August, 2010

Multiperiod Remanufacturing Planning With Uncertain Quality of Inputs

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December 5, 2007

Abstract

In this paper we consider production planning of remanufactured products when inputs have different and uncertain quality levels, and there are capacity constraints. This situation is typical of most remanufacturing environments, where inputs are product returns (also called cores). Production (remanufacturing) cost increases as the quality level decreases, and any unused cores may be salvaged at a value that increases with their quality level. Decision variables include, for each period and under a certain probabilistic scenario, the amount of cores to grade, the amount to remanufacture for each quality level and the amount of inventory to carry over for future periods for un-graded cores, graded cores, and finished remanufactured products. Our model is grounded with data collected at a major OEM that also remanufactures. We formulate the problem as a stochastic program, and illustrate how the deterministic version of the problem yields solutions that cannot be implemented in practice. The stochastic program, although a large linear program, can be solved easily using Cplex. We provide a numeric study to generate insights into the nature of the solution.

Keywords: Remanufacturing, Production Planning, Stochastic Programming

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1 Introduction

Remanufacturing is the process of restoring used products to a "like-new' condition. According to Hauser and Lund (2003), the size of the remanufacturing sector in the U.S. is \$53 billion, with over 70,000 firms and 480,000 employees. In addition, remanufacturing contributes to energy and material conservation, and provides employment solutions to low skill workers. Thus, remanufacturing has both economic, environmental, and social benefits. Similar to the planning of production of new products, developing a production plan to remanufacture returned products (cores) typically involves determining the quantity of units to process each period under time varying demand and a capacity constraint on the number of units that can be processed each period. Despite these benefits and similarities, developing a remanufacturing operation often poses significant challenges to firms who are more accustomed to planning production of new products in the forward chain.

The most common method used to develop these plans (for both new and remanufactured products) is to treat the demand as deterministic and solve for the plan using a linear programming formulation. Uncertainty in the demand stream is typically accounted for by solving the production plan on a rolling horizon, where the demand forecasts are frequently updated, the linear program is resolved, and each solution only impacts the current period's production quantity as a new plan will be generated with updated information before the quantities in the future periods are produced. While this method has worked reasonably well for many years and for many firms who only produce new products, there are particular characteristics of the remanufacturing industry that create problems when the traditional production planning methods are used to plan a remanufacturing process. Two of the main characteristics that differentiate remanufacturing from new product production are uncertainty in the number of returned products and uncertainty in the *quality* of the returns.

When producing new products, as long as proper planning has been done to procure the components that go into the new products, any quantity (up to the capacity of the facility) can be produced in any period. In addition, there is typically little variation in the amount of capacity required to produce one new unit from another of the same make and model. Now consider a remanufacturing operation. The number of units remanufactured each period can not exceed the number of returned cores available, even if there is capacity at the facility to produce more. In addition, a returned shipment of used cores will typically vary in the quality of the cores, with some cores requiring significantly more capacity to bring the unit up to a required quality standard than others. As an example, IBM's remanufacturing facility in Raleigh, N.C. may receive a shipment of end-of-lease laptops from one of their customers. Each laptop must be remanufactured to a pre-determined configuration and quality level before being offered for sale on IBM's website. A random draw of two laptops from the incoming shipment may result in one of the laptops simply requiring testing, cleaning, formatting of the harddrive, and the loading of the standard software configuration; jobs requiring approximately 15 minutes of production capacity at this facility. The second laptop, however, may have a broken latch, a worn keyboard, and a non-standard memory configuration. This laptop will require 45 minutes of production capacity before it matches the quality and configuration standards, a difference of 300% in the capacity required of the first laptop. There are very few production environments for new products that have this much variation in the processing times of individual units of the same product.

Based on our conversations with the managers of remanufacturing facilities, the uncertainty in the quality of the returns presents a much larger problem than the uncertainty in the quantity of returns. Many firms who are actively involved in remanufacturing recover their used cores by either leasing their new products (ensuring the used units will be returned at the end of the lease) or by offering trade-in credits on the return of an old unit when a new product is purchased. Product leasing results in the most predictable return stream and is a popular option for firms that also remanufacture. In fact, product leasing corresponded to 28%, or $199 billion, of all capital expenditures by American businesses in the U.S. in 2004 (ELA 2005). An example of a firm that remanufactures products off-lease is Pitney-Bowes (Ferguson et al. 2007). Firms that choose to recover their cores through a trade-in program are also able to accurately forecast their return stream, as the quantity of returned cores is highly correlated with the sales forecast for the firm's new products. The main issue that both sets of firms (firms who lease and firms who offer trade-in credits) still struggle with is the variation in the quality of the cores once they arrive. As the preceding example demonstrates, this issue makes the production planning process much more complex compared to production planning for new products.

To address this problem, we formulate a stochastic program for a multi-period setting where the quality levels of a lot of returned cores may take on any configuration of bad to good quality levels based on pre-established probabilities. Stochastic programming (SP) is an attractive methodology for this problem because (unlike a regular linear programming formulation) it guarantees a feasible solution under all possible quality level realizations.

SP also allows for a capacity constraint and simply requires the solution of a (large) linear program. We demonstrate how SP can be applied to the remanufacturing planning problem, provide an example of how the solution evolves over multiple periods, and test the method in an extensive numerical analysis. By analyzing the results of our numerical analysis, we perform a sensitivity analysis to determine the parameter values that have the largest impact on the solution. The results of this sensitivity analysis provides guidance for firms seeking to make investments to improve their remanufacturing operations. The SP formulation has some history in the new product production planning problem but is new to the remanufactured product planning problem, as discussed in the next section.

2 Literature Review

Our paper is positioned among two main streams of literature. The first concerns the consideration of uncertainty in planning production for "regular" (or forward) manufacturing, and the second is related to planning production in a remanufacturing environment. Regarding the first stream, the literature recognizes the pitfalls of not considering uncertainty when planning production; the main sources of uncertainty considered are demand and environmental (which may impact multiple parameters). For a comprehensive review, see Mula et al. (2006); we focus here on work most relevant to this paper, namely, research that uses SP in multi-period settings. SP (see Birge 1997 for a review) is an attractive methodology in production planning problems under uncertainty because of its ability to incorporate multiple constraints (e.g., capacity) that are common in most production environments. Production planning problems are typically formulated as two-stage SP problems where a random outcome (e.g., random demand, state of the economy, etc.) occurs only once, at the beginning of second stage. Examples include Gupta and Maranas (2003) who model demand uncertainty; Eppen et al. (1989), and Leung et al. (2006), who model environmental uncertainty (which impacts demand and cost parameters). In all three examples, decisions are made in the first stage—production decisions in the first, plant selection in the latter two—before the random outcome is realized. A multi-stage SP approach has been proposed for the more strategic problem of capacity planning for semiconductor manufacturing over a several-year planning horizon (Christie and Wu 2002) under different demand and technology scenarios. Multi-stage SP approaches to (tactical) production planning problems are not commonly found in the literature, perhaps due to their size and computational complexity. They are sometimes used, however, to provide a benchmark of optimality to heuristic approaches, such as hierarchical production planning (Dempster et al. 1981). We propose a SP model to solve for a multi-period production plan in a remanufacturing environment, which has the unique (to remanufacturing) characteristic of non-uniformity of inputs (cores). The model is intuitive, allows for uncertainty in the quality levels of the cores, and is computationally manageable (we solved realistic scenarios in an average of 79 seconds on a shared Unix server). We next address the second stream of reseach on production planning for remanufacturing.

There is a growing body of literature addressing production planning for remanufacturing. For comprehensive overviews, see Inderfurth et al. (2004), Inderfurth and Teunter (2003), Guide (2000), and Fleischmann et al. (1997). Many papers consider a single-period problem where there are only two categories of cores, good and bad; good units are all remanufactured *at the same cost* whereas bad units are salvaged (e.g., Zikopoulos and Tagaras 2007, Bakal and Akcali 2006, Ferrer 2003). Other papers acknowledge different remanufacturing costs for cores of different quality, but still under a single-period setting (e.g., Galbreth and Blackburn 2006, Aras et al. 2004). In contrast, we consider a multi-period problem where cores have different qualities, and there are disposal and inventory carrying options across periods. Multi-period production planning problems have been addressed by Ferguson et al. (2007), Guide et al. (2001), and Golany et al. (2001). All of the papers mentioned above, however, assume a deterministic quality of cores, or rather, consider the *expected values* of cores of a given quality level at each period to formulate the optimization problem.

