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Multiple crack weight for solution of multiple interacting cracks by meshless numerical methods

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SUMMARY

We devise a multiple crack weight (MCW) method for the accurate and effective solution of strongly interacting cracks by meshless numerical methods. The MCW method constructs weight functions around cracks so that they simultaneously characterize all the cracks present in the single nodal domain of influence. This approach reduces the number of nodes necessary to achieve sufficient accuracy and consequently it decreases the computational effort. Numerical examples demonstrate that the method allows an accurate solution of multiple cracks problems. Convergence of the method is analysed and discussed. Copyright © 2006 John Wiley & Sons, Ltd.

KEY WORDS: meshless methods; multiple crack weight; multiple interacting cracks

INTRODUCTION

Interaction between multiple cracks is one of the most important but less investigated phenomenon in fracture mechanics. Stress corrosion cracking, hydrogen embrittlement, creep micro-cracking and other common fracture mechanisms are characterized by systems of interacting flaws. Theoretical research of this class of problems is hindered by limitations of the existing numerical methods.

The numerical solution by the traditional finite element method (FEM), of fracture mechanics problems with multiple cracks requires an enormous mesh refinement near each crack tip, including the embedding of many singular elements. Moreover, solution by FEM of dynamic cracks is limited to simple cases. This is because modelling of growing discontinuities requires time-consuming remeshing at every time step. For this reason an adaptive FEM has become essential. However adaptive remeshing and mapping of variables is a difficult, computationally expensive task and is a source of cumulative numerical errors. The development of meshless methods [1–9] and the extended finite elements method (X-FEM) [10] in recent years has

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enabled the solution of problems with growing cracks without remeshing. Nevertheless, these methods continue to be computationally expensive when solving multiple cracks because of the large nodal densities needed in meshless methods and the fine meshes needed in X-FEM for an accurate solution. Therefore, there is a continual effort to improve the accuracy without increasing the degrees of freedom.

We focus on improvements to meshless methods for the solution of fracture mechanics problems. Although X-FEM has recently received greater attention than meshless methods, they remain an efficient and accurate approach to solve fracture mechanics problems. Recent developments of meshless methods for the solution of different classes of problems such as multiple interacting cracks [11], 3D cracks [12], and cracks in elastic–plastic materials [13] improve these numerical methods, make them more attractive to the user.

There are two main approaches, in meshless methods, for modelling discontinuities and capturing singular stresses at the crack tip. The first one is based on the incorporation of a jump function along the discontinuity and a specific near crack-tip displacement solution in the extrinsic basis [14]. This approach was adopted from X-FEM and has similar limitations. The enrichment area is limited when multiple cracks are densely distributed or when crack tips are close to the boundaries. Modelling of moving cracks in dynamic problems requires the incorporation of different near crack-tip solutions which depends on the crack velocity. In addition, enrichment for developing cracks in elastic–plastic materials is not yet established.

Many of these limitations can be avoided by using another approach which is based on a special modification of the weight functions at the crack tip. For this purpose, several methods have been devised: *the visibility* method [1, 15], *the* '*see-through*' method [16], *the transparency* method [6, 17], *the wedge model* [18] and *the diffraction* method [6, 17]. The first developed schemes were the *visibility* and '*see-through*' methods. They provide an accurate solution only when very large nodal densities are used. In the visibility method, this is because the weight and shape functions are discontinuous near the crack tip and the size of the discontinuity is a function of the nodal spacing. Although, the 'see-through' method provides continuous approximations, it effectively shortens the crack and does not properly capture the singular stress at the crack tip. The transparency method and the wedge model provide more accurate results, however they have a restriction on the position of nodes limiting their use in dynamic crack problems.

Duflot and Nguyen-Dang [19] proposed an *enriched weight function* method. In this method the diffraction weight functions of three nodes near the crack tip are multiplied by the square root of the distance from the crack tip leading to more accurate capture of the singular stresses. However, no analysis of the displacement and the stress field at the crack tip was performed to demonstrate that only three enriched nodes are sufficient to capture the singular stresses and to enforce the zero displacement condition at the crack tip. Nevertheless, calculated stress intensity factors showed greater accuracy using the enriched weight functions compared with an ordinary diffraction approximation. In the current formulation, the enriched nodes are moved together with the crack tip. Hence, the application of this method appears to be limited to static and quasi static cases.

Most recently, *the spiral weight* method [20] for the construction of weight functions around crack tips was developed to increase the accuracy of meshless approximations for the practically important case of a linear basis. This takes into account the advantages and drawbacks of other methods that modify weight function shape around cracks. The spiral weight functions are constructed to preserve the discontinuity along the entire crack length. Numerical examples show that the spiral weight method is more accurate than the diffraction method when using a linear basis.

