.134	66
Tunisia (TUN)	6	5	5	2	0	0	0.038	49
Turkey (TUR)	19	19	22	6	0	9	0.090	57
Turkmenistan (TKM)	5	5	4	2	0	0	0.171	80
Uganda (UGA)	5	5	5	2	1	1	0.000	13
Ukraine (UKR)	10	8	12	3	3	11	− 0.005	12
United States (USA)	39	41	33	12	41	33	− 0.396	6
Uzbekistan (UZB)	7	6	7	2	2	2	0.014	41
Venezuela (VEN)	8	6	7	3	3	0	0.143	71
Total	940	848	1,006	340	338	402	–	–

is much higher. The exception is USA, whose ideal values are equal to the observed values. Hence, for each medal type, the sum of the ideal values of all the DMUs exceeds the total number of medals awarded. In other words, the ideal point is not a valid FSO target. The computed FSO_WT targets are valid FSO targets that are as close to the ideal point as possible (measured by the weighted Tchebychef distance) and, moreover, as a secondary goal, as close to the observed values as possible (measured by the weighted rectilinear distance). As regards

the performance index ξ_r, this is negative for 12 NOCs. These are the NOCs that won more medals than would have been expected if all NOCs had performed at their best level. Hence, to compensate for the increase in the number of medals that the inefficient NOCs would win if they performed at their best level, these super-efficient NOCs must lower the number of medals they would obtain. This is necessary since the total number of medals of each type awarded in the Games is fixed. In particular, the super-efficient NOCs are JAM, AUS, CUB, NZL, JPN, USA, GBR, ROC, BAH, NED, HUN, and UKR. Not surprisingly, all these countries belong to the set E of efficient NOCs according to the conventional DEA approach. In this list, we can find NOCs that always perform well in the Olympic Games, in certain cases surprisingly well, given their relatively small size. This list also includes JPN, which performed exceptionally well in Tokyo 2020 (Ramchandani 2020). This so-called home advantage effect has also been observed in other Olympic Games (e.g., Lozano et al., 2002).

There are also a number of countries whose target coincides with the observed values (and therefore have $\xi_r = 0.000$), namely CHN, BRA, KOR, NOR, SUI, BEL, BER, BUL, CRO, ECU, EST, FIJ, GEO, GRN, IRL, KOS, LAT, QAT, SLO, SMR, SRB, SWE, DEN, UGA. Note that the closest-target criterion adopted for the secondary goal incentivizes DMUs to maintain their number of medals and thus join this neutral-performance group, which acts as a sort of effective reference frontier, generally below the conventional efficient frontier (although three of these countries, namely BER, GRN, and SMR belong to the set E of efficient NOCs). Note that most of NOCs are labelled as underperformers, (i.e., they have $\xi_r > 0.000$), with ARM, BUR, KGZ, MDA, SYR, BOT, CIV, COL, KUW, LTU, and MGL obtaining the worst performance indices.

Before proceeding further with the presentation and discussion of the results, it may be of interest to clarify a question rightly raised by one of the reviewers and it is that the computed targets do not necessarily comply with the expected monotonicity in inputs and outputs. Thus, for example, it should be expected that since Thailand (THA) consumes more inputs than Australia (AUS) its target outputs should be larger than (or at least equal to) those of AUS. As it can be seen in Table 4, this not what happens, with the target of AUS (5, 6, 22) being larger than that of THA (5, 0, 1). To understand why this may happen one more piece of information needs to be considered and that is the number of medals actually won by THA and AUS and which are (1, 0, 1) and (17, 7, 22), respectively. Hence, AUS would probably not complain for having a target that is more ambitious than that of THA. On the contrary, AUS might feel that its target is not ambitious enough as it implies winning fewer medals than they have won. Hence, target setting in FSO DEA is very different and more challenging than in conventional DEA. In FSO DEA, the limitation on the total amount of outputs greatly constraints and distorts the target setting process. Thus, apart from the large number of alternative FSO efficient targets, some DMUs (labelled super-efficient) cannot be allocated as many medals as they have actually won, i.e., their targets are set below the observed performance. This is contrary to what occurs in conventional DEA and that is why sound concepts like the monotonicity between input and output are less important in this context than, for example, the distance to the observed outputs. This paradoxical reduction in outputs expected of some DMUs, which is unthinkable in conventional DEA, is however required in FSO DEA in order for underperforming DMUs to be able to raise their output targets.

