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MULTIOBJECTIVE OPTIMIZATION IN STRUCTURAL DESIGN: THE MODEL CHOICE PROBLEM

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Existing and potential applications of multi-optimization techniques to structural design are reviewed.

Two approaches are available to formulate a multiobjective structural design problem. The first approach starts with a classical design, say minimize weight subject to cost, reliability, risk and other constraints; and then some of the quantities included in the constraints, in particular cost and reliability, are used to define additional objectives. Thus, if $\underline{X}$ denotes the design or decision variable vector, $W(\underline{X})$, $K(\underline{X})$ and $R(\underline{X})$ the weight, cost and reliability objective functions, respectively, and $G(\underline{X})$ is a set of non-negativity constraints, the multiobjective problem is written as

Problem P1:

$$\min_{\underline{X}} \underline{Z}(\underline{X}) = (W(\underline{X}), K(\underline{X}), 1-R(\underline{X})) \quad (1)$$

subject to

$$G(\underline{X}) \geq 0$$

Note that $G(\underline{X})$ usually includes constraints on the objectives themselves such as allowable maximum cost or minimum reliability.

The second approach consists in modeling the design problem directly in multiobjective form. This formulation may lead not only to problem P1, but also to the inclusion of qualitative objectives into the analysis, expressed by criteria such as aesthetics $A(\underline{X})$ and employment $M(\underline{X})$. Since qualitative (ordinal) objectives are usually defined on a discrete scale, it is convenient to consider a discrete set of alternatives as well. Accordingly, let $\underline{X} = \{\underline{X}(i): i=1,2,\ldots, J\}$ be a discrete set of alternative designs. Then the jth alternative design is evaluated by the criterion vector

$$\underline{C}(j) = (W(j), K(j), 1-R(j), A(j), M(j))$$

and the multiobjective problem now becomes:

Problem P2:

Find an alternative $\underline{X}(j)$ that constitutes a satisfactory trade off between the elements of criterion vector $\underline{C}(j)$.

For example, consider the standard steel-floor design (1) as described in (2), in which cost is to be minimized. The optimization technique used in that model is geometric programming (3), (4). Alternatives may be obtained by minimizing weight, or by probabilistic design (5), (6). Table 1 shows five alternatives obtained from the basic model of (2). Design I represents the original problem, Design IV corresponds to a minimum weight formulation, Designs II, III, V are probabilistic. These alternatives have been obtained by changing constraints into objectives, which means that Table 1 stems from Problem P1.

TABLE I Alternative Steel Floor Designs Versus Four Objectives

	DESIGN I	II	III	IV	V
Cost	3850	3085	2774	4780	4162
V-Ratio	1:1	1:2	1:2	1:1	1:2
Reliability	1.00	0.80	0.95	1.00	.95
Applied Weight	1000	720	570	1000	570

Design I Standard (Deterministic)
Design II Probabilistic, R = .28
Design III Probabilistic, R = .43
Design IV Minimum U_T
Design V Minimum U_T and Probabilistic R = .43

For both problems P1 and P2, a trade-off solution, also called "satisfactum", is to be sought among the set of non-dominated solutions or Pareto-optimum set. Alternative k dominates alternative j if $\underline{C}^*(k) \geq \underline{C}(j)$; the dominance is strict if at least one element of vector $\underline{C}(k)$ is greater than the corresponding element of $\underline{C}(j)$.

Once the multiobjective problem has been formulated as either problem P1 or P2, a solution technique which matches with the type of problem and desiratas of the decision-maker is to be chosen. This model choice problem is examined in a systematic manner and illustrated by setting up a problem with a choice between eleven multiobjective techniques, respectively:

1. Compromise Programming (7), (8)
2. Goal Programming (9), (10)
3. Cooperative Game Theory (11), (12)
4. Multiattribute Utility Theory (13), (14)
5. Surrogate Worth Trade-off (15)
6. ELECTRE (16), (17), (18)
7. Q-analysis (19), (20)
8. Dynamic Compromise Programming (21), (22), (23)
9. PROTRADE (24), (25)
10. STEP Method (26)
11. Local Multiattribute Utility Functions (27).

These techniques can be categorized by means of five binary classification criteria:

a. Marginal versus non-marginal difference between alternatives; are only marginal differences between alternatives being considered? If yes, formulation P1 is applicable; if not, that is, if major differences between alternatives are possible, say an arch versus a gravity dam, then formulation P2 may be preferable. A parallel classification criterion would be design versus maintenance problem.

b. Qualitative versus quantitative criteria: are there qualitative criteria which cannot or should not be quantified? If so, formulation P2 may be more appropriate than formulation P1.

c. Prior versus progressive articulation of preferences: at which point of the analysis is the decision-maker required to express his preference function, if at all?

d. Interactive versus non-interactive: has the technique been explicitely designed for an interactive mode of application?

e. Comparison of alternatives to a given solution point or to each other; in the former case, the solution point may be an aspiration level, corresponding to a feasible solution, or a goal point, corresponding to a non-feasible (often ideal) solution.

To these five classification criteria are added other criteria describing the characteristics of the problem (size, uncertainty, number of objectives...), the decision maker (level of understanding, time available for interaction) and the techniques themselves (robustness, partial versus complete ranking provided, ease of use...). This procedure leads to defining four categories of choice criteria (23):

1. mandatory binary criteria: for example, under formulation P1, a technique able to solve only discrete problems would be eliminated from further consideration

2. non-mandatory binary criteria: for example, comparison to an aspiration level versus comparison of a goal point

3. technique-dependent criteria: time required from decision-maker, robustness

4. application-dependent criteria: number of objectives, formulation P1 or P2.