We design the multiple crack weight method. This method defines the nodal weight functions so that they simultaneously characterize all the cracks and their tips that are present in the domain of influence. This approach reduces the number of nodes necessary for accurate solutions.

MULTIPLE CRACK WEIGHT METHOD

We develop an algorithm for the construction of weight functions to handle multiple interacting cracks, when the distance between cracks can be smaller than the domain of influence of the nodes. This algorithm extends the application of methods that modify the shape of weight functions near a crack to the case of multiple cracks. Among the methods that can be extended for the solution of multiple cracks according to the algorithm developed below are the spiral weight, diffraction, transparency and visibility methods. The algorithm creates a nodal weight function that simultaneously characterizes all the crack tips located in the nodal domain of influence. For simplicity of presentation, the algorithm will be presented for the example of the diffraction method. We call this method the *multiple crack weight* (MCW) method.

Weight functions used in meshless approximations may have different shapes of domain of influence. The most common are the circle and the rectangle. For the circular domain of influence a frequently used weight function is the quadratic spline:

$$w(d_{\mathrm{I}}) = \begin{cases} 1 - 6\left(\dfrac{d_{\mathrm{I}}}{d_{m\mathrm{I}}}\right)^2 + 8\left(\dfrac{d_{\mathrm{I}}}{d_{m\mathrm{I}}}\right)^2 - 3\left(\dfrac{d_{\mathrm{I}}}{d_{m\mathrm{I}}}\right)^4, & d_{\mathrm{I}} \leqslant d_{m\mathrm{I}} \\ 0, & d_{\mathrm{I}} > d_{m\mathrm{I}} \end{cases} \tag{1}$$

where $d_{\mathrm{I}} = \|x - x_{\mathrm{I}}\|$ is the distance between the point x and node point x_{I} and $d_{m\mathrm{I}}$ is the domain of influence of node x_{I}.

The general process of calculating the weight function $w_{\mathrm{I}}(x)$ for node x_{I} and sampling point x by MCW is given by

1. When the line (x_{I}, x) does not intersect any crack, then point x is called *visible*. Then $w_{\mathrm{I}}(x)$ is calculated by formula (1), where $d_{\mathrm{I}} = \|x - x_{\mathrm{I}}\|$.
2. Otherwise, the point x is *invisible* and then
   - 2.1. Look for all crack lines that are crossed by the line (x_{I}, x). We call these *crossed* cracks (cracks $1, 2, 3$ in Figure 1(a)).
   - 2.2. For all n *crossed* cracks, we look for the shortest path connecting the node point x_{I} with the sample point x going through one of the two tips of each of the *crossed* cracks (Figure 1(a) and (b)).

     The optimal path may skip part of the *crossed* cracks tips (crack number 2 in Figure 1(a)) if no segment of this path crosses other *crossed* cracks. There is a possibility that two or more cracks may have common tips. In that case, such tips cannot be included in the path (Figure 1(c)). We mark the tips of the *crossed* cracks that the path goes through them as x_c^i, $i = 1 \ldots n$.

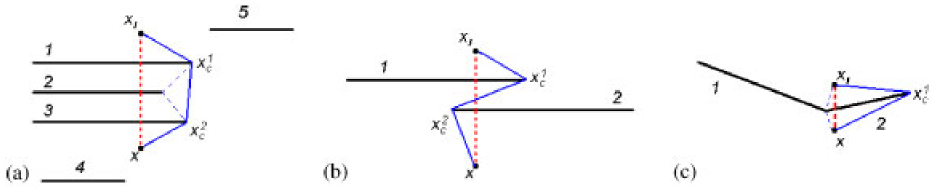

Figure 1. Paths $(x_I, x_c^{(1)}, \ldots, x_c^{(n)}, x)$ for various crack configurations.

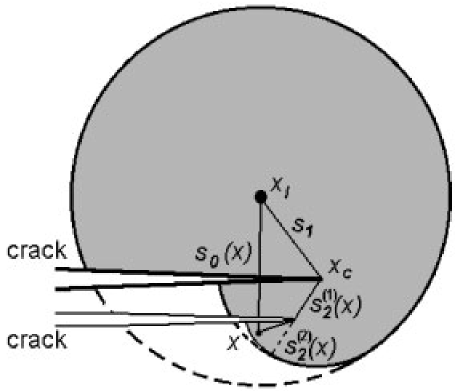

Figure 2. The weight functions modified by MCW algorithm.

