

## Multiple Filamentation of Circularly Polarized Beams

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We derive a new system of equations that describes the propagation of circularly polarized laser beams in a Kerr medium. Analysis and simulations of this system show that multiple filamentation is suppressed for circularly polarized beams.

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In recent years there has been a growing interest in the propagation of laser beams whose power is many times the threshold power  $P_c$  for self-focusing [1–5]. At such high intensities self-focusing dynamics is considerably more complex than at powers moderately above  $P_c$ , as it is accompanied by additional nonlinear phenomena such as multiple filamentation (MF), i.e., beam breakup into several long and narrow filaments [2,6], multiphoton ionization, and supercontinuum generation. Experiments have revealed considerable differences in these nonlinear phenomena between circularly and linearly polarized beams. For example, Meyer [7] observed two-photon induced Raman scattering with linearly polarized input beams but not with circularly polarized ones; Zilio *et al.* [3] showed that circular polarization suppresses photoassociative ionization much more than linear polarization; Sandhu *et al.* [4] showed that supercontinuum generation is highly suppressed with circular polarization, and Petit *et al.* [5] showed that multiphoton ionization is less efficient with circular polarization. It has been shown theoretically that the ionization potential for circularly polarized beams is considerably higher than for linearly polarized beams [8]. At present, however, there is no satisfactory theory that can explain all the above differences between self-focusing of circularly and linearly polarized beams. In this Letter we point out the key role played by the following difference between the two polarization states: Linear polarization breaks up the beam's cylindrical symmetry, whereas circular polarization does not. We show that, as a result, circular polarization is more likely to suppress MF than linear polarization. Because MF limits the power that each filament can carry, it is possible that our results on MF can shed light on other differences between linear and circular polarizations. In addition, our results suggest that it may be beneficial to use circularly polarized beams in applications where there is a need to suppress MF.

The propagation of intense cw laser beams in a Kerr medium is governed by the vectorial nonlinear Helmholtz equations [see Eqs. (2) and (3)]. Under the assumption that the beam is linearly polarized, i.e., that its electric field  $\vec{\mathcal{E}} = (\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3)$  can be described by  $\vec{\mathcal{E}} = (\mathcal{E}_1(x, y, z), 0, 0)$ , using the slowly varying envelope

approximation  $\mathcal{E}_1 = A_1(x, y, z)e^{ik_0z}$  and the paraxial approximation  $(A_1)_{zz} \ll k_0(A_1)_z$ , beam propagation is governed by the dimensionless nonlinear Schrödinger equation (NLS),

$$\begin{aligned} i(A_1)_z + \Delta_{\perp}A_1 + |A_1|^2A_1 &= 0, \\ A_1(x, y, z = 0) &= A_1^0(x, y), \end{aligned} \quad (1)$$

where  $\Delta_{\perp} = \partial_{xx} + \partial_{yy}$ . In 1965 Kelley used the NLS (1) to predict the existence of a threshold power  $P_c$ , such that when the input beam power  $P(0) = \int |A_1^0(x, y)|^2 dx dy$  is above  $P_c$ , the beam would collapse after a finite propagation distance [9]. The existence of a threshold power was confirmed experimentally, providing support to the validity of the NLS model. This model was, however, less successful in explaining the phenomena of MF for the following reason. Because the NLS is isotropic in the transverse  $(x, y)$  plane, when the input beam is cylindrically symmetric, i.e.,  $A_1(x, y, z = 0) = A_1^0(r)$  where  $r = \sqrt{x^2 + y^2}$ , then according to the NLS the beam would remain cylindrically symmetric during propagation. In contrast, during MF the beam's cylindrical symmetry completely breaks down. Therefore, it is natural to ask what physical mechanism is responsible for the breakup of cylindrical symmetry that leads to MF. For over 35 years the standard (and only) explanation for MF, due to Bespalov and Talanov [10], has been that MF is initiated by *random* noise in the input beam profile. In [11,12] we pointed out that at the vectorial Helmholtz level a linearly polarized input beam cannot be cylindrically symmetric as a vectorial entity, because it has a preferred direction in the transverse plane, namely, the direction of linear polarization. We also showed that this vectorial-induced symmetry breaking can lead to MF even when the linearly polarized input beams are cylindrically symmetric, i.e., when  $\mathcal{E}_1(x, y, z = 0) = \mathcal{E}_1^0(r)$  and  $\mathcal{E}_2(x, y, z = 0) \equiv 0$ . This raises the question whether these vectorial effects can also lead to MF of circularly polarized input beams, since in that case the polarization state does not induce a preferred direction.