Figures 2 and 3 show the number of medals of the different types that are transferred from super-efficient NOCs to underperforming NOCs. It can be observed that most such transfers are carried out in gold medals and by some countries (e.g., USA, JPN, GBR, ROC, and AUS). The corresponding increases in the number of medals is less concentrated, with many
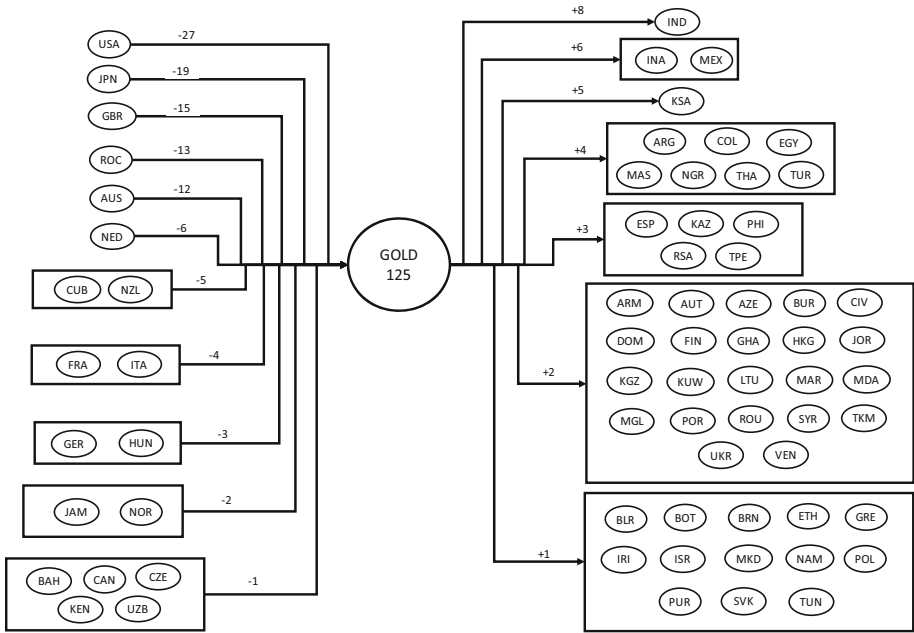
Fig. 2 Changes (reductions and increases) in the number of gold medals computed by proposed approach
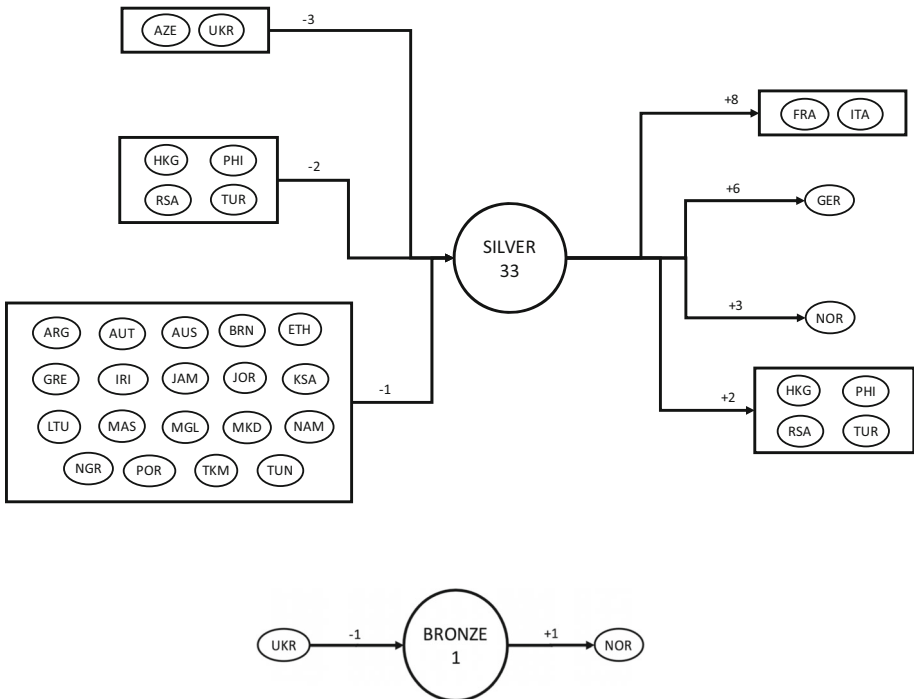
Fig. 3 Changes (reductions and increases) in the number of silver and bronze medals computed by proposed approach

underperformers benefitting from the medals released by the super-efficient countries. Note also that the countries that reduce their number of gold medals do not need to additionally reduce their number of medals of the other types. In total, 125 gold medals should change hands versus only 33 silver medals and just 1 bronze medal.

Figures 4 and 5 show the weighted value of medals actually won by each NOC (horizontal axis) versus the weighted value of the corresponding FSO_WT target (vertical axis). Figure 5 is a zoom-in of the bottom left-hand corner of Fig. 4, that is, the region corresponding to low values of both the observed weighted number of medals and the weighted output targets. Most NOCs actually lie in this region. In Fig. 5, it can be more clearly observed how there are several overlapping NOCs (e.g., GHA, CIV, BUR, KUW, SYR, and MDA). This corresponds

Fig. 4 Weighted observed outputs versus weighted target outputs

Fig. 5 Weighted observed outputs versus weighted target outputs (Zoom in)

to countries that won the same number of medals and are also projected on the same target. This type of coincidence is a result of the fairness criterion employed in projecting the NOCs, which treats similar countries in a similar way as regards their targets, taking into account, also, their Population and GDP. This does not necessarily mean that the same performance index is assigned to these countries though.

For both Figs. 4 and 5, since, according to (10), the corresponding performance index of a NOC is proportional to the difference between its vertical and its horizontal coordinates, the NOCs that lie above the bisector line are therefore underperformers while those that lie below are super-efficient. The NOCs on the bisector are neutral in terms of performance, that is, $\xi_r = 0.000$. The further the vertical distance of a country to the bisector line, the greater the weighted change needed in the number of medals to achieve the target. This change can actually be an increase, that is, winning more medals or medals of higher value, or it can be a decrease, that is, winning fewer medals or medals of lower value. The managerial implications of the former case, that is, an underperformer NOC that needs to improve its performance, is the usual case except that the computed targets are much more realistic and achievable than the targets computed by the conventional non-FSO DEA approach. The other two scenarios are more innovatory. Thus, for NOCs of neutral performance, the proposed approach implies a maintenance objective, that is, the current level of performance is acceptable and maintaining it should be the priority. Finally, the target reduction prescribed for super-efficient countries should not be interpreted to mean that the country should not strive to win as many medals as possible, but rather that the observed super-efficient performance is above what can reasonably be expected. This leads to a strategy that identifies and prioritizes those sports where their performance was strongest in order to secure those medals.