2.3. Modify d_I according to the selected method for modification of the weight function near the crack tip (diffraction method in this example):

$$d_I = \left(\frac{s_1 + s_2(x)}{s_0(x)}\right)^{\lambda} s_0(x) \tag{2}$$

where $s_0(x) = \|x - x_I\|$, λ is the diffraction parameter (for problems with equally spaced nodal distributions and a linear basis, $\lambda = 2$. For the problems with a basis enrichment, λ is equal to either to 1 or 2) and s_1, $s_2(x)$ are modified according to the following rule (see Figure 2):

$$s_1 = \|x_I - x_c^{(1)}\| \tag{3}$$

$$s_2(x) = s_2^{(1)}(x) + s_2^{(2)}(x) + \cdots + s_2^{(n)}(x) \tag{4}$$

where

$$s_2^{(k)}(x) = \|x_c^{(k+1)} - x_c^{(k)}\| \quad \text{for } k = 1 \ldots n-1 \tag{5}$$

and

$$s_2^{(n)}(x) = \|x - x_c^{(n)}\| \tag{6}$$

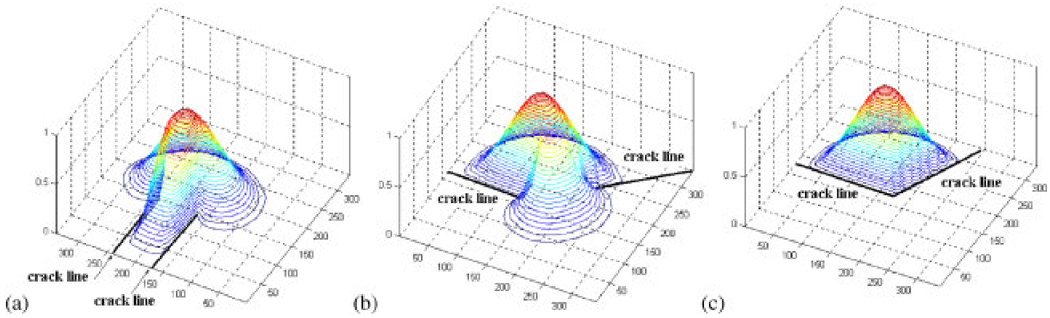

Figure 3. Spline weight functions by the diffraction method ($\lambda = 2$) modified by MCW algorithm: (a) two parallel cracks; (b) two-angled and spaced cracks; and (c) two-angled connected cracks.

2.4. If the modified d_I satisfies $d_\mathrm{I} \leqslant d_{m\mathrm{I}}$ then the $w_\mathrm{I}(x)$ is calculated by (1), otherwise $w_\mathrm{I}(x) = 0$.

In Figure 3, the weight functions for three common configurations of two cracks calculated by the diffraction method are presented.

NUMERICAL EXAMPLES AND DISCUSSION

This section presents a study of the reliability and accuracy of the MCW method for the solution of multiple cracks problems. We consider three numerical examples of interacting and intersecting cracks. They are solved using the element free Galerkin (EFG) meshless numerical method [1–9]. The stress intensity factors are calculated and compared to available reference solutions provided by other numerical methods. Problems involving double-edge collinear cracks, star-shaped cracks and a system of four cracks are chosen to illustrate the main aspects of solution of multiple crack problems. These include construction of the weight functions by MCW for intersecting and interacting cracks when a number of cracks lie in the domain of influence of a single node, construction of the mesh, integration scheme, etc. The convergence of the stress intensity factors as a function of the number of nodes is analysed and discussed.

In all the calculations, the nodal distribution is equally spaced with additional nodes along the cracks surfaces and an additional node at the free tips of the cracks. In the second example of star-shaped cracks, a star-shaped array of nodes around the free tips of the cracks was used to enhance the accuracy. The radius of the outer ring of star-shaped additional nodes was 0.75 of the distance between regular nodes. The nodal domain of influence, $d_{m\mathrm{I}}$ was calculated as a product of constant $d_{\max}$ and the nodal spacing parameter c_I. $d_{\max} = 2.5$ was used in all the calculations. This value of $d_{\max}$ was shown in Reference [17] as optimal for various static and dynamic fracture mechanics problems. Parameter c_I is the nodal spacing, which is at a distance to the second nearest node for equally spaced nodes and the distance to the third nearest node for other nodal distributions.

The above-mentioned regular nodal distribution was also used as a mesh for the numerical integration using the Gauss quadrature rule with 16×16 in the first example and 12×12 Gauss quadrature points in each cell in the second and the third examples. This significant number of