Let us first consider an ideal circularly polarized cylindrically symmetric input beam, i.e., when  $\mathcal{E}_+(x, y, z = 0) = \mathcal{E}_+^0(r)$  and  $\mathcal{E}_-(x, y, z = 0) = \mathcal{E}_-^0 \equiv 0$ , where

$\mathcal{E}_\pm = (\mathcal{E}_1 \pm i\mathcal{E}_2)/\sqrt{2}$  are the left (+) and right (-) circular components. Since such an input beam has no preferred direction in the  $(x, y)$  plane, according to the vector Helmholtz model the beam will remain cylindrically symmetric during propagation in an isotropic medium. Thus, unlike cylindrically symmetric linearly polarized input beams, circularly polarized ones would not undergo MF. This conclusion, however, does not imply that circularly polarized beams cannot undergo MF, because in practice an input beam is never perfectly circularly polarized nor is it perfectly cylindrically symmetric. Therefore, in the following we study whether small ellipticity of the input polarization or small imperfections in the input beam profile can lead to MF of circularly polarized beams.

Our starting point is the vector nonlinear Helmholtz equations for the propagation of cw laser beams in a Kerr medium [13]

$$\begin{aligned} \Delta \vec{\mathcal{E}}(x, y, z) - \nabla(\nabla \cdot \vec{\mathcal{E}}) + k_0^2 \vec{\mathcal{E}} &= -\frac{k_0^2}{k_0^2 \epsilon_0 n_0^2} \vec{\mathcal{P}}_{\text{NL}}, \\ \nabla \cdot \vec{\mathcal{E}} &= -\frac{1}{\epsilon_0 n_0^2} \nabla \cdot \vec{\mathcal{P}}_{\text{NL}}, \end{aligned} \quad (2)$$

where  $\vec{\mathcal{P}}_{\text{NL}}$  is the nonlinear polarization vector,  $\epsilon_0$  is vacuum permittivity, and  $n_0$  is the linear index of refraction. When the Kerr medium is isotropic and homogeneous, the nonlinear polarization vector is given by [13,14]

$$\begin{aligned} i(A_+)_z + \Delta_\perp A_+ + \frac{1}{1+\gamma} |A_+|^2 A_+ &= -\frac{1+2\gamma}{1+\gamma} |A_-|^2 A_+ - \frac{1}{4} f^2 (A_+)_{zz} \\ &\quad - \frac{f^2}{2(1+\gamma)} [4|\nabla_\perp A_+|^2 A_+ + (\nabla_\perp A_+)^2 A_+^* + |A_+|^2 \Delta_\perp A_+ + A_+^2 \Delta_\perp A_+^*], \end{aligned} \quad (5a)$$

$$i(A_-)_z + \Delta_\perp A_- + \frac{1+2\gamma}{1+\gamma} |A_+|^2 A_- = 0, \quad (5b)$$

where  $|\nabla_\perp A_+|^2 = |A_{+,x}|^2 + |A_{+,y}|^2$  and  $(\nabla_\perp A_+)^2 = A_{+,x}^2 + A_{+,y}^2$ . The left-hand side of Eq. (5a) is the NLS for  $A_+$ . The right-hand side terms of (5a), from left to right, correspond to the effects on  $A_+$  of the  $O(f^4, \epsilon^2)$  coupling to  $A_-$ , the  $O(f^2)$  nonparaxiality, and the  $O(f^2)$  coupling to  $A_3$ .

We note that previous studies of circularly polarized beams followed Close *et al.* [16] who neglected  $\mathcal{E}_3$  and the grad-div term in (2). Most studies also applied the paraxial approximation and obtained the coupled  $(A_+, A_-)$  system

$$\begin{aligned} i(A_\pm)_z + \Delta_\perp A_\pm + \frac{1}{1+\gamma} [|A_\pm|^2 + (1+2\gamma)|A_\mp|^2] A_\pm \\ = 0. \end{aligned} \quad (6)$$

