Multiple-Model Probability Hypothesis Density Filter for Tracking Maneuvering Targets

K. PUNITHAKUMAR, Student Member, IEEE

T. KIRUBARAJAN, Member, IEEE

A. SINHA
McMaster University

Tracking multiple targets with uncertain target dynamics is a difficult problem, especially with nonlinear state and/or measurement equations. With multiple targets, representing the full posterior distribution over target states is not practical. The problem becomes even more complicated when the number of targets varies, in which case the dimensionality of the state space itself becomes a discrete random variable. The probability hypothesis density (PHD) filter, which propagates only the first-order statistical moment (the PHD) of the full target posterior, has been shown to be a computationally efficient solution to multitarget tracking problems with a varying number of targets. The integral of PHD in any region of the state space gives the expected number of targets in that region.

With maneuvering targets, detecting and tracking the changes in the target motion model also become important. The target dynamic model uncertainty can be resolved by assuming multiple models for possible motion modes and then combining the mode-dependent estimates in a manner similar to the one used in the interacting multiple model (IMM) estimator. This paper propose a multiple-model implementation of the PHD filter, which approximates the PHD by a set of weighted random samples propagated over time using sequential Monte Carlo (SMC) methods. The resulting filter can handle nonlinear, non-Gaussian dynamics with uncertain model parameters in multisensor-multitarget tracking scenarios. Simulation results are presented to show the effectiveness of the proposed filter over single-model PHD filters.

I. INTRODUCTION

Tracking a maneuvering target using multiple models has been shown to be highly effective in many applications. In single-model filters, if the model used by the filter does not match the actual system dynamics, the filter will tend to diverge since the actual errors fall outside the range predicted by the filter using its error covariance [7]. Maneuvering targets might switch between different modes of operation, and tracking using a single-model filter might fail since the filter may be accurate for only one mode of operation. In multiple-model approaches, several filters, each matched to a different target motion mode, operate in parallel, and then the overall state estimate is given by a weighted sum of the estimates from each filter. In many target tracking problems with linear, Gaussian systems, the interacting multiple model (IMM) estimator [7–9], in which a bank of different hypothetical target motion models is used, has been proven to have better performance than the (single-model) Kalman filter. For example, in the target tracking benchmark problem [6], in which performances of different algorithms for tracking highly maneuvering targets were compared, the IMM estimator outperformed the Kalman and α–β filters and yielded one of the best solutions. Extensions of multiple models to particle filters [12, 30] have also resulted in better results for nonlinear, non-Gaussian systems than single-model particle filters.

Tracking multiple maneuvering targets is much more difficult than tracking a single maneuvering target since there is a challenge in correctly associating the measurements with the targets. In the literature, extensions of the IMM estimator to work with standard tracking algorithms have resulted in the IMMJPDA (IMM estimator with the joint probabilistic data association algorithm) [5, 10, 14], IMM/MHT (IMM estimator with the multiple hypothesis tracking algorithm) [16, 23] and the joint IMMPDA particle filter [11]. A drawback of these algorithms is that all of them involve the model-data association problem and the problem is further complicated in a cluttered environment.

The fully Bayesian perspective of multitarget problems, using techniques like grid approaches, are very computationally challenging and not of practical interest when the number of targets is large. One alternative, computationally efficient solution to multiple target tracking problems that avoids model-data association difficulties is the probability hypothesis density (PHD) filter [27]. The PHD is the first-order statistical moment of the full joint multitarget posterior distribution. The recursive PHD filter completely characterizes the statistics of the dynamic Poisson point process under the assumption that the predicted multitarget density is Poisson

distributed. Discussion about dynamic Poisson point processes in parametric estimation context can be found in [15, 21, 33]. The mathematical derivations of the PHD filter equations based on probability generating functionals and functional derivatives are given in [28]. The PHD is different from the ordinary probability density function (pdf) and it does not necessarily integrate to unity over the whole state space. In contrast, the integral of PHD over a region gives the expected number of targets in that region.

The PHD filter has been shown to be a computationally tractable method for unified group target detection, tracking and classification [24] and for cluster tracking [26]. Further, the PHD filter resembles the general single-target single-sensor Bayesian filtering, hence it can be implemented using any nonlinear filtering method that can handle multi-modal densities. For example, [17] describes a spectral compression-based PHD filter. A sequential Monte Carlo (SMC) implementation of the PHD filter [3] has been shown to be efficient in a cluttered environment, and a similar implementation [32] handled missed detections well. Particle system implementations of the PHD filter can also be found in [29, 35, 36].