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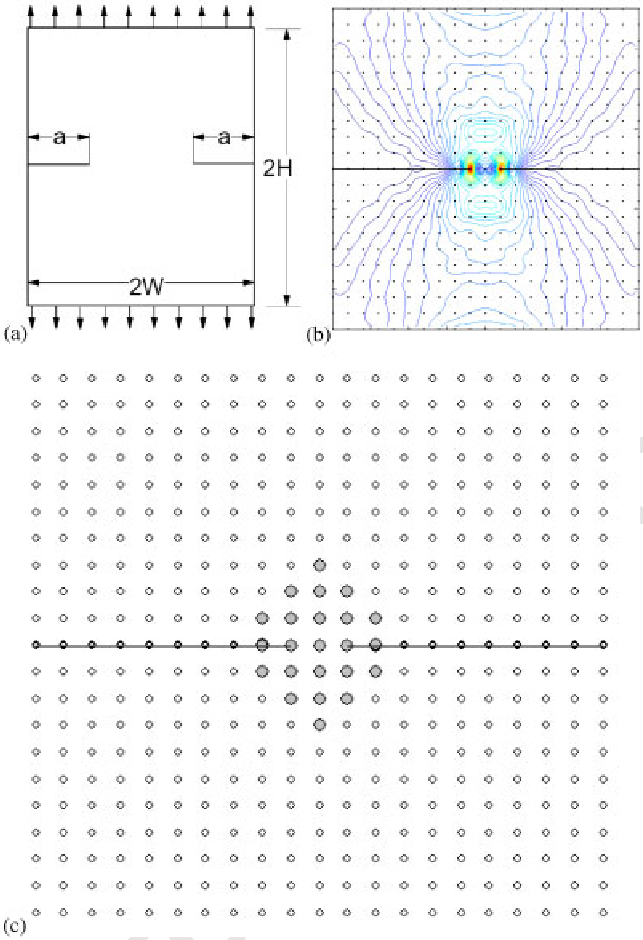

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Figure 4. (a) Double-edge collinear cracks in finite plate; (b) Von Mises stress distribution for $a/W = 0.9$ and $H/W = 1$; and (c) Nodal distribution for $a/W = 0.9$ and $H/W = 1$. Filled nodes represent nodes whose domains of influence intersect both cracks.

Gaussian points was chosen to minimize a numerical error from integration. The fully enriched basis was coupled with a linear basis at the crack tips. A plane strain condition was assumed. The MCW algorithm was used for the modification of the weight functions calculated by the spiral weight when a number of crack tips lay in the nodal domain of influence.

Double-edged collinear cracks in finite plate under normal load

To demonstrate the accuracy of the method, we start with the solution of interacting double-edged collinear cracks in a finite plate, see Figure 4(a). In this example, we calculate the stress intensity factors for two crack geometries $a/W = 0.8$ and 0.9, and for two plate geometries $H/W = 1$ and 3. Meshes with 21×21 and 21×61 regular nodes and additional nodes around

Table I. Normalized stress intensity factors for a double-edged crack in a finite plate.

	F_{I}	F_{I} ref 1	F_{I} ref 2	F_{I} ref 3	E_1, %	E_2, %	E_3,%	N
$a/W=0.8$, $H/W=1$	1.6111	1.5806	1.5962	1.6432	1.93	0.93	1.96	11
$a/W=0.8$, $H/W=3$	1.5497	1.5649	—	—	0.97	—	—	11
$a/W=0.9$, $H/W=1$	2.1326	2.1133	—	—	0.91	—	—	25
$a/W=0.9$, $H/W=3$	2.1016	2.1133	—	—	0.55	—	—	25

$E_{1,2,3}$ represents the percent difference of normalized stress intensity factors, F_{I} with reference solutions F_{I} ref 1, 2, 3. We note that for the two last cases, the normalized stress intensity factors F_{I} ref 1 were identical despite the difference in the geometry of the specimen ($H/W=1$ and 3). N represents the number of nodes whose domain of influence crosses both cracks.

Table II. Values of the function $\eta(a/W, H/W)$.

a/W	0.8	0.9
$H/W=1$	1.01	1.00
$H/W=3$	1.00	1.00

the crack tips were used for the cases $H/W=1$ and 3, respectively. In all four cases, the constructed mesh had a number of nodes with domains of influence that intersected two cracks (Figure 4(c) and Table I, column N). Normalized stress intensity factors $F_{\mathrm{I}}=K_{\mathrm{I}}/\sigma\sqrt{\pi a}$ were calculated and compared to three available reference solutions, see Table I.

In the first solution (Table I ref 1) by Bowie [21], approximate stress intensity factors were calculated by

$$K_{\mathrm{I}}=\sigma\sqrt{2W\ \tan\frac{\pi a}{2W}\eta\left(\frac{a}{W},\frac{H}{W}\right)},\quad K_{\mathrm{II}}=0 \tag{7}$$

The values of the function $\eta(a/W, H/W)$ are presented in Table II.

The accuracy of Bowie's solution is unknown. However, in the case of a semi-infinite plate the accuracy of his solution was about of 99% compared with the approximating solution of Irwin, see Reference [22].

Two other solutions for the function $\eta(a/W, H/W)$ are provided by the method of Denda and Dong [23] for the cases $a/W=0.8$ and $H/W=1$. These values were 1.02 by the *whole crack element* and 1.05 by the *crack tip element*. Applying formula (7), the normalized stress intensity factors were 1.5962 (Table I ref 2) and 1.6432 (Table I ref 3), respectively.