Comparison with (5) shows that system (6) might lead to wrong conclusions regarding MF and other aspects of self-focusing of circularly polarized beams (e.g., polarization

$$\begin{aligned} \vec{\mathcal{P}}_{\text{NL}}(\vec{\mathcal{E}}) &= \frac{4\epsilon_0 n_0 \bar{n}_2}{1+\gamma} [(\vec{\mathcal{E}} \cdot \vec{\mathcal{E}}^*) \vec{\mathcal{E}} + \gamma(\vec{\mathcal{E}} \cdot \vec{\mathcal{E}}) \vec{\mathcal{E}}^*] \\ &= \frac{4\epsilon_0 n_0 \bar{n}_2}{1+\gamma} [(|\mathcal{E}_+|^2 + |\mathcal{E}_-|^2 + |\mathcal{E}_3|^2) \vec{\mathcal{E}} \\ &\quad + \gamma(2\mathcal{E}_+ \mathcal{E}_- + \mathcal{E}_3^2) \vec{\mathcal{E}}^*], \end{aligned} \quad (3)$$

where  $\vec{\mathcal{E}}^*$  is the complex conjugate of  $\vec{\mathcal{E}}$ ,  $\bar{n}_2$  is the Kerr coefficient, and  $\gamma$  is a constant whose value depends on the physical origin of the Kerr effect [15]. Because of the grad-div term in Eq. (2) and the vectorial Kerr relation (3),  $\mathcal{E}_+$  is both linearly and nonlinearly coupled to  $\mathcal{E}_-$  and to  $\mathcal{E}_3$ .

Let us consider the case where the input beam is nearly left-circularly polarized, i.e.,  $\mathcal{E}_-^0/\mathcal{E}_+^0 = O(\epsilon)$ , where  $\epsilon \ll 1$  is the ellipticity parameter. We nondimensionalize the variables according to  $\tilde{x} = x/r_0$ ,  $\tilde{y} = y/r_0$ ,  $\tilde{z} = z/2L_{\text{DF}}$ , and  $(\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3) = (2r_0 k_0)^{-1} \sqrt{n_0/\bar{n}_2} (A_1, A_2, A_3) e^{ik_0 z}$ , where  $r_0$  is the input beam width and  $L_{\text{DF}} = k_0 r_0^2$  is the diffraction length. We also define  $A_\pm(x, y, z) = (A_1 \pm iA_2)/\sqrt{2}$ . Because  $\epsilon \ll 1$  and  $f = 1/r_0 k_0 \ll 1$ , we can use a systematic perturbation analysis, similar to the one in [12], to show that over propagation distances of a few diffraction lengths

$$\mathcal{E}_3/\mathcal{E}_+ = O(f) \quad \text{and} \quad \mathcal{E}_-/\mathcal{E}_+ = O(f^2, \epsilon). \quad (4)$$

Therefore, we conclude that to leading order the beam would remain circularly polarized during propagation. Relations (4) can be used to show that to leading order the vector nonlinear Helmholtz system (2) and (3) reduces to the coupled  $(A_+, A_-)$  system

stability, collapse, etc). For example, our analysis shows that if  $\epsilon \ll f$  then  $\mathcal{E}_3 \gg \mathcal{E}_-$ , which means that in this case (6) includes the weak effect of coupling to  $\mathcal{E}_-$  while neglecting the stronger effect of coupling to  $\mathcal{E}_3$ . Similarly, solutions of (6) can collapse, whereas the  $O(f^2)$  terms in (5) arrest collapse [12,17].

Returning to the issue of MF, we note that the  $(A_+, A_-)$  system (5) is *isotropic* in the transverse plane. Therefore, when the left- and right-circular components of the input beam are each cylindrically symmetric, i.e.,  $A_\pm^0(x, y) = A_\pm^0(r)$ , then according to (5) the beam would remain cylindrically symmetric during propagation (see Fig. 1a with  $\epsilon = 0.05$ ). Indeed, our calculations show that the symmetry breaking terms that were neglected in (5a) scale like  $\epsilon f^2$ . For comparison, when the input beam is linearly polarized, the corresponding equation for  $A_1$  has  $O(f^2)$  symmetry breaking terms that result from the couplings between  $\mathcal{E}_1$  and  $\mathcal{E}_3$  [12]. Thus, the symmetry breaking terms in the case of nearly circularly polarized beams are

