Multi-Objective Evolutionary Algorithms in Aeronautical and Aerospace Engineering

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***Abstract*—Nowadays, the solution of multi-objective optimization problems in aeronautical and aerospace engineering has become a standard practice. These two fields offer highly complex search spaces with different sources of difficulty, which are amenable to the use of alternative search techniques such as metaheuristics, since they require little domain information to operate. From the several metaheuristics available, multi-objective evolutionary algorithms (MOEAs) have become particularly popular, mainly because of their availability, ease of use and flexibility. This paper presents a taxonomy and a comprehensive review of applications of MOEAs in aeronautical and aerospace design problems. The review includes both the characteristics of the specific MOEA adopted in each case, as well as the features of the problems being solved with them. The advantages and disadvantages of each type of approach are also briefly addressed. We also provide a set of general guidelines for using and designing MOEAs for aeronautical and aerospace engineering problems. In the final part of the paper we provide some potential paths for future research, which we consider promising within this area.**

I. Introduction

Optimal design in aeronautical/aerospace engineering is, by nature, a multiobjective-multidisciplinary and highly difficult problem. Aerodynamics, structures, propulsion, acoustics, manufacturing and economics, are some of the disciplines involved in this type of problems. Even if a single discipline is considered, many design problems have competing objectives (e.g., to optimize a wing's lift and drag or a wing's structural strength and weight). During the last three decades, the process of engineering design has been clearly improved because of the dominant role that computational simulations have played in this area [87] e.g., Computational Fluid Dynamics (CFD) simulations to perform aerodynamic analysis [67] and Computational Structural Dynamics/Mechanics (CSD/M) through the use of the Finite Element Method (FEM) to process structural analysis [169]. The increasing demand for optimal and robust designs, driven by economic and environmental constraints, along with an increasing computing power, has improved the role of computational simulations, from being just analytical tools until becoming design optimization tools.

In spite of the fact that gradient-based numerical optimization methods have been successfully applied in a variety of aeronautical/aerospace design problems [63], [153][1], their use is considered a challenge due to the following difficulties found in practice:

1) The design space is frequently multimodal and highly non-linear.
2) Evaluating the objective function (performance) for the design candidates is usually time consuming, due mainly to the high-fidelity and dimensionality required in the simulations.
3) By themselves, single-discipline optimizations may provide solutions which not necessarily satisfy objectives and/or constraints considered in other disciplines.
4) The complexity of the sensitivity analyses in Multidisciplinary Design Optimization (MDO[2]) increases as the number of disciplines involved becomes larger.
5) In MDO, a trade-off solution, or a set of them, are searched for.

Based on the previously indicated difficulties, designers have been motivated to use alternative optimization techniques such as Evolutionary Algorithms (EAs) [34], [86], [122]. Multi-Objective Evolutionary Algorithms (MOEAs) have gained an increasing popularity as numerical optimization tools in aeronautical and aerospace engineering during the last few years [4], [87], [120]. These population-based methods mimic the evolution of species and the survival of the fittest, and compared to traditional optimization techniques, they present the following advantages:

- **Robustness:** In practice, they produce good approximations to optimal sets of solutions, even in problems with very large and complex design spaces. Instead of a single-point search with gradient information, MOEAs use a population of design candidates (i.e., they perform a multi-point search) and are less prone to get trapped in local optima. Additionally, they can manage non-differentiable, mixed real-discrete and highly non-linear objective functions/fitness landscapes.
- **Multiple solutions per run:** As MOEAs use a population of candidates, they are designed to generate multiple trade-off solutions in a single run. Evidently, the generation of more solutions also involves a higher computational time when dealing with expensive applications. Thus, the number of solutions to be generated by a MOEA in the applications discussed in this paper tends to be low, unless surrogate models are adopted.
- **Easy to parallelize:** The design candidates in a MOEA population, at each generation, can be evaluated in paral-

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[1]It is worth noting that most of the applications using gradient-based methods have adopted them to find global optima or a single compromise solution for multi-objective problems.

[2]Multidisciplinary Design Optimization, by its nature, can be considered as a multi-objective optimization problem, where each discipline aims to optimize a particular performance metric.

lel using diverse paradigms. This can be useful in problems involving objective functions that are costly to evaluate (something common in aeronautical and aerospace applications).

- **Simplicity:** MOEAs use only the objective function values for each design candidate. They do not require a substantial modification or complex interfacing for using a CFD or CSD/M code. This situation substantially reduces the cost related to code writing and tuning every time a new application is envisaged. Furthermore, designers can easily make use of in-house developed and/or commercial codes previously validated.
- **Easy to hybridize:** Along with the simplicity previously stated, MOEAs also allow an easy hybridization with alternative methods, e.g., memetic algorithms, which additionally introduce specifities to the implementation, without influencing the MOEA simplicity.
- **Novel solutions:** In many cases, gradient-based optimization techniques converge to designs which have little variation even if produced with very different initial setups. In contrast, the inherent explorative capabilities of MOEAs allow them to produce, some times, novel and non-intuitive designs.

The important volume of information that has been published on the use of MOEAs in aeronautical and aerospace engineering applications (mainly motivated by the advantages previously addressed) has led us to write this paper, which provides a review of this work in an organized and classified manner. As we will see later on, MOEAs have been used in a variety of design stages and in diverse problems.

The remainder of this paper is organized as follows. In Section II, some basic concepts on multi-objective optimization are presented. Section III briefly describes some of the MOEAs that have been most commonly used in the specialized literature. Section IV presents a taxonomy of applications of MOEAs in aeronautical and aerospace engineering. Such applications are explained in more detail in Section V. After that, in Section VI, possible future research paths are highlighted. Finally, Section VII presents the main conclusions of this review.

II. Basic Concepts

A Multi-Objective Optimization Problem (MOP) can be mathematically defined as follows[3]:

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})] \tag{1}$$

subject to:

$$g_i(\vec{x}) \leq 0 \ , \quad i = 1, 2, \ldots, m \tag{2}$$

$$h_i(\vec{x}) = 0 \ , \quad i = 1, 2, \ldots, p \tag{3}$$

where $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ is the vector of decision variables, $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., m$, $j = 1, ..., p$ are the constraint functions of the problem.

The set of constraints of the problem defines the feasible region in the search space of the problem. Any vector of variables $\vec{x}$ which satisfies all the constraints is considered a feasible solution. In their original version, an EA (and also a MOEA) lacks a mechanism to deal with constrained search spaces. This has motivated a considerable amount of research regarding the design and implementation of constraint-handling techniques for both EAs and MOEAs [23], [108].

Regarding optimal solutions in MOPs, the following definitions are provided:

Definition 1. A vector of decision variables $\vec{x} \in \mathbb{R}^n$ dominates another vector of decision variables $\vec{y} \in \mathbb{R}^n$, (denoted by $\vec{x} \prec \vec{y}$) if and only if $\vec{x}$ is partially less than $\vec{y}$, i.e., $\forall i \in \{1, \ldots, k\} : f_i(\vec{x}) \leq f_i(\vec{y}) \wedge \exists i \in \{1, \ldots, k\} : f_i(\vec{x}) < f_i(\vec{y})$.

Definition 2. A vector of decision variables $\vec{x} \in \mathcal{X} \subset \mathbb{R}^n$ is **nondominated** with respect to $\mathcal{X}$, if there does not exist another $\vec{x}' \in \mathcal{X}$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$.

Definition 3. A vector of decision variables $\vec{x}^* \in \mathcal{F} \subset \mathbb{R}^n$ ($\mathcal{F}$ is the feasible region) is **Pareto-optimal** if it is nondominated with respect to $\mathcal{F}$.

Definition 4. The **Pareto optimal set** $\mathcal{P}^*$ is defined by:

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} | \vec{x} \text{ is Pareto-optimal}\}$$

Definition 5. The **Pareto front** $\mathcal{PF}^*$ is defined by:

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^k | \vec{x} \in \mathcal{P}^*\}$$

The goal on a MOP consists on determining the Pareto optimal set from the set $\mathcal{F}$ of all the decision variable vectors that satisfy (2) and (3).

Thus, when solving a MOP, we aim to find not one, but the set of solutions representing the best possible trade-offs among the objectives (the so-called Pareto optimal set).

III. Multi-Objective Evolutionary Algorithms

It is worth indicating that traditional EAs require some modifications in order to deal with multi-objective optimization problems. The main two are the following:

1) All the nondominated solutions should be considered equally good by the selection mechanism. This means that a different notion of fitness is required for dealing with multi-objective optimization problems. The most popular mechanism to deal with this problem is called Pareto ranking and was introduced by Goldberg [51]. This approach assigns a rank to each solution based on its Pareto dominance, such that nondominated solutions

[3]Without loss of generality, minimization is assumed in the following definitions, since any maximization problem can be transformed into a minimization one.

are all sampled at the same rate. However, in the early days of MOEAs, several mechanisms not based on Pareto optimality were adopted with EAs [24].
2) EAs tend to converge to a single solution if run long enough, because of stochastic noise [51]. Therefore, a mechanism to maintain diversity is required. This component is known as the *density estimator*. Fitness sharing [52] was the earliest density estimator, but many others have been proposed over time, including clustering [189], entropy [41], adaptive grids [81] and crowding [32], among others.

MOEAs can be classified in several ways [24]. However, for the purposes of this survey, we decided to adopt a simple high-level classification that considers only two types of MOEAs: (a) Non-Pareto-based and (b) Pareto-based. The first group contains MOEAs that do not adopt the concept of Pareto optimality in their selection mechanism, whereas the second comprises those MOEAs that adopt Pareto optimality in their selection mechanism. Some of the most popular non-Pareto-based MOEAs are the following:

- **Lexicographic method**: The user ranks the objectives of the problem in a decreasing order and the optimization proceeds from higher to lower order objectives, one at a time. Once an objective is optimized, the aim is to improve as much as possible the following objective(s) without decreasing the quality of the previous one(s) [24]. This sort of approach normally generates a single nondominated solution, but if instead of using a fixed objective as the most important, it is randomly chosen, several solutions can be generated in one run.
- **Aggregating functions**: All the objectives are added up into a single (scalar) value which constitutes the objective to be optimized. Since objectives tend to be defined in very different ranges, a normalization is normally required. Also, weights tend to be assigned to each objective in order to define preferences from the user [24]. Varying the weights during the run allows, in general, the generation of different nondominated solutions in one run [59], [71].
- **Population-based methods**: A number of sub-populations (usually as many as the number of objective functions of the problem) are generated from a main population of an EA. Each sub-population optimizes a single objective function and then all the sub-populations are merged and mixed. The aim is that, when performing crossover, individuals that are good in one objective will recombine with individuals that are good in another one [149]. This sort of approach produces several nondominated solutions in a single run, but it typically misses good compromises among the objectives because of the way in which individuals are selected in each population [24].

Among the Pareto-based methods, there are two sub-classes: the non-elitist MOEAs and the elitist MOEAs. Non-elitist MOEAs do not retain the nondominated solutions that they generate and could, therefore, lose them after applying the evolutionary operators. Elitist MOEAs retain these solutions either in an external archive or in the main population.

The most representative non-elitist MOEAs are the following:

- **Nondominated Sorting Genetic Algorithm** (NSGA): It was proposed by Srinivas and Deb [160]. It is based on several layers of classifications of the individuals. Before selection is performed, the population is ranked on the basis of nondomination: all nondominated individuals are classified into one category (with a dummy fitness value, which is proportional to the population size, in order to provide an equal reproductive potential for these individuals). To maintain the diversity of the population, these classified individuals are shared with their dummy fitness values. Then this group of classified individuals is ignored and another layer of nondominated individuals is considered. The process continues until all individuals in the population are classified. Since individuals in the first front have the maximum fitness value, they always get a higher selection probability than the rest of the population.

- **Niched-Pareto Genetic Algorithm** (NPGA): Proposed in [62]. It uses a tournament selection scheme based on Pareto dominance. The basic idea of the algorithm is the following: Two individuals are randomly chosen and compared against a subset from the entire population (typically, around 10% of the population). If one of them is dominated (by the individuals randomly chosen from the population) and the other is not, then the nondominated individual wins. When both competitors are either dominated or nondominated (i.e., there is a tie), the result of the tournament is decided through fitness sharing [52].

- **Multi-Objective Genetic Algorithm** (MOGA): Proposed in [46]. In this approach, the rank of a certain individual corresponds to the number of individuals in the current population by which it is dominated. All nondominated individuals are assigned the lowest possible rank (i.e., one), while dominated ones receive as rank the number of individuals that dominate them plus one.

Among the most popular Pareto-based elitist MOEAs, we have the following:

- **Strength Pareto Evolutionary Algorithm** (SPEA): Introduced in [189]. It uses an archive containing nondominated solutions previously found (the so-called external nondominated set). At each generation, nondominated individuals are copied to the external nondominated set, removing the dominated solutions. For each individual in this external set, a *strength* value is computed. This strength is similar to the ranking value of MOGA, since it is proportional to the number of solutions to which a certain individual dominates. The fitness of each member of the current population is computed according to the strengths of all external nondominated solutions that dominate it. In SPEA, instead of using niches based on distance (as MOGA and NPGA), Pareto dominance is adopted to ensure that the solutions are properly distributed along the Pareto front. Although no niche radius is required, the effectiveness of this approach relies on the size of the external nondominated set, since such a set participates in the selection process of SPEA. Because of this, the authors decided to adopt a technique that prunes the contents of the external nondominated set so that its size remains below a certain threshold. The approach adopted for this sake was a clustering technique called "average linkage method" [112].

- **Strength Pareto Evolutionary Algorithm 2** (SPEA2): SPEA2 has three main differences with respect to its predecessor [188]: (1) it incorporates a fine-grained fitness assignment strategy which, for each individual, takes into account both the number of individuals to which it dominates and the number of individuals that dominate it; (2) it uses a nearest neighbor density estimation technique which guides the search more efficiently, and (3) it has an enhanced archive truncation method that guarantees the preservation of boundary solutions.
- **Pareto Archived Evolution Strategy** (PAES): This algorithm was introduced in [83]. PAES consists of a (1+1) evolution strategy (i.e., a single parent that generates a single offspring) in combination with a historical archive that records the nondominated solutions previously found. This archive is used as a reference set against which each mutated individual is being compared. Such a historical archive is the elitist mechanism adopted in PAES. However, an interesting aspect of this algorithm is the procedure used to maintain diversity which consists of a crowding procedure that divides objective space in a recursive manner. Each solution is placed in a certain grid location based on the values of its objectives (which are used as its "coordinates" or "geographical location"). A map of such grid is maintained, indicating the number of solutions that reside in each grid location. Since the procedure is adaptive, no extra parameters are required (except for the number of divisions of the objective space).
- **Nondominated Sorting Genetic Algorithm II** (NSGA-II): This approach was introduced in [32] as an improved version of the NSGA. In the NSGA-II, for each solution one has to determine how many solutions dominate it and the set of solutions to which it dominates. The NSGA-II estimates the density of solutions surrounding a particular solution in the population by computing the average distance of two points on either side of this point along each of the objectives of the problem. This value is the so-called *crowding distance*. During selection, the NSGA-II uses a crowded-comparison operator which takes into consideration both the nondomination rank of an individual in the population and its crowding distance (i.e., nondominated solutions are preferred over dominated solutions, but between two solutions with the same nondomination rank, the one that resides in the less crowded region is preferred). The NSGA-II does not use an external memory as the other MOEAs previously discussed. Instead, the elitist mechanism of the NSGA-II consists of combining the best parents with the best offspring obtained (i.e., a $(\mu + \lambda)$-selection). Due to its clever mechanisms, the NSGA-II is much more efficient (computationally speaking) than its predecessor, and its performance is so good, that it has become very popular in the last few years, becoming a landmark against which other MOEAs have to be compared [187].

There are several other multi-objective metaheuristics available. The two following are discussed here because they are adopted by some of the applications discussed here:

- **Particle Swarm Optimization:** This metaheuristic is inspired on the choreography of a bird flock which aim to find food [77]. It can be seen as a distributed behavioral algorithm that performs (in its more general version) a multidimensional search. The implementation of the algorithm adopts a population of particles, whose behavior is affected by either the best local (i.e., within a certain neighborhood) or the best global individual. Particle swarm optimization (PSO) has been successfully used for both continuous nonlinear and discrete binary optimization [40]. For extending PSO to deal with MOPs, the main issues are: (1) how to select particles (to be used as leaders) in order to give preference to nondominated solutions over those that are dominated?, (2) how to retain the nondominated solutions found during the search process in order to report solutions that are nondominated with respect to all the past populations and not only with respect to the current one?, and 3) how to maintain diversity in the swarm in order to avoid convergence to a single solution? Normally, mechanisms very similar to those adopted with MOEAs (namely, Pareto-based selection and external archives) have been adopted in multi-objective particle swarm optimizers (MOPSOs). However, the addition of other mechanisms (e.g., a mutation operator) is also relatively common in MOPSOs. An important number of multi-objective versions of PSO currently exist (see for example [140]), and this remains as a very active area of research.
- **Differential Evolution:** This metaheuristic was proposed by Kenneth Price and Rainer Storn [130], [161] to optimize problems over continuous domains. The core idea is to use vector differences for perturbing a vector population, and it aims to estimate the gradient in a region (rather than in a point). Differential Evolution (DE) performs mutation based on the distribution of the solutions in the current population. In this way, search directions and possible step sizes depend on the location of the individuals selected to calculate the mutation values. Several DE variants are possible, and they differ in the way in which the parents are selected and in the form in which recombination and mutation takes place (see [130] for more information on DE). The high success of DE in single-objective optimization has made it an interesting candidate for solving MOPs. The main issues for extending DE to multi-objective optimization

are very similar to those of PSO (i.e., how to select parents, how to store nondominated solutions and how to maintain diversity in the population). As with MOPSOs, very similar mechanisms to those adopted by MOEAs have been use with multi-objective differential evolution (MODE). A variety of MODE approaches currently exist (see for example [110]), and this also remains as a very active area of research. It is worth noting that MODEs are often considered MOEAs [24].