The reasons and explanations behind the results of the performance assessment (super-efficient, neutral, underperformer) carried out by the proposed approach are not straightforward. It must first be taken into account that the performance index is derived from the computed targets, and, second, that these are determined using a centralized FSO perspective, that is, the target of any single country cannot be determined independently of those of the other countries. The combinatorial character of the corresponding Pareto optimal set requires the specification of suitable criteria for the selection of those targets. The proposed approach uses a lexicographic approach with two criteria. The main criterion is a Minmax criterion that guarantees that no country is left behind (i.e., treated unfavourably) in terms of the weighted relative distance to its ideal point. This ideal point represents the maximum number of medals of each type that a country could aspire to win in the best possible scenario and takes into account both its population and GDP. The secondary criterion used is an overall closest-target projection, that is, regardless of whether a country is either super-efficient and should reduce its number of medals or an underperformer that should increase its number of medals, the sum of these medal changes should be minimized, and remain lexicographically subject to the optimality of the main criterion.

Figure 6 shows the conventional WA DEA efficiency score σ_r versus the proposed performance index ξ_r. Note that the former is non-negative (zero for technically efficient countries) while the latter can be negative (super-efficient NOCs), positive (underperforming NOCs), or exactly zero (neutral-performance NOCs). The super-efficient countries therefore lie on the lower side of the left-hand vertical axis. There are also some points on the (0,0) coordinates, that is, technically efficient countries whose performance is neutral. All the other points on the horizontal axis are also neutral-performance countries, even though some of them (such as BRA and KOR) have relatively large values of the inefficiency scores. Finally, note that, to improve the visibility of the figure, this does not include the data corresponding to MEX,
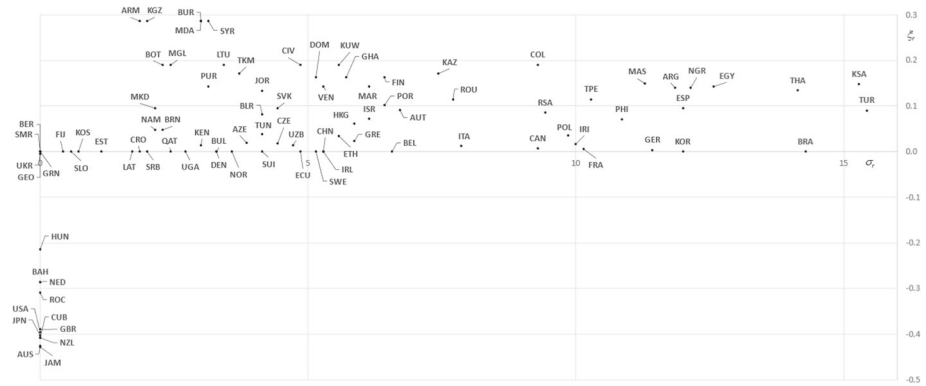
Fig. 6 WA efficiency score (σ_r) versus proposed performance index (ξ_r)

INA, and IND, whose inefficiency score σ_r is around or above 20 (with performance index ξ_r around 0.15).

It can be observed above that the efficiency ranking computed by the proposed FSO_WT approach clearly differs from that of the conventional WA DEA approach (Spearman's rank correlation coefficient = 0.462). This indicates that the FSO efficiency assessment, while taking into account the FSO constraints, has an influence. This is confirmed in Table 5, which shows the inverse efficiency scores and associated ranking of the radial non-FSO approaches of Lozano et al., (2002) and Wu et al., (2010), labelled BCC_AR and BCC_AR_Int, respectively. Spearman's rank correlation coefficients of the proposed FSO_WT approach with these two non-FSO methods are 0.852 and 0.794, respectively. Note that the NOCs in Table 5 are ordered according to their FSO_WT rank.

For comparison, Table 5 also shows the inverse efficiency scores and associated ranking of the ZSG DEA approaches (with proportional output reduction strategy) of Lins et al., (2003), Bi et al., et al., et al., (2014), and Bouzidis and Karagiannis (2022), labelled ZSG-Lins, ZSG-Bi, and ZSG-BK, respectively. In all three cases, the same relative weights of the medals (i.e., $w_G = \beta w_S = \beta^2 w_B$ with $\beta = 2$) have been used. Note that, although the efficiency scores of ZSG-Lins and ZSG-BK fail to coincide, they lead to the same ranking of the DMUs. Spearman's rank correlation coefficient of the proposed FSO_WT approach with these two FSO methods is 0.835, larger than Spearman's rank correlation of FSO_WT with ZSG-Bi (0.620). It can therefore be observed that the proposed approach differs from ZSG DEA approaches, even though they do take the FSO constraints into account. This difference is caused by the different way in which this is carried out in ZSG DEA and in the proposed FSO_WT. For example, ZSG DEA uses a radial metric and computes neither ideal values nor FSO targets. For the sake of comparison, the rank correlation coefficient of the proposed FSO_WT approach with the official Lexicographic ranking is 0.675.

Figure 7 shows the NOCs ranked in increasing order of their performance index. First, with negative ξ_r values, are the super-efficient NOCs. Interestingly, no ties occur in this group. There is large group of neutral-performance NOCs that together define a sort of neutral-reference frontier that separates the super-efficient NOCs from the underperforming NOCs. Among the latter there are also several ties, especially in the last positions, that is, among the NOCs that have underperformed most significantly.

Finally, a sensitivity analysis of the results of the proposed approach with respect to the value of parameter β has been carried out and it was found that, on the condition that beta

Table 5 Inverse efficiency scores and ranking of different methods versus those of proposed approach