To conclude, the advantages of a multiobjective formulation over a single objective one with a sensitivity analysis is that more alternatives can be explored and that explicit trade-offs between criteria can be made. Furthermore, given any problem involving trade-offs between quantitative or even qualitative criteria, an appropriate multiobjective technique can usually be found by following the proposed model choice procedure. The potential use of multiobjective techniques in structural design thus looks quite promising.

References

(1) Rozvany, G. I. N., Optimal Design of Flexural System, Pergamon Press, Oxford, New York, Toronto, Sydney, Paris, Frankfurt, 1976.

(2) Templeman, A. B., Structure Design for Minimum Cost Using the Method of Geometric Programming, Proc. of the Institute of Civil Engineering 46, pp. 459-472, August 1970.

(3) Duffin, R. J., E. L. Peterson and C. M. Zener, Geometric Programming, Wiley, New York, 1967.

(4) Beightler, C. S., D. T. Phillips and D. J. Widel, Foundation of Optimization, Prentice-Hall, Inc., New Jersey, 1979.

(5) Haugen, E., Probabilistic Approaches to Design, Wiley, London, New York, Sydney, 1968.

(6) Ghiocal, D. and D. Lungu, Wind, Snow and Temperature Effects on Structures Based on Probability, Abacus Press, Kent, England, 1975.

(7) Zeleny, M., Compromise Programming, in Multiple Criteria Decision Making, M. K. Starr and M. Zeleny, eds., University of South Carolina Press, Columbia, 1973.

(8) Zeleny, M., Multiple Criteria Decision Making, McGraw-Hill Book Company, New York, 1982.

(9) Lee, S., Goal Programming for Decision Analysis, Auerbach, Philadelphia, 1972.

(10) Ignizio, J., Goal Programming and Extensions, Heath, Lexington, Mass., 1976.

(11) Szidarovszky, F., I. Bogardi and L. Duckstein, Use of Cooperative Games in a Multiobjective Analysis of Mining and Environment, Proc. of the 2nd International Conference on Applied Numerical Modeling, Madrid, Sept. 11-15, 1978.

(12) Szidarovszky, F., M. Gershon and A. Bardossy, A Goal Programming Approach for Dynamic Multiobjective Decision Making, Presented at the CORS-TIMS-ORSA Joint National Meeting, Toronto, Canada, May 3-6, 1981.

(13) Keeney, R. and H. Raiffa, Decisions with Multiple Objectives: Preferences and Value Tradeoffs, Wiley, New York, 1976.

(14) Krzysztofowicz, R. and L. Duckstein, Preference Criterion for Flood Control Under Uncertainty, Water Resources Research, Vol. 15, No. 3, pp. 513-520, June 1979.

(15) Haimes, Y. Y., W. Hall and H. Freedman, Multiobjective Optimization in Water Resources Systems: The Surrogate Worth Tradeoff Method, Elsevier, Amsterdam, 1975.

(16) Benayoun, R., B. Roy and B. Sussman, ELECTRE: Une Methode pour Guider le Choix en Presence de Points de Vue Multiple, Direction Scientifique, Note de travail No. 49, SEMA, Paris, 1966.

(17) Roy, B., Problems and Methods with Multiple Objective Functions, Mathematical Programming, Vol. 1, No. 2, pp. 239-268.

(18) Gershon, M., L. Duckstein and R. McAniff, Multiobjective River Basin Planning with Qualitative Criteria, to appear, Water Resources Research, 1981.

(19) Duckstein, L. and J. Kempf, Multicriteria Q-analysis for Plan Evaluation, Preprint, presented at the 9th Meeting of the EURO Working Group on MCDM, Amsterdam, April 1979.

(20) Pfaff, R. and L. Duckstein, Ranking Alternative Plans that Manage the Santa Cruz River Basin by Using Q-analysis as a Multicriteria Decision Making Aid, Proceedings of the Joint AZ Sect., AWRA, and Hydrology Sect., AZ-Nevada Acad. of Science, May 1-2, 1981, Tucson.

(21) Opricovic, S., An Extension of Compromise Programming to the Solution of Dynamic Multicriteria Problems, 9th IFIP Conference on Optimization Techniques, Warsaw, Poland, September, 1979.

(22) Szidarovszky, F., Some Notes on Multiobjective Dynamic Programming, Working paper No. 79-1, Dept. of Systems & Industrial Engineering, University of Arizona, Tucson, 1979.

(23) Gershon, M., Model Choice in Multiobjective Decision Making in Natural Resource Systems, Unpublished Ph.D Dissertation, Dept. of Systems & Industrial Engineering, University of Arizona, Tucson, 1981.

(24) Goicoechea, A., L. Duckstein and M. Fogel, Multiobjective Programming in Watershed Management: A Study of the Charleston Watershed, Water Resources Research, 12(6), pp. 1085-1092, 1976.

(25) Goicoechea, A., D. Hansen and L. Duckstein, Introduction to Multiobjective Analysis with Engineering and Business Applications, to appear, Wiley, New York, 1981.

(26) Benayoun, R., J. de Montgolfier, J. Tergny and O. Larichev, Linear Programming with Multiple Objective Functions: STEP Method (STEM), Mathematical Programming 1(3), pp. 366-375, 1971.

(27) Oppenheimer, K. R., A Proxy Approach to Multiattribute Decision Making, Management Science 24(6), pp. 675-689, 1978.

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