Although many other MOEAs exist (see for example [25], [186]), it is not the intention of this paper to be comprehensive. The interested reader may refer to [24], [31] for more information on this topic.

The main advantages of MOEAs are their generality, ease of use and the fact that they require little or no specific domain information to operate. Also, they are less susceptible to the specific features of the problem (e.g., shape or continuity of the Pareto front) than traditional mathematical programming techniques [24].

Although the performance of MOEAs has been traditionally assessed using a variety of quantitative measures (see for example [24], [190]), few of them have been adopted in the applications discussed in this paper. This is probably due to the high computational cost of these applications and the few nondominated solutions that are normally produced. This is the reason why the use of such performance measures is not discussed in the applications reviewed here, except if one of them is adopted in the selection process (e.g., SMS-EMOA adopts a selection mechanism based on a performance measure called *hypervolume* [10]).

IV. A Taxonomy of Applications

Aeronautical/aerospace engineering design process comprise three phases: (i) *Conceptual design*, (ii) *Preliminary design*, and (iii) *Detailed design* [13]. In each of these phases, design concepts are analyzed to determine their compliance with the performance requirements, as well as their manufacturability and economical viability. The design process cannot be considered as serial, but as a cyclic process, in which many design iterations are required. This iterative process is mainly executed between the first two phases. Applications surveyed in this article cover the spectrum of *Conceptual design* and *Preliminary design* where numerical optimization has its greatest impact, and where the goal of optimization is to refine the design, prior to the *Detailed design* phase in which design production is initiated (see Figure 1).

Although very interesting ways of classifying complex MOPs have been proposed in the past (see for example the approach described in [73]), the taxonomy adopted in this article aims to reflect the optimization problem complexity degree in terms of three main features: (i) physics-model fidelity, (ii) the number of disciplines involved, and (iii) the associated computational cost needed to perform the optimization process. The classes considered are the following:

1) **Conceptual design optimization**: Being this the earliest phase of the design process, it has an emphasis on finding the best *Design Concepts*, ensuring designers that they are heading into the correct design path, guaranteeing to meet all design's performance requirements.
2) **2D geometries and airfoil shape optimization**: In these applications the dimensionality of the problem is reduced, and the physics for the simulations can be considered as two-dimensional.
3) **3D complex physics/shape optimization**: 3D complex physics, 3D complex geometries or the combination of both are considered in this class of applications.
4) **Structural optimization**: Considering the design of lighter and stronger structures as the premise of aeronautical/aerospace design, this class of application looks for the best trade-off between these two objectives, clearly in conflict.
5) **Multidisciplinary design optimization**: These applications cover those where two or more disciplines are involved, each one with specific objectives to accomplish or to optimize.
6) **Aerospace system optimization**: Applications focused on space systems such as spacecrafts and satellites.
7) **Control system design**: These applications are used for parametric design in different control laws.

The different approaches in each one of these classes will be described in the following section. It is worth mentioning that this review of the state-of-the-art is focused on Pareto-based MOEAs. This decision was made based on the fact that the number of references of non-Pareto-based approaches would not allow a careful description of each approach.

V. Applications

A. Conceptual design optimization

Traditionally, the aeronautical/aerospace *Conceptual Design* phase has been conducted with the help of databases, statistics, and regression/low-order engineering models as well as company's/designer's accumulated experience. The main outcome of this design phase has been to determine a few promising *Design Concepts* to be further analyzed in the *Preliminary Design* phase, in which numerical simulations or experimental setups are developed to verify and refine the design. Additionally, tradeoff analyses are performed in order to identify unreasonable or conflicting requirements. This latter task has been limited because of the large design spaces that need to be explored, and a holistic (multidisciplinary) vision of the design is required when multiple disciplines are involved in the design. Nowadays, with the increasing computing power available, low-cost/fidelity numerical simulations have spread toward the *Conceptual Design* phase, making it possible to benefit from the *exploration* of large design spaces with reduced time and low computational cost. Additionally, it is possible to envision performing *trade-off* analysis of the multi-objective and/or multidisciplinary designs. Both of these characteristics are inherent in the use of MOEAs for the present class of applications reported next:

- Oyama and Liou [124] addressed the conceptual design of rocket engine pumps, for a centrifugal single and multi-stage pump design. In both cases two objectives were de-

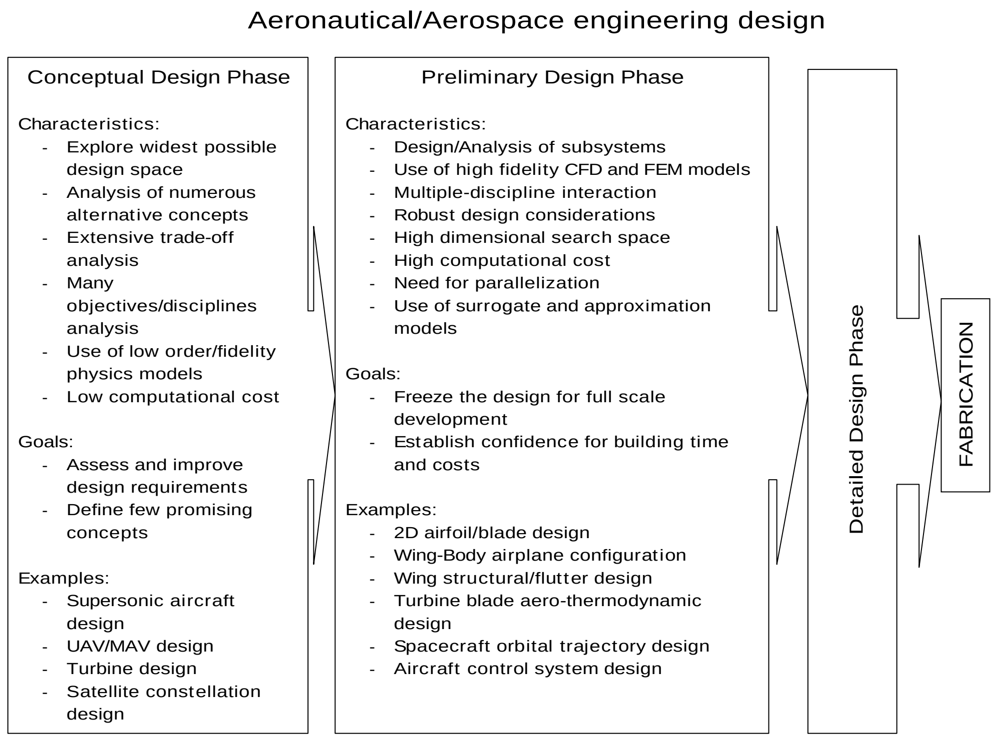

Fig. 1. Graphical representation of the three stages of design in aeronautical/aerospace engineering

fined: (i) maximization of total head in the pump, and (ii) minimization of the pump input power. Side constraints were considered for the design variables range, defining the pump geometry. An additional operating constraint was imposed for the static pressure at the rotor tip in order to detect the inception of cavitation, being crucial to prevent this condition for the optimal design. The authors adopted MOGA with fitness sharing [52], blended crossover (BLX-α) and uniform random mutation. Conceptual designs were evaluated using a one dimensional meanline pump flow-modeling method, which provides a fast modeling of turbopumps for rocket engines at very low computational cost. For the first conceptual design case, a total of 498 different nondominated solutions were obtained, while 660 were found in the second case. Authors noted that improvements in the objective functions were within 1% in both objectives with respect to a reference design.

- Buonanno and Mavris [15] addressed the conceptual design of a small supersonic aircraft, considering seven objectives: (i) weight, (ii) range, (iii) takeoff balanced field length, (iv) loudness, (v) overpressure, (vi) flight Mach number, and (vii) cabin size. Some of them were minimized, while others were maximized. An application example presented by the authors comprised a set of up to 64 design variables (both continuous and discrete variables were considered), describing the aircraft geometry and the mission requirements. The authors used a parallel hybrid subjective/quantitative MOEA, in which the fitness of an individual was a combination of both quantitative and qualitative metrics, with the latter being defined by a human evaluator. A parallel-MOEA) (pMOEA), based on the injection island genetic algorithm [36], was adapted for this MOP. The strategy consisted on assigning one objective function per island and solving a two-objective optimization problem. The second objective for each island was constructed as a goal attainment metric based on the mission requirements for the aircraft. In this way, each island obtained a set of solutions excelling in its assigned objective and representing a trade-off with respect to the project goals. After a certain number of generations, the nondominated solutions from the islands were sent to a central island which solved the seven-objective problem formulated as a goal attainment problem. Each island used SPEA2. The nondominated solutions from the central island were transferred back to each of the islands and the process was repeated until satisfactory solutions were obtained. The authors used physics-based analysis tools for performance prediction. Low-order/fidelity models were used for the involved disciplines: aerodynamics, propulsion, stability and control, economics, aeroelasticity, manufacturing and acoustics, along with modules for weight estimation and geometry parameterization.

- Valliyappan and Simpson [175] solved a conceptual design optimization for a general aviation aircraft product family of small propeller driven GAA (General Aviation Aircraft) to be scaled around the 2, 4, and 6 seats configurations, and which can cruise from 150 to 300 knots and have a range from 800 to 1000 miles. The aim of this study was to explore the design space in order to find the trade-off between platform commonality and individual product performance within the aircraft family. The MOP comprised four objective functions which were defined by means of a goal programming formulation, where the deviations of each goal from their targets were minimized. For this sake, a set of 7 goals (aspiration levels), and a set of 7 constraints were defined. The first two objectives measured the technical and economical related goals within the family, respectively; while the third objective measured the total constraint violation for the whole family; finally, objective four measured the variance index or degree of commonality in variables within the product family. Design candidates were defined with a set of 14 continuous/discrete design variables, and the evaluation of the aircraft performance was done via NASA's GASP (General Aviation Synthesis Program). The authors used the NSGA-II. A special encoding was adopted in order to contain a set of commonality controlling genes (one gene per variable), followed by a concatenation of genes defining the design variables of each product in the product family.
- Rajagopal et al. [135] investigated an Unmanned Aerial Vehicle (UAV) conceptual design. Two objectives were considered: (i) the maximization of the endurance (the time an airplane can fly given a payload and a given fuel weight) and (ii) the minimization of the wing weight. Six design variables were used, four of them being wing-geometry related parameters (aspect ratio, wing loading, taper ratio, thickness to chord ratio) and the other two being UAV's operational parameters (loiter velocity and altitude). Additionally, constraints were imposed on the performance parameters of the UAV design. These included: (1) wing weight, (2) rate of climb, (3), stall speed, and (4) maximum speed at sea level condition. NSGA-II with real-numbers encoding and the SBX crossover operator was adopted. This MOEA was coupled to Raymer's RDS software, which is based on the design methods described in [138], in order to evaluate the performance of each design candidate. The authors reported that a Pareto front was obtained with a total of 11 solutions.
- Kuhn et al. [88] developed a multidisciplinary conceptual design methodology for its application to hybrid airship design (aerostatic lift and aerodynamic lift). Two objectives were considered: (i) minimization of the total mass, and (ii) maximization of the payload. Thirteen constraints were imposed, related to stress levels in the components. A set of 18 mixed real/discrete variables were used to represent the geometry of the airship and its structural properties. The optimization tool adopted was a MOEA called GAME (Genetic Algorithm for Multicriteria Engineering) [90], which is based on Evolution Strategies (ES). The evaluation of the objective functions was done with models varying in fidelity, ranging from interpolation models to FEM models. The latter was used for the structural analysis using a FEM commercial software. A Hybrid Universal Ground Observer (HUGO) airship demonstrator was designed, with a total of 10,000 design candidates being evaluated.
- Jing and Shuo [74] presented the conceptual design of an air-breathing hypersonic cruise vehicle. Five design objectives were considered: (i) maximization of the lift-to-drag ratio, (ii) minimization of the stagnation temperature, (iii) maximization of the thrust-to-drag ratio, (iv) maximization of the airframe volume, and (v) minimization of the Radar Cross Section (RCS). Constraints were imposed on variables ranges, flow flux and Mach number at inlet conditions, trimmed angle of attack and rolling angle, and static stability and maneuverability margins as well. 21 design variables were used to define the geometry of the design candidates. The authors adopted MOGA with the following features: real numbers encoding, arithmetic crossover, Gaussian mutation, steady-state reproduction and fitness sharing. Constraint handling was done by an accurate penalty strategy. Additionally, for further improvement of the solutions, a simulated annealing algorithm[4] was adopted as a local search engine. The objectives were evaluated using simplified models with reduced computational cost. Only three globally non-dominated solutions could be generated. Such solutions were further evaluated and compared against a reference design. The authors noted that these solutions were better in all the objectives than the reference design (i.e., they dominated it).
- Xiaoqing et al. [184] evaluated the multiobjective optimization of hypersonic waverider shape generation. Three objectives were considered: (i) lift-to-drag ratio, (ii) vehicle's volume, and (iii) vehicle's volumetric ratio. No information is given, concerning constraints, thus it is assumed that only side constraints on variable ranges are considered. The base section of the waverider was defined by means of analytical shape functions (i.e., fourth-order polynomials), keeping to a minimum the number of design variables. The authors explored two different techniques: (a) cone derived waverider, and (b) osculating cone derived waverider. The authors adopted the NSGA-II with an improved crowding mechanism.
- Theisinger and Braun [170] identified hypersonic entry aeroshell shapes in order to find trade-off designs with increased landed mass capabilities. Three objectives

[4]Kirkpatrick et al. [79] pointed out the analogy between an "annealing" process and optimization: a system state is analogous to the solution of an optimization problem; the free energy of the system (to be minimized) corresponds to the cost of the objective function to be optimized; the slight perturbation imposed on the system to change it to another state corresponds to a movement into a neighboring position (with respect to the local search state); the cooling schedule corresponds to the control mechanism adopted by the search algorithm; and the frozen state of the system corresponds to the final solution generated by the search algorithm (using a population size of one). These analogies led to the development of the so-called *simulated annealing* algorithm.

were considered: (i) drag-area, (ii) static stability and (iii) volumetric efficiency. This particular spacecraft design problem was driven by planetary entry-descent-landing performance requirements and thermal/structural limitations, which are naturally conflicting. All objectives were maximized and two constraints were imposed to the volumetric efficiency and on the lift-to-drag ratio. Side constraints were applied to the design variables in order to obtain designs fitting with the current launch systems. Aeroshell shape was described by a bi-parametric, cubic by quadratic, non-uniform rational B-spline 3D surface, allowing them to define the optimization problem with 20 design variables, including the aeroshell angle of attack. The authors adopted the version of the NSGA-II available in the *iSIGHT* commercial software. Additionally, the objective function evaluations were performed with the estimated flowfield around the aeroshell using a physics-based simulation, namely the Newtonian impact theory. The Mars Science Laboratory Aeroshell was adopted as a reference design. The authors found several design candidates that performed better than the reference design in the three objectives under consideration.

Analysis of the use of MOEAs in conceptual design:

Table I summarizes the application of MOEAs in conceptual design optimization problems. From this table and the previous review, it can be observed that the NSGA-II is the most frequently adopted approach. The common use of Pareto-based approaches seems to corroborate the hypothesis from some authors regarding the suitability of Pareto optimality to drive the search at the preliminary stages of design [181]. It should be clear that the use of MOEAs is computationally expensive, which is the reason why analytic and/or low-order engineering models are adopted in most cases. Only in a few applications, researchers seem to rely on low-order physics-based models [15], and variable-fidelity physics-based models [88]. Nevertheless, we believe that in the near future, MOEAs will become a standard practice, as the computing power available continues to increase each year. It is also worth noting that MOEAs are flexible enough as to allow their coupling to both engineering models and low-order physics-based models without major changes. They can also be easily parallelized, since MOEAs normally have low data dependency. Finally, it is worth indicating the advantage of incorporating a subjective evaluation scheme for cases in which the search must be controlled, disallowing the generation of impractical design solutions as reported by Buonanno and Mavris [15].