NOC r	ξ_r (Rank) FSO_WT	(Efficiency)$^{-1}$ BCC_AR	(Efficiency)$^{-1}$ BCC_AR_Int	(Efficiency)$^{-1}$ ZSG–Lins	(Efficiency)$^{-1}$ ZSG–Bi	(Efficiency)$^{-1}$ ZSG–BK
JAM	− 0.429 (1)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	0.984 (1)
AUS	− 0.425 (2)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	0.984 (1)
CUB	− 0.408 (3)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	0.984 (1)
NZL	− 0.408 (3)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	0.984 (1)
JPN	− 0.402 (5)	1.000 (1)	1.000 (1)	1.085 (13)	1.000 (1)	0.985 (13)
USA	− 0.396 (6)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	0.984 (1)
GBR	− 0.390 (7)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	0.984 (1)
ROC	− 0.310 (8)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	0.984 (1)
BAH	− 0.286 (9)	1.000 (1)	1.000 (1)	1.000 (1)	1.000 (1)	0.984 (1)
NED	− 0.286 (10)	1.016 (13)	1.000 (1)	1.036 (11)	1.000 (1)	0.984 (11)
HUN	− 0.214 (11)	1.169 (15)	1.688 (22)	1.174 (15)	1.000 (1)	0.986 (15)
UKR	− 0.005 (12)	2.311 (32)	2.250 (34)	2.277 (31)	1.000 (1)	0.998 (31)
CHN	0.000 (13)	1.026 (14)	1.026 (17)	1.141 (14)	1.026 (16)	0.985 (14)
BRA	0.000 (13)	3.133 (39)	1.000 (1)	3.024 (36)	2.829 (55)	1.006 (36)
KOR	0.000 (13)	3.060 (38)	2.333 (36)	2.955 (35)	2.253 (44)	1.005 (35)
NOR	0.000 (13)	1.822 (22)	2.000 (27)	2.188 (29)	1.822 (34)	0.997 (28)
SUI	0.000 (13)	2.214 (30)	3.000 (38)	2.186 (28)	1.614 (30)	0.997 (29)
BEL	0.000 (13)	3.424 (42)	3.333 (43)	3.786 (43)	3.424 (61)	1.014 (43)
BER	0.000 (13)	1.000 (1)	1.000 (1)	1.079 (12)	1.000 (1)	0.985 (12)
BUL	0.000 (13)	2.210 (30)	1.000 (1)	2.538 (33)	2.210 (43)	1.000 (33)

Table 5 (continued)

NOC r	ξ_r (Rank) FSO_WT	(Efficiency)$^{-1}$ BCC_AR	(Efficiency)$^{-1}$ BCC_AR_Int	(Efficiency)$^{-1}$ ZSG–Lins	(Efficiency)$^{-1}$ ZSG–Bi	(Efficiency)$^{-1}$ ZSG–BK
CRO	0.000 (13)	1.660 (19)	1.667 (21)	1.763 (19)	1.577 (28)	0.992 (19)
ECU	0.000 (13)	3.710 (46)	3.500 (45)	4.491 (48)	3.709 (63)	1.021 (48)
EST	0.000 (13)	2.992 (37)	1.000 (1)	3.111 (37)	2.402 (45)	1.006 (37)
FIJ	0.000 (13)	1.855 (23)	1.111 (18)	1.946 (24)	1.682 (32)	0.994 (23)
GEO	0.000 (13)	1.198 (16)	1.958 (26)	1.361 (16)	1.000 (1)	0.988 (16)
GRN	0.000 (13)	1.000 (1)	2.000 (27)	1.000 (1)	1.900 (35)	0.984 (1)
IRL	0.000 (13)	3.528 (44)	4.000 (49)	4.661 (50)	3.527 (62)	1.023 (50)
KOS	0.000 (13)	1.449 (18)	2.000 (27)	1.889 (22)	1.448 (25)	0.993 (22)
LAT	0.000 (13)	3.491 (43)	2.000 (27)	3.834 (45)	3.047 (59)	1.014 (45)
QAT	0.000 (13)	2.404 (34)	2.083 (32)	3.295 (39)	2.404 (46)	1.009 (39)
SLO	0.000 (13)	1.278 (17)	2.026 (31)	1.475 (17)	1.278 (19)	0.989 (17)
SMR	0.000 (13)	1.000 (1)	2.115 (33)	1.000 (1)	1.000 (1)	0.984 (1)
SRB	0.000 (13)	1.866 (24)	2.275 (35)	1.854 (21)	1.088 (17)	0.993 (21)
SWE	0.000 (13)	2.113 (29)	2.500 (37)	2.526 (32)	1.428 (23)	1.000 (32)
DEN	0.000 (13)	2.070 (28)	1.000 (1)	2.048 (26)	1.618 (31)	0.995 (26)
UGA	0.000 (13)	2.806 (36)	3.000 (38)	2.946 (34)	2.750 (53)	1.005 (34)
GER	0.003 (37)	2.041 (26)	1.000 (1)	1.975 (25)	1.472 (26)	0.995 (25)
FRA	0.005 (38)	1.985 (25)	1.156 (19)	1.929 (23)	1.711 (33)	0.994 (24)
CAN	0.006 (39)	2.311 (32)	1.556 (20)	2.249 (30)	1.942 (38)	0.997 (30)
ITA	0.013 (40)	1.720 (21)	1.938 (25)	1.680 (18)	1.134 (18)	0.991 (18)
UZB	0.014 (41)	2.623 (35)	3.000 (38)	3.403 (41)	2.623 (49)	1.010 (41)
KEN	0.014 (41)	1.695 (20)	1.789 (24)	1.826 (20)	1.548 (27)	0.993 (20)

Table 5 (continued)