This paper proposes a multiple-model PHD (MMPHD) filter to address the problem of tracking multiple maneuvering targets using the SMC approach. SMC approaches have the advantages of computational tractability [31] and provable convergence properties [4, 20], and are applicable under the most general circumstances since no assumptions need to be made on the form of the underlying probability density. The SMC approximation of the MMPHD filter is applicable to track multiple maneuvering targets with nonlinear, non-Gaussian dynamics. The implementation of the proposed filter is detailed, and an application is considered. The results show the effectiveness of the proposed filter over single-model PHD filters.

This paper is organized as follows. Section II summarizes the PHD filter. Section III describes the problem of tracking multiple maneuvering targets, and includes an illustration of the MMPHD filter. In Section IV, a target tracking example is presented with simulation results to compare the performance of the MMPHD filter with that of single-model PHD filters. Finally, conclusions are given in Section V.

II. PHD FILTER

In tracking multiple targets, if the number of targets is unknown and varying with time, it is not possible to compare states with different dimensions using ordinary Bayesian statistics of fixed dimensional spaces. However, the problem can be addressed by using finite set statistics (FISST) [18, 25] to incorporate comparisons of state spaces of different dimensions. FISST facilitates the construction of multitarget densities from multiple-target transition functions by computing set derivatives of belief-mass functions [25], which makes it possible to combine states of different dimensions. The main practical difficulty with this approach is that the dimension of the full state space becomes large when many targets are present, which increases the computational load exponentially in the number of targets. Since the PHD is defined over the state space of one target in contrast to the full posterior distribution, which is defined over the state space of all the targets, the computational cost of propagating the PHD over time is much lower than propagating the full posterior density. A comparison of multitarget filtering using the complete FISST particle filter and the PHD particle filter in terms of computation and estimation accuracy is given in [32].

In general, a PHD-based multitarget tracker will experience more difficulty in resolving closely-spaced targets than the tracker based on the full target posterior. However, if the pdfs of individual targets is highly concentrated around their means compared with the target separation, so that the individual target pdfs do not overlap significantly, it will become possible to resolve targets using the PHD filter as well. A theoretical explanation about the capability of the PHD filter to resolve closely-spaced targets in Gaussian context is given in [27].

By definition, the PHD $D_{k|k}(\mathbf{x}_k \mid Z_{1:k})$, with single-target state vector $\mathbf{x}_k$, and given all the measurements up to time step k, is the density whose integral on any region S of the state space is the expected number of targets $N_{k|k}$ contained in S. That is,

$$N_{k|k} = \int_S D_{k|k}(\mathbf{x}_k \mid Z_{1:k}) d\mathbf{x}_k. \qquad (1)$$

Since this property uniquely characterizes the PHD and the first-order statistical moment of the full target posterior distribution possesses this property, the first-order statistical moment of the full target posterior is indeed the PHD. The first moment of the full target posterior, or the PHD, given all the measurement $Z_{1:k}$ up to time step k, is given by the set integral [27]

$$D_{k|k}(\mathbf{x}_k \mid Z_{1:k}) = \int f_{k|k}(\{\mathbf{x}_k\} \cup Y \mid Z_{1:k}) \delta Y. \qquad (2)$$

The reader is refered to [27] for detailed mathematical explanations about the PHD filter. The approximate expected target states are given by the local maxima of the PHD. The following section gives the prediction and update steps of one cycle of the PHD filter.

A. Prediction

In a general scenario of interest, there are target disappearances, target spawning, and entry of new targets. We denote the probability that a target with state $\mathbf{x}_{k-1}$ at time step $(k-1)$ will survive at time step k by $e_{k|k-1}(\mathbf{x}_{k-1})$, the PHD of spawned targets at time step k from a target with state $\mathbf{x}_{k-1}$ by $b_{k|k-1}(\mathbf{x}_k \mid \mathbf{x}_{k-1})$, and the PHD of newborn spontaneous targets at time step k by $\gamma_k(\mathbf{x}_k)$. Then, the predicted PHD is given by

$$D_{k|k-1}(\mathbf{x}_k \mid Z_{1:k-1}) = \gamma_k(\mathbf{x}_k)$$

$$+ \int [e_{k|k-1}(\mathbf{x}_{k-1})f_{k|k-1}(\mathbf{x}_k \mid \mathbf{x}_{k-1}) + b_{k|k-1}(\mathbf{x}_k \mid \mathbf{x}_{k-1})]$$

$$\times D_{k-1|k-1}(\mathbf{x}_{k-1} \mid Z_{1:k-1})d\mathbf{x}_{k-1} \qquad (3)$$

where $f_{k|k-1}(\mathbf{x}_k \mid \mathbf{x}_{k-1})$ denotes the single-target Markov transition density. The prediction equation (3) is lossless since there are no approximations.