An aspect that is important to emphasize is the poor scalability of Pareto-based MOEAs as we increase the number of objectives [82]. Many of the applications previously described considered a low number of conflicting objectives (two or three in most cases). Although MOEAs can still be used in high-dimensional objective spaces, it is required to use mechanisms different from the traditional Pareto-based selection [64]. This issue, however, does not seem to be a major concern in most of the applications reviewed above. A remarkable exception is the work reported in [15] in which the authors deal with a problem having seven objectives. The authors adopt in this case a parallel MOEA based on the concepts of co-evolution of multiple populations. This approach seems to produce acceptable results in this high-dimensional objective search space. Another issue that seems to be a common concern in this first group of applications is the encoding of the decision variables. Since this sort of application normally has mixed decision variables (e.g., discrete and continuous), authors tend to propose their own *ad-hoc* encodings, which also require specialized crossover and mutation operators associated to them. It should also be evident that in this first type of applications, authors paid little or no attention to the fine-tuning of parameters of their MOEAs. This may be due to the obvious difficulties to perform a careful statistical analysis when dealing with very expensive objective functions. However, other possible alternatives such as self-adaptation or on-line adaptation have not been properly addressed by researchers in this area yet [174]. If such self-adaptation and on-line adaptation mechanisms are unaffordable, at least the use of relatively high mutation rates is suggested, combined with a plus selection mechanism that combines the population of parents with the population of offspring and retains the best half. This will increase the selection pressure but will maintain enough diversity as to avoid premature convergence. Finally, it is worth mentioning the use of external files (or archives) as a viable alternative to reduce objective function evaluations and perform a more accurate search. This sort of mechanism can be particularly useful when combined with relaxed forms of Pareto dominance such as ϵ-dominance [94], which allows to regulate convergence, and has not been adopted by researchers working in this first group of applications.

B. 2D geometries and airfoil shape optimization

Aeronautic and aerospace systems are, in general, complex engineering systems. Their analysis and design is a very complex task. There exist, however, many enginering design cases where this complexity can be tackled by analyzing basic components of the complete system, on which reduced/simplified models can be used as the basis for analyzing the whole system. Examples of these conditions are the design of 3D complex shapes such as wings and turbine blades, where the analysis of their 2D building sections (airfoils) is frequently performed prior to the analysis of the complete 3D geometry. In other cases, the geometry for the system can be such that its operating conditions can be estimated by analyzing its sectional properties. Examples of this latter condition are the aircraft engine inlets/nozzles, where the flow can be assumed as two-dimensional or axisymmetrical. In this section, some applications of MOEAs for these types of problems are presented.

- Yamaguchi and Arima [185] dealt with the optimization of a transonic compressor stator blade in which three objectives were minimized: (i) pressure loss coefficient, (ii) deviation outflow angle, and (iii) incidence toughness.

Ref	NObj	NCons	NVars	VarType	Algorithm	Operators	Physics Model	NPop	Gmax	Remarks
[124]	2	s.c.	11	Continuous	MOGA	Fitness sharing, BLX-α crossover, uniform random mutation, Best-N selection	Mean line pump flow modeling	120	30	None
[15]	7	s.c.	64	Mixed continuous/discrete	SPEA2	Hierarchical crossover operator	Multiple disciplines low order/fidelity models	N/A	N/A	Island based parallel interactive GA with subjective evaluation
[175]	4	s.c.	14	Mixed continuous/discrete	NSGA-II	SBX crossover and polynomial mutation	Low order models	20	150	Objectives defined by means of goal programming technique
[135]	2	4	6	Continuous	NSGA-II	SBX crossover and polynomial mutation	Multiple disciplines, low order and database models	N/A	N/A	None
[88]	2	s.c.	18	Mixed continuous/discrete	GAME	Evolution strategies' mutation operator	Multiple disciplines with low fidelity and FEM models	400	25	None
[74]	5	6	21	Continuous	MOGA	Arithmetic crossover, gaussian mutation, fitness sharing and steady-state reproduction	Multiple disciplines simplified models	300	300	Constraint handling using exact penalty method, and simulated annealing as a local search operator
[184]	3	s.c.	5	Continuous	NSGA-II	SBX crossover, polynomial mutation and improved crowding mechanism	Inviscid flow model	N/A	N/A	None
[170]	3	2	20	Continuous	NSGA-II	SBX crossover and polynomial mutation	Newton impact theory	N/A	N/A	None

NObj = Number of objectives; NCons = Number of constraints; NVars = Number of design variables; VarType = Type of variables; NPop = Population size; Gmax = Maximum number of generations; N/A = Not available; s.c. = Only side constraints are adopted.

TABLE I
SUMMARY OF MOEAS APPLIED TO CONCEPTUAL DESIGN OPTIMIZATION PROBLEMS.

The last objective function can be considered as a robust condition for the design, since it is computed as the sum of the pressure loss coefficients at two off-design incidence angles. The airfoil blade geometry was defined by twelve design variables. The authors adopted MOGA with real-numbers encoding, fitness sharing and intermediate crossover. Aerodynamic performance evaluation for the compressor blade was done using Navier-Stokes CFD simulations. The optimization process was parallelized, using 24 processors in order to reduce the computational time required. In order to promote diversity, during the first few generations, parents were selected from individuals with the first two lowest rank values (i.e., dominated individuals were also selected) and later on, only nondominated individuals were selected.

- Benini and Toffolo [9] addressed the development of high-performance airfoils for its application in axial flow compressors. They minimized two objectives: (i) nondimensional pressure ratio, and (ii) the pressure loss coefficient reduced from the unit value. Constraints were imposed on the design conditions, and were evaluated at 5 different flow-field points, in order to obtain airfoils being at least equal in performance to the reference airfoils adopted by the authors. The airfoil geometry was defined using three Bézier curves. In total 9 designs variables were used to define the airfoil geometry, its length, pitch, and incidence. A special procedure was used to avoid generating either useless or invalid airfoil geometries. The MOEA used by the authors is based on an elitist $(\mu + \mu)$ evolution strategy, which adopted binary encoding. In their implementation, μ offspring were generated using crossover and were mutated with a random-based mechanism. Repeated solutions (clones) were replaced by randomly-generated individuals. In the selection process, the combined population of parents and offspring were Pareto-ranked but considering also a diversity metric defined as a function of the minimal normalized Euclidean distance (in decision variable space) of each individual to its closest neighbor. The best μ individuals were retained as members of the following generation. The evaluation of the objective functions was done by means of CFD simulations with a high computational cost. The nondominated solutions generated by the authors were found to be superior in performance to the reference airfoils, using NACA 65 family airfoils.
- Naujoks et al. [113] addressed an airfoil design problem in which extreme Pareto optimal solutions were defined for two operational design points (two competing objectives): one for high lift performance at low speed condition and the other one for low drag performance at high speed condition. The airfoil was represented by two Bézier curves, and a total of 12 design variables were adopted. No constraints were defined, other than side constraints (upper and lower limits for the design variables). The authors used an approach called MODES (Multi Objective Derandomized Evolution Strategy). In this case a (1+10)-DES (Derandomized Evolution Strategy) was adopted, which means that only one parent was used to produce the offspring. The aerodynamic evaluation of the design candidates is performed using a CFD Navier-Stokes simulation with a high computational cost. It is worth noting, however, that for the examples presented by the authors, a budget of only 1000 evaluations was

considered. Although this was a very small number of objective function evaluations, the authors reported the generation of good approximations of the Pareto front. In a further paper, Naujoks et al. [114] proposed to use a (20+20)-MODES strategy, along with an additional selection mechanism inspired on the NSGA-II. The results presented with this additional selection mechanism were very similar to those obtained before, both in terms of quality of the Pareto approximation and in terms of the spread of the nondominated solutions along the Pareto front.

- Beume et al. [10] poposed the SMS-EMOA (SMS stands for S-metric[5] selection) strategy. The approach was used to solve a multi-objective airfoil design problem. As in the previous case, Pareto extreme solutions were defined by three operational conditions for lift, drag and pitching moment coefficients. The optimization problem was to find trade-off solutions minimizing the drag values for the three flow conditions, while not losing lift and keeping the pitching moment within a 2% range from the reference design points. Additionally, geometrical constraints were included for the airfoil shape. These last constraints were treated in a direct manner, discarding all infeasible solutions, previous to a CFD simulation. Results for this application were presented and compared with those obtained by using NSGA-II, in both cases with a limited budget of 1,000 function evaluations.
- Rai [133] dealt with the robust optimal aerodynamical design of a turbine blade airfoil shape, taking into account the performance degradation due to manufacturing uncertainties. Two objectives were considered: (i) to minimize the variance of the pressure distribution over the airfoil's surface, and (ii) to maximize the probability of constraint satisfaction. Only one constraint was considered, related to the minimum thickness of the airfoil shape. The constraint-handling technique adopted was the one developed by the same author and reported in [132]. The airfoil shape parameterization consisted of eight decision variables but in the experiments presented, only two of them were used for perturbing one airfoil side (the pressure side). The author adopted a multi-objective differential evolution (MODE) approach [130]. Its main features included a mechanism to reduce the set of nondominated solutions in case its size exceeded a certain (pre-defined) threshold. This was done to promote diversity in the population. It also adopted an intermediate population whose size was twice as large as the original and which was Pareto ranked so that only the first half was retained for the next generation. The author used a high-fidelity CFD simulation on a perturbed airfoil geometry in order to evaluate the aerodynamic characteristics of the airfoil generated by MODE. The simulation follows a probability density function that is observed for manufacturing tolerances. This process required a high computational cost, which the author attempted to reduce by using an artificial neural network [150] Response Surface Model (RSM).
- Ray and Tsai [136] considered an airfoil shape design optimization problem with two objectives to be minimized: (i) the ratio of the drag-to-lift squared coefficients, and (ii) the squared moment coefficient. Constraints were imposed on the flow Mach number and angle of attack. Airfoil shapes were defined by the PARSEC representation [158]. This airfoil representation allowed to define the geometry of an airfoil with 11 design variables which are more related to its aerodynamic performance than in other type of airfoil representations. The optimizer used is a multi-objective particle swarm optimizer (MOPSO) [3]. A particular feature of this application was that the particle swarm scheme was based on movements for the particles of one position to another in the design space, rather than on an update of an individual's velocity as done in the standard particle swarm optimization algorithm. The aim of this scheme was a reduction in the number of user-defined inputs. The flow solver utilized corresponds to an Euler code which was able to capture nonlinearities in the flow such as shock waves. In their results, the authors obtained a set with 32 nondominated solutions. In a related work, Ray and Tsai [137] presented a parallel implementation of this MOPSO for airfoil shape optimization. This approach was also hybridized with a gradient-based algorithm. Contrary to standard hybridization schemes where gradient-based algorithms are used to improve the nondominated solutions obtained (i.e., as a local search engine), in this approach the authors used the gradient information to repair solutions not satisfying the equality constraints. This repairing algorithm was based on the Marquardt-Levenberg algorithm [100], [106]. During the repairing process, a subset of the design variables was used, instead of the whole set, in order to reduce the dimensionality of the optimization problem to be solved.
- Obayashi et al. [117] studied the aerodynamic design of cascade airfoils shapes. The problem considered three objective functions: (i) pressure rise, (ii) flow turning angle, and (iii) total pressure loss. The first two objectives were maximized and the third one was minimized. The authors used a real-coded MOGA. Objective evaluation was performed using a 2D Navier-Stokes code for flow evaluation. The same MOEA was also used for the design of a four-stage compressor [117], [123]. In this second application, two objective functions were maximized: (i) total pressure ratio and (ii) isentropic efficiency. The MOP consisted of 80 design variables, and one constraint on the flow conditions, in order to avoid designs with flow separation. The evaluation was done using flow simulations based on the streamline curvature method in which solutions are obtained iteratively, causing a high computational cost even when an engineering model is used. The nondominated solutions obtained by the

[5]The **hypervolume** (also known as the S metric or the Lebesgue Measure) of a set of solutions measures the size of the portion of objective space that is dominated by those solutions collectively. It has been proved that the maximization of this performance measure is equivalent to finding the Pareto optimal set [45], and this has also been empirically verified by some researchers [38].

authors outperformed a baseline design in both objective functions by an amount of 1%.

- D'Angelo and Minisci [29] solved a subsonic airfoil shape optimization problem, in which two objective functions were minimized: (i) drag force coefficient, and (ii) lift force coefficient difference with respect to a reference value. The airfoil geometry was parameterized using Bézier curves both for its camber line and for its thickness distribution. Five design variables were used and constraints were imposed on the extreme values of the objective functions. The authors adopted MOPED (Multi-Objective Parzen-based Estimation of Distribution) [27], which uses the Parzen method to build a probabilistic representation of the nondominated solutions, with multivariate dependencies among the decision variables. The authors included three modification to improve MOPED: (a) the use of a Kriging model by which solutions were evaluated without resorting to costly computational simulations, (b) the use of evolution control to keep the evolution from converging to false Pareto fronts, and (c) the hybridization of the algorithm with some mechanisms from NSGA-II (selection and ranking of solutions). Aerodynamic evaluations were performed by using a CFD simulation code, tailored for aerodynamic airfoil analysis. The authors indicated that this subsonic airfoil shape optimization problem presented difficulties associated to more complex problems: The true Pareto front was discontinuous and partially converged solutions (when divergence was detected, the iterative process was stopped) from the aerodynamic simulation code introduced irregularities in objective function space. The approximation model reduced the number of objective function evaluation in a significant manner (to one sixth of their original value).
- Bing et al. [11] presented the aerodynamic shape optimization for a 2D Hypersonic inlet and 2D SERN (Single-Expansion-Ramp Nozzle) used in scramjet engines. Two applications were presented, one with two objectives and the other with three objectives. For the first optimization example a 2D Hypersonic engine inlet was considered, and the aim was to maximize the two following objectives: (i) pressure recovery, and (ii) static pressure rise. Constraints on the design variables, inlet geometry and flow condition at exit, were imposed. The inlet geometry was defined using four decision variables. The evaluation of the design performance required high fidelity CFD Navier-Stokes simulations since the flow physics was highly nonlinear for the operating flow conditions indicated. The results of both the NSGA-II and the Neighborhood Cultivation Genetic Algorithm (NCGA) [182] were compared. The second problem considered the same inlet design previously defined, with the additional objective of minimizing the inlet drag coefficient. From the results presented by the authors, in both cases, the NCGA algorithm performed better than NSGA-II, obtaining more nondominated solutions with a better spread along the Pareto front.
- Brown et al. [14] addressed the optimization design of a scramjet inlet considering two objectives: (i) total pressure recovery factor, and (ii) variation of pressure recovery factor for a $\pm$ 5% change in free stream Mach number. The first objective was maximized, while the second was minimized. According to the design problem, geometric constraints were defined in order to remove physically unrealistic solutions. Additionally, operational flow constraints were considered to guarantee the auto-ignition in the engine. This condition required a certain range for pressure, temperature and Mach number in the flow at specific locations. The inlet was considered as a 2-D geometry and consisted of three flat ramps and a cowl at the combustion chamber inlet. In this case, 12 design variables were adopted. The MOEA adopted used a selective breeding process that ranked solutions according to the constraints, and also on the basis of the desirability of the values of the objectives (according to the user's preferences). The objective functions consisted of hypersonic flow conditions in which strong shock waves were present. The authors did not report the cardinality of the set of nondominated solutions that they obtained, but they reported the generation of a considerably high number of nondominated solutions.
- Congedo et al. [26] dealt with the airfoil shape optimization for transonic flows of Bethe-Zel'dovich-Thompson (BZT) fluids. In this case, two design conditions were explored, both for a non-lifting airfoil, and for a lifting airfoil. In the second case, the MOP considered two design objectives: (i) maximization of lift at BZT subcritical conditions, and (ii) minimization of wave drag while maximizing lift for supercritical BZT flow conditions. The geometry of the airfoil shape was represented with a Bézier curve with 16 2D control points, i.e., 32 decision variables, from which 10 are constants used to control the leading edge and trailing edge positions as well as the leading edge slope. Thus, the problem consisted of 22 variables. The only constraint included was the thickness to chord ratio of the airfoil, which was adjusted to its specified value, once a design was generated, and prior to the flow solution. The authors used the NSGA with a sigma-share formula given in [131], which takes into account the population size and the number of objectives. They chose parameters such that less than 1,000 objective function evaluations were performed. The authors reported that all the solutions that they obtained outperformed the baseline design as well as the designs obtained using traditional design methods.
- Shimoyama et al. [156] developed a novel optimization approach for robust design. In their approach, a design for multi-objective six-sigma (DFMOSS) [155] was applied for the robust aerodynamic airfoil design of a Mars exploratory airplane. The core of the design methodology was, on the one hand, the concept of *Robust Design*[6] and,

[6]Robust design takes into account the fact that in real-world engineering designs, performance of a design can vary from its expected value, due mainly to errors and uncertainties in the design and/or manufacturing process, and/or in the operating conditions. Therefore, the aim is to find the trade-off between the optimality of the design and its robustness.

on the other, its multi-objective nature. The idea of the DFMOSS methodology was to incorporate a MOEA to simultaneously optimize the mean value of an objective function, while minimizing its standard deviation due to the uncertainties indicated above. The airfoil shape optimization problems considered two cases: a robust design of (a) airfoil aerodynamic efficiency (lift-to-drag ratio), and (b) airfoil pitching moment constraint. In both cases, only the variability in the flow Mach number was taken into account. The authors adopted MOGA. The airfoil geometry was defined using Bézier curves both for the upper and for the lower surfaces. 6 control points were used, resulting in 12 design variables. The aerodynamic performance of the airfoil was evaluated by CFD simulations using the Favre-Averaged compressible thin-layer Navier-Stokes equations. Eighteen robust nondominated solutions were obtained in the first test case. From this set, almost half of the population attained the 6σ condition. In the second test case, more robust nondominated solutions were found, and they satisfied a sigma level as high as 25σ.