NOC r	ξ_r (Rank) FSO_WT	(Efficiency)$^{-1}$ BCC_AR	(Efficiency)$^{-1}$ BCC_AR_Int	(Efficiency)$^{-1}$ ZSG–Lins	(Efficiency)$^{-1}$ ZSG–Bi	(Efficiency)$^{-1}$ ZSG–BK
IRI	0.017 (43)	4.534 (53)	5.875 (63)	4.917 (55)	4.371 (74)	1.026 (55)
CZE	0.018 (44)	2.041 (26)	1.750 (23)	2.163 (27)	1.938 (37)	0.996 (27)
AZE	0.019 (45)	3.964 (48)	3.700 (48)	3.916 (46)	1.439 (24)	1.015 (46)
GRE	0.024 (46)	4.168 (49)	4.000 (49)	4.725 (53)	4.167 (70)	1.024 (53)
ETH	0.034 (47)	6.133 (63)	4.727 (58)	6.031 (60)	3.790 (66)	1.038 (60)
POL	0.036 (48)	3.149 (40)	7.000 (67)	3.252 (38)	2.694 (51)	1.008 (38)
TUN	0.038 (49)	5.390 (60)	15.000 (78)	6.128 (62)	4.836 (77)	1.039 (62)
BRN	0.048 (50)	8.726 (72)	3.077 (41)	9.691 (73)	2.740 (52)	1.077 (73)
NAM	0.048 (50)	7.591 (66)	4.250 (53)	9.342 (70)	2.639 (50)	1.074 (70)
HKG	0.061 (52)	4.943 (58)	5.273 (61)	4.857 (54)	2.980 (58)	1.025 (54)
PHI	0.071 (53)	8.748 (73)	8.500 (73)	9.439 (71)	0.169 (83)	1.075 (71)
ISR	0.071 (53)	4.261 (51)	4.000 (49)	5.425 (58)	4.260 (72)	1.032 (58)
BLR	0.082 (55)	3.369 (41)	4.375 (55)	3.327 (40)	1.937 (36)	1.009 (40)
RSA	0.086 (56)	7.591 (66)	17.500 (82)	9.140 (69)	0.199 (78)	1.072 (69)
TUR	0.090 (57)	6.317 (64)	9.800 (74)	6.040 (61)	0.398 (47)	1.038 (61)
AUT	0.091 (58)	5.321 (59)	3.667 (47)	5.220 (57)	2.007 (39)	1.029 (57)
ESP	0.095 (59)	3.540 (45)	33.000 (92)	3.439 (42)	0.488 (41)	1.010 (42)
MKD	0.095 (59)	7.740 (69)	5.000 (59)	9.000 (68)	3.077 (60)	1.070 (68)
SVK	0.095 (59)	4.560 (54)	8.000 (71)	4.695 (52)	2.945 (56)	1.024 (52)
POR	0.102 (62)	6.872 (65)	20.000 (83)	6.742 (64)	4.562 (75)	1.046 (64)
ROU	0.114 (63)	5.470 (61)	7.500 (69)	6.542 (63)	0.336 (57)	1.044 (63)
TPE	0.114 (63)	4.429 (52)	3.429 (44)	4.296 (47)	0.364 (53)	1.020 (47)

Table 5 (continued)

NOC r	ξ_r (Rank) FSO_WT	(Efficiency)$^{-1}$ BCC_AR	(Efficiency)$^{-1}$ BCC_AR_Int	(Efficiency)$^{-1}$ ZSG–Lins	(Efficiency)$^{-1}$ ZSG–Bi	(Efficiency)$^{-1}$ ZSG–BK
JOR	0.133 (65)	11.154 (75)	6.625 (66)	11.016 (74)	4.071 (68)	1.092 (74)
THA	0.134 (66)	17.056 (80)	29.333 (89)	20.087 (80)	0.059 (93)	1.189 (81)
ARG	0.139 (67)	22.308 (82)	17.000 (81)	21.554 (82)	0.119 (89)	1.205 (82)
NGR	0.139 (67)	30.475 (88)	27.333 (88)	29.408 (88)	0.082 (91)	1.290 (88)
EGY	0.143 (69)	9.948 (74)	6.250 (65)	9.595 (72)	0.192 (79)	1.077 (72)
MAR	0.143 (69)	8.262 (71)	20.000 (83)	12.437 (75)	8.260 (88)	1.107 (75)
PUR	0.143 (71)	4.662 (56)	7.000 (67)	7.066 (65)	4.662 (76)	1.049 (65)
VEN	0.143 (71)	4.232 (50)	4.000 (49)	4.934 (55)	0.454 (42)	1.026 (56)
KSA	0.148 (73)	45.464 (93)	44.000 (93)	54.007 (93)	0.068 (92)	1.554 (93)
MAS	0.150 (74)	28.206 (87)	21.750 (86)	27.292 (87)	0.088 (90)	1.267 (87)
IND	0.152 (75)	16.174 (78)	16.000 (79)	15.050 (77)	0.151 (86)	1.136 (77)
INA	0.156 (76)	16.961 (79)	16.889 (80)	16.017 (79)	0.130 (87)	1.146 (79)
DOM	0.163 (77)	5.697 (62)	4.273 (54)	5.611 (59)	0.495 (40)	1.033 (59)
FIN	0.163 (77)	24.490 (84)	5.200 (60)	24.027 (84)	0.266 (64)	1.232 (84)
GHA	0.163 (77)	41.993 (91)	5.500 (62)	41.299 (91)	0.160 (84)	1.417 (91)
KAZ	0.171 (80)	7.748 (70)	7.500 (69)	7.580 (66)	0.725 (22)	1.055 (66)
MEX	0.171 (80)	34.162 (89)	25.500 (87)	32.399 (89)	0.176 (81)	1.322 (89)
TKM	0.171 (80)	13.007 (77)	30.000 (91)	15.174 (78)	5.233 (80)	1.136 (78)
BOT	0.190 (83)	20.022 (81)	3.500 (45)	19.867 (80)	3.768 (65)	1.187 (80)
CIV	0.190 (84)	38.536 (90)	4.400 (56)	37.952 (90)	5.733 (82)	1.381 (90)
COL	0.190 (84)	7.642 (68)	6.143 (64)	8.185 (67)	0.394 (48)	1.061 (67)
KUW	0.190 (84)	42.142 (92)	8.000 (71)	41.443 (91)	6.423 (85)	1.419 (92)

Table 5 (continued)