To our knowledge, ours is the first paper to consider stochastic quality of cores, with its associated capacity and cost implications, for production planning in a multi-period setting. We do this by using a stochastic programming formulation. Although there is a significant stream of research in stochastic inventory models that include remanufacturing options (e.g., DeCroix 2006, Ferrer and Ketzenberg 2004, Inderfurth et al. 2001, Toktay et al. 2000, van der Laan et al. 1999), the Markov decision process approach used in these papers prevent modeling complexities such as capacity constraints and multiple quality grades. Although our approach does not yield closed-form solutions, we illustrate through an example in §3 how it always results in *implementable* solutions, as the model formulation guarantees feasibility for each random outcome in the planning horizon. This desirable property does not hold when either (i) expected values are used in other mathematical programming formulations, or (ii) capacity constraints and/or the existence of multiple quality grades are ignored. Regarding (ii), consider the fact that cores of worse quality require more production resources than those of better quality; given that the mix changes in each period (depending on the outcome of the grading process), then capacity is also impacted.

3 Model

Consider a remanufacturing environment where end-of-lease returned products (cores) are graded and grouped into I different quality levels. We consider a single product type, so our problem is an aggregate production planning problem; the model can be easily extended to multiple product types (although computational complexity would increase significantly). Graded cores are then remanufactured to meet demand for remanufactured products. The total amount of cores is known in advance, however, their quality levels are uncertain. The demand for remanufactured products is forecasted over a planning horizon of T periods. Remanufactured products are identical and independent of the quality of the original cores used, however, remanufacturing costs differ across quality grades. Cores can be kept in stock to be graded in the future and graded cores can be kept in stock to be remanufactured in the future. The holding cost for a remanufactured product is higher than for a core. Graded cores can also be salvaged at any time; higher quality cores have higher salvage values corresponding to the opportunity to harvest higher quality cores for spare parts. The problem is to determine how many of the available cores to grade, how many of the graded cores to be remanufactured, and how many of them to be salvaged each period. Due to capacity constraints, the firm may not be able to meet the non-stationary demand, as lower quality cores consume more capacity than higher quality cores; backlogs are allowed. The objective is to find a plan that maximizes total expected profit.

We formulate our problem as a stochastic program as follows. The random variable in each period is the outcome (r_{it}^j, as defined below) of the grading process. For example, suppose there are two quality grades (good and bad), and two possible grading outcomes. In outcome 1, which happens with probability p_1, the inspection process yields 70% of good quality cores, whereas in outcome 2, the inspection process yields 40% of good quality cores. We define scenarios for each time period t as follows. Each time period t has $J(t)$ scenarios, each corresponding to a realization of all grading outcomes in periods $1, \ldots, t$. So, for example, period 2 has four possible scenarios ($J(2) = 4$) pertaining to the percentage of good quality cores each period ((0.7,0.7), (0.7,0.4), (0.4,0.7), (0.4,0.4)), whereas period 4 has 16 scenarios ((0.7,0.7,0.7,0.7),. . . .(0.4,0.4,0.4,0.4)). Period 0 (the period prior to the start of the planning horizon) has only one scenario, since all random variables have not been observed prior to the planning horizon. This is shown in Figure 1. Note the "nested" structure of the scenarios: scenario 1 of period 1 is nested in scenarios 1 and 2 of period 2, which are nested in scenarios 1, 2, 3 and 4 of period 3. When necessary and to avoid confusion, we name scenario j in period t as scenario (j, t). For a given time period t, define $\text{pred}(j, t)$ as the scenario, in period $t - 1$ that is the predecessor of the scenario in period t. In the example above, for period 2, pred(1,2) = pred(2,2) = (1,1), pred (3,2) = pred(4,2) = (2,1), and so forth. Our notation is defined in Table 1.

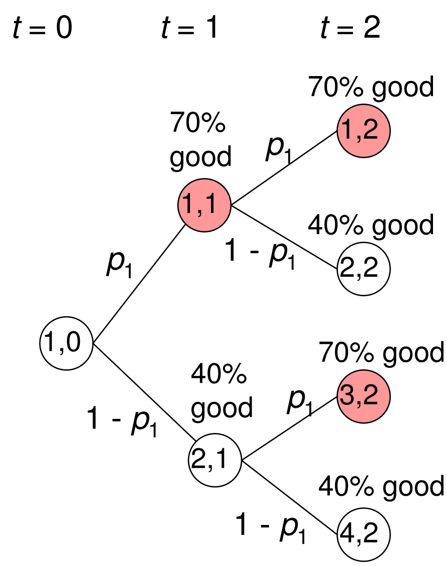

Figure 1: Construction of Scenarios

In the description of our model below, we assume demand to be deterministic. Stochastic demand can be easily incorporated into the model, at least from a theoretical standpoint, by re-defining a random outcome in a period as a combination of the random realization of the grading process *and* the random demand realization; we expand upon this point later. From a model formulation perspective, this is an easy model extension. From a practical implementation standpoint, however, it presents significant challenges because the number of variables increases exponentially with the number of random outcomes in a period. A model with both random quality of cores and random demand can feasibly be solved only for a limited number of scenarios and time periods. Stochastic demand has been explored in previous production planning literature (see §2), and since the novelty of our paper resides in the peculiarity of the remanufacturing environment with its non-uniform input, we maintain in our numerical analysis the assumption of deterministic demand (i.e. no forecast error). Further, the impact of the deterministic demand assumption can always be mitigated by solving the model using a rolling horizon.

The sequence of events in a period is as follows:

1. Cores B_t arrive. The firm decides upon the amount of cores to grade $x_t^{\text{pred}(j)}$.
2. The outcome of the grading process r_{it}^j is observed
3. Based on demand D_t, and the outcome of grading process, the firm decides upon the amount of cores to refurbish z_{it}^j and the amount to salvage v_{it}^j for each quality grade i.
4. Demand is filled, inventories are updated, holding costs are incurred.

Our problem (assuming zero initial inventories) can thus be formulated as:

$$\begin{aligned}\max_{x_t^j, z_{it}^j, v_{it}^j} \Pi \quad &= \quad \sum_{t=1}^{T}\sum_{j=1}^{J(t)} p_t^j \left\{ \sum_{i=1}^{I} ((p_r - c_{it}) z_{it}^j + s_{it} v_{it}^j - h_{it} u_{it}^j) - h_r y_t^{j+} - \pi y_t^{j-} \right\} \qquad (1)\\ &\quad - \sum_{t=1}^{T}\sum_{j=1}^{J(t-1)} p_t^j \left(g x_t^j - h b_t^j \right)\end{aligned}$$

subject to

$$\begin{aligned}(y_{t-1}^{\text{pred}(j,t)+} - y_{t-1}^{\text{pred}(j,t)-}) - (y_t^{j+} - y_t^{j-}) + \sum_{i=1}^{I} z_{it}^j &= D_t, \quad \forall t; j = 1, \ldots, J(t) &(2)\\ b_t^j + x_t^j - b_{t-1}^{\text{pred}(j,t)} &= B_t, \quad \forall t; j = 1, \ldots, J(t-1) &(3)\\ z_{it}^j + u_{it}^j - u_{i,t-1}^{\text{pred}(j,t)} + v_{it}^j &= r_{it}^j x_t^{\text{pred}(j,t)}, \quad \forall i; \forall t; j = 1, \ldots, J(t) &(4)\\ \sum_{i=1}^{I} a_i z_{it}^j &\leq C_t, \quad \forall t; j = 1, \ldots, J(t) &(5)\\ y_T^j &= 0, \quad j = 1, \ldots, J(T) &(6)\\ y_t^{j+}, y_t^{j-}, z_{it}^j, v_{it}^j, u_{it}^j, x_t^j &\geq 0, \quad \forall i, t, j. &(7)\end{aligned}$$