- Szöllös et al. [162] addressed the aerodynamic shape optimization of the airfoil geometry of a standard-class glider, considering three objectives: (i) maximize gliding ratio at high flight speed, (ii) maximize gliding ratio at average weather conditions, and (iii) minimize sink rate at low turning speeds. All these objectives are specified in terms of airfoil's aerodynamic lift and drag coefficients as well as flight operating conditions in terms of the Reynolds number (Re) and the Mach number (M). Constraints are considered for: (a) airfoil's maximal lift coefficient at landing flight conditions, (b) maximum airfoil's thickness to chord ratio, (c) trailing edge thickness, and (d) pitching moment coefficient (C_m) which is required not to be worse than a reference airfoil design. The authors introduced a new MOEA called *multi-objective micro-genetic algorithm with range adaptation, based on ϵ-dominance* or $\epsilon\mu$ARMOGA. This approach is inspired on the Adaptive Range Multi-Objective Genetic Algorithm (ARMOGA) [143]. ARMOGA incorporates two archiving techniques: a global archive, which stores all the best solutions obtained so far, and a recent archive, which stores the best solutions of the past previous generations. Solutions from the second archive participate in the parent selection process. $\epsilon\mu$ARMOGA introduces two additional mechanisms. The first corresponds to the use of a small population size (i.e. the use of a micro-genetic algorithm as in [25], [85]), coupled with the use of an external file for storing the nondominated solutions obtained so far. The second mechanism corresponds to the use of the concept of ϵ-dominance [95], which is a relaxed form of Pareto dominance that has been used as an archiving strategy that allows to regulate convergence. The authors initialized the population using a Latin Hypercube Sampling (LHS) technique, and the main population was reinitialized at every certain number of generations, based on the average and standard deviation of the decision variables. The objective functions were evaluated using a CFD simulation code. The authors obtained feasible solutions with improvements on the order of 10%, 8% and 7-10% for the first, second and third objectives, respectively, with respect to a reference airfoil design.

Analysis of the use of MOEAs in 2D geometries and airfoil shape optimization:

Table II summarizes the application of MOEAs in 2D geometries and airfoil shape optimization problems. From this table and the previous discussion, we can see that, as before, a wide variety of Pareto-based elitist MOEAs have been used in this domain. It is also worth noting the use of MOEAs in *robust design*, in which solutions are evaluated with off-design operating conditions and manufacturing tolerances. Such solutions are thus representing more realistic designs. Several authors report improved designs when adopting MOEAs, but unsuccessful cases have also been reported. The cases in which MOEAs fail to produce improved designs seem to be associated to situations in which the baseline design had been already improved in a significant manner, or when the search space is so highly constrained that it is difficult to move to better regions. Again, the high computational cost associated to the use of MOEAs is evident. In spite of the advantages of Pareto-based MOEAs, it is also evident that, when dealing with expensive objective functions such as those of the above applications, the use of careful statistical analysis of parameters is unaffordable. Thus, the parameters of the MOEAs discussed in this section were simple guesses or taken from values suggested by other researchers. It is also important to note that some researchers have suggested clever approaches that allow the use of very small population sizes, although surrogate models have also been employed, as in the previous section. Nevertheless, the use of other simpler techniques such as fitness inheritance or fitness approximation [139] seems to be uncommon in this domain and could be a good alternative when dealing with high-dimensional problems. Additionally, the authors of this group of applications have relied on very simple constraint-handling techniques, most of which discard infeasible individuals. Alternative approaches exist, which can exploit information from infeasible solutions and can make a more sophisticated exploration of the search space when dealing with constrained problems (see for example [108]) and this has not been properly studied yet. Finally, it is worth emphasizing that, in spite of the difficulty of these problems and of the evident limitations of MOEAs to deal with them, most authors report finding improved designs when using MOEAs, even when in all cases a fairly small number of fitness function evaluations was allowed. This clearly illustrates the high potential of MOEAs in this domain.

C. 3D complex physics/shape optimization

Sophisticated aeronautical/aerospace systems possess in most cases, complex three-dimensional shapes and/or are designed to operate in complex physical environments. Examples of such complex three-dimensional shapes are those of

Ref	NObj	NCons	NVars	VarType	Algorithm	Operators	Physics Model	NPop	Gmax	Remarks
[185]	3	s.c.	12	Continuous	MOGA	Intermediate crossover and fitness sharing	Navier-Stokes	100	30	Robust design optimization
[9]	2	5	9	Discrete	$(\mu + \mu)$-ES	Gaussian mutation, Goldberg's Pareto ranking, crowding based on euclidian distance in decision space	Navier-Stokes	100	200	None
[113]	2	s.c.	12	Continuous	(1+10)-MODES	Adaptive derandomized mutation strategy, selection based on the NSGA-II	Navier-Stokes	1	N/A	Use of a maximum of 1,000 designs
[10]	3	2	12	Continuous	SMS-EMOA	Adaptive derandomized mutation strategy, steady-state selection based on hypervolume measure	Navier-Stokes	20	N/A	Use of a maximum of 1,000 designs
[133]	2	1	8	Continuous	MODE	DE's crossover and mutation operators	Navier-Stokes	10	25	Robust design optimization, use of ANN RSM
[136]	2	2	11	Continuous	MOPSO	N/A	Euler model	100	50	None
[117]	3	s.c.	N/A	Continuous	MOGA	N/A	Navier-Stokes	64	75	None
[123]	2	1	80	Continuous	MOGA	N/A	Streamline curvature method	300	1000	None
[29]	2	s.c.	5	Continuous	MOPED	N/A	Coupled boundary layer potential flow panel method	N/A	N/A	Use of Kriging model
[11]	3	2	4	Continuous	NCGA	N/A	Parabolized Navier-Stokes	100	50	None
[14]	2	N/A	12	Continuous	N/A	Elitist selective interbreeding, ranking of solutions according to constraints and user defined preferences, weighted variable recombination	Navier-Stokes	100	100	None
[26]	2	1	22	Continuous	NSGA-II	SBX crossover and polynomial mutation	Euler flow with thermodynamical model for dense gases	36	24	None
[156]	2	s.c.	12	Continuous	MOGA	Stochastic universal sampling, blended crossover, uniform mutation, best-N selection	Favre-Averaged compressible thin-layer Navier-Stokes	64	100	Robust design optimization based on 6σ
[162]	3	4	12	Continuous	$\epsilon\mu$ARMOGA	SBX crossover, no mutation is used, external file storage based on ϵ-dominance	Coupled boundary layer potential flow panel method	4	2000	Reinitialization of population is used for diversity preserving, instead of mutation

NObj = Number of objectives; NCons = Number of constraints; NVars = Number of design variables; VarType = Type of variables; NPop = Population size; Gmax = Maximum number of generations; N/A = Not available; s.c. = Only side constraints are adopted.

TABLE II
SUMMARY OF MOEAS APPLIED IN 2D GEOMETRIES AND AIRFOIL SHAPE OPTIMIZATION PROBLEMS

turbine/propeller blades, and complete aircraft configurations. Complex three-dimensional physics are present for high speed flow over wings and turbine/propeller blades, in which shock waves can arise, affecting the design performance. For these cases, the MOP cannot be simplified by the use of reduced models, such as two-dimensional simulations, as done in the applications of the previous section. Next, we will discuss applications of MOEAs in which their authors deal with these 3D complex physics/shape optimization problems.

- Sasaki et al. [145] and Obayashi et al. [118] solved a multi-objective aerodynamic wing shape optimization problem in which they minimized three objectives: (i) drag coefficient for transonic cruise, (ii) drag coefficient for supersonic cruise, and (iii) bending moment at the wing root for supersonic cruise condition. The set of constraints comprised lift coefficient at both transonic and supersonic cruise conditions, wing area and maximum airfoil thickness. The variables for this design were 66 in total, and defined the wing planform shape, airfoil chord and thickness distribution at several wing stations, as well as wing twist angles at the same airfoil locations. The authors adopted MOGA and the design candidates were evaluated by a high-fidelity Navier-Stokes CFD flow simulation. The evaluation process was parallelized using the master-slave paradigm. In a further paper, Sasaki et al. [146] used the same algorithm for the aerodynamic optimization of a supersonic transport wing-body configuration. In this application, two objectives were considered: (i) drag coefficient and (ii) difference in Darden's equivalent area distribution. Constraints on the lift coefficient were imposed during the optimization, and on the length and volume of the fuselage. The aim of the second objective was to achieve low sonic boom characteristics. For this problem, the number of variables increased to 131, as the fuselage geometry was added in this case. The aerodynamic evaluation for the first objective was performed by an Euler CFD simulation to considerably reduce the computational time with respect to the use of a Navier-Stokes CFD simulation. Nonetheless, the optimization process was parallelized using the master-slave paradigm. Two test cases were considered, each one having different upper/lower limits for the section nearby the wing-body intersection.

- Sasaki and Obayashi [147] solved a problem similar to the previous one [146] and obtained analogous results, but in this case, the ARMOGA algorithm was used. Also, and in order to incorporate constraints, an extended Pareto ranking method based on constraint-dominance was used [47].
- Ng et al. [115] addressed a multiobjective wing platform and airfoil shape optimization problem. The MOP aimed to redesign the reference ONERA M6 wing minimizing two objectives: (i) W/Wo, which is the ratio for the design wing weight with respect to the reference ONERA M6 wing weight, and (ii) CD/CDo, which is the ratio of the design wing drag coefficient with respect to that of the reference wing. The first objective was evaluated using a semi-empirical equation, while the second was obtained from a multigrid Euler CFD simulation. Constraints were imposed on the flow Mach number and constant lift coefficient. No special constraint handling technique was used, but the CFD code was instructed to vary the angle of attack, subjected to a tolerance, in order to satisfy this equality constraint. This technique can be seen as a mechanism to repair solutions. The wing platform was represented by 5 design variables: (a) taper ratio, (b) wing sweep angle, (c) twist angle, (d) aspect ratio, and (e) thickness-to-chord ratio. The airfoil used for the wing corresponded to the symmetric airfoil used in the ONERA M6 wing, and was the same across the wing. The optimizer used was based on the PSO algorithm described in Ray et al. [136]. The authors presented results for two test cases: the first with 4 steps and the second with 8 steps. In the first case 10 nondominated solutions were obtained, while 11 were found in the second case. In both cases, all the nondominated designs were better in the first objective function compared to the reference wing, and for the second objective, almost half of the population were better while the rest were worse, with respect to the reference wing. An Adaptive Search Space Operator (ASSO) technique was used by the authors to give the algorithm the possibility of adapting decision variables bounds by shrinking/expanding the boundaries of the design space.
- Lian and Liou [101] addressed the optimization of a three-dimensional rotor blade, namely the redesign of the NASA rotor 67 compressor blade, a transonic axial-flow fan rotor, which was the first of a two-stage compressor fan. Two objectives were considered in this case: (i) maximization of the stage pressure rise, and (ii) minimization of the entropy generation. Constraints were imposed on the mass flow rate to have a difference less than 0.1% between the new one and the reference design. The blade geometry was constructed from airfoil shapes defined at four span stations, with a total of 32 design variables. The authors adopted MOGA. The optimization process was coupled to a second-order RSM, which was built with 1,024 design candidates using the Improved Hypercube Sampling (IHS) algorithm. 12 design solutions were selected from the RSM-Pareto front obtained, and such solutions were verified with a high fidelity CFD simulation. The objective function values slightly differed from those obtained by the approximation model, but all the selected solutions were better in both objective functions than the reference design. Similar work was presented by Lian and Liou [102] but minimizing the blade weight instead of the entropy generation. Similar performance results were obtained with lighter blades. More recently, Kim and Liou [78] presented the design of three new MOEAs, including addtitional mechanisms to the basic MOGA algorithm indicated before. Such mechanisms included: an elite-preserving approach (EP-MOGA), a modified sharing function (EP-MOGAS), and a gradient-based directional operator (EP-MOGAS-D).
- Holst [61] presented the aerodynamic optimization of a wing-body configuration in which two objective functions were maximized: (i) lift-to-drag ratio, and (ii) configuration volume. Constraints were imposed on the operating flow condition at transonic Mach number and at a fixed lift. The problem had 66 decision variables which controlled the wing geometry, its position along the fuselage and the section shape of the fuselage at some specified fuselage stations. The author adopted MOGA. The proposed approach was able to reduce the fuselage cross section in the vicinity of the wing-fuselage juncture, which is a common practice in aerodynamic design for the transonic flow regime.
- Sasaki et al. [142] solved an aerodynamic MOP for a turbine compressor stage. The main aim was to improve three aerodynamic objectives, by identifying the trade-offs among them in the baseline condition: (i) isentropic efficiency, (ii) blockage, and (iii) flow loss. Equality constraints on the design were imposed, intended mainly to maintain the flow and operating conditions similar to those of the baseline geometry: Stage loading, mass flow rate, stage exit whirl angle and pressure ratio. Such equality constraints were transformed into inequalities, and thresholds were reduced as the optimization proceeded. The three-dimensional shape of the blade was re-designed from the baseline geometry, by defining parameters that allowed: (a) axial movement of sections along the engine axis, (b) circumferential movement of sections, (c) solid body rotation of sections based on trailing edge position, and (d) control on the number of blades. In total, 28 design variables were used per compressor stage. The authors adopted ARMOGA. The aerodynamic evaluation was performed with high fidelity Reynolds-Averaged Navier-Stokes CFD tools to analyze a compressor stage. The CFD analysis comprised the rotor/stator interaction. The authors presented two application examples, the first of which had a fixed number of rotor/stator blades. The optimization process was able to improve the baseline design while 8 designs satisfied all the constraints. Efficiency was improved within 1%,

even when infeasible solutions were considered. After analyzing the trade-off among the objectives from the first test case, a second test case was proposed, considering the number of rotor/stator blades as an additional variable, and changing the approximation function in the radial direction. In this case, a B-spline function was used instead of the cubic-spline adopted in the previous case. Results from this second test case achieved an efficiency improvement of 1.5%. In this case, 14 feasible designs were generated, from which only 4 were nondominated.