NOC r	ξ_r (Rank) FSO_WT	(Efficiency)$^{-1}$ BCC_AR	(Efficiency)$^{-1}$ BCC_AR_Int	(Efficiency)$^{-1}$ ZSG− Lins	(Efficiency)$^{-1}$ ZSG− Bi	(Efficiency)$^{-1}$ ZSG− BK
LTU	0.190 (84)	11.813 (76)	11.000 (76)	13.509 (76)	3.790 (66)	1.118 (76)
MGL	0.190 (84)	4.713 (57)	12.000 (77)	4.677 (51)	1.369 (21)	1.023 (51)
ARM	0.286 (89)	3.836 (47)	3.231 (42)	3.809 (44)	1.329 (20)	1.014 (44)
BUR	0.286 (89)	24.433 (84)	4.400 (56)	24.200 (85)	4.254 (71)	1.233 (85)
KGZ	0.286 (89)	4.561 (54)	10.667 (75)	4.528 (49)	1.607 (29)	1.022 (49)
MDA	0.286 (89)	22.927 (83)	20.000 (83)	22.723 (83)	4.096 (69)	1.217 (83)
SYR	0.286 (89)	25.378 (86)	29.000 (89)	25.126 (86)	4.353 (73)	1.243 (86)
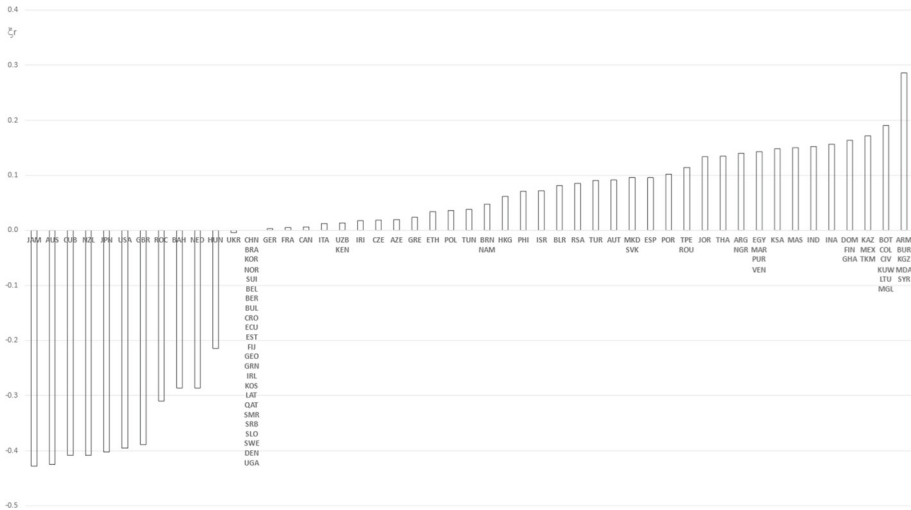
Fig. 7 NOCs ranked in increasing order of performance index ξ_r

is not too close to unity (which would correspond to giving the same weight to all medal types), the results are not very sensitive.

5 Conclusions

In this paper, an innovative way is proposed of assessing the efficiency, ranking, and setting targets in the Olympic Games. It uses a centralized DEA perspective that takes into account the FSO character of the total number of medals won by all the NOCs. The problem is formulated in a multiobjective optimization framework, and utilizes the ideal-point concept and the WTM solution. Since the latter generally has alternative optima, a secondary goal is proposed to project the DMUs onto their closest targets. In this second step, a rectangular distance is applied. The final targets thus computed involve winning more medals in the case of inefficient NOCs and, in order to compensate, winning fewer medals in the case of other NOCs that have performed super-efficiently. The proposed performance index is simply a weighted sum of the changes in the number of medals, and it can be positive, negative, or zero, depending on the NOC.

As strengths of the proposed method, its FSO character should be highlighted (i.e., the total number of medals available is respected), as should the fairness principle implied by the WTM solution (i.e., for all NOCs and all medal types, the targets become as close as possible to their ideal values) and the closest-target principle implied by the secondary goal (i.e., from among the alternative FSO_WT efficient targets, the target that minimizes the rectangular distance to the observed outputs is determined). Another feature of the proposed approach is that, unlike all previous approaches, it computes not only performance indices (akin to a type of FSO super-efficiency score) but also, and more importantly, FSO_WT efficient targets. Furthermore, the computed performance indices enable the NOCs to be classified into three categories corresponding to underperforming, neutral, and super-efficient DMUs. The latter category refers to those NOCs that have won more medals than they would have done if all the NOCs had performed at their best level.

Apart from the mathematical modelling aspects, the proposed approach represents a significant departure from existing FSO DEA approaches, whose sole objective is that of efficiency estimation. These approaches employ variables that represent the changes in the number of medals of the DMUs (a necessity in order to impose the FSO constraints) but they fail to report the corresponding targets. This is due to the existence of many alternative FSO efficient targets and, for those only interested in computing efficiency scores, unrealistic and indefensible targets may result. The proposed approach follows a different path whereby, instead of ignoring the existence of alternative FSO efficient targets, it tackles the challenging problem of computing reasonable targets for the DMUs. In other words, instead of focusing on computing efficiency scores, we focus on computing targets, and determine the performance indices of the DMUs ex post, based on those targets.

These performance indices ξ_r are derived as a subproduct of the computed targets and represent an empirical way of measuring the distance of a DMU to its corresponding target and, as such, of the effort to reach said target. Therefore, $\xi_r > 0$ implies winning more medals or medals of a more valuable type: more effort in either case. In contrast, $\xi_r < 0$ means that the number and/or value of the medals won exceeds what could be expected given the inputs (i.e., the resources) of the country. In other words, such a strong performance is unsustainable in the event that all DMUs perform at their best level. Finally, $\xi_r = 0$ indicates a neutral performance, in line with expectations.