The results of the numerical example show that the proposed method provides solution which is in 99% agreement with the reference solutions in all four cases with relatively small nodal density. This is due to the ability to simultaneously characterize several discontinuities in a single domain of influence. It is important to note that solution of the same problem with the standard EFG method requires significantly dense nodal distribution with smaller domain of influence to avoid more than one discontinuity in a domain of influence. For example, in the case $a/W=0.9$ and $H/W=1$, one forced to use at least 71×71 nodes for solution by the regular EFG method. The new method with 21×21 regular nodes has similar accuracy to EFG with 71×71 nodes and requires a factor of 14 less computer time than EFG. The necessity of the fine meshes is dictated by limitations in the construction of the weigh functions and is

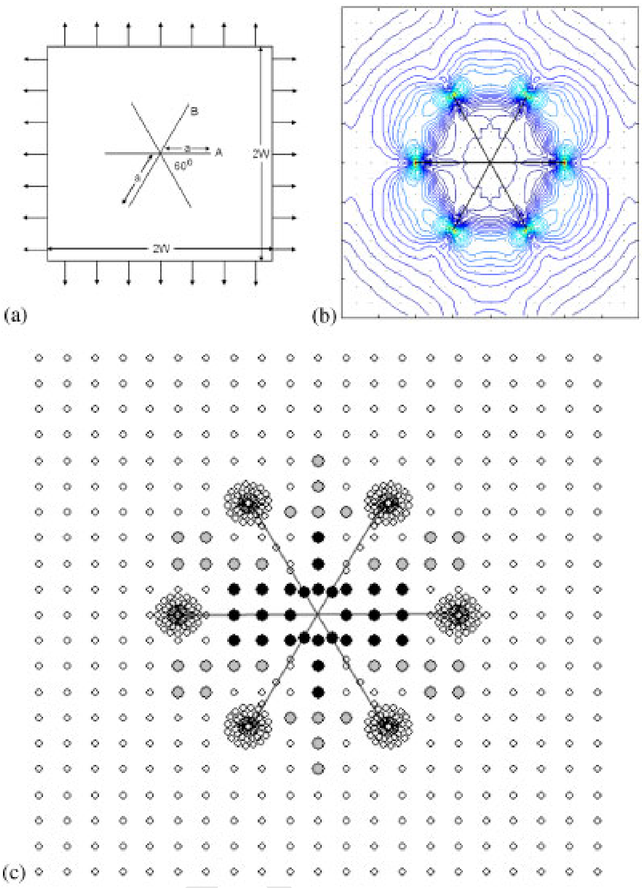

Figure 5. (a) Finite-size plate with star-shaped crack under bi-axial loading; (b) Von Mises stress distribution for $a/W = 0.5$; and (c) Mesh distribution for $a/W = 0.5$. Filled dark and grey nodes represent nodes whose domains of influence intersect six and two cracks, respectively.

not due to the ability of the method to provide an accurate solution with a smaller number of nodes.

Star-shaped crack in finite plate under bi-axial load

We next consider six intersecting cracks that were used to model a star-shaped crack (Figure 5(a)). The normalized stress intensity factors $F_{\mathrm{I}}^{A} = K_{\mathrm{I}}^{A}/\sigma\sqrt{\pi a}$, $F_{\mathrm{I}}^{B} = K_{\mathrm{I}}^{B}/\sigma\sqrt{\pi a}$ and $F_{\mathrm{II}}^{B} = K_{\mathrm{II}}^{B}/\sigma\sqrt{\pi a}$ were calculated using the domain form of the interaction integral for $a/W = 0.5$ and several ratios of a/h, where a is the crack length and h is the average nodal spacing. The results presented in Figure 6 show the convergence of the solution as the mesh is refined. The stress intensity factors are oscillating while they converge to their limiting values. The amplitude of the oscillations vanishes as the ratio a/h increases. The relative differences between the stress intensity factors, F_{I}^{A}, F_{I}^{B}, F_{II}^{B} calculated with $a/h = 5$ mesh and those calculated with $a/h = 10$ were 0.23, 0.20, 0.75%, respectively.