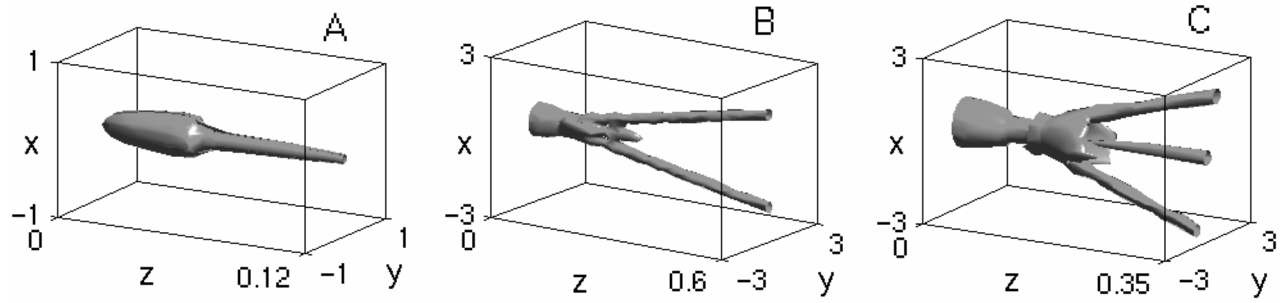


FIG. 1. Isointensity plots of  $A_+(x, y, z)$ . (a) Equations (5) with  $\gamma = 1/2$ ,  $f = 0.01$  and  $A_{\pm}^0(r) = ce^{-r^2}(1 \pm e^{0.05i})$  with  $P(0) = 5P_c$ . (b) Equation (7) with  $\gamma = 1/2$ ,  $f = 0.05$  and  $A^0(x, y) = ce^{-(0.9x)^2 - y^2}$  with  $P(0) \approx 8.3P_c$ . (c) Same as (b) with  $A^0(x, y) = ce^{-x^2 - y^2}[1 + 0.1 \times \text{noise}(x, y)]$  with  $P(0) \approx 10P_c$ .

considerably weaker than the corresponding ones for linearly polarized beams. For this reason we conclude that small ellipticity is unlikely to lead to MF of circularly polarized beams.

We now study whether imperfections in the input profile can lead to MF of circularly polarized beams. To do that, we note that when the input beam is perfectly circularly polarized (i.e.,  $\varepsilon = 0$ ) then (4) shows that  $\mathcal{E}_-/\mathcal{E}_+ = O(f^2)$ . Hence, system (5) reduces to the scalar equation

$$i(A_+)_{z} + \Delta_{\perp}A_+ + \frac{1}{1 + \gamma} |A_+|^2 A_+ = -\frac{1}{4} f^2 (A_+)_{zz} - \frac{f^2}{2(1 + \gamma)} [4|\nabla_{\perp}A_+|^2 A_+ + (\nabla_{\perp}A_+)^2 A_+^* + |A_+|^2 \Delta_{\perp}A_+ + A_+^2 \Delta A_+^*]. \quad (7)$$

Simulations of Eq. (7) show that MF can occur as a result of either input beam astigmatism (Fig. 1b) or input beam noise (Fig. 1c). We recall that input beam astigmatism and/or noise can lead to MF of linearly polarized beams. However, since the effective Kerr index of circularly polarized beams is smaller by  $(1 + \gamma)$  from that of linearly polarized ones, the threshold power for noise/astigmatism induced MF is somewhat higher for circularly polarized beams. In addition, the simulations in [12] suggest that the threshold power for MF of linearly polarized beams induced by vectorial effects is considerably lower than for noise/astigmatism induced MF. Therefore, we conclude that the threshold power for MF of circularly polarized beams is considerably higher than for linearly polarized ones.

Table I summarizes the possibility of MF under various input beam characteristics. To recap, ideal cylindrically symmetric circularly polarized input beams will not un-

dergo MF. Small ellipticity of the input polarization is unlikely to lead to MF of circularly polarized beams, whereas input beam noise/astigmatism can lead to MF. Therefore, suppression of MF of circularly polarized beams should focus on producing a cylindrically symmetric input beam, rather than on producing perfect circular polarization. In contrast, one cannot suppress MF of linearly polarized beams by producing a clean cylindrically symmetric input beam. Finally, circularly polarized beams are less likely to undergo MF than linearly polarized beams.

TABLE I. Possibility of MF under various input beam characteristics.

	Linear Polarization	Circular Polarization
Perfect polarization state and cylindrically symmetric profile	yes (deterministic)	no
Small deviation from preferential polarization state	yes	unlikely
Profile imperfections (noise/astigmatism)	yes	yes

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