The objective function (2) maximizes total profit: revenue from remanufactured products plus salvage revenue minus costs—holding cost for graded cores, holding cost for remanufactured products, backlogging cost, grading cost, and holding cost for ungraded cores. Constraint (2) is the inventory balancing constraint for remanufactured products under each

Table 1: Notation

Indexes	
i	Quality category, $i = 1$ (best),..., I (worst)
t	Time period, $t = 1, \ldots, T$
j	Scenario j of time period t, $j = 1, ..., J(t)$; $J(0) = 1$
Parameters	
D_t	Demand for remanufactured products at period t
B_t	Quantity of cores arriving at beginning of period t
C_t	Available remanufacturing capacity in period t
a_i	Unit remanufacturing resource usage for quality i cores
p_r	Remanufactured product price
c_{it}	Unit remanufacturing cost for quality-i cores at time t
s_{it}	Unit salvage value for quality-i cores at time t
h_{it}	Unit holding cost for quality-i cores at time t
h_r	Unit holding cost for a remanufactured product
h	Unit inventory holding cost for un-graded cores
π	Unit backlogging cost per period
g	Unit grading cost
p_t^j	Probability of scenario j of time period t
r_{it}^j	Fraction of items of quality i under scenario j of period t
Decision Variables	
x_t^j	Amount of cores graded under scenario j of period t
z_{it}^j	Quantity of quality-i cores remanufactured at period t under scenario j
v_{it}^j	Quantity of quality-i cores salvaged at end of period t under scenario j
Auxiliary Variables	
u_{it}^j	Inventory of quality-i cores at end of period t under scenario j
y_t^{j+}	Inventory of remanufactured products at end of period t under scenario j
y_t^{j+}	Backlog of remanufactured products at end of period t under scenario j
b_t^j	Amount of un-graded cores at the end of period t under scenario j of period t.

scenario j of time period t: for a given scenario j, one can meet demand from the current period's production and starting inventory of remanufactured products (all scenarios j with a common predecessor share the same starting inventory $y_t^{\text{pred}(j)}$), or demand can be backlogged. Constraints (3) and (4) are the inventory balance constraints for un-graded and graded cores, respectively; note that in (4), all scenarios j of period t that share the same predecessor have the same amount of graded cores $x_t^{\text{pred}(j)}$. Constraint (5) regards the capacity constraint for period t, which should be met for any scenario. Finally, constraint (6) ensures that the firm does not produce in excess of demand over the planning horizon.

If demand is stochastic, we simply redefine an outcome in any period to be the joint realization of the random grading process and random demand in that period. For example, if there are 3 possible grading realizations (good, medium, bad) and two possible demand realizations (high, low), then each period has $3 \times 2 = 6$ outcomes. The only change in the above formulation is that D_t is replaced with D_t^j in (2).

Thus, the stochastic programming approach generates a solution for each scenario in the grading process, and guarantees its feasibility (if there is enough capacity such that the problem is feasible). We illustrate some important properties of this problem in Example 1 below.

Example 1: The planning horizon has three periods, so $T = 3$. There are two quality grades, good ($i = 1$) and bad ($i = 2$). A good unit consumes one unit of capacity whereas a bad unit consumes 30% more, and thus $a_1 = 1$, $a_2 = 1.3$. There are two possible outcomes for the grading process: in outcome A, which happens with 35% probability, there are 10% good units; in outcome B, which happens with 65% probability, there are 90% good units. Demands are 200, 280 and 220 for periods 1 through 3; cores are 250, 330, and 270 for periods 1 through 3; capacity is 320 units per period. All cost and revenue parameters are time invariant, and are as follows: $p_r = 100$, $g = 1$, $h = 0.5$, $h_1 = h_2 = 1$, $h_r = 1.5$, $\pi = 50$ (in essence, backlogs are to be avoided if possible), $c_1 = 30$, $c_2 = 50$, $s_1 = 30$, $s_2 = 20$, where the time subscripts have been removed. Period 1 has two scenarios, corresponding to the two possible grading outcomes; period 2 has four scenarios, corresponding to two outcomes for each of the two scenarios in period 1; period 3 has eight scenarios. The scenarios, respective probabilities, and the optimal solution to the problem are listed in Table 2 below; the optimal solution also calls for grading all returned products in each period, and thus no un-graded inventory is carried from period to period. The total expected profit (objective function) is \$47,290.

Table 2: Solution for Example 1

Scenarios and Probabilities						*Optimal Solution*			
t	Scen. j	pred(j)	p_t^j	r_{1t}^j	r_{2t}^j	z_{1t}^j	z_{2t}^j	v_{1t}^j	v_{2t}^j
1	(1,1)	(1,0)	0.3500	0.1	0.9	25.0	201.2	0.0	23.8
1	(2,1)	(1,0)	0.6500	0.9	0.1	225.0	1.2	0.0	23.8
2	(1,2)	(1,1)	0.1225	0.1	0.9	33.0	220.8	0.0	76.2
2	(2,2)	(1,1)	0.2275	0.9	0.1	253.8	0.0	0.0	33.0
2	(3,2)	(2,1)	0.2275	0.1	0.9	33.0	220.8	0.0	76.2
2	(4,2)	(2,1)	0.4225	0.9	0.1	253.8	0.0	0.0	33.0
3	(1,3)	(1,2)	0.0429	0.1	0.9	27.0	193.0	0.0	50.0
3	(2,3)	(1,2)	0.0796	0.9	0.1	220.0	0.0	23.0	27.0
3	(3,3)	(2,2)	0.0796	0.1	0.9	70.2	149.8	0.0	93.2
3	(4,3)	(2,2)	0.1479	0.9	0.1	220.0	0.0	66.2	27.0
3	(5,3)	(3,2)	0.0796	0.1	0.9	27.0	193.0	0.0	50.0
3	(6,3)	(3,2)	0.1479	0.9	0.1	220.0	0.0	23.0	27.0
3	(7,3)	(4,2)	0.1479	0.1	0.9	70.2	149.8	0.0	93.2
3	(8,3)	(4,2)	0.2746	0.9	0.1	220.0	0.0	66.2	27.0

Notice that the optimal solution for period 2 only depends on the grading outcome in that period. Specifically, the firm remanufactures 33 and 220.8 units of good and bad cores respectively under outcome A, and 253.8 and 0 respectively under outcome B. In period 3, however, the optimal solution not only depends on the outcome of the grading process in that period, but also on the history. For example, under outcome A in period 3, the firm's optimal solution depends on previous outcomes: under scenarios (1,3), and (5,3), the firm remanufactures 27 and 193 units of good and bad cores respectively; these are the scenarios where the firm also had the A outcome in period 2. In scenarios (3,3) and (7,3), however, the optimal solution is to remanufacture 70.2 and 149.8 of good and bad cores respectively; these are the scenarios where the firm had a B outcome in period 2.

Now, consider the deterministic version of this problem, which is the approach taken by most multi-period planning models in the literature. The problem is formulated as in (2)-(7), except that there are no scenarios. Consequently, all variables do not have a superscript j, but rather use expected values: $r_{1t} = 0.65(0.1) + 0.35(0.9) = 0.62$, and $r_{2t} = 0.38$ for all

t. Solving this deterministic LP results in a solution where the firm grades all cores in all periods, as in the stochastic formulation. The amount of cores suggested to remanufacture, however, are $z_{11} = 155$, $z_{12} = 204.6$, and $z_{13} = 167.4$ for quality grade 1; $z_{21} = 45$, $z_{22} = 75.4$, and $z_{23} = 52.6$ for quality grade 2; and to salvage 50 units of graded bad quality cores in all periods. The total expected profit is $47,690. Notice, however, that this solution is not implementable. Consider what happens in period 1, for example, if outcome A happens. The firm can only remanufacture 25 good units (as opposed to the deterministic problem's solution of $z_{11} = 155$); if outcome B happens, the firm can only remanufacture 25 bad units. Thus, the solution is not implementable under all quality realizations. The probability of a non-implementable solution only increases in future periods, as successive random outcomes in previous periods alter the firm's inventory position.