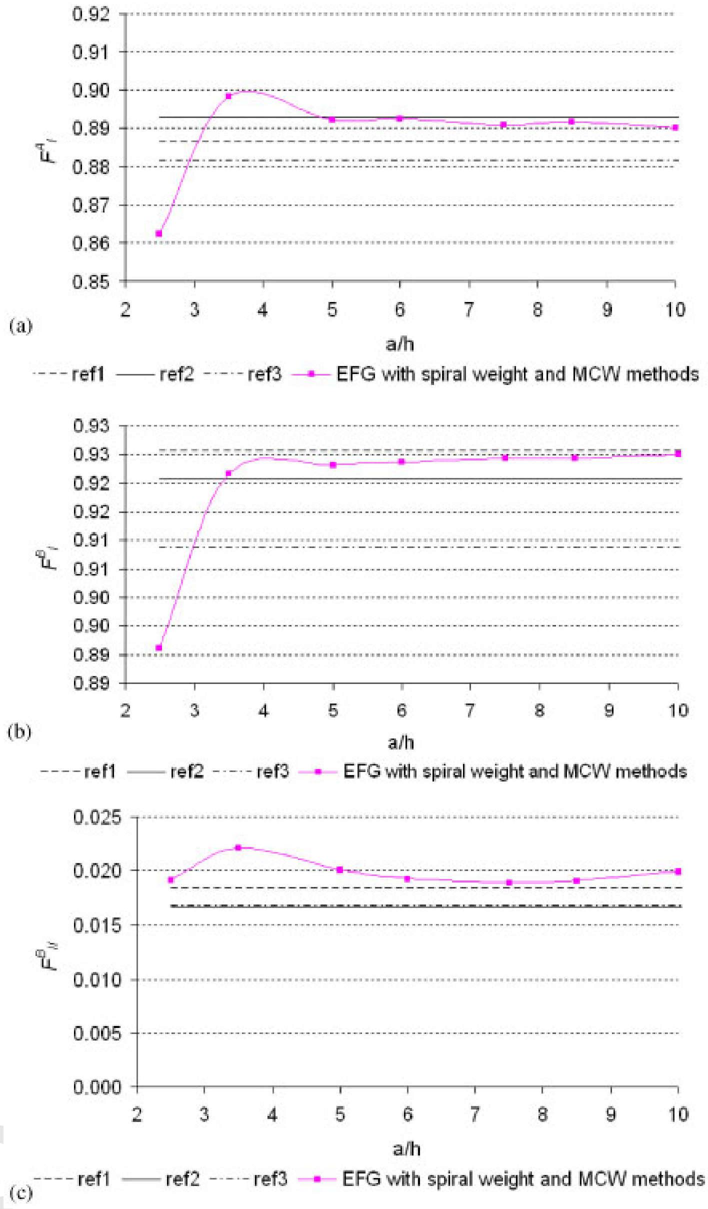

Figure 6. Convergence of the normalized stress intensity factors: (a) F_{I}^{A}; (b) F_{I}^{B}; and (c) F_{II}^{B} as a function of average nodal spacing. The results of ref 2 and ref 3 in (c) are identical.

Comparing these results with the reference solutions [24, 25], one sees that the accuracy of the solution is acceptable even for the relatively small ratio of $a/h = 5$ (mesh with 21×21 regular nodes) and that the results agree satisfactorily with those in References [24, 25] (Table III, case $a/W = 0.5$). This is despite the fact that the small regular nodal distribu-

Table III. Normalized stress intensity factors for the problem in Figure 5.

		F_{I}	F_{I} ref 1	F_{I} ref 2	F_{I} ref 3	E_1, %	E_2,%	E_3, %	N
$a/W=0.2$	F_{I}^{B}	0.7690	0.7683	—	0.7578	0.09	—	1.48	80
$a/W=0.2$	F_{II}^{B}	0.0007	0.0005	—	0.0004	*	—	*	80
$a/W=0.2$	F_{I}^{A}	0.7691	0.7670	0.7746	0.7570	0.27	0.71	1.60	80
$a/W=0.3$	F_{I}^{B}	0.7994	0.7983	0.7973	0.7884	0.14	0.26	1.40	50
$a/W=0.3$	F_{II}^{B}	0.0020	0.0021	0.0021	0.0022	*	*	*	50
$a/W=0.3$	F_{I}^{A}	0.7970	0.7931	0.7942	0.7846	0.49	0.35	1.58	50
$a/W=0.4$	F_{I}^{B}	0.8527	0.8466	0.8466	0.8365	0.72	0.72	1.94	68
$a/W=0.4$	F_{II}^{B}	0.0077	0.0080	0.0064	0.0070	*	*	*	68
$a/W=0.4$	F_{I}^{A}	0.8352	0.8287	0.8332	0.8255	0.78	0.24	1.18	68
$a/W=0.5$	F_{I}^{B}	0.9232	0.9255	0.9208	0.9087	0.25	0.26	1.60	72
$a/W=0.5$	F_{II}^{B}	0.0201	0.0184	0.0168	0.0168	*	*	*	72
$a/W=0.5$	F_{I}^{A}	0.8921	0.8864	0.8928	0.8815	0.64	0.08	1.20	72
$a/W=0.6$	F_{I}^{B}	1.0405	1.0445	1.0401	1.0182	0.38	0.04	2.19	88
$a/W=0.6$	F_{II}^{B}	0.0451	0.0364	0.0350	0.0388	*	*	*	88
$a/W=0.6$	F_{I}^{A}	0.9749	0.9673	0.9760	0.9758	0.79	0.11	0.09	88
$a/W=0.7$	F_{I}^{B}	1.2384	1.2367	1.2369	1.1936	0.14	0.12	3.75	88
$a/W=0.7$	F_{II}^{B}	0.0622	0.0593	0.0614	0.0529	*	*	*	88
$a/W=0.7$	F_{I}^{A}	1.1022	1.0971	1.1120	1.1142	0.46	0.88	1.08	88
$a/W=0.8$	F_{I}^{B}	1.5577	1.5624	1.5593	—	0.30	0.10	—	88
$a/W=0.8$	F_{II}^{B}	0.0804	0.0864	0.0826	—	*	*	—	88
$a/W=0.8$	F_{I}^{A}	1.3454	1.3423	1.3581	—	0.23	0.94	—	88
$a/W=0.9$	F_{I}^{B}	2.1605	2.1927	2.1659	—	1.47	0.25	—	90
$a/W=0.9$	F_{II}^{B}	0.0906	0.0868	0.088	—	*	*	—	90
$a/W=0.9$	F_{I}^{A}	1.9146	1.9037	1.9578	—	0.57	2.21	—	90