B. Update

The predicted PHD can be corrected with the availability of measurements Z_k at time step k to get the updated PHD. We assume that the number of false alarms is Poisson distributed with the average rate of λ_k and that the probability density of the spatial distribution of false alarms is $c_k(\mathbf{z}_k)$. Let the detection probability of a target with state $\mathbf{x}_k$ at time step k be $p_D(\mathbf{x}_k)$. Then, the updated PHD at time step k is given by

$$D_{k|k}(\mathbf{x}_k \mid Z_{1:k}) \cong \left[\sum_{\mathbf{z}_k \in Z_k} \frac{p_D(\mathbf{x}_k)f_{k|k}(\mathbf{z}_k \mid \mathbf{x}_k)}{\lambda_k c_k(\mathbf{z}_k) + \psi_k(\mathbf{z}_k \mid Z_{1:k-1})} + (1 - p_D(\mathbf{x}_k)) \right]$$

$$\times D_{k|k-1}(\mathbf{x}_k \mid Z_{1:k-1}) \qquad (4)$$

where the likelihood function $\psi(\cdot)$ is given by

$$\psi_k(\mathbf{z}_k \mid Z_{1:k-1}) = \int p_D(\mathbf{x}_k)f_{k|k}(\mathbf{z}_k \mid \mathbf{x}_k)D_{k|k-1}(\mathbf{x}_k \mid Z_{1:k-1})d\mathbf{x}_k$$

$$(5)$$

and $f_{k|k}(\mathbf{z}_k \mid \mathbf{x}_k)$ denotes the single-sensor/single-target likelihood. The update equation (4) is not lossless since approximations are made on predicted multitarget posterior to obtain the closed-form solution (4). The reader is referred to [27] for further explanations.

III. MULTIPLE-MODEL PHD FILTER

This section first describes the mathematical formulation of the problem of tracking multiple maneuvering targets and the solution to it using the MMPHD algorithm. This is followed by the

implementation details of the algorithm using an SMC approach.

A. Problem Formulation

The general parameterized target dynamics of the jth target are given by

$$\mathbf{x}_k^j = a_{r,k}(\mathbf{x}_{k-1}^j, \mathbf{w}_{r,k-1}, r_k^j), \qquad j = 1, \ldots, N_k^X \qquad (6)$$

where $\mathbf{x}_k^j$ is the target state vector at time step k, N_k^X is the number of targets at time step k, $a_{r,k}$, in general, is a nonlinear function, $\mathbf{w}_{r,k-1}$ is the mode-dependent process noise vector of known statistics, and $r_k^j \in \{1, \ldots, N_r\}$ is the model index parameter governed by an underlying Markov process with the model transition probability

$$f_{k|k-1}(r_k = q \mid r_{k-1} = p) = h_{pq}. \qquad (7)$$

We denote the PHD of the mode-dependent full target state distribution at time step k by $D_{k|k}(\mathbf{x}_k, r_k \mid Z_{1:k})$. Once again we consider target survival, target spawning, and appearance of completely new targets given by $e_{k|k-1}(\cdot)$, $b_{k|k-1}(\cdot)$, and $\gamma_k(\cdot)$, respectively.

The measurements originate from either targets or clutter. We denote the total number of measurements at time step k by N_k^Z. The target-originated measurement due to the jth target on ith sensor is given by

$$\mathbf{z}_k^{i,j} = g_{r,k}(\mathbf{x}_k^j, \mathbf{v}_{r,k}, r_k^j) \qquad (8)$$

where $g_{r,k}$, in general, is a nonlinear function and $\mathbf{v}_{r,k}$ is a mode-dependent measurement noise vector of known statistics. Some targets may not be detected at time step k. We denote the probability that the target with state $\mathbf{x}_k$ is detected at time step k by $p_D(\mathbf{x}_k)$.

B. Multiple-Model Approach to Target Tracking

The multiple-model approach to tracking maneuvering targets by detecting maneuvers and identifying the appropriate model has been shown to be highly effective. In this approach, a finite number of filters operate in parallel, and the target motion is assumed to follow one of the models in the mode set of the tracker. Considering the compromise between complexity and performance, the IMM approach has been shown to be the most effective of the known multiple-model approaches, including the generalized pseudo-Bayesian (GPB) [1, 13] algorithms. A GPB algorithm of order n (GPB$_n$) requires N_r^n filters in its bank, where N_r is the number of models. The IMM estimator performs nearly as well as GPB$_2$, but requires only N_r filters operating in parallel. Thus, it has significantly less computational complexity, which is almost the same as that of GPB$_1$. Further, the IMM estimator does not require maneuver detection

decisions as in the case of variable state dimension (VSD) filter [7] algorithms, and undergoes a soft switching between models based on the updated mode probabilities.