Now, suppose that, the production capacity is instead $C_t = 300$ for all t, and backlogs *are not* allowed. The deterministic version of the problem appears to be feasible and yields a similar solution to the case where $C_t = 320$ except that $z_{21} = 47.0$, $z_{22} = 73.4$, and the firm salvages two less bad-quality units in the first period but two more in the second period due to the tighter capacity constraint (as in the previous example, this solution is not implementable under certain quality realizations). The stochastic version of the problem, however, accurately shows the problem is infeasible: if the outcome of the grading process is "B" in both periods 1 and 2, for example, there is no feasible production plan that is able to meet demand in period 2, since the bad quality units consume 30% more capacity than the good-quality units. In summary, this simple example illustrates the issues associated with traditional production planning problems in remanufacturing operations: it yields non-implementable solutions and it may suggest a plan that appears to be feasible even though no feasible plan exist for all possible random variable realizations.

3.1 Computational Considerations

The formulation we present for grading and remanufacturing planning is a multistage stochastic linear program (MS-SLP) which may become difficult to solve for the size of problems found in practice. The size of the problem grows exponentially with respect to the number of quality grades and number of periods. The structure of the problem, however, can be exploited to develop algorithms in case a general-purpose linear programming code fails to provide solutions in a reasonable computational time. One such approach can be the use of decomposition methods which use the block-angular structure of the MS-SLP formulations. The L-shaped Method, first suggested by Van Slyke and Wett (1969) for two stage problem and then generalized to multiple stages as a nested decomposition method (Birge 1985), should provide better computational results.

In this paper, however, we were able to solve problem of a realistic size in very reasonable computation times based on data grounded in a real remanufacturing environment, without the need to resort to such a special algorithm. We consider five outcomes for the quality grades and six planning periods (next section). This choice of the parameter values also leads to an easier interpretation of the results. The solution provides a course of action for each outcome in each period. When the number of possible outcomes grows very large, tracing the solution paths, which differ from each other with very small probabilities, for implementation may be impractical. The same applies to the number of periods in the planning horizon. Furthermore, we recommend that the problem is solved on a rolling horizon basis and a six-period planning horizon serves the purpose of preventing myopic decisions. In our computational experiments, the solution times we observed (about 79 seconds on average) with a general-purpose linear programming software were very reasonable and should enable the problem to be solved on a rolling horizon basis. It seems that the sparse factorization schemes embedded in most general purpose software take into account the structure of the problem to a certain extent. In the next section, we provide details about our numerical study, and interpret the results.

4 Numerical Analysis

The objective of this section is to investigate how the various problem parameters (costs, demand pattern, capacity) impact the firm's optimal solution (total cost, amount of cores graded, remanufactured, inventoried, and salvaged). This is done through a large-scale numerical study.

4.1 Experimental Design

The choice of parameter values for our numerical study is largely taken from existing research (see Ferguson et al. 2007), which is based on actual data provided by Pitney-Bowes and other remanufacturers; we provide justification for additional parameters when necessary. Costs are normalized by choosing the remanufactured product's selling price to be $p_r = 100$. We choose a common production planning horizon of six months, so $T = 6$. We consider three demand patterns, as shown in Table 3; these demand patterns are slightly modified from the 12-month data of Ferguson et al. (2007) to fit a six-period planning horizon. The variability of demand type 1 is evenly distributed throughout the planning horizon, while demand type 2 has two months with significantly higher than average demand. Finally, demand type 3 is constant during the first three months, but ramps up to about double this level in the last two months of the planning horizon.

Table 3: Demand Data for Numerical Study

Period	Type 1	Type 2	Type 3
1	395	240	245
2	385	280	245
3	495	565	245
4	360	610	335
5	215	235	545
6	310	230	545
Total	2160	2160	2160
Mean ($\bar{D}$)	360	360	360
Std. Dev.	93	178	148

Returns typically exceed demand for remanufactured products (Ferguson et al. 2007, Galbreth and Blackburn 2006). Thus, we set the number of incoming cores, across all quality grades, at 150% of the average demand per period; this corresponds to 540 incoming cores per period. The quality of an incoming core q is the realization of a random variable Q between 0 to 1, where $q = 0$ represents the worst possible core and $q = 1$ represents the best possible core . For simplicity, we assume that $Q \sim U[0, 1]$. The cost to remanufacture a core of quality q is given by $c(q) = \alpha_0 + (\alpha_1 - \alpha_0)q^\beta$, where α_0, α_1 and β are parameters; this functional form represents considerable diversity of curve shapes depending on the chosen parameter values. The parameter α_0 is the remanufacturing cost for the worst possible quality core; likewise, α_1 is the remanufacturing cost for the best possible quality core. Similarly to Ferguson et al., we use $\alpha_0 = 60$, $\alpha_1 = 25$ (recall that $p_r = 100$), and consider $\beta \in \{0.3, 1.0, 3.0\}$; this results in the curves shown in Figure 2. Under this setup, if there are I quality grades distributed uniformly over [0,1], then $c_i = I\int_{1-\frac{i}{I}}^{1-\frac{i-1}{I}} c(q)dq$, which results in $c_i = \alpha_0 + \frac{\alpha_1-\alpha_0}{(1+\beta)I^\beta}\left((I-i+1)^{1+\beta} - (I-i)^{1+\beta}\right)$.

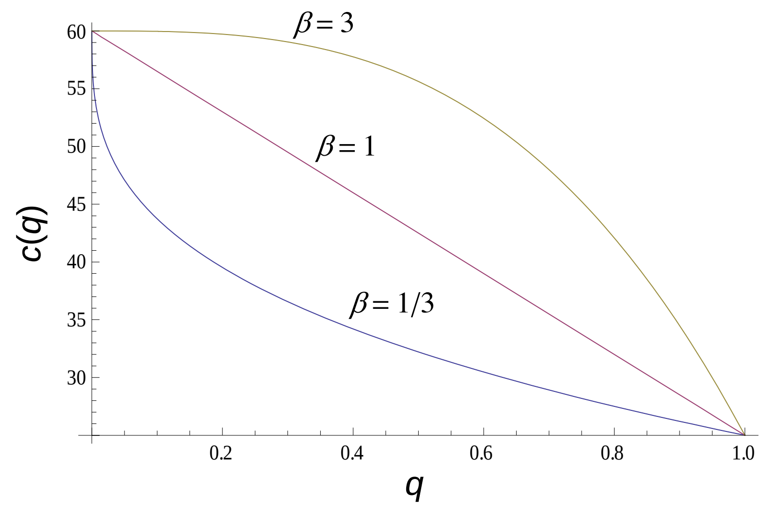

Figure 2: Curves for $c(q)$ with $\alpha_0 = 60$, $\alpha_1 = 25$

The salvage value for a core comprises its value in re-usable parts and material; the higher the quality of a core the higher its salvage value. Because remanufacturing is a value-added operation, the salvage value can be viewed as a fraction $0 \le \theta < 1$ of the net remanufacturing profit margin, that is, $s_i = \theta(p_r - c_i)$, where in our design we allow $\theta \in \{0.10, 0.40, 0.70\}$. (If $\theta > 1$, then remanufacturing is not profitable, and the optimal solution is trivial: salvage all units.) The value $\theta = 0.10$ represents the case where cores are primarily salvaged for materials recovery, whereas the value $\theta = 0.70$ represent a case where a significant value can be recovered from the harvesting of spare parts. The three levels for unit holding cost for cores, which is the same for all quality categories ($h_i = h_j, \forall i, j$), are $h_i \in \{1, 2, 3\}$. These values correspond to 12%, 24%, and 36% of the remanufactured product's price per year; they represent typical holding cost values, from low time sensitivity to high time sensitivity products (we account for market price depreciation in the product's holding cost value). Further, we set $h = 0.5h_i$ and $h_r = 1.5h_i$, so the holding cost for an ungraded core and for a remanufactured product are 50% less and more, respectively, than the holding cost for a graded return. We set unit grading cost to be 10%, 40% or 70% of the remanufacturing cost for the best possible quality core, that is, $g \in \{0.1\alpha_1, 0.4\alpha_1, 0.7\alpha_1\}$.