- Benini [8] extended a previous work from Benini and Toffolo [9] for a three-dimensional transonic compressor rotor design optimization problem in which two objective functions were maximized: (i) total pressure ratio, and (ii) adiabatic efficiency. Constraints were imposed on the design conditions as to obtain the mass flow of a reference design, the NASA Rotor 37. The blade geometry used in the transonic compressor rotor was parameterized by Bézier curves defining the mean camber line and the thickness distribution. Three profiles along the blade span were defined: at hub, midspan and tip. A total of 23 decision variables defined the 3D compressor rotor geometry. The author used the MOEA described in [9], which is based on evolution strategies. The performance evaluation of the designs was done using high fidelity Navier-Stokes CFD simulations. The authors noted that the nondominated solutions produced were clustered around the reference design point, due to a tight constraint imposed on the flow mass rate, which did not allow the algorithm to explore a wider region of the search space. Nevertheless, the author was able to obtain improvements in both objective functions using the proposed approach.
- Chiba et al. [17] explored the trade-offs among four aerodynamic objective functions in the optimization of a wing shape for a Reusable Launch Vehicle (RLV). The objective functions were: (i) the shift of the aerodynamic center between supersonic and transonic flight conditions, (ii) pitching moment in the transonic flight condition, (iii) drag in the transonic flight condition, and (iv) lift for the subsonic flight condition. The first three objectives were minimized while the fourth was maximized. These objectives were selected for attaining control, stability, range and take-off constraints, respectively. The RLV definition comprised 71 design variables to define the wing platform, wing position along the fuselage and airfoil shape at prescribed wingspan stations. The authors adopted ARMOGA, and the aerodynamic evaluation of the RLV was done with a Reynolds-Averaged Navier-Stokes CFD simulation. A trade-off analysis was conducted with 102 nondominated individuals generated by the MOEA.
- Song and Keane [159] performed the shape optimization of a civil aircraft engine nacelle. The primary goal of the study was to identify the trade-off between aerodynamic performance and noise effects associated with various geometric features for the nacelle. For this, two objective functions were defined: i) scarf angle, and ii) total pressure recovery. The nacelle geometry was modeled using 40 parameters, from which 33 were considered design variables. The authors adopted the NSGA-II with a commercial CFD software for evaluating the three-dimensional flow characteristics. Due to the large size of the design space to be explored, as well as the simulations being time consuming, a Kriging-based surrogate model was adopted in order to keep the number of designs being evaluated with the CFD tool to a minimum. The authors reported difficulties in obtaining a reliable Pareto front (there were large discrepancies between two consecutive Pareto front approximations). They attributed this behavior to the large number of variables in the design problem, and also to the associated difficulties to obtain an accurate Kriging model for these situations. In order to alleviate this situation, they performed an analysis of variance (ANOVA) test to find the variables that contributed the most to the objective functions. After this test, they presented results with a reduced surrogate model, employing only 7 decision variables. The authors argued that they obtained a design similar to a reference one, but requiring a lower computational cost because of the use of this reduced Kriging model.
- Jeong et al. [69] investigated the improvement of the lateral dynamic characteristics of a lifting-body type re-entry vehicle in transonic flight condition. Two objectives were minimized: (i) the derivative of the yawing moment, and (ii) the derivative of the rolling moment. The MOP involved four design variables, and two solutions were sought: The first one without constraints, and the second one constraining the lift-to-drag ratio for the lifting-body type re-entry vehicle. The authors adopted the Efficient Global Optimization for Multi-Objective Problems (EGOMOP) algorithm developed by Jeong et al. [68]. This algorithm was built upon the ideas of the EGO and ParEGO Algorithms from Jone et al. [76] and Knowles et al. [80], respectively. For the exploration of the nondominated solutions, the authors adopted MOGA. Due to the geometry of the lifting body and the operating flow condition of interest, namely high Mach number and strong vortex formation, the evaluation of the objectives was done by means of a full Navier-Stokes solver. Since the objectives were actually derivatives, multiple flow solutions were required to determine their values in a discrete manner, considerably increasing the total computational time due to a large number of calls of the CFD code. The authors were able to find better geometry configurations than the baseline one, with better lateral dynamic characteristics, both for the unconstrained and for the constrained instances.
- Lee et al. [98] presented the robust design optimization of an ONERA M6 wing shape. The robust optimization was based on the concept of the Taguchi method in which the optimization problem is solved considering uncertainties in the design environment, in this case, the flow Mach number. The problem had two objectives: (i) minimization of the mean value of an objective function with respect to variability of the operating conditions, and (ii) minimization of the variance of the objective function

of each candidate solution, with respect to its mean value. In the sample problems, the wing was defined by means of its planform shape (sweep angle, aspect ratio, taper ratio, etc.) and of the airfoil geometry, at three wing locations (each airfoil shape was defined with a combination of mean lines and camber distributions), using a total of 80 design variables to define the wing designs. Geometry constraints were defined by upper and lower limits of the design variables. The authors adopted the Hierarchical Asynchronous Parallel Multi-Objective Evolutionary Algorithm (HAPMOEA) [54], which is based on evolution strategies, incorporating the concept of Covariance Matrix Adaptation (CMA). The aerodynamic evaluation was done with a CFD simulation. It is worth noting that HAPMOEA uses, during the evolutionary process, a hierarchical set of CFD models, varying the grid resolution of the solver (three levels are used), as well as different population sizes (depending on the grid resolution). The authors presented two solutions, with and without uncertainties. In the latter case the problem considered two design points (at two different operating conditions), and the algorithm found the trade-off solutions between these two design points. For the case of the design with uncertainties, the optimization problems found the trade-off solutions considering the minimization for the mean value of the objective function (the inverse of the lift-to-drag ratio for the wing) and its variance with respect to the mean value. From the results presented by the authors, the Pareto fronts were continuous and exhibited a concave geometry for the trade-off solutions. 12 solutions were obtained in the robust design of the wing and all the nondominated solutions presented a shock-free flow both at the upper and at the lower surface of the wing. Additionally, the nondominated solutions showed a better behavior, in terms of aerodynamic performance (lift-to-drag ratio) with a varying Mach number, as compared to the baseline design. In these examples, the authors used three grid-levels (model resolution): fine, intermediate, and coarse. During the evolutionary process, the individuals were moved from the coarse to the fine levels and viceversa. A total of 1100 individuals were evaluated.

- Oyama et al. [126] applied a design exploration technique to extract knowledge information from a flapping wing MAV (Micro Air Vehicle). The flapping motion of the MAV was analyzed using multi-objective design optimization techniques in order to obtain nondominated solutions which were analyzed with Self Organizing Maps (SOMs) in order to extract knowledge about the effects of the flapping motion parameters on the objective functions. The conflicting objectives considered were: (i) maximization of the time-averaged lift coefficient, (ii) maximization of the time-averaged thrust coefficient, and (iii) minimization of the time-averaged required power coefficient. The problem had five design variables and the geometry of the flying wing was kept fixed. Constraints were imposed on the averaged lift and thrust coefficients so that they were positive. The authors adopted MOGA. Due to the nature of the complex flow in this problem, the objective functions were obtained by means of CFD simulations, solving the unsteady incompressible Navier-Stokes equations. Objective functions were averaged over one flapping cycle. The purpose of the study was to extract trade-off information from the objective functions and the flapping motion parameters such as plunge amplitude and frequency, pitching angle amplitude and offset, and phase difference. In order to minimize the turnaround computational time, the evaluation of the objective functions was parallelized using a cluster of workstations. From the results obtained, the authors extracted extreme nondominated solutions which were further analyzed to understand their flow physics for each objective in particular.
- Arabnia and Ghaly [5] presented the aerodynamic shape optimization of turbine stages in three-dimensional fluid flow, so as to minimize the adverse effects of three-dimensional flow features on the turbine performance. Two objectives were considered: (i) maximization of isentropic efficiency for the stage, and (ii) minimization of the streamwise vorticity. Additionally, constraints were imposed on: (1) inlet total pressure and temperature, (2) exit pressure, (3) axial chord and spacing, (4) inlet and exit flow angles, and (5) mass flow rate. The blade geometry, both for rotor and stator blades, was based on the E/TU-3 turbine which is used as a reference design to compare the optimization results. The multi-objective optimization consisted of finding the best distribution of 2D blade sections in the radial and circumferential directions. For this, a quadratic rational Bézier curve, with 5 control points was used for each of the two blades. The authors adopted NSGA. Both objective functions were evaluated by using a 3D CFD flow simulation. The authors adopted an artificial neural network (ANN) based RSM. The ANN model with backpropagation, contained a single hidden layer with 50 nodes, and was trained and tested with 23 CFD simulations, sampling the design space using the LHS technique. The optimization process was undertaken by using the ANN model to estimate both the objective functions, and the constraints. Finally, the nondominated solutions obtained were evaluated with the actual CFD flow simulation. The authors indicated that they were able to obtain design solutions which were better than the reference turbine design.
- Tani et al. [168] solved a rocket engine turbopump blade shape optimization design which considered three objective functions: (i) shaft power, (ii) entropy rise within the stage, and (iii) angle of attack of the next stage. The first objective was maximized while the others were minimized. The design candidates defined the turbine blade aerodynamic shape and consisted of 58 design variables. The authors adopted MOGA. The objective function values were obtained from a CFD Navier-Stokes flow simulation. The authors reported solutions that were better than a baseline design turbopump blade shape. Indeed, improvements on the three objective functions were of 8%, 30% and 40%,

respectively, as compared to the baseline design.

Analysis of the use of MOEAs in 3D complex physics/shape optimization:

Table III summarizes the application of MOEAs in 3D complex physics/shape optimization problems. For this group of applications, a common point is that 3D complex shapes and/or complex physics models are considered, which requires, in most cases, the use of high dimensional design space and/or sophisticated simulation tools. For both cases, the design optimization search becomes highly computationally expensive (some authors report times in the order of days or even months for the problems that they solved). Such applications require approaches that can minimize their high computational cost. Some authors relied on parallelization techniques for this sake (see for example [145]). An interesting parallel approach is the one reported by Lee et al. [98], in which the evaluation of the objectives is done in an asynchronous manner, with a scheme that resembles an island model [24]. Such asynchronous parallel MOEAs are uncommon in the specialized literature, in spite of their high potential in the sort of applications reported in this section. Another alternative is the use of surrogate models, which are adopted by a number of works reported in this section. For example, Lian and Liou [101], [102], used a second order RSM, Song and Keane [159] used a Kriging-based model, Lee et al. [98] adopted hierarchical CFD models (i.e., models with varying mesh sizes, which produce approximations at a reduced computational cost), and Arabnia and Ghaly [5] adopted an artificial neural network. The use of approximate models can be seen as an advantage, but also presents drawbacks, for example, for large dimensional design spaces, as indicated by Song and Keane [159]. Another alternative is to adopt simpler approximation mechanisms such as fitness inheritance [109] and fitness approximation [164]. Another aspect worth emphasizing is that most authors adopted MOEAs with real-numbers encoding, rather than with binary encoding. This is relatively common when dealing with engineering applications having a high number of decision variables. The lack of modern diversity maintenance approaches such as archiving techniques (see for example [60], [94], [151]) is also evident within the applications of this section, although there are some interesting exceptions. For example, Sasaki and Obayashi [147] adopted two external archives for their MOEA. Also interesting is the proposal of Holst [61] of using "bins" (this approach is similar in its operation to the adaptive grid adopted in PAES [83]). However, it is worth noting that both, Sasaki & Obayashi's and Holst's approaches quickly degrade their performance as the number of objectives increases.

An interesting area worth exploring is the design of mechanisms that allow a better (i.e., more intelligent) exploration of the search space. For example, Sasaki [143], and Ng et al. [115] use statistics gathered from the population in order to guide the search. Such approach, however, requires a good diversity maintenance mechanism in order to avoid an excessive selection pressure that would produce premature convergence.

In spite of the large number of constraint-handling techniques currently available for evolutionary algorithms [23], [108], in most of the works reported in this section there is a noticeable lack of them. The use of good constraint-handling techniques is particularly useful when the optimum solutions lie on the boundary between the feasible and the infeasible regions, which is normally the case in multi-objective optimization [24]. Their use can contribute to a better (i.e., more efficient and effective) exploration of the search space in the presence of constraints.

D. Structural optimization

Since its origins, aeronautical and aerospace engineering design has adopted, as a premise, the design of lighter and stronger structures, which are two objectives that are clearly in conflict. The applications of MOEAs in structural optimization that are reviewed in this section make evident that these design goals are still pursued by researchers in aeronautical and aerospace engineering.[7]

- Langer et al. [91] applied an integrated approach using Computer Aided Design (CAD) modeling with a MOEA for structural shape and topology optimization problems. The authors dealt with the structural optimization of a typical instrument panel of a satellite in which two objectives were defined: (i) minimizing the instrument panel mass, and (ii) maximizing the first eigenfrequency. The problem had eight constraints, which were defined in terms of operating conditions, mainly given by stress, temperature and eigenfrequency levels, as well as geometric constraints. The problem had 17 design variables from which 2 were discrete and the rest were mixed (continuous/discrete). The discrete variables considered the number of stringers to use in the panel, and the plate and stringer materials. The MOEA developed by the authors had the following features: it used a mix of real/integer representation for continuous and discrete variables, respectively, and crossover and mutation operators were applied diferently for each type of variable. Besides, the algorithm used Pareto dominance-based ranking to assign fitness to an individual and the "goals and priorities" strategy [47] as a constraint-handling technique. The nondominated solutions obtained at each generation were stored in an external file, which constituted, at the end of the evolutionary process, the approximation of the Pareto optimal set generated by the MOEA. In their application examples, the authors solved the optimization problem for three shape and topology optimization cases: (a) panel without instruments, (b) panel with instruments at fixed positions, and (c) panel with instrumental placing. The evaluation of the objective functions comprised four load cases: (a) quasi-static acceleration, (b) modal analysis, (c) sinusoidal vibration loads, and (d) 'pseudo temperature' load. This latter load case, restricted the positioning of the instruments on the panel, due to limiting operating temperature for a specific instrument. The first three load

[7]For a good survey of the use of MOEAs in structural optimization, the interested reader is referred to [73].

Ref	NObj	NCons	NVars	VarType	Algorithm	Operators	Physics Model	NPop	Gmax	Remarks
[145]	3	4	66	Continuous	MOGA	Fitness sharing, BLX-α crossover, best N selection	Navier-Stokes	64	30	None
[118]	3	4	66	Continuous	MOGA	Fitness sharing, averaged crossover, best N selection	Navier-Stokes	64	70	None
[146]	2	3	131	Continuous	MOGA	Fitness sharing, BLX-α crossover, best N selection	Euler/Navier-Stokes	64	20	None
[147]	2	3	131	Continuous	ARMOGA	Fitness sharing, BLX-α crossover, best N selection	Euler/Navier-Stokes	64	20	Design variables ranges are adapted every M generations, based on the statistics of the archive and current population
[115]	2	2	5	Continuous	MOPSO	Adaptive Search Spacing Operator (ASSO)	Euler	N/A	N/A	The ASSO operator allows to extend the initial design space
[101]	2	1	32	Continuous	MOGA	Fitness sharing, BLX-α crossover, best N selection, random uniform mutation	Reynolds-Averaged Navier-Stokes	N/A	N/A	Use of RSM
[61]	2	2	66	Continuous	MOGA	Masking array to activate/deactivate the design variables, selection based on bins of the non-dominated archive, random average crossover, local and global mutation operators	Potential flow	34	N/A	None
[142]	3	4	28	Continuous	ARMOGA	Stochastic universal sampling, SBX crossover, polynomial mutation, best N selection, Pareto ranking incorporating constraints	Reynolds-Averaged Navier-Stokes	16	20	Grid-enabled parallel computation
[8]	2	1	23	Discrete	$(\mu+\mu)$-ES	Gaussian mutation, Goldberg's Pareto ranking, crowding based on Euclidian distance in the decision space	Navier-Stokes	20	100	None
[17]	4	s.c.	71	Continuous	ARMOGA	Fitness sharing, BLX-α crossover, best N selection	Reynolds-Averaged Navier-Stokes	8	30	None
[159]	2	s.c.	33	Continuous	NSGA-II	SBX crossover, polynomial mutation	Navier-Stokes	60	20	Use of Kriging model
[69]	2	1	4	Continuous	MOGA	N/A	Navier-Stokes	N/A	N/A	None
[98]	2	s.c.	80	Continuous	HAPMOEA	ES mutation operator with Covariance Matrix Adaptation (CMA-ES), distance dependent mutation, tournament selection	Navier-Stokes	*	N/A	* Population sizes are 20, 40 and 60 for fine, medium and coarse CFD mesh grids, 1100 design candidates evaluated
[126]	3	2	5	Continuous	MOGA	Fitness sharing, roulette wheel selection,BLX-α crossover, random uniform mutation, Pareto based constraint handling	Navier-Stokes	N/A	N/A	Knowledge extraction from the multi-objective optimization process
[5]	2	5	10	Continuous	NSGA	N/A	Navier-Stokes	N/A	N/A	ANN model
[168]	3	s.c.	58	Continuous	MOGA	Stochastic universal sampling, BLX-α crossover, best N selection	Navier-Stokes	16	50	None

NObj = Number of objectives; NCons = Number of constraints; NVars = Number of design variables; VarType = Type of variables; NPop = Population size; Gmax = Maximum number of generations; N/A = Not available; s.c. = Only side constraints are adopted.

TABLE III

SUMMARY OF MOEAS APPLIED IN 3D COMPLEX PHYSICS/SHAPE OPTIMIZATION PROBLEMS cases were evaluated in parallel using a FEM simulation on a cluster of workstations. In the first application example, the Pareto front was approximated and presented small regions of discontinuity. For the second example, the Pareto front changed radically its shape,[8] with more regions of discontinuity. Finally, for the third case, the authors did not present a Pareto front but indicated that this case presented difficulties to generate feasible solutions, due to the tight constraints defined. This condition was alleviated by introducing a solution repairing algorithm. Langer et al. [92] extended the previous MOEA using RSM in order to reduce its associated computational cost. One important feature in this application is that a clustering technique was used to build multiple response surfaces over continuous subspaces of the complete design space.