$E_{1,2,3}$ represents the percent difference of normalized stress intensity factors F_{I}^{A}, F_{I}^{B} with reference solutions ref 1, 2, 3. *, Percent difference is not representative in this case since the calculated and reference values of F_{II}^{B} are small or close to zero. We note that in this case there is even a significant percent difference between the three reference solutions. N represents number of nodes whose domains of influence crosses two or more cracks.

tion cannot match the inclined crack lines properly, several cells of the integration mesh are crossed by cracks and two crack tips are in the nodal domain of influence of many nodes (Figure 5(c) and Table III, column N). This demonstrates that the EFG method combined with the spiral weight and MCW methods is able to solve accurately multiple crack problems with even relatively small nodal distributions.

Next we solve the star-shaped crack problem for different a/W ratio. For the ratio a/W equal to either 0.5, 0.6, 0.7 or 0.8, the regular mesh had 21×21 nodes. For smaller values of a/W, we preserve the ratio $a/h=5$. The calculated normalized stress intensity factors (Table III) were compared with those calculated by X-FEM in Reference [24] for two different meshes distributions (Table III ref 1 and ref 2) and by Cheung *et al*. [25]. Our results show good agreement with the reference results and are closer to those provided by X-FEM than to results of Cheung *et al*.

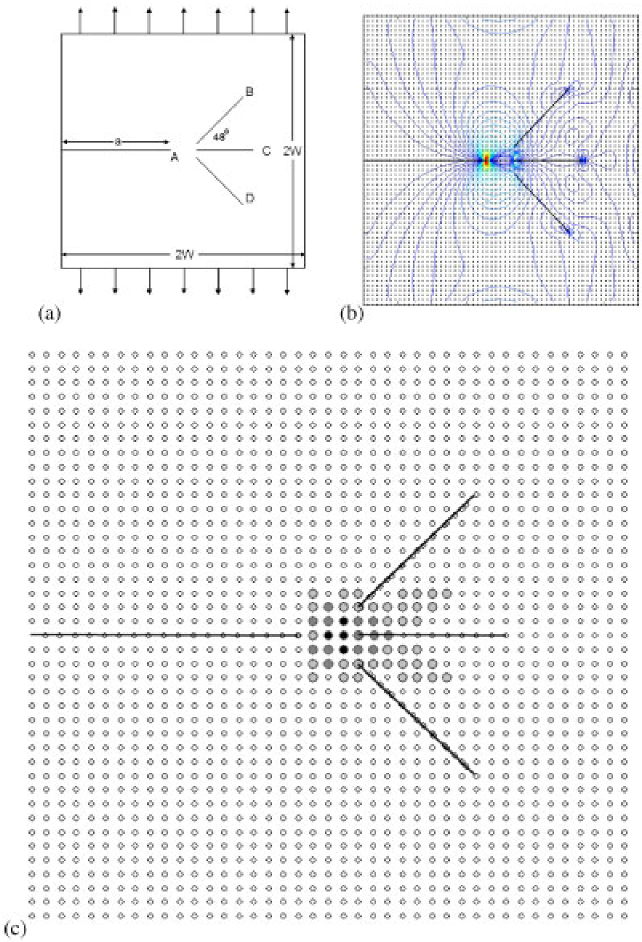

Figure 7. (a) Finite-size plate with four cracks under normal loading; (b) Von Mises stress distribution for 61×61 mesh; and (c) Mesh distribution with 41×41 regular nodes. Filled black, dark grey, grey nodes represent nodes whose domains of influence intersect four, three and two cracks, respectively. Defining the origin in the middle point of the specimen in (a) and W equal 2, then the coordinates of cracks A, B, C and D tips are (0.2,0)–$(-2, 0)$; (0.2,0.2)–(1,1); (0.2,0,1.2,0) and $(0.2, -0.2)$–$(1, -1)$, respectively.