We consider unit backlogging cost of $\pi \in \{10, 20, 40\}$. Given that h_r varies between 1.5 and 4.5, the ratio $\frac{\pi}{h_r}$ varies between 2.2 and 26.7; numbers of this magnitude have been used in previous research (e.g., Cachon 1999). Further, π can be interpreted as a percent discount off the remanufactured product's price if the firm postpones the due date for meeting demand by one month. For example, $\pi = 10$ means that the firm gives the customer a 10% discount off the list price if delivery is postponed by one month. We normalize resource usage for the best quality grade to $a_1 = 1$. Resource usage for the worst quality grade core is $a_I = 1 + A$, where $A \in \{0.25, 0.50, 0.75\}$; resource usage for intermediate quality grades are a linear interpolation of these two points such that $a_i = 1 + A\frac{i-1}{I-1}$. Capacity availability is set at 20%, 60% and 100% of excess capacity over average demand if the firm only remanufactures high quality cores, thus we consider $\frac{C_t}{D} \in \{1.2, 1.6, 2.0\}$.

We complete our experimental design by describing the grading outcome. We assume three quality grades (good, medium, bad), and five outcomes for the grading process (worst, worse, average, better, best). The probabilities are described in Table 4. As an example, the "best" case scenario resulting from the grading process would result in 66.7% good quality cores, 33.3% medium quality cores, and 0% bad quality cores.

Table 4: Grading Outcome and Associated Probabilities

Outcome	Prob.	% good	% medium	% bad
worst	0.1	0.000	0.333	0.667
worse	0.2	0.167	0.333	0.500
average	0.4	0.333	0.333	0.333
better	0.2	0.500	0.333	0.167
best	0.1	0.667	0.333	0.000

Our experimental design is summarized in Table 5 below. In total, there are 3^8 = 6,561 experiments. For each cell, we solve the resulting linear program in Cplex 9.0, using the Cplex callable library. Each cell results in a linear program with 222,647 variables and 101,556 constraints. Run time for one cell ranges between 31 and 488 seconds, averaging 79 seconds on a shared Sun V480 computing environment. For each optimal solution, we also compute the associated values of the decision variables. Thus, our *metrics* include the optimal objective function value (Π), and the *expected values* per period for each of the problem's decision variables (optimal solution values weighted by the respective probabilities in each scenario). That is, we compute the expected number of graded cores (denoted in short by $\frac{\bar{x}_t}{T}$), expected number of remanufactured $\frac{\bar{z}_{it}}{T}$ and salvaged units $\frac{\bar{v}_{it}}{T}$ for each quality category, expected inventory for ungraded cores $\frac{\bar{b}_t}{T}$, graded cores $\frac{\bar{u}_{it}}{T}$, remanufactured units $\frac{\bar{y}_t^+}{T}$, and expected backlogs $\frac{\bar{y}_t^-}{T}$.

Table 5: Experimental Design for Numerical Study ($p_r = 100$,$\alpha_0 = 60$,$\alpha_1 = 25$)

Factor description	Symbol	Factor Levels
Demand pattern types	D_t	1, 2, 3 (from Table 3)
Unit backlogging cost per month	π	10, 20, 40
Shape parameter for cost curve	β	$\frac{1}{3}$, 1, 3
Salvage value as a fraction of margin	$\theta = \frac{s_i}{p_r - c_i}$	0.10, 0.40, 0.70
Holding cost for cores per month	h_i	1, 2, 3
Normalized unit grading cost	$\frac{g}{\alpha_1}$	0.1, 0.4, 0.7
Extra capacity usage for worst quality	A	25%, 50%, 75%
Capacity relative to avg. demand	$\frac{C_t}{\bar{D}}$	1.2, 1.6, 2.0

4.2 Numerical Results

In Table 6 we report the descriptive statistics for each of the 14 metrics of interest. To better understand the numbers, recall that average demand per period is 360 units; average arriving cores are 540 units per period or 180 of each of the three quality grades on average. On average, the firm remanufactures 174, 143, and 42 units, and it salvages 2, 33, and 133 units of good, average and bad quality cores per period respectively. Thus, about 97% of all good quality cores are remanufactured (this corresponds to about half the overall demand), with the remainder half of demand being met mostly by average, and to a less extent, bad quality cores. In contrast, for incoming bad quality cores, 23% are remanufactured, 74% are salvaged, and the remaining 3% are kept in inventory as ungraded cores, on average. The minimum value of $\frac{\bar{z}_{1t}}{T}$ is 120 units; this happens when the firm only grades the strictly necessary amount of cores to meet demand due to the combination of a high grading cost and low salvage value (minimum value of $\frac{\bar{x}_t}{T} = 360$ units). Because incoming cores exceed demand by 50%, there is not a single instance where the firm remanufactures all average and bad quality cores; the maximum values for $\frac{\bar{z}_{2t}}{T}$ and $\frac{\bar{z}_{3t}}{T}$ are 168 and 120 units, respectively.

The firm does not carry much inventory of graded cores, as demonstrated by median values for $\frac{\bar{u}_{it}}{T}$ of 13, 5 and 0 units for good, average and bad quality cores respectively. As we

Table 6: Descriptive Statistics for Experimental Design

	Π	$\frac{\bar{z}_{1t}}{T}$	$\frac{\bar{z}_{2t}}{T}$	$\frac{\bar{z}_{3t}}{T}$	$\frac{\bar{v}_{1t}}{T}$	$\frac{\bar{v}_{2t}}{T}$	$\frac{\bar{v}_{3t}}{T}$	$\frac{\bar{u}_{1t}}{T}$	$\frac{\bar{u}_{2t}}{T}$	$\frac{\bar{u}_{3t}}{T}$	$\frac{\bar{y}_t^+}{T}$	$\frac{\bar{y}_t^-}{T}$	$\frac{\bar{x}_t}{T}$	$\frac{\bar{b}_t}{T}$
mean	129,494	174	143	42	2	33	133	20	8	0	44	13	527	35
stdev	26,200	11	11	19	2	16	28	19	11	5	55	26	36	99
min	54,861	120	114	15	0	0	0	0	0	0	0	0	360	0
25th perc	110,165	175	136	29	0	23	130	7	0	0	1	0	540	0
median	130,760	178	141	43	1	36	137	13	5	0	21	1	540	0
75th perc	148,593	180	153	49	3	43	150	26	12	0	65	9	540	20
max	189,462	180	168	120	10	66	165	134	106	156	227	104	540	779

discuss later, the firm only carries inventory of remanufactured products because of capacity constraints coupled with a non-stationary demand pattern; the median value is 21 units, with a mean of 44 units. Similarly, the median value of backlogged units is 1 unit but the average is 13 units; backlogs only occur if there is insufficient capacity, as is the case when $A = 0.75$ and $\frac{C_t}{D} = 1.2$. On average, the firm grades 527 of the 540 incoming cores, although in most cases (83% of cases), the firm grades all 540 cores in all periods under all scenarios. In most cases, the firm prefers not to carry inventory of ungraded cores (median value is 0 units); instead, the firm prefers to grade them right away because they can then be salvaged in the same period. When salvage values are low and grading cost is high, however, the firm only grades the number of cores strictly necessary to meet demand, and consequently carries inventory of ungraded cores (maximum expected inventory of 779 units).

We now discuss which factors in the experimental design have the largest impact on the metrics just discussed. Note that there is variance in the metrics, as evidenced by the standard deviation and minimum and maximum values reported in Table 6. Thus, it is of interest to understand which factors have the largest impact on a given performance metric of the optimal solution. To accomplish this, we perform simple regressions, one for each metric and experimental factor. In each regression, the dependent variable is the metric of interest (e.g., Π), the independent variable is the experimental factor (e.g., β), and there are 6,561 observations, corresponding to the experimental cells. The value of R^2 for each regression provides a measure of the impact of the respective factor value on the respective metric (Wagner 1995). The results are reported in Table 7. The dummy variable d_i takes the value of 1 if demand pattern i is used and 0 otherwise. For example, the regression Π vs. β has $R^2 = 0.45$, so the parameter β explains 45% of all the variance in profit.