- Voutchkov et al. [180] solved a robust structural design of a simplified FEM jet engine model. This application aimed at finding the best jet engine structural configuration minimizing the variation of reacting forces under a range of external loads, the mass for the engine and the engine's fuel consumption. The authors considered the minimization of four objectives: (i) standard deviation of the internal reaction forces, (ii) mean value of the internal reaction forces, (iii) engine's mass, and (iv) mean value of the specific fuel consumption. The FEM model comprised a set of 22 groups of shell elements, and the thickness corresponding to 15 of these groups were considered as design variables. The authors adopted the NSGA-II. The evaluation of the structural response was done in parallel by means of FEM simulations. The computational time was reduced using a Kriging-based RSM. The first two objectives were computed over 200 external load variations. The authors reported finding a good compromise (and robust) design.
- Todoroki and Sekishiro [173] proposed a new optimization method for composite structural components. The problem consisted of two objectives: (i) minimize the structural weight of a hat-stiffened wing panel, subject to buckling load constraints, and (ii) maximize the probability of satisfying a predefined buckling load. The problem was described by a set of mixed real/discrete design variables. The real variables corresponded to the stiffener geometry definition, while the discrete variables were related to the number of plies for the composite panel. Constraints were imposed on the dimensions of the stiffener, but they were automatically satisfied in the definition of the decision variables' ranges. The authors adopted MOGA coupled to a Kriging model, in order to reduce the number of objective function evaluations, and to a Fractal Branch and Bound (FBB) method [172] for the stacking sequence optimization needed in laminar composites structures. The authors noted that the first objective was not computationally costly, since it could be computed once the geometry of the design candidates was defined. On the other hand, the buckling load constraint demanded a large computational cost, since it needed a FEM simulation. For this reason, a Kriging model was adopted and initialized with sampling points obtained by the LHS technique. The optimization cycle consisted of two layers. The upper layer was driven by MOGA and the Kriging model, where the optimization of the structural dimensions took place. At the lower layer, the stacking sequences of the stiffener and panels were optimized by means of the FBB method. From the results obtained, a comparison of different designs was made. The solution obtained with the evolutionary algorithm was 3% heavier than a previous design obtained with a conventional (deterministic) method, but required only 301 FEM analysis compared to the tens of thousands required by the previous design.
- Olympio and Gandhi [121] applied a hybrid MOEA to generate a constrained topology optimization design for morphing aircraft structures. The problem consisted mainly on finding the trade-off for cellular structures with voids, meeting the following four objectives: (i) high recoverable strain capability to allow several cycles of morphing, (ii) low work necessary to morph for minimal additional need on the actuation system, (iii) high bending stiffness to reduce out-of plane deformation due to surface pressure and (iv) low mass. Constraints were defined on local strains in order to prevent plastic deformations or material failure. In this application, comprising the distribution of material in the structural element, a FEM analysis was performed to evaluate the objective functions related to strain and stiffness for the material. Mesh elements were considered as the design variables which are discrete in nature. Special techniques were used to suppress non-connected regions of material. The cardinality of the design variables vector depended on the discretization of the finite element mesh used in solving the problem. The authors adopted the ϵ-NSGA-II of Kollat and Reed [84]. This MOEA can be seen as an improved version of the NSGA-II, which incorporates ϵ-dominance [94], dynamic population sizing, and an automatic termination criterion. The ϵ-NSGA-II was hybridized with a local search procedure, which consisted of flipping elements adjacent to actual structural elements and evaluating its sensitivity. This can be seen as a specialized operator which acts only on void elements adjacent to structural elements. In the application examples, this local search procedure was limited to a user-defined number of iterations, and was incorporated after a specific number of generations. Additionally, the authors proposed the use of a variable mutation rate, in which the mutation rate was increased or decreased from its current value, depending on the improvement of the solutions. The authors presented two application examples. The first corresponded to a one-dimensional flexible skin using a mesh grid size of 20×20 elements (400 design variables), and the second example corresponded to a

[8] If the problem was transformed into one that considered the minimization of both objectives, this change in geometry for the Pareto front would correspond to a change from a convex to a concave shape. This type of geometry is challenging to traditional mathematical programming techniques.

shear-compression flexible skin using the same mesh size.

Analysis of the use of MOEAs in structural optimization:

Table IV summarizes the application of MOEAs in structural optimization problems. The problems presented in this section are characterized by the use of mixed variable types, which in some cases required that the MOEA adopted special representations and operators. There were also several problems that involved the solution of a combinatorial optimization problem. It is worth emphasizing that traditional MOEAs such as NSGA-II do not necessarily perform well in multi-objective combinatorial optimization problems, since they were originally designed to solve continuous optimization problems. Additional elements such as a good local search engine are normally required when solving combinatorial optimization problems. In fact, there is a wide variety of MOEAs that have been designed to solve multi-objective combinatorial optimization problems (see for example [37], [48]). However, many of them do not support mixed problems such as those described in this section. This seems to indicate that the solution of multi-objective structural optimization problems such as those described in this section is a research line that is worth exploring in the future. The use of the so-called multi-objective memetic algorithms [50], which hybridize MOEAs with powerful local search engines seems to be an obvious choice to tackle the problems described in this section, but they have been scarcely used in this field until now.

Another interesting topic is the use of advanced archiving techniques that allow us to limit the number of nondominated solutions to be stored in a clever way. The ϵ-NSGA-II of Kollat and Reed [84] is an example of such clever archiving techniques. However, other alternatives exist which have not been properly exploited in the context of aeronautical and aerospace engineering (see for example [151]).

E. Multidisciplinary design optimization

As indicated before, aeronautical and aerospace designs are typically multidisciplinary, involving disciplines such as aerodynamics, structures, propulsion, acoustics, manufacturing and economics, among others. Normally, each of the disciplines involved aims at optimizing one specific performance metric, which makes multidisciplinary design multi-objective in nature. Next, we present some applications of MOEAs in multidisciplinary design optimization (MDO).

- Obayashi et al. [119], [120] and Takahashi et al. [165] addressed the MDO of a wing platform. Three objectives were considered: (i) aerodynamic drag, (ii) wing weight, and (iii) fuel weight. Constraints were imposed on lift and on wing structural strength. No special constraint-handling mechanism was adopted, and for any solution that violated the constraints, its rank was lowered, by using a constant penalty value of 10. Three design variables were considered for the wing planform: sweep angle, chord length at the kink and chord length at the tip. Other variables such as the wingspan, root chord length and position of the kink took a fixed value. The authors adopted MOGA. Two disciplines were considered: aerodynamics and structures. The aerodynamic evaluation was performed with the potential CFD solver FLO27, from which only induced and wave drag could be obtained. For the wing weight, an algebraic model was used, and for the last objective, the volume of the wing was calculated to estimate the amount of fuel that could be stored in the wing tanks. The first two objectives were minimized while the third was maximized. The structural analysis, evaluated the skin thickness required, as well as the stress distributions which was considered as a constraint in the problem.
- Choi et al. [20] solved a MDO problem involving Supersonic Business Jet design. The goal was to obtain a trade-off design having good aerodynamic performances while minimizing the intensity of the sonic boom signature at the ground level. Three objectives were considered: (i) the aircraft drag coefficient, (ii) initial pressure rise (boom overpressure), and (iii) ground perceived noise level. In this case, the disciplines involved were aerodynamics and aeroacoustics. Constraints were imposed on some geometrical parameters, and on aircraft's operational conditions. No special constraint-handling mechanism was used other than discarding infeasible candidates. The geometry of the aircraft was defined by 17 design variables, allowing the modification of the wing platform, its position along the fuselage, and some cross sections and camber for the fuselage. The authors adopted the NSGA-II. For evaluating the objective functions, a high-fidelity Euler simulation was obtained with a very fine grid close to the aircraft's surface. In order to reduce the computational time required by the optimization cycle, Kriging models were employed, one for each objective function. Its initial definition was formed with a LHS of the design space with 232 initial solutions including both, feasible and infeasible candidates. The authors were able to find solutions that were better than a baseline design.
- In related publications, Chung and Alonso [21] and Chung et al. [22] solved the same MDO problem described before, but using the μ-GA algorithm from Coello and Toscano [25]. This change aimed at reducing the total number of function evaluations performed during the optimization process. The μ-GA algorithm uses a population of only 4 individuals, an external file and a reinitialization process. In one study [21], the design cycles were performed using a Kriging model. Two design cycles were executed, each consisting of 150 solution candidates using the LHS technique, around a base design in the first cycle. The second cycle was performed around the best solution obtained in the previous cycle aiming to improve it. In the other study [22], the authors proposed and tested the Gradient Enhanced Multiobjective Genetic Algorithm (GEMOGA). The basic idea of this MOEA is to enhance the nondominated solutions obtained by a genetic algorithm with a gradient-based local search procedure. One important feature of this approach was that the gradient information was obtained from the Kriging model. Therefore, the computational cost was not

Ref	NObj	NCons	NVars	VarType	Algorithm	Operators	Physics Model	NPop	Gmax	Remarks
[91]	2	8	17	Mixed continuous/discrete	N/A	Arithmetic crossover and Gaussian mutation for continuous variables, two-point and uniform crossover for discrete variables, Pareto ranking	FEM structural analysis	200	20	Topological shape optimization, use of external archive for keeping non-dominated solutions
[180]	4	s.c.	15	Discrete	NSGA-II	SBX crossover and polynomial mutation	FEM structural analysis	N/A	N/A	Robust design optimization, use of Kriging model
[173]	2	s.c.	7	Mixed continuous/discrete	MOGA	Fitness sharing, SBX crossover and polynomial mutation for continuous variables. Two-point crossover and uniform mutation for discrete variables	FEM structural analysis	100	300	Use of Kriging model
[121]	4	s.c.	400	Discrete	ϵ-NSGA-II	Dynamic population sizing, variable mutation rate, SBX crossover and polynomial mutation	FEM structural analysis	N/A	N/A	Topological shape optimization, use of local search procedure for improving solutions

NObj = Number of objectives; NCons = Number of constraints; NVars = Number of design variables; VarType = Type of variables; NPop = Population size; Gmax = Maximum number of generations; N/A = Not available; s.c. = Only side constraints are adopted.

TABLE IV
SUMMARY OF MOEAS APPLIED IN STRUCTURAL OPTIMIZATION

considerably increased. In both studies, the authors reported obtaining very good approximations of the Pareto optimal set.

- Kumano et al. [89] addressed the MDO of the wing shape of a small jet aircraft. In this study, four objectives were minimized: (i) drag at the cruise condition, (ii) drag divergence between cruising and off-design condition, (iii) pitching moment at the cruising condition, and (iv) structural weight of the main wing. Additionally, two constraints were considered, related to the wing's rear spar heights, and the strength and flutter margins. The wing geometry was defined by airfoil sections at four wingspan stations, and wing twist at five wing stations. A total of 109 design variables were required. The authors adopted MOGA. Aerodynamics and structures were the two disciplines needed for evaluating the objective functions. Since high-fidelity CFD and CSD simulations were used, demanding a very high computational time, the optimization process was performed by means of a Kriging model. The authors were able to obtain an improved design with respect to a reference solution.
- Chiba et al. [16] performed a MDO design exploration. The aim of this study was to find the trade-offs for the design of a wing for its use in a silent supersonic transport application. Five objectives were considered: minimization of (i) pressure drag (ii) friction drag, (iii) boom intensity at supersonic condition, and (iv) composite structural weight of the wing; and maximization of (v) lift at subsonic condition. In these objectives, aerodynamics and structural dynamics were the main disciplines under consideration. The constraints of this problem were mainly geometrical, and no special constraint-handling mechanism was required other than discarding any solution that violated the geometrical constraints. The geometry of the wing was defined by 58 design variables. The authors adopted a hydrid MOEA consisting of a combination of two algorithms: ARMOGA and a MOPSO. The motivation of this hybridization was to exploit, on the one hand, the ability for performing global search of ARMOGA, and, on the other hand, the ability of the MOPSO for performing local search. Both algorithms used real-coded design variables. One half of the population was handled by ARMOGA, with a further subdivision, assigning one quarter of the population to each crossover method indicated above. The other half of the population at each generation was handled by the MOPSO. The evaluation of the aerodynamic properties was done via an Euler solution with TAS-Code, coupled to a simplified model for estimating the friction drag, reducing in this way the computational cost of this discipline. The structural properties (composite strength and modal analysis) were verified with the commercial code NASTRAN. Finally, the intensity of the sonic boom was also evaluated. The authors obtained 75 nondominated solutions on which a data mining method was applied, using ANOVA and SOM methods, in order to reduce them to a set containing only 24 solution from which the designer was able to select only one.
- Chiba et al. [18] addressed the MDO problem of a wing shape for a transoic regional-jet aircraft. In this case, three objective functions were minimized: (i) block fuel for a required airplane's mission, (ii) maximum takeoff weight, and (iii) difference in the drag coefficient between transonic and subsonic flight conditions. Additionally, five constraints were imposed, three of which were related to the wing's geometry and two more to the operating conditions in lift coefficient and to the fuel volume required for a predefined aircraft mission. The wing geometry was defined by 35 design variables. The authors adopted ARMOGA. The MDO process was done with high fidelity CFD/CSD simulations. The disciplines involved included aerodynamics and structural analysis and

during the optimization process, an iterative aeroelastic solution was generated in order to minimize the wing weight, with constraints on flutter and strength requirements. Also, a flight envelope analysis was done, i.e., obtaining high-fidelity Navier-Stokes solutions for various flight conditions. The population (consisting of only eight individuals) was reinitialized at every 5 generations for range adaptation. In spite of the use of such a reduced population size, the authors were able to find several nondominated solutions outperforming the initial design. They also noted that during the evolution, the wing-box weight tended to increase, but this degrading effect was redeemed by an increase in aerodynamic efficiency, given a reduction in the block fuel of over one percent, which would be translated in significant savings for an airline's operational costs.

- Sasaki et al. [144] solved a MDO for a supersonic wing shape. In this case, four objective functions were minimized: (i) drag coefficient at transonic cruise, (ii) drag coefficient at supersonic cruise, (iii) bending moment at the wing root at supersonic cruise condition, and (iv) pitching moment at supersonic cruise condition. The problem was defined by 72 design variables. Constraints were imposed on the variables ranges and on the wing section's thickness and camber, all of them being geometrical constraints. Thus, no special constraint-handling techniques were required, other than discarding any infeasible solution, and generating a new one using the genetic operators, until a valid solution was obtained. The authors adopted ARMOGA, and the aerodynamic evaluation of the design solutions, was done by high-fidelity Navier-Stokes CFD simulations. No aeroelastic analysis was performed, which considerably reduced the total computational cost. The objective associated with the bending moment at wing root was evaluated by numerical integration of the pressure distribution over the wing surface, as obtained by the CFD analysis. The authors indicated that among the nondominated solutions there were designs that were better in all four objectives with respect to a reference design.
- Lee et al. [97] utilized a generic framework for MDO [53] to explore the improvement of aerodynamic and radar cross section (RCS) characteristics of an Unmanned Combat Aerial Vehicle (UCAV). In this application, two disciplines were considered, the first concerning the aerodynamic efficiency, and the second one, dealing with the visual and radar signature of an UCAV airplane. In this case, three objective functions were minimized: (i) inverse of the lift-to-drag ratio at ingress condition, (ii) inverse of the lift-to-drag ratio at cruise condition, and (iii) frontal area. The number of design variables was 100 and only side constraints were considered in the design variables. The first two objective functions were evaluated using a potential flow CFD solver (FLO22) coupled to FRICTION code to obtain the viscous drag. The authors adopted the Hierarchical Asynchronous Parallel Multi-Objective Evolutionary Algorithm (HAPMOEA). The authors reported a processing time of 200 hours for their approach, on a single 1.8 GHz processor. It is important to consider that HAPMOEA operates with different CFD grid levels (i.e., approximation levels): coarse, medium, and fine. In this case, the authors adopted different population sizes for each of these levels. Also, solutions were allowed to migrate from a low/high fidelity level to a higher/lower one in an island-like mechanism.
- In further work, Lee et al. [96] solved the same previously defined UCAV MDO problem, but considering a robust design methodology (the Taguchi method [163]) to incorporate uncertainties in the operation environment of the UCAV. The MDO problem considered two cases, each with three objectives. The first case corresponded to a mono-static RCS and its aerodynamic shape optimization, and the objectives to be minimized were: (i) radar cross section for the mono-static case, (ii) mean value for the inverse of the lift-to-drag ratio, and (iii) variance for the inverse of the lift-to-drag ratio with respect to its mean value. The second case was a mono/bi-static RCS and aerodynamic shape optimization, with the following objectives to be minimized: (i) mono-static RCS (ii) bi-static RCS, and (iii) both, the mean value of the inverse lift-to-drag ratio, and its variance. For this latter objective, an aggregating function was used, instead of extending the optimization problem to one with four objectives. In both cases, the robust design considered uncertainties in operating conditions such as flying Mach number, angle of attack and radar signal orientation with respect to the UCAV. In both test cases, the authors adopted HAPMOEA. The MDO problem comprised more than 100 design variables, with constraints imposed on the thickness of the airfoil sections for structural concern. From the results, a set of 15 nondominated solutions was obtained in the first case and a set of 10 solutions was obtained in the second case. From these solutions, the designers were able to select one which had superior performance in all the objectives with respect to a baseline design (this happened for the two cases considered).
- Pagano et al. [127] presented an application for the MDO of an aircraft propeller. The aim was to improve the propeller performance. Basically, two conflicting objectives were considered: (i) minimizing the noise emission level, and (ii) maximizing aerodynamic propeller efficiency. For this industrial problem, several disciplines were considered: aerodynamics, structures, and aeroacoustics. For each of these disciplines, specialized computer physics-based simulation codes were employed. Each design solution evaluation comprised an iterative procedure among these simulation codes in order to evaluate a more realistic operating condition. Therefore, the optimization process was computationally demanding. In order to reduce the burden of this high computational cost, the authors opted for the use of design of experiments techniques, and RSM for efficiently exploring the design space. The geometry for the propeller blade was considered as the output for this optimization process, and was parameterized using 14 design variables which included blade twist, sectional chord and leading edge line

definition, all, at several prescribed blade radial stations. The MDO problem contained constraints on the geometry design variables, and on propeller shaft power at two flight conditions: takeoff and cruise, respectively. The authors adopted The Nondominated Sorting Evolutionary Algorithm+ (NSEA+[9]) as implemented in the OPTIMUS commercial software. The authors were able to obtain design solutions which performed better than a reference propeller design. Approximately 20 nondominated solutions were obtained, all of which were better than the reference design in both objectives.