The proposed approach has been applied to the Tokyo 2020 Olympic Games and compared with conventional (i.e., non-FSO) DEA approaches as well as with other FSO approaches. Although there is a certain ranking similarity with the alternative approaches (higher in the case of BCC_AR, BCC_AR_Int, ZSG-Lins, and ZSG-BK and lower in the case of ZSG-Bi, Lexicographic ranking, and WA DEA), it is clear that, due to its differentiated character, the proposed approach provides a ranking distinct from the other methods. The NOCs that are ranked highest by the proposed approach are JAM, AUS, CUB, NZL, JPN, USA, GBR, ROC, BAH, NED, HUN, and UKR. All of them are super-efficient and have to reduce the number of medals won by a significant percentage which, for example, in the case of USA amounts to 27 fewer gold medals, and in the case of JPN and GBR, this entails 15 and 13 fewer gold medals, respectively. These reductions in the number of medals won corresponds to correcting the serial correlation that complicates efficiency assessment in this type of competition. As regards the countries that exhibited the worst relative performance in Tokyo 2020 according to the proposed approach, these are ARM, BUR, KGZ, MDA and SYR. In addition to these two extremes of most- and least-performing countries, it is important to identify a set of neutral-performance countries that can be said to have matched their expectations, while taking into account their Population and GDP as well as the FSO character of the competition.

As regards the managerial implications of the proposed approach, information regarding the ideal number of medals a country could aspire to win based on its size and wealth is of interest. Moreover, the rationale that states that the attainment of the ideal number of medals is generally impossible leads to targets of a more realistic nature being set, which take into account the FSO character of the Games. Furthermore, since the proposed approach uses the current situation as a reference, the computed target could be considered as a first step towards the ideal target.

As a continuation of this research, we envisage its extension to include NDEA approaches towards the Olympic Games. Furthermore, other multiobjective optimization methods, particularly interactive multiobjective optimization methods (e.g., Soltani & Lozano, 2020), can be applied to this centralized DEA scenario.

Appendix

This appendix contains the list of the acronyms used in the text.

AR	Assurance Region.
BCC_AR	Banker, Charnes and Cooper + Assurance Region.
BCC_AR_Int	Banker, Charnes and Cooper + Assurance Region + Integer data.
CRS	Constant Returns to Scale.
CSW	Common set of weights.
DEA	Data Envelopment Analysis.
DMU	Decision-Making Unit.
DMU	Decision-Making Unit.
EEF	Equilibrium Efficient Frontier.
FSO	Fixed-sum output.
EEF	Equilibrium Efficient Frontier.
	FSO_WTFixed-sum output + Weighted Tchebychef Method.
GDP	Gross Domestic Product.
GEEF	Generalized Equilibrium Efficient Frontier.
MILP	Mixed-Integer Linear Program.
NDEA	Network DEA.
NOC	National Olympic Committee.
VRS	Variable Returns to Scale.
WTM	Weighted Tchebychef Method.
ZSG	Zero-Sum Gains approach.
ZSG-Bi	Zero-Sum Gains approach proposed by Bi et al., (2014)
ZSG-BK	Zero-Sum Gains approach proposed by Bouzidis and Karagiannis (2022)
ZSG-Lins	Zero-Sum Gains approach proposed by Lins et al., (2003).

References

Amirteimoori, A., Masrouri, S., Yang, F., & Kordrostami, S. (2017). Context-based competition strategy and performance analysis with fixed-sum outputs: An application to banking sector. *Journal of the Operational Research Society, 68*, 1461–1469.

Aparicio, J., Ruíz, J. L., & Sirvent, I. (2007). Closest targets and minimum distance to the Pareto-efficient frontier in DEA. *Journal of Productivity Analysis, 28*, 209–218.

Arana-Jiménez, M., Sánchez-Gil, M. C., & Lozano, S. (2020). Efficiencya Assessment and Target Setting Using a Fully Fuzzy DEA Approach. *International Journal of Fuzzy Systems, 22*, 1056–1072.

Azizi, H., & Wang, Y. M. (2013). Improved DEA models for measuring interval efficiencies of decision-making units. *Measurement, 46*, 1325–1332.

Banker, R. D., & Morey, R. (1986). Efficiency analysis for exogenously fixed inputs and outputs. *Operations Research, 34*, 513–521.

Bi, G., Feng, C., Ding, J., Liang, L., & Chu, F. (2014). The linear formulation of the ZSG-DEA models with different production technologies. *Journal of the Operational Research Society, 65*, 1202–1211.

Bouzidis, T., & Karagiannis, G. (2022). An alternative ranking of DMUs performance for the ZSG-DEA model. *Socio-Economic Planning Sciences, 81*, 101179.

Calzada-Infante, L., & Lozano, S. (2016). Analysing Olympic Games through dominance networks. *Physica a: Statistical Mechanics and Its Applications, 462*, 1215–1230.

Chiang, C. I., Hwang, M. J., & Liu, Y. H. (2011). Determining a common set of weights in a DEA problem using a separation vector. *Mathematical and Computer Modelling, 54*, 2464–2470.

Churilov, L., & Flitman, A. (2006). Towards fair ranking of Olympics achievements: The case of Sydney 2000. *Computers & Operations Research, 33*, 2057–2082.

Collier, T., Johnson, A. L., & Ruggiero, J. (2011). Measuring Technical Efficiency in Sports. *Journal of Sports Economics, 12*, 579–598.

Cooper, W. W., Pastor, J. T., Aparicio, J., & Borras, F. (2011). Decomposing profit inefficiency in DEA through the weighted additive model. *European Journal of Operational Research, 212*, 411–416.

Despotis, D. K., Sotiros, D., & Koronakos, G. (2016). A network DEA approach for series multi-stage processes. *Omega, 61*, 35–48.

Ebrahimnejad, A., & Tavana, M. (2014). An interactive MOLP method for identifying target units in output-oriented DEA models: The NATO enlargement problem. *Measurement, 52*, 124–134.