Table 7: $R^2(\times 100)$ Values for Simple Linear Regressions Where the Dependent Variable is Metric and the Independent Variable is Experimental Factor

	Π	$\frac{\bar{z}_{1t}}{T}$	$\frac{\bar{z}_{2t}}{T}$	$\frac{\bar{z}_{3t}}{T}$	$\frac{\bar{v}_{1t}}{T}$	$\frac{\bar{v}_{2t}}{T}$	$\frac{\bar{v}_{3t}}{T}$	$\frac{\bar{u}_{1t}}{T}$	$\frac{\bar{u}_{2t}}{T}$	$\frac{\bar{u}_{3t}}{T}$	$\frac{\bar{y}_t^+}{T}$	$\frac{\bar{y}_t^-}{T}$	$\frac{\bar{x}_t}{T}$	$\frac{\bar{b}_t}{T}$
d_1	0	0	0	0	1	0	0	23	1	0	27	4	0	0
d_2	0	1	0	1	29	0	1	0	1	0	0	1	0	0
d_3	0	1	1	2	19	0	1	21	0	0	20	11	0	1
π	0	0	0	0	0	0	0	0	0	0	1	0	0	0
β	45	0	7	3	1	3	1	4	4	0	0	0	0	0
θ	24	15	8	0	14	29	5	8	11	1	0	0	19	14
h_i	0	1	4	0	2	4	0	6	11	1	0	0	1	2
$\frac{g}{a_1}$	24	15	0	6	1	8	11	0	0	1	0	0	15	11
A	0	0	7	6	1	3	3	2	0	0	3	8	0	0
$\frac{C_t}{D}$	1	2	36	25	8	15	13	1	1	0	28	33	1	3

The cost parameters β and $\frac{g}{a_1}$, and revenue parameter θ, have the most impact on profit, explaining 45%, 24% and 24% of the variance, respectively; the remaining parameters play a minor role. In terms of the optimal solution (not necessarily the optimal profit), all parameters have some impact, with the highest impacts (as judged by the sum of the R^2 values across the columns) being, in decreasing order, relative capacity $\frac{C_t}{D}$, relative salvage value θ, and relative grading cost $\frac{g}{a_1}$. Extra capacity usage by the bad cores, A, has a significant impact on the amount of backlogs because it directly affects the amount of demand the firm can meet. Essentially, the firm only backlogs when there is insufficient capacity to meet demand, so the combination of high A and low $\frac{C_t}{D}$ produces backlogs. The demand pattern significantly impacts inventory and salvaging of good quality cores, inventory of remanufactured product and backlogs; this is due to demand spikes coupled with limited capacity. Holding cost h_i has less of an impact in the optimal solution except for the inventory of good and average quality cores. In general, the only incentive for the firm to carry inventory is due to capacity constraints and non-stationary demand. Clearly, this hierarchy holds for the ranges of levels for the factors employed in our experimental study, which we were careful to choose among low, average and high values that are found in practice.

As we previously mentioned, the firm typically remanufactures all good quality cores; the exception is when grading cost is high relative to salvage value. This is the case when the firm only grades the minimum amount of cores to meet demand, and as a result, the variance explained for the amount of good cores remanufactured $\frac{\bar{z}_{1t}}{T}$ is significant only for the parameters θ and $\frac{g}{\alpha_1}$ at 15% each. Capacity constraints explain the bulk of the variance in $\frac{\bar{z}_{2t}}{T}$ and $\frac{\bar{z}_{3t}}{T}$ at 36% and 25%, respectively. Regarding the amount of good cores salvaged $\frac{\bar{v}_{1t}}{T}$, 29%, 19%, 14% and 8% of the variance is explained by d_2, d_3, θ and $\frac{C_t}{D}$, respectively. As we will see later (Figure 3), when the firm has excess capacity and salvage values are high, it can afford to salvage good quality cores to boost revenue and use the excess capacity for capacity-hungry average and bad cores; this strategy works only when there are periods of significantly lower demand than average, which is the case with demand patterns 2 and 3. A similar logic holds for the amount of average $\frac{\bar{v}_{2t}}{T}$ and bad quality cores salvaged $\frac{\bar{v}_{3t}}{T}$. In this case, higher grading cost also plays a significant role, explaining 8-11% of the variance because higher $\frac{g}{\alpha_1}$ implies less cores salvaged (average and bad quality cores are typically the ones salvaged); the demand pattern plays no role here because, again, the average and bad cores are usually salvaged.

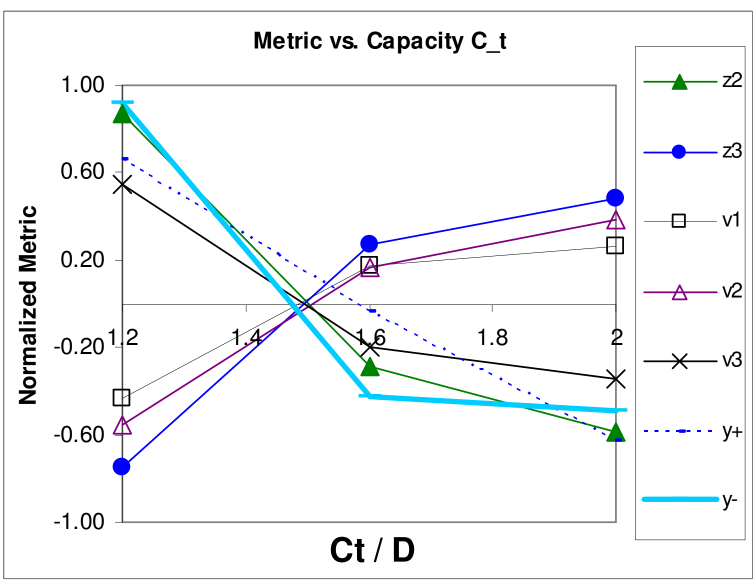

Figure 3: Normalized Metric Value vs. Relative Capacity $\frac{C_t}{D_t}$

The variance in the amount of good quality cores $\frac{\bar{u}_{1t}}{T}$ carried in inventory is explained primarily by demand patterns 1 and 3 at 23% and 21% respectively. The firm carries inventory of graded cores except for the case when there is a single spike in the middle of the demand pattern (as in demand pattern 2); in that case the extra core inventory would be of no use due to the lack of capacity to remanufacture in that high-demand period (see also Figure 6). The same effect also shows in the amount of remanufactured units $\frac{\bar{y}_t^+}{T}$ carried in inventory for the exact same reason; but here the capacity constraint shows as the most important reason (28% of variance explained) why the firm carries inventory of remanufactured products. The amount of backlogs is primarily explained by capacity $\frac{C_t}{D}$, demand pattern 3, and capacity usage A, at 33%, 11% and 8% respectively.

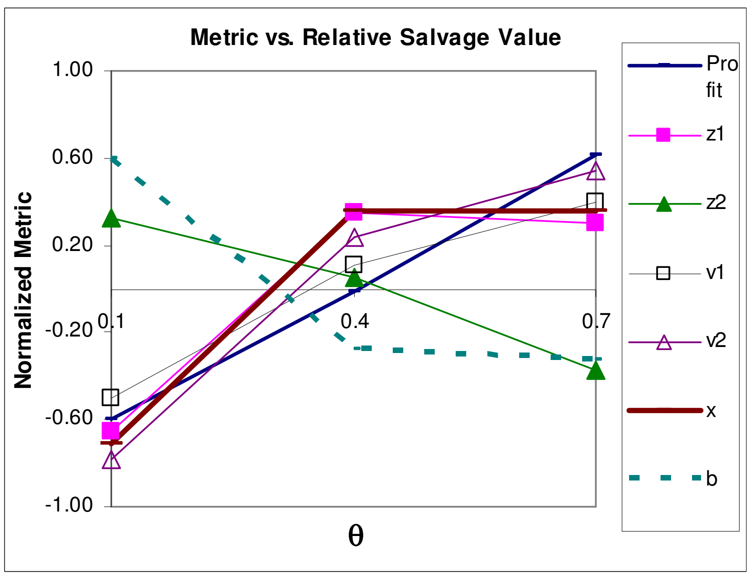

Figure 4: Normalized Metric Value vs. Relative Salvage Value

Knowing which factors most impact the optimal solution, we now focus on the direction of impact. To that end, we compute, for each factor level (e.g., $\theta = 0.1$) the average value of the metric of interest across all respective experimental cells, and normalize it by subtracting the mean (across all cells) and dividing by the standard deviation (as reported in Table 6). This normalization is done so that all metrics are measured on the same scale, i.e., relative to its mean value. We plot only the most significant relationships as discussed before in the R^2 analysis, and these are shown in Figures 3-7 (the entire set of results is available in Tables 8-9 in the appendix). In all charts, the scale in the vertical axis is the same so that the reader can relate the results with those of Table 7—a higher R^2 in Table 7 results in a curve with a higher slope in Figures 3-7. We also use the same marker and line pattern for each metric across charts.