- Nikbay et al. [116] presented a coupling of techniques for multidisciplinary analysis and optimization, particularly addressing the aeroelastic optimization problem including aerodynamics and structures as the main disciplines. The authors adopted the NSGA-II and a MDO problem which aimed to improve the reference experimental wing AGARD 455.6. For this problem, the wing geometry was defined in terms of wing taper ratio and wing quarter chord swept angle, which were considered as the design variables. The objective functions were: (i) maximization of the lift-to-drag ratio and (ii) minimization of the wing's weight. Also, one constraint was included in the maximal aeroelastic wing's tip deformation, which was prescribed as a function of the wingspan. In this approach, both the aerodynamic and the structural simulation were performed with high fidelity CFD and CSD commercial codes. A special iterative process was defined in order to couple the multiple-discipline effects presented in the optimization, i.e., exchanging parametric CAD definition, pressure loads and deformations, between the software used for each discipline. From their application example, the authors obtained 14 nondominated solutions, from which the extremes of the Pareto front were extracted.
- Johnson et al. [75] performed a MOEA-based MDO for the aerodynamic and heat transfer performances of heat shields for blunt body reentry vehicles. The authors were interested in obtaining trade-offs among the performance parameters (objectives) of the vehicles, which included: (i) peak heat flux, (ii) total head load, and (iii) maximum cross range. The optimization was performed with the University of Maryland Parallel Trajectory Optimization Program (UPTOP), which is based on a differential evolution scheme, and allows the analysis of reentry trajectory vehicles with three degrees of freedom to be coupled with the analysis for vehicle's aerodynamic and heat transfer performances. Even when three objectives were considered in the problem, the authors performed experiments with only two of them at a time. The design variables for the optimization problems were 13 or 14, depending on the definition of the axial profile of the vehicle (three different axial profiles were used). The constraints set consisted of nine constraints, which considered trajectory design limits, theory limitations, and aerodynamic moments limits. The crossover and mutation rates were varied randomly from zero to one, with the aim of maximizing the range of the nondominated solutions produced. From the results presented, the authors selected optimal trajectory/vehicle configurations for two reentry conditions.
- Rajagopal and Ganguli [134] addressed the MDO preliminary design of an UAV wing. In their study, the authors aimed at optimizing two conflicting objectives: UAV endurance and wing's structural weight. In this case, the involved disciplines are aerodynamics and structural analysis. Two objective functions were considered: (i) the maximization of the endurance (the time an airplane can fly given a payload and a given fuel weight) and (ii) the minimization of the wing weight. A total of ten design variables were used for defining the wing's geometry as well as its structural properties. Constraints were imposed on the aerodynamic performance and geometry, both for the airfoil shape and for the complete wing. Also, constraints were imposed on the minimal structural strength and stiffness of the wing. The authors adopted the NSGA-II, and the objective functions were evaluated using CFD and CSD simulation codes. This required a very high computational cost, which led the authors to the use of Kriging-based models. The authors reported finding only 5 feasible nondominated solutions.
- Jagdale et al. [65] applied a MOEA for the conceptual multidisciplinary design of a bendable UAV wing. Such types of wings, constructed from composite materials, have two conflicting structural requirements: first, the complete wing must be able to be folded for its storage in a container and, second, it must be stiff enough to withstand the aerodynamic loads during flight operation, in order to avoid buckling, due to an excesive material strength and large deformation. For the multidisciplinary design, two major analysis disciplines were considered: aerodynamics and structures. Two objectives were considered: (i) maximize the lift-to-drag ratio of the wing, (ii) maximize the wing's buckling speed. Additionally, a set of four constraints was included comprising: a minimum cruising speed, a positive lift coefficient, a stability margin, and the desired rolled wing diameter range. Both objective functions were evaluated using CFD and CSD simulation codes. The authors adopted the NSGA-II. Ten design variables were used: seven to define the wing geometry and three to define its composite plies orientation. The wing geometry related variables are continuous, but the authors indicated that they used a discretization for them. The authors reported finding trade-off solutions that were able to outperform a reference design in both objectives.

Analysis of the use of MOEAs in multidisciplinary design optimization:

Table V summarizes the application of MOEAs in multidisciplinary design optimization. A common feature of the applications discussed in this section was the interaction of two

[9]NSEA+ adopts the selection mechanism of the NSGA-II and the mutation operator of the evolution strategies.

Ref	NObj	NCons	NVars	VarType	Algorithm	Operators	Physics Model	NPop	Gmax	Remarks
[120] [119] [165]	3	2	3	Continuous	MOGA	Weighted averaged crossover, Pareto ranking, fitness sharing, best N selection	Potential flow model and FEM model	100	30	Penalty-based constraint handling
[20]	3	N/A	17	Continuous	NSGA-II	SBX crossover and polynomial mutation	Euler and aeroacoustic models	N/A	N/A	Use of Kriging model
[21]	3	N/A	17	Continuous	μ-GA	N/A	Euler and aeroacoustic models	N/A	N/A	None
[22]	3	N/A	17	Continuous	GEMOGA	N/A	Euler and aeroacoustic models	N/A	N/A	Use of Kriging for gradient calculation
[89]	4	2	109	Continuous	MOGA	N/A	Navier-Stokes CFD and FEM structural model	N/A	N/A	Use of Kriging model
[16]	5	N/A	58	Continuous	ARMOGA and MOPSO	BLX-α and PCA-BLX-α crossover operators, fitness sharing, Pareto ranking	Euler CFD flow and FEM models	20	12	Population is divided among the algorithms used as well as the crossover operators
[18]	3	5	35	Continuous	ARMOGA	Fitness sharing, Pareto Ranking, best N selection	Navier-Stokes CFD and FEM models	8	20	None
[144]	4	2	72	Continuous	ARMOGA	Fitness sharing, BLX-α crossover, Pareto ranking, best N selection	Navier-Stokes CFD and simplified structural models	64	30	None
[97]	3	N/A	100	Continuous	HAPMOEA	ES mutation operator with Covariance Matrix Adaptation (CMA-ES), distance dependent mutation, tournament selection	Potential flow CFD and Radar Cross Section (RCS) estimation models	*	N/A	* Population sizes are 40, 40 and 60 for fine, medium and coarse CFD mesh grids, 1550 design candidates evaluated
[96]	2	1	100	Continuous	HAPMOEA	ES mutation operator with Covariance Matrix Adaptation (CMA-ES), distance dependent mutation, tournament selection	Potential flow CFD and Radar Cross Section (RCS) estimation models	*	N/A	* Population sizes are 15, 40 and 60 for fine, medium and coarse CFD mesh grids, 1100 design candidates evaluated. Robust design optimization
[127]	2	2	14	Continuous	NSEA+	ES mutation operators and NSGA-II selection mechanism	Simplified aerodynamics, FEM and aeroacoustic models	20	17	Use of RSM
[116]	2	1	2	Continuous	NSGA-II	N/A	Navier-Stokes CFD and structural FEM models	12	17	None
[75]	3	9	14	Continuous	DE based	N/A	Aerodynamic and thermodynamic models	130	N/A	Random variation of mutation rate
[134]	2	4	10	Continuous	NSGA-II	SBX crossover and polynomial mutation	Simplified aerodynamics and structural FEM models	50	100	Use of Kriging model
[65]	2	4	10	Discrete	NSGA-II	N/A	Simplified aerodynamics and structural FEM models	30	70	None

NObj = Number of objectives; NCons = Number of constraints; NVars = Number of design variables; VarType = Type of variables; NPop = Population size; Gmax = Maximum number of generations

TABLE V
SUMMARY OF MOEAS APPLIED IN MULTIDISCIPLINARY DESIGN OPTIMIZATION

or more disciplines in the evaluation of the objective functions. This was combined in some cases with a high-dimensional search space, leading to very costly computer simulations that required the use of surrogate models and/or parallelization techniques. The need for highly efficient MOEAs specially tailored for this sort of problems is quite evident. Although some authors reported using very small population sizes (including micro-genetic algorithms), the computational cost of the MOEAs adopted remains as their main limitation. Thus, this area clearly needs further research aimed at producing efficient and effective MOEAs that can produce good approximations of the Pareto optimal set requiring only a very low number of objective function evaluations. The use of advanced archiving techniques can also be advantageous [151]. Finally, the lack of properly designed constraint-handling techniques is also evident. Such approaches can help to reduce the overall computational cost of the evolutionary process, but has not been properly addressed yet (see for exampe [108]).

F. Aerospace system optimization

Apart form atmospheric flight, aerospace engineering deals with the design of spacecraft and space systems such as satellites. The use of MOEAs in these applications is reviewed next.

- Hartman et al. [57] and Coverstone-Carroll et al. [28] presented the application of a MOEA to the design of low-thrust spacecraft trajectories. The authors considered two study cases: a) Earth-Mars rendevouz [28], [57], and b) Earth-Mercury rendevouz [28]. The authors adopted the NSGA [160] and considered three objectives: i) maximize spacecraft mass delivery at rendevouz, ii) minimize the spacecraft mission flight time, and iii) maximize the spacecraft heliocentric revolutions. Three constraints were also imposed on the MOP, from which two were related to the minimum and maximum values for the heliocentric revolutions (i.e., they constrain the range value that the third objective can attain). The third constraint was the convergence error that results from solving a two-point boundary value problem (TPBVP), which includes two sets of seven nonlinear and coupled differential equations each. Since for this case there is no closed form solution, a numerical approximation, based on the calculus of variations was used. In fact, this latter process corresponds to an optimization process by itself, since it involves computing the optimal spacecraft thrust schedule as well as the thrust orientation, along with the optimal orbit that maximizes the delivered weight at the rendevouz point, with its specific constraints at launch/rendevouz points as well as along the transfer orbit. This last optimization process corresponds to the objective function evaluation, which is computationally intensive, since many of the solutions generated by the MOEA might not be feasible. The NSGA was hybridized with a local search procedure [10] based on a gradient method implemented in NASA's JPL SEPTOP (Solar Electric Propulsion Trajectory Optimization Program) software. So, the MOEA (NSGA in this case) was used for the global search, and the parameters obtained for each individual in the population, were used as input parameters for the SEPTOP software. It is interesting to note that, as reported by Hartmann [58], after applying the local search, the individuals are not updated in their parameters, but only in their fitness values (i.e., the authors adopt a Baldwinian learning strategy). Thus, the authors argue that diversity is preserved in the population. They also adopted a penalty function to handle the constraints of the problem. The authors were able to find several families of optimal trajectories for the two spacecraft missions analyzed, including some novel trajectories.

- Lee et al. [99] addressed a low-thrust orbit transfer from a geostationary orbit to a retrograde Molnya-type orbit. The challenge in this problem is that it requires to modify five out of six orbital parameters, which is performed with low-thrust applied during long periods of time. The authors considered two objectives: i) minimize the required propellant mass, and ii) minimize the toal flight time. The authors relied on the Q-law (a Lyapunov feedback control law) theory, which requires the tuning of 13 control parameters defining the decision vector. Three different MOEAs were adopted: 1) NSGA [160], 2) The Pareto-based Ranking Genetic Algorithm[11] (PRGA), and 3) the Strength Pareto Genetic Algorithm[12] (SPGA). The results obtained by these three MOEAs were compared based on two performance measures: the size of the dominated space, and the coverage of two Pareto fronts. For each candidate solution in the MOEA's population, an optimal orbital transfer was estimated, using the Q-law, such that it satisfied the orbital's initial and final boundary conditions, while minimizing the total flight time. Once the schedule and orientation of the thrust along the orbit are obtained, the required propellant mass, and the flight time, allow to evaluate the two objective functions previously indicated. From their comparative study, the authors concluded that both NSGA and SPGA had a similar performance with respect to the measures adopted. These two MOEAs outperformed PRGA. It is worth noting, however, that the authors performed only three runs with each algorithm, because of the high computational cost involved in the evaluation of the objective functions of this problem.
- Luo et al. [104] solved the problem of rendez-vous trajectory parameter optimization. In this case, three objective functions were considered: (i) the time of flight for the spacecrafts to accomplish the rendez-vouz, (ii) the total velocity characteristic which is a function of multiple impulses performed by the chaser spacecraft, and (iii) the trajectory safety performance index, which is a measure of the distance the chaser spacecraft attains in "free path" with respect to the target spacecraft, in case the thrust control ceases. A simplified model (linearized) was adopted for solving the trajectory of the rendez-vouz problem. The problem consisted of a decision vector that could vary in size due to the number of impulses considered in the optimization problem. In the application problems presented, the authors used either three or four impulses, originating decision vectors of seven or eight variables, respectively. Constraints were imposed on the times of applying the impulse and the interval time between two consecutive impulses. The authors adopted the NSGA-II. The constraint-handling mechanism incorporated into the NSGA-II was adopted without any changes. The evaluation of the objective functions was obtained by an iterative method, i.e., a set of differential equations, governing the spacecraft motion. The example problems presented by the authors were for three and four impulses rendez-vouz trajectory optimization. In each case 10 runs were performed and a "global" Pareto front was constructed considering the Pareto fronts obtained in each execution. The authors did not report the number of nondominated solutions obtained in any case.
- In a similar work, Luo et al. [103] extended their application for the multiple-impulse rendez-vouz trajectory optimization problem, but in this case using a more sophisticated model (non-linear) for evaluating the objective functions. Additionally, constraints on the path were included to solve a problem with more realistic operational conditions. As before, the NSGA-II was adopted. The problems that were solved corresponded to a three and four impulses rendez-vouz trajectory optimization. In both cases, trade-offs were obtained among the time of flight, the propellant cost, and the trajectory safety for rendez-vouz missions, with and without path constraints.
- Ferringer et al. [43] addressed the problem of satellite

[10] In Hartmann [58] the approach is called **NSMA** which stands for Nondominated Sorting Memetic Algorithm.

[11] The description of this algorithm provided by the authors corresponds to MOGA [46].

[12] This is really SPEA [189].

constellation design. The authors looked for a three-satellite constellation which minimized two objectives: (i) Maximum Revisit Time (MRT), and (ii) Average Revisit Time (ART). Both objectives were influenced by satellite orbital parameters: (a) inclination, (b) right ascension of the ascending node, and (c) mean anomaly, which were used as design variables. Orbital height was not treated as a variable but fixed at an altitude guaranteeing horizon-horizon visibility among satellites. The evaluation of the objective functions was obtained by modeling satellite constellation visibility to ground locations, defined by discrete grid points and overlaying the land area of interest. The authors adopted the NSGA-II with binary encoding.

- In more recent work, Ferringer et al. [44] addressed the problem of satellite constellation reconfiguration using a MOEA. The problem solved by the authors considered the Global Positioning System (GPS) constellation for two degrading cases: (a) loss of one satellite, out of 24 comprising the constellation, and (b) loss of one plane of satellites, out of a total of six planes (loss of 4 satellites). The GPS constellation was designed to provide global average coverage greater than 99.9% in ideal operating conditions, and greater than 96.9% in the worst case. This coverage was calculated by considering a visibility of at least 4 satellites above a 5° angle over the Earth's horizon. For the application problem, a total of six objective functions were defined: (i) four-fold average daily visibility time, (ii) four-fold worst-case-point daily visibility time, (iii) total time of flight, (iv) maximum ΔV[13] required by any maneuvered satellite, (v) sum of the ΔV variance of the maneuvered satellites, and (vi) satellites maneuvered. All these objectives comprised constellation performance objectives, constellation reconfiguration costs, and satellite maneuver risk. The first two objectives were maximized, while the others were minimized. The authors adopted the ϵ-NSGA-II algorithm of Kollat and Reed [84]. Additionally, the authors indicated the use of a technique called time continuation which was applied during several runs of the algorithm. When using this mechanism, the initial population for every successive run was formed by keeping 25% of nondominated solutions of the previous run and the other 75% solutions were created randomly. The optimization problem was defined with a vector of 24 or 21 design variables depending on the degrading cases indicated above. The design variables corresponded to the mean anomaly and integer phasing orbits for the satellites in the constellation.