Estellita Lins, M. P., Angulo-Meza, L., & Moreira da Silva, A. C. (2004). A multi-objective approach to determine alternative targets in data envelopment analysis. *Journal of the Operational Research Society, 55*, 1090–1101.

Flegl, M., & Andrade, L. A. (2018). Measuring countries' performance at the Summer Olympic Games in Rio 2016. *Opsearch, 55*, 823–846.

Gutiérrez, E., & Lozano, S. (2016). Efficiency assessment and output maximization possibilities of European small and medium sized airports. *Research in Transportation Economics, 56*, 3–14.

Jablonsky, J. (2018). Ranking of countries in sporting events using two-stage data envelopment analysis models: A case of Summer Olympic Games 2016. *Central European Journal of Operations Research, 26*, 951–966.

Korhonen, P., Stenfors, S., & Syrjänen, M. (2003). Multiple objective approach as an alternative to radial projection in DEA. *Journal of Productivity Analysis, 20*, 305–321.

Lei, X., Li, Y., Xie, Q., & Liang, L. (2015). Measuring Olympics achievements based on a parallel DEA approach. *Annals of Operations Research, 226*, 379–396.

Li, Y., Lei, X., Dai, Q., & Liang, L. (2015). Performance evaluation of participating nations at the 2012 London Summer Olympics by a two-stage data envelopment analysis. *European Journal of Operational Research, 243*, 964–973.

Li, Y., Liang, L., Chen, Y., & Morita, H. (2008). Models for measuring and benchmarking Olympics achievements. *Omega, 36*, 933–940.

Li, Y., Liu, J., Ang, S., & Yang, F. (2021). Performance evaluation of two-stage network structures with fixed-sum outputs: An application to the 2018winter Olympic Games. *Omega, 102*, 102342.

Lins, M. P. E., Gomes, E. G., Soares de Mello, J. C. C. B., & Soares de Mello, A. J. R. (2003). Olympic ranking based on a zero sum gains DEA model. *European Journal of Operational Research, 148*, 312–322.

Lovell, C. A. K., & Pastor, J. T. (1995). Units invariant and translation invariant DEA models. *Operations Research Letters, 18*, 147–151.

Lozano, S. (2023). Bargaining approach for efficiency assessment and target setting with fixed-sum variables. *Omega, 114*, 102728.

Lozano, S., & Adenso-Díaz, B. (2018). Network DEA-based biobjective optimization of product flows in a supply chain. *Annals of Operations Research, 264*, 307–323.

Lozano, S., & Khezri, S. (2021). Network DEA smallest improvement approach. *Omega, 98*, 102140.

Lozano, S., Soltani, N., & Dehnokhalaji, A. (2020). A compromise programming approach for target setting in DEA. *Annals of Operations Research, 288*, 363–390.

Lozano, S., Villa, G., Guerrero, F., & Cortés, P. (2002). Measuring the performance of nations at the Summer Olympics using data envelopment analysis. *Journal of the Operational Research Society, 53*, 501–511.

Malekmohammadi, N., Hosseinzadeh Lotfi, F., & Jaafar, A. B. (2011). Target setting in data envelopment analysis using MOLP. *Applied Mathematical Modelling, 35*, 328–338.

Marler, R. T., & Arora, J. S. (2004). Survey of multi-objective optimization methods for engineering. *Structural Multidisciplinary Optimization, 26*, 369–395.

Ramchandani, G., (2022). Home advantage in the Summer Olympic Games: evidence from Tokyo 2020 and prospects for Paris 2024, https://olympicanalysis.org/section-4/home-advantage-in-the-summer-olympic-games-evidence-from-tokyo-2020-and-prospects-for-paris-2024/ (accessed May 16th 2022)

Sekitani, K., & Zhao, Y. (2021). Performance benchmarking of achievements in the Olympics: An application of data envelopment analysis with restricted multipliers. *European Journal of Operational Research, 294*, 1202–1212.

Soares de Mello, J. C. C. B., Angulo-Meza, L., & Branco Da Silva, B. P. (2009). A ranking for the Olympic Games with unitary input DEA models. *IMA Journal of Management Mathematics, 20*, 201–211.

Soares de Mello, J. C. C. B., Angulo-Meza, L., & Lacerda, F. G. (2012). A DEA model with a non discritionary variable for Olympic evaluation. *Pesquisa Operacional, 2*, 21–29.

Soltani, N., & Lozano, S. (2020). Interactive multiobjective DEA target setting using lexicographic DDF. *RAIRO-Operations Research, 54*, 1703–1722.

Wu, J., Liang, L., & Chen, Y. (2009a). DEA game cross-efficiency approach to Olympic rankings. *Omega, 37*, 909–918.

Wu, J., Liang, L., & Yang, F. (2009b). Achievement and benchmarking of countries at the Summer Olympics using cross efficiency evaluation method. *European Journal of Operational Research, 197*, 722–730.

Wu, J., Zhou, Z., & Liang, L. (2010). Measuring the performance of nations at Beijing summer Olympics using integer-valued DEA model. *Journal of Sports Economics, 11*, 549–566.

Yang, F., Wu, D. D., Liang, L., & O'Neill, L. (2011). Competition strategy and efficiency evaluation for decision making units with fixed-sum outputs. *European Journal of Operational Research, 212*, 560–569.

Yang, M., Li, Y. J., & Liang, L. (2015). A generalized equilibrium efficient frontier data envelopment analysis approach for evaluating DMUs with fixed-sum outputs. *European Journal of Operational Research, 246*, 209–217.

Zhang, D., Li, X., Meng, W., & Liu, W. (2009). Measuring the performance of nations at the Olympic Games using DEA models with different preferences. *Journal of the Operational Research Society, 60*, 983–990.

Zhu, Q., Song, M., & Wu, J. (2020). Extended secondary goal approach for common equilibrium efficient frontier selection in DEA with fixed-sum outputs. *Computers & Industrial Engineering, 144*, 106483.