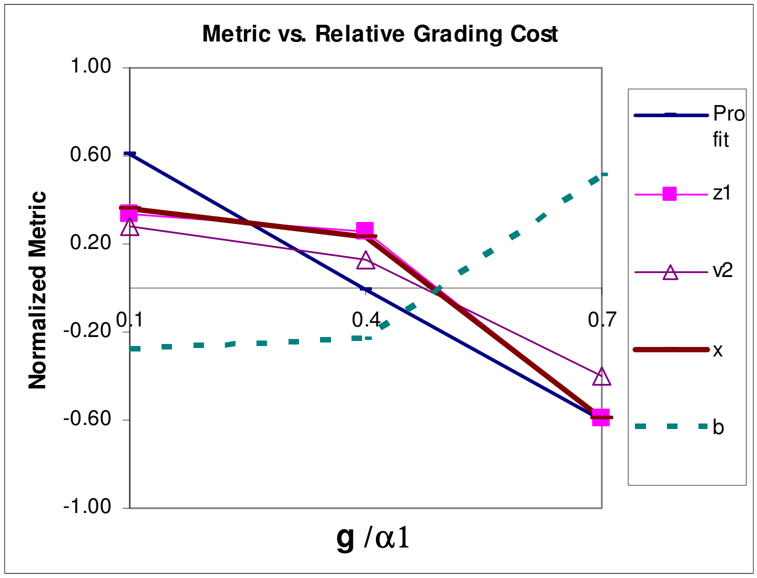

Figure 5: Normalized Metric Value vs. Relative Grading Cost

First, consider in Figure 3 the factor with the highest impact in most metrics, capacity relative to demand. With excess capacity, the firm can afford to salvage good and average quality cores (thus the increase in v_1 and v_2 and decrease in z_2) to boost revenue and use the excess capacity for capacity-intensive bad cores (increasing z_3 and consequently decreasing v_3). More capacity also reduces the need to carry inventory of remanufactured products y^+, and reduces the amount of backlogs (y^-). The largest impact on the metrics occurs as $\frac{C_t}{D}$ increases from 1.2 to 1.6; it can be shown that $\frac{C_t}{D} = 1.6$ provides sufficient capacity to meet demand with expected (average) mix of good, average and bad cores. Nonetheless, there is still some improvement in the metrics as $\frac{C_t}{D}$ increases from 1.6 to 2.0 because of the stochastic nature of the grading process. As an example, with many bad outcomes in a row, which happens with a positive probability, the firm does not have enough capacity to meet demand if $\frac{C_t}{D} = 1.6$ and $A = 0.75$.

Now, consider in Figures 4-5 the factors with the second and third largest impact in the metrics of interest, relative salvage value and relative grading cost. Again, only the most important relationships are plotted to avoid cluttering. Typically, as discussed before, the firm grades all cores (and remanufactures all graded high quality cores); the exception

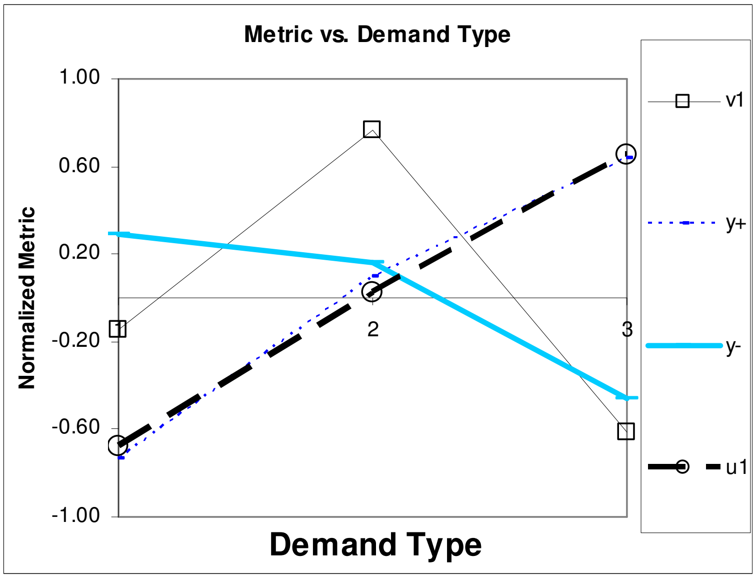

Figure 6: Normalized Metric Value vs. Demand Type

is when salvage values are low (Figure 4) and grading cost is high (Figure 5). Low salvage values, relative to grading costs, cause the amount of grading, x, to decrease (which increases the inventory of ungraded cores, b); the decreased availability of good quality cores decreases the absolute number of good quality cores remanufactured, z_1, at the same rate (since the firm remanufactures almost all good quality cores it grades). However, a lower salvage value for average quality cores decreases their amount salvaged, v_2, and consequently increases their amount remanufactured, z_2. Everything else being equal, a higher salvage value or a lower grading cost per unit increases expected profit, and the relationship is approximately linear.

Now, consider in Figure 6 the impact of the demand pattern. Demand type 2 has the "spike in the middle", and demand type 3 has a spike at the end, which necessitates more carrying of inventory, y^+, from earlier periods with excess capacity. The amount of good quality cores carried in inventory, u_1, is also higher for demand patterns 2 and 3 because they have periods of low demand when the excess capacity can be used to remanufacture good quality cores graded earlier in the planning horizon. Figure 7 shows the impact of extra capacity usage, A, on the optimal solution. Essentially, a higher value of A means less bad quality cores, z_3, remanufactured, with more of them salvaged, v_3; the firm remanufactures more of the average quality cores, z_2 (with less of them salvaged), and some of the demand

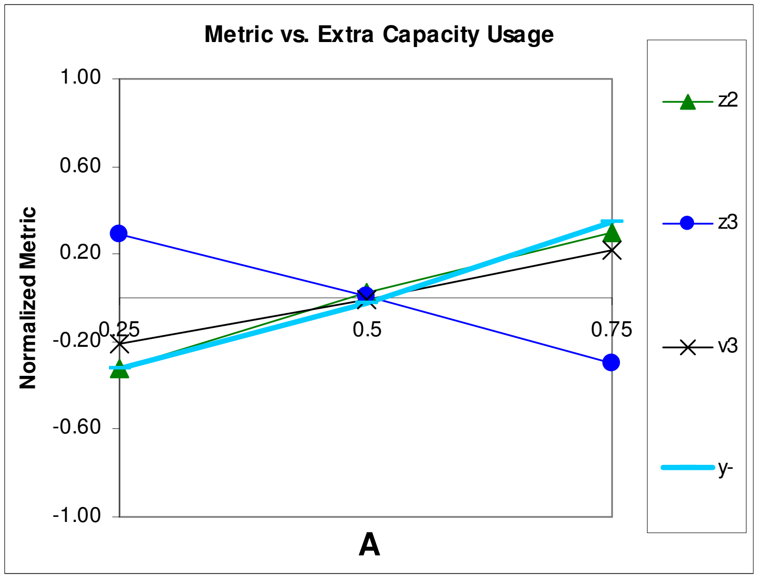

Figure 7: Normalized Metric Value vs. Cost Shape Parameter

is backlogged because of insufficient capacity. Good quality cores are not impacted by A because the firm remanufactures most of them. The remanufacturing cost shape parameter β has the most impact on the objective function ($R^2 = 0.45$, Table 7), with less of an impact on the optimal solution per se; in the interest of space we omit this chart as the relationship is obvious—higher cost, lower profit.