System of four cracks in finite plate under normal load

In the next example, we consider a system of four interacting cracks (Figure 7). In contrast to the previous case, these cracks are not intersecting. They are positioned so that their tips are located in close proximity to each other. Such crack configuration poses a significant challenge to numerical methods because all four adjacent crack tips are located in the stress singularity dominated area of each other. Moreover, application of different mesh refinement and enrichment techniques is limited in this case. Also a huge number of nodes (131×131 regular nodes) are required to avoid more then one crack in any nodal domain of influence

Table IV. Normalized stress intensity factors for problem in Figure 7.

Mesh	a/h	F_{I}	E, %	N
41×41	22.22	2.787641	0.4819	59
51×51	27.78	2.813881	0.4549	50
61×61	33.33	2.810926	0.3494	46
71×71	38.89	2.801350	0.0075	39
81×81	44.44	2.801140		32

E represents the percent difference of normalized stress intensity factors F_{I}, with reference F_{I} calculated for 81×81 nodes. N represents number of nodes whose domain of influence intersect two, three or four cracks.

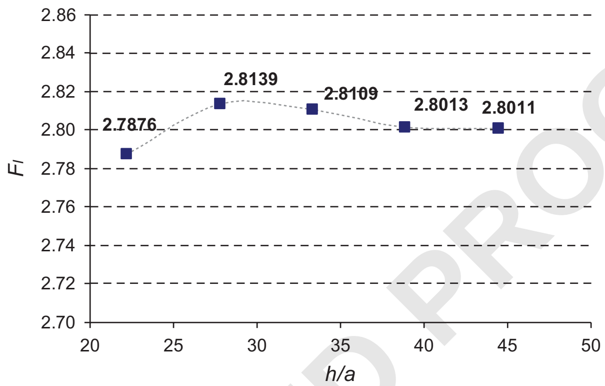

Figure 8. Convergence of the normalized stress intensity factors F_{I} as a function of average nodal spacing.

and to provide enough degrees of freedom to accurately capture adjacent crack singularities. This makes implementation of the solution by regular EFG method on a personal computer almost impractical.

Nevertheless, these difficulties can be minimized by MCW method. For the solution of the problem we use meshes with 41×41, 51×51, 61×61, 71×71 and 81×81 (case 1, 2, 3, 4 and 5, respectively) equally spaced nodes. In all five cases, there are several nodes that have two, three or four crack tips in their domain of influence (Figure 7(c), Table IV, column N). The results of calculations show that the normalized stress intensity factor $F_{\mathrm{I}} = K_{\mathrm{I}}/\sigma\sqrt{\pi a}$ for cases 1–4 are within 99.5% agreement with the results obtained in case 5 with the largest number of nodes (Figure 8, Table IV). Considering Von Mises stress distribution (Figure 7(b)) one can see strong interaction between four adjacent crack tips which are located in the stress singularity dominated area of each other. The singularities at the crack tips are accurately captured and stresses are smooth despite complex crack geometry and relatively small number of nodes. Thus, the results of the numerical example shows that the number of nodes necessary to achieve an accurate solution with MCW method is far below minimal number of nodes (131×131) required by the regular EFG method in order to avoid more than one discontinuity in nodal domain of influence.

CONCLUSION

We create the multiple crack weight method for the construction of nodal weight functions around cracks. They simultaneously characterize all cracks present in the nodal domain of influence. This allows the solution of strongly interacting static and dynamic cracks without enormous mesh refinement and significantly reduces the computational efforts. In the first numerical example, the computational time for the MCW method was 14 times less compared with the EFG method. This occurred because coarser nodal distributions are required compared with the standard EFG method for comparable accuracy. The time difference is even larger when the system of many strongly interacting cracks covers a significant part of the domain.

The reliability and the accuracy of the new technique for analysing multiple crack interactions was demonstrated by solving several problems and comparing the calculated normalized stress intensity factors with available reference solutions. The solution for the case of the double-edge collinear cracks in a finite plate reveals good agreement with three available reference solutions (Table I). For all the four considered cases, the solution was in 99% agreement with at least one of the references solutions for that case. Calculations for a star-shaped crack in a finite plate under bi-axial loading give good agreement compared with reference results obtained by X-FEM in Reference [24] and by Cheung *et al*. [25] (Table III). There is less than a 0.5% difference between the present calculation and at least one of the three reference values for the normalized stress intensity factors F_{I}^{A} and F_{I}^{B}, in all the cases with one exception each for F_{I}^{A} and F_{I}^{B}. In the third example, the solution of a complex problem about four interacting cracks showed convergence and accurate capturing of stress singularities using EFG and MCW methods with a number of nodes which is significantly smaller than one required by regular EFG method.

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