- Vasile and Croisard [107] addressed the robust preliminary and multidisciplinary design for an interplanetary spacecraft mission, namely, the *BepiColombo* mission. The robust design considered uncertainties in several design parameters, and aims at reducing the impact of these on the optimal value for the design criteria. Unlike other approaches presented above, which make use of the Taguchi method as the robust design framework, in this case, the authors make use of Evidence theory [33], [154]. This allows to model both, stochastic and epistemic uncertainties (i.e., the authors assume a poor or incomplete knowledge of the design parameters). The latter situation is commonly present in the preliminary design phase of the spacecraft mission considered. The authors considered two objectives in this case: i) maximize the Cumulative Belief Function (CBF) (i.e, a measure of the maximum confidence that a design is better than a certain threshold, in the cost function), and ii) minimize a given cost function, which in the examples presented, corresponds to minimizing the wet mass (related to the mass of propellant required to perform the low-thrust transfer) of the spacecraft being designed. The MOEA used by the authors was the NSGA-II [32]. In the solution of robust design problems, design candidates are not evaluated at fixed values of the design parameters, but considering uncertainties in them. In this case, three uncertain parameters were considered with four threshold intervals and a corresponding BPA (Basic Probability Assignment) each. Thus, for evaluating the CBF, a total of 64 *Focal Elements* (intersection threshold regions for all the uncertain parameters with different BPAs each), had to be searched for. In each of these threshold regions, a local optimizer was used to estimate the maximum of the system's function. Thus, if the whole evolutionary process is considered, it is evident that this is a computationally expensive application. Furthermore, the authors reported the use of a Kriging model for approximating the relation between the spacecraft maximum thrust and the power to be generated by the solar arrays, with the Delta budget (ΔV), which is an important value for the objective function evaluation. The authors compared the use of the NSGA-II to a reference (nearly optimal) solution, and concluded that their hybrid approach was very useful for estimating the optimum and for narrowing down the search in the presence of uncertainties.
- Minisci et al. [111] dealt with the robust multi-disciplinary preliminary design of a small scale Unmanned Space Vehicle (USV), which was planed to be used for space re-entry operations. In this case, the simultaneous optimization of both, the spacecraft shape, and its trajectory control profile, are required. The authors considered three objectives: i) minimize the mean value of heat flux in the USV, ii) minimize the mean value of the estimated internal spacecraft temperature, and iii) minimize the weighted sum of the variances of the first two objectives, which were evaluated along the re-entry trajectory, considering uncertainties in two aerodynamic forces (lift and drag), and in the thermal conductivity and the specific heat of the material used for building the spacecraft. Two constraints were also included in the maximum attainable values for the variance of the heat flux and in the estimated internal spacecraft temperature.

[13] In orbital mechanics, ΔV, corresponds to the impulse or change in velocity needed to make an orbital change of the satellite or any spacecraft. This ΔV is given by the propulsion system.

The authors adopted an approach called MOPED [27], which is based on an Estimation of Distribution Algorithm (EDA) [93]. MOPED makes use of nondominated sorting and crowding (taken from NSGA-II [32]) and was used to search on the spacecraft geometry parameters (six in total). Additionally, an optimal control subproblem was solved for finding the optimal re-entry trajectory (i.e., to determine the angle of attack profile along the trajectory), from a set of dynamic equations, formulated by nonlinear differential equations and a set of initial/boundary conditions that had to be satisfied. The authors adopted variable fidelity meta-models, or surrogates, whith the aim of reducing its high computational cost. Artificial neural networks (ANNs) were used as meta-models, at the beginning of the evolutionary process, being trained only with a low fidelity analytical aerodynamic model. Towards the end of the evolutionary process, the ANNs were traided with high fidelity CFD solutions.

Analysis of the use of MOEAs in aerospace system optimization:

Table VI summarizes the application of MOEAs in aerospace system optimization.

From the above applications described, it is worth noticing that MOEAs applied to aerospace systems cover a wide variety of problems, including multiple disciplines and the use of robust design techniques. Also, it is important to emphasize that most of the applications discussed in this section involve the use of a coupled global-local search optimization scheme. This is to say that a MOEA is used to find a set of good solutions, perhaps at a coarse granularity (e.g., without considering all the decision variables), which are further improved using a local search engine (gradient-based techniques are normally used for this sake). For example in [111], the MOEA is used at an upper level, with a subset of the decision variables and without incorporating any constraints, while the constraints and all the decision variables are considered and solved at a lower level, in which a gradient-based optimization process is used to find feasible solutions. Although memetic MOEAs have existed for several years in the specialized literature [50], the development of specific MOEA-based approaches that properly combine a global and a local search scheme in an efficient and effective way when dealing with space applications, is still an open research area. Issues such as how to couple the global search engine with the local search engine, how to handle the constraints (particularly when dealing with large scale applications having many nonlinear constraints), how to handle mixed problems that combine, for example, integer and real-numbers decision variables (which could be handled separately or at different granularities by the global the local search engines), how to make the search less expensive (computationally speaking) are some of the possible paths for future research in this area. In this regard, Vasile and Zuiani [178] have recently proposed an interesting approach based on the collaboration of multiple agents. This approach blends a number of metaheuristics, including particle swarm optimization and differential evolution. This approach, has been succesfully applied to the design of multi-impulse trajectories [178], to the robust design optimization of low-thrust transfers, and aerocapture manoeuvres [176]; to the design of an integrated space and terrestrial solar power plant [177] and to the design of satellite formation [105]. Another interesting issue that arises in the problems discussed in this section is the size of the feasible region, which can be very small with respect to the entire search space. In this sense, some techniques for pruning the search space have been proposed [152] and have been succesfully applied in the context of low-thrust gravity-assist trajectory design. This constitutes another promising research topic, to be considered when designing MOEAs for space applications.

G. Control system design

In this final group, the applications are those in which MOEAs are used to find the parameters involved in control systems.

- Chipperfield and Fleming [19] described the use of a MOEA in the design of a control system for gas-turbine aero-engines. This application evaluated populations of candidate control systems and modes, aiming at selecting sensors and defining a suitable controller for a manoeuvre about a particular operating point while meeting a set of strict design criteria including stability, sensitivity and the accommodation of degradation with engine ageing. The application example presented by the authors considered attaining nine design objectives comprising the engine's time response, thrust level, and turbine blade temperature, among other criteria, in response to a change in thrust demand. The control system was evaluated using a linearized model of a reference engine. The authors adopted MOGA with mating restrictions[14] and fitness sharing in objective function space. From their results, the authors obtained trade-off information which allowed them to look into the positive and/or negative aspects of different control schemes.
  In a similar research work, Thompson et al. [171] used the same version of MOGA previously indicated for the multi-objective optimization of an aircraft engine controller architecture, particularly for a military aircraft engine, where many inputs and outputs were duplicated, increasing considerably the number of sensors and actuators inputs (240 approximately) to be considered in the controller design.
- Aranda et al. [6] used a MOEA for the design of an aircraft flight control system. The application concerned the design of control laws, which were further used for evaluating control designs. The MOEA adopted was based on Pareto ranking. The flight controller took several input signals, and after their evaluation it returned system performance in a vector of control response metrics. This vector comprised 21 parameters.

[14]Several researchers within evolutionary multi-objective optimization have experimented with schemes that impose rules on the individuals that can be recombined. However, there is no clear evidence of the superiority of this sort of mating scheme with respect to the use of a traditional one in which no restrictions are imposed on the individuals to be recombined [24].

Ref	NObj	NCons	NVars	VarType	Algorithm	Operators	Physics Model	NPop	Gmax	Remarks
[28], [57]	3	3	8	Continuous	NSGA	Single point crosover, uniform mutation, stochastic universal sampling, fitness sharing in decision space, and Pareto ranking	Orbital mechanics and rocket equation models	150	30	Use of binary encoding, a local search mechanism and a Baldwinian learning strategy.
[99]	2	N/A	13	Continuous	NSGA, PRGA (MOGA), and SPGA (SPEA)	N/A	Orbital mechanics and rocket equation models	1000	200	None
[104]	3	N/A	7/8	Continuous	NSGA-II	Arithmetical crossover and nonuniform mutation	Linearized orbital mechanics model	100	200	None
[103]	3	N/A	7/8	Continuous	NSGA-II	Arithmetical crossover and nonuniform mutation	Nonlinear orbital mechanics model	100	200	None
[43]	2	s.c.	3	Discrete	NSGA-II	SBX crossover and polynomial mutation	Orbital mechanics model	32	400	Island-based parallel implementation of NSGA-II
[44]	6	s.c.	21/24	Continuous	ϵ-NSGA	SBX crossover and polynomial mutation	Orbital mechanics model	48	250	None
[107]	2	N/A	N/A	Continuous	NSGA-II	SBX and polynomial-based mutation	Orbital mechanics and rocket equation models	20	N/A	Results are presented for 100,000, 500,000, and 1,000,000 total function evaluations.
[111]	3	2	6	Continuous	MOPED	Nondominated sorting, crowding	Analytical, structural, and CFD models	60	50	Use of ANNs as meta-model.

NObj = Number of objectives; NCons = Number of constraints; NVars = Number of design variables; VarType = Type of variables; NPop = Population size; Gmax = Maximum number of generations; N/A = Not available; s.c. = Only side constraints are adopted.

TABLE VI
SUMMARY OF MOEAS APPLIED IN AEROSPACE SYSTEM OPTIMIZATION

The design variables for the optimization were the elements of the two control law gain matrices (2×5 and 2×2 matrices) for the inner loop controller. 14 design variables were considered, being these variables floating point numbers. The authors presented results for both, longitudinal and lateral flight controllers.

Analysis of the use of MOEAs in control system design:

Table VII summarizes the application of MOEAs in control system design. The applications described in this section are also computationally inexpensive, allowing the use of more elaborate MOEAs and archiving techniques which, apparently, have not been used so far within this domain. However, another interesting feature of the problems described here is that the approaches developed to solve them may be extrapolated to other domains, since control systems are commonly used in a wide variety of engineering disciplines (see for example [66]). This should motivate the development of more research within this area.

VI. FUTURE RESEARCH PATHS

As evidenced in this survey, the use of MOEAs for solving aeronautical and aerospace engineering optimization problems is already a mature area which has spread over a broad range of application subdomains. Most of the applications reviewed in this paper are based on a genetic algorithm, being MOGA and NSGA-II the most frequently used (both of them with diverse modifications). All the applications reviewed in this paper represent real-world application problems, which require, in many cases, the use of expensive computational simulations to evaluate the objective functions. Additionally, the problems analyzed are typically very high dimensional, having large, complex and poorly understood search spaces, which make them intractable using traditional mathematical programming techniques. In fact, the high computational cost associated to some of these problems makes the use of MOEAs infeasible, unless alternative techniques are adopted. The most common ones are the use of response surface models (or approximation models), the use of parallel programming (mainly to evaluate the population's fitness values), and the use of other meta-heuristics that are better suited for continuous optimization than genetic algorithms (e.g., differential evolution, evolution strategies and particle swarm optimization). Additionally, other authors have hybridized their MOEAs with gradient-based methods, aiming to combine the strengths of the global search performed by an evolutionary algorithm with the local search performed by a gradient-based technique.

From the applications analyzed in this paper, the following salient issues have been identified as requiring further research:

- **Alternative chromosome encodings**: Most of the applications analyzed here mention the use of specific chromosome representations but, in general, it is assumed that vectors of real numbers or binary numbers are normally adopted (with a set of associated crossover and mutation operators). However, other encodings exist, which could probably help to improve the performance of a MOEA. Such alternative encodings include the use of matrix or structured/hierarchical representations (see for example [30], [179]), which could be particularly useful for 3D complex geometries (see for example [12]).
- **Use of small population sizes**: One possible choice for reducing the total number of objective function evaluations performed by a MOEA is to use very small

Ref	NObj	NCons	NVars	VarType	Algorithm	Operators	Physics Model	NPop	Gmax	Remarks
[19]	9	s.c.	6	Discrete	MOGA	Structured chromosome representation, mating restrictions, fitness sharing	Control mode analysis	70	N/A	None
[6]	20	s.c.	14	Continuous	MOGA	Binary tournament selection, multiple crossover operators, Pareto ranking, fitness sharing	Control mode analysis	N/A	N/A	None

NObj = Number of objectives; NCons = Number of constraints; NVars = Number of design variables; VarType = Type of variables; NPop = Population size; Gmax = Maximum number of generations; N/A = Not available; s.c. = Only side constraints are adopted.

TABLE VII
SUMMARY OF MOEAS APPLIED IN CONTROL SYSTEM DESIGN

population sizes with proper mechanisms to maintain diversity. This is normally not done because the use of such small population sizes normally causes premature convergence of EAs due to a sudden loss of diversity [72], [129]. However, with carefully designed mechanisms that can maintain diversity, it is possible to use very small population sizes. An example of this are the micro-genetic algorithms for multi-objective optimization, which have been already used in aeronautical engineering [21], [162]. It is worth noting, however, that several other metaheuristics that have a high potential in aeronautical engineering have been only scarcely used with very small population sizes (e.g., differential evolution and evolution strategies).

- **Use of techniques to improve efficiency**: The use of response surface models presents difficulties as the number of decision variables increases, mainly because the number of samplings required for obtaining a high fidelity model increases, too. A possible way of dealing with this problem is to build local response surface models as proposed by Emmerich et al. [39] and Giannakoglou [49]. and to use them for a pre-screening process in the selection process (i.e., to select promising members at each generation which will be evaluated by the exact model, reducing, in consequence, the overall computational cost). Another possible option for improving efficiency is to adopt knowledge extraction techniques and then reuse this information during the evolutionary search. Although such techniques have been normally used in an *a posteriori* manner (adopting self-organizing maps and ANOVA, as in [18], [125], [126], [159]), it is also possible to use them as an *a priori* technique. For example, Gräning et al. [55] successfully applied this type of approach to the single-objective optimization of 3D turbine blade geometries. The extension of this type of approach to aeronautical/aerospace multi-objective optimization problems is, indeed, a very promising research path.
There are, however, other approaches that can reduce the number of objective function evaluations without having to build an approximate model of the problem. Perhaps the most well-known choices within the evolutionary algorithms literature are fitness inheritance [157] and fitness approximation [70]. Both of them have been used with MOEAs (see for example [139]), but their use in real-world applications is still scarce (see for example [128]), mainly because practitioners are either not aware of them, or do not trust their reliability in highly nonlinear search spaces [35]. It is also worth remarking that several other approaches exist for improving the efficiency of a MOEA, but most of them remain unused in real-world applications (see for example [1], [166]).
- **Efficient constraint-handling techniques**: Most of the applications reviewed in this paper dealt with problems subject to constraints. In most cases, infeasible solutions were discarded and generated again, or a simple external penalty function was adopted. However, many other constraint-handling approaches exist, which could be very useful in multi-objective optimization, since they can explore the boundary between the feasible and the infeasible region in a more efficient way than traditional penalty functions (see for example [108], [148]).It would also be interesting to design approaches that can efficiently deal with problems having many nonlinear constraints.
- **Alternative selection schemes**: Most modern MOEAs rely on Pareto-based ranking [51]. However, this sort of selection scheme has certain limitations, from which its poor scalability is perhaps the most remarkable [82]. Recently, and mainly motivated by this scalability problem, a number of alternative selection schemes for MOEAs have been introduced in the specialized literature. From them, perhaps the most remarkable approaches are those based on a performance measure known as *hypervolume* (see for example [38]) and those based on relaxed forms of Pareto dominance (see for example [42]). Such approaches have been scarcely used in aeronautical/aerospace engineering (see for example [10]).
- **Alternative parallelization techniques**: Due to the high computational cost required by many aeronautical and aerospace engineering optimization problems, the use of parallelism is relatively common. However, more elaborate parallelization techniques based, for example, on coevolution [167], cellular computing [2], GPU-based computing [183] and asynchronous techniques [7] are still scarce in this area and more work in that direction is expected in the next few years. These techniques have been adopted in other costly applications arising in areas such as genetic programming [56].

VII. Conclusions

This paper has presented a survey of applications of MOEAs in aeronautical and aerospace engineering. A taxonomy of approaches together with a a short review of applications in each of the categories contained in it, have been presented.

The main conclusion from this review is that MOEAs are widely accepted as an alternative numerical optimization tool in this area, mainly because of their ease of use and their effectivity (several authors reported finding solutions that improved the reference design).

The main drawback of MOEAs is clearly the high computational cost associated to applications in which these algorithms must be coupled to complex physical simulations such as CFD and CSM. Although several authors report using surrogate models and parallelization techniques in such costly applications, new approaches are required, as indicated in the final part of this paper in which some possible alternatives to deal with this problem have also been provided. Finally, another issue that certainly deserves attention is the need for stronger theoretical foundations for MOEAs. Issues such as not being able to (mathematically) prove that the solution produced by some specific MOEA is optimal may be seen with skepticism by some researchers in this area. Although some important work has been done in this regard (see for example [141]), much more work is still needed.

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