5 Conclusion

In this paper, we consider a production planning problem where the inputs (returned units) have different and uncertain quality levels, and there are capacity constraints; a situation typical of remanufacturing operations. From a production planning perspective, this scenario has several implications: the firm needs to grade returned units (cores) at a possible cost; the grading operation yields an uncertain outcome with respect to the makeup of core quality; different quality grades have different cost and processing requirements, and graded cores can be salvaged at potentially different salvage values depending on their quality. In addition, uncertainty in the quality levels of the cores creates problems for the planning of remanufactured product production because the varying quality grades typically require different levels of capacity to bring the processed units up to a pre-determined configuration and acceptable quality level for the market. Thus, remanufacturing higher quality cores can be accomplished at a lower cost and lower capacity usage than remanufacturing lower quality cores. We propose a stochastic programming formulation to solve the remanufacturing production planning problem Our model determines the quantity of cores of each quality grade that should be remanufactured, held in inventory for a future period, or salvaged each period in a finite horizon planning horizon. Unlike a deterministic production planning model that uses linear programming with inputs based on the expected values of the returned units' quality levels, our SP approach guarantees an implementable solution for all realizations of the quality levels.

Through an extensive numerical study based on parameter values either previously reported in the literature or observed in practice, we measured the relative impact of each parameter on the firm's profit (our objective function) as well as each of the corresponding decision variables. From the results of this test, we conclude several interesting insights on the remanufacturing production planning problem. First, the firm's profit is most heavily influenced by the shape of the production cost curve as it relates to the quality of the core (convex increasing, linear, or concave increasing in the lower quality of the core), the salvage value of an unprocessed core, and the cost of grading. Thus, firms should take extra care in estimating these values. The per unit holding cost of the cores, as well as unit backlogging cost, on the other hand, have a smaller impact on the optimal solution. This finding is encouraging as firms often have a difficult time estimating these parameter values. As one would expect, it is often optimal for a firm to remanufacture all of the highest quality cores. The exception occurs when the cost of grading is high and the revenue from salvaging is low. In these cases, the firm is better off just grading and remanufacturing enough units to meet demand (as opposed to grading all the returns and only remanufacturing the highest quality cores).

While our model formulation and numerical study was based on the assumptions of deterministic demand (no forecast error) and a known return stream of cores, the model can be easily modified to accommodate uncertainties in both of these inputs. For instance, the set of scenarios evaluated in the stochastic program could be nested to include a given quality set for a given return amount and a given demand realization. The trade-off, of course, is an exponential increase in the number of outcomes, and consequently, on the computational requirement of solving the linear program. Future research could provide guidance on the best way to handle demand and supply uncertainties in the planning of a remanufacturing process. The alternatives are to include the uncertainties in the model formulation with the subsequent increase in computational times or to solve the model frequently, on a rolling horizon, with forecast updates for both the demand and the return stream of cores.

Appendix: Impact of Factors on Metrics

Tables 8-9 below presents the results for the impact of factor levels on the normalized metrics (average for factor level minus total average divided by standard deviation), from which Figures 3-7 were built.

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Table 8: Impact of Factor Levels on Normalized Metrics - Part 1

Factor	Level	Π	$\frac{\bar{z}_{1t}}{T}$	$\frac{\bar{z}_{2t}}{T}$	$\frac{\bar{z}_{3t}}{T}$	$\frac{\bar{v}_{1t}}{T}$	$\frac{\bar{v}_{2t}}{T}$	$\frac{\bar{v}_{3t}}{T}$
	1	-0.01	0.01	-0.05	0.04	-0.15	0.03	-0.03
Dem Type	2	-0.03	-0.17	-0.06	0.15	0.76	0.01	-0.12
	3	0.03	0.16	0.11	-0.19	-0.62	-0.04	0.15
	10	0.04	0.01	0.01	-0.02	-0.04	-0.01	0.01
π	20	0.01	0.00	-0.01	0.01	0.01	0.00	0.00
	40	-0.05	-0.01	-0.01	0.02	0.03	0.01	-0.01
	$\frac{1}{3}$	0.83	-0.09	0.12	-0.02	0.19	-0.13	-0.01
β	1	0.04	0.13	0.31	-0.27	-0.07	-0.14	0.22
	3	-0.87	-0.04	-0.42	0.28	-0.12	0.27	-0.21
	0.1	-0.60	-0.65	0.32	0.20	-0.50	-0.78	-0.44
θ	0.4	-0.01	0.35	0.05	-0.24	0.11	0.23	0.31
	0.7	0.61	0.30	-0.37	0.04	0.40	0.54	0.12
	1	0.03	-0.10	0.25	-0.09	-0.14	-0.28	0.01
h_i	2	0.00	0.03	-0.03	0.00	-0.03	0.04	0.01
	3	-0.02	0.07	-0.22	0.08	0.17	0.24	-0.01
	0.1	0.61	0.34	-0.01	-0.20	0.17	0.28	0.28
$\frac{g}{a_1}$	0.4	-0.01	0.25	0.07	-0.20	-0.05	0.13	0.23
	0.7	-0.60	-0.59	-0.06	0.40	-0.12	-0.40	-0.52
	0.25	0.05	-0.07	-0.33	0.29	0.13	0.21	-0.21
A	0.5	0.01	0.00	0.03	0.01	-0.01	-0.02	-0.01
	0.75	-0.06	0.07	0.30	-0.30	-0.12	-0.19	0.22
	1.2	-0.15	0.20	0.87	-0.75	-0.44	-0.55	0.55
$\frac{C_t}{D}$	1.6	0.06	-0.09	-0.28	0.27	0.17	0.17	-0.20
	2.0	0.09	-0.11	-0.59	0.48	0.27	0.39	-0.35

Table 9: Impact of Factor Levels on Normalized Metrics - Part 2

Factor	Level	$\frac{\bar{u}_{1t}}{T}$	$\frac{\bar{u}_{2t}}{T}$	$\frac{\bar{u}_{3t}}{T}$	$\frac{\bar{y}_t^+}{T}$	$\frac{\bar{y}_t^-}{T}$	$\frac{\bar{x}_t}{T}$	$\frac{\bar{b}_t}{T}$
	1	-0.68	0.10	0.02	-0.73	0.29	-0.02	-0.10
Dem Type	2	0.03	-0.15	0.04	0.10	0.17	-0.04	-0.05
	3	0.65	0.05	-0.06	0.64	-0.46	0.06	0.14
	10	0.01	-0.03	0.00	-0.09	0.04	0.00	0.01
π	20	0.01	0.00	0.00	0.01	-0.01	0.00	0.00
	40	-0.01	0.04	0.01	0.09	-0.03	0.00	-0.01
	$\frac{1}{3}$	0.07	0.05	0.00	0.00	0.00	-0.05	0.06
β	1	0.25	0.26	-0.09	-0.03	0.00	0.11	-0.11
	3	-0.32	-0.31	0.10	0.03	0.00	-0.06	0.05
	0.1	0.30	0.42	0.19	0.01	0.00	-0.71	0.60
θ	0.4	0.08	-0.02	-0.09	-0.01	0.00	0.36	-0.27
	0.7	-0.38	-0.40	-0.09	0.00	0.00	0.36	-0.32
	1	0.34	0.46	0.15	0.06	-0.02	-0.12	0.17
h_i	2	-0.06	-0.12	-0.06	0.00	0.00	0.02	-0.02
	3	-0.28	-0.34	-0.09	-0.06	0.03	0.10	-0.14
	0.1	0.02	-0.03	-0.09	-0.01	0.00	0.36	-0.28
$\frac{g}{a_1}$	0.4	0.04	0.03	-0.09	-0.01	0.00	0.24	-0.23
	0.7	-0.05	-0.01	0.19	0.03	-0.01	-0.59	0.51
	0.25	-0.19	0.08	0.05	-0.20	-0.32	-0.05	0.09
A	0.5	0.03	-0.09	-0.01	0.02	-0.03	0.00	-0.01
	0.75	0.16	0.01	-0.04	0.19	0.35	0.05	-0.08
	1.2	-0.22	0.08	0.08	0.66	0.92	0.12	-0.22
$\frac{C_t}{D}$	1.6	0.22	-0.34	-0.04	-0.04	-0.43	-0.05	0.04
	2.0	0.00	0.26	-0.04	-0.62	-0.49	-0.06	0.18

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