The multiple-model approach used in the proposed MMPHD algorithm is structurally similar to that of the IMM estimator in the mixing and combination stages. The MMPHD filter also requires, implicitly, only N_r PHD filters to operate in parallel when there are N_r models describing target dynamics. Further, the MMPHD filter does not require a maneuver detection decision and undergoes a soft switching between the models. The IMM estimator differs from the multiple-model approach presented here in that the former uses only first- and second-order statistics of the target densities in the the mixing and combination stages. The techniques used in the IMM estimator cannot be applied to combine the mode-dependent PHD filter outputs since the densities are not necessarily Gaussian. Further, the densities might be multi-modal when they represent multiple targets; thus, it is not reasonable to approximate them by only first- and second-order statistics. Thus, the multiple-model approach presented here uses the branched true densities, i.e., the complete densities conditioned on each model, in the mixing and update stages. This approach can handle multi-modal target densities at the expense of increased computational load compared with the IMM estimator. One cycle of the recursive MMPHD algorithm can be described in three stages as follows.

1) *The Mixing Stage*: In this stage, each mode-matched filter is fed with a different density that is a combination of the previous mode-dependent densities. The initial density $\tilde{D}_{k|k-1}(\mathbf{x}_{k-1}, r_k = q \mid Z_{1:k-1})$ fed to the PHD filter, which is matched to the target model q, is calculated on the basis of Markovian model transition probability matrix $[h_{pq}]$ and mode-dependent prior density $D_{k-1|k-1}(\mathbf{x}_{k-1}, r_{k-1} = p \mid Z_{1:k-1})$, i.e.,

$$\tilde{D}_{k|k-1}(\mathbf{x}_{k-1}, r_k = q \mid Z_{1:k-1})$$
$$= \sum_{p=1}^{N_r} D_{k-1|k-1}(\mathbf{x}_{k-1}, r_{k-1} = p \mid Z_{1:k-1}) h_{pq},$$
$$q = 1, \ldots, N_r. \quad (9)$$

Target spawning, birth, and disappearance are not considered at the mixing stage and they are considered in the prediction stage. The densities $\tilde{D}_{k|k-1}(\cdot)$ and $D_{k-1|k-1}(\cdot)$ in (9) are similar to probability densities except that they do not integrate to unity. The mixing described in (9) is then similar to total probability theorem.

2) *The Prediction Stage*: Once the initial density for the PHD filter that is matched to target model q is calculated, the mode-dependent predicted density is calculated as

$$D_{k|k-1}(\mathbf{x}_k, r_k = q \mid Z_{1:k-1}) = \gamma_k(\mathbf{x}_k, r_k = q)$$
$$+ \int [e_{k|k-1}(\mathbf{x}_{k-1}) f_{k|k-1}(\mathbf{x}_k \mid \mathbf{x}_{k-1}, r_k = q)$$
$$+ b_{k|k-1}(\mathbf{x}_k \mid \mathbf{x}_{k-1}, r_k = q)]$$
$$\times \tilde{D}_{k|k-1}(\mathbf{x}_{k-1}, r_k = q \mid Z_{1:k-1}) d\mathbf{x}_{k-1}. \quad (10)$$

The prediction of the mode-dependent PHD cannot be just applied to that of the single-model PHD, which is matched to the target model, because the PHD of spontaneous target birth $\gamma_k(\cdot)$ and target spawning $b_{k|k-1}(\cdot)$ are also mode dependent. The integral of the mode-dependent PHD $D_{k|k-1}(\mathbf{x}_k, r_k = q \mid Z_{1:k-1})$ over a region gives the expected (predicted) number of targets in that region assuming all the targets are travelling with the target dynamics described by model q.

3) *The Update Stage*: With the availability of measurements at time step k, the mode-dependent updated density is calculated as (for $q = 1, \ldots, N_r$)

$$D_{k|k}(\mathbf{x}_k, r_k = q \mid Z_{1:k})$$
$$\cong \left[\sum_{\mathbf{z}_k \in Z_k} \frac{p_D(\mathbf{x}_k) f_{k|k}(\mathbf{z}_k \mid \mathbf{x}_k, r_k = q)}{\lambda_k c_k(\mathbf{z}_k) + \psi_k(\mathbf{z}_k \mid Z_{k-1})} + (1 - p_D(\mathbf{x}_k)) \right]$$
$$\times D_{k|k-1}(\mathbf{x}_k, r_k = q \mid Z_{1:k-1}) \quad (11)$$

where the likelihood function $\psi(\cdot)$ is given by

$$\psi_k(\mathbf{z}_k \mid Z_{1:k-1})$$
$$= \int p_D(\mathbf{x}_k) f_{k|k}(\mathbf{z}_k \mid \mathbf{x}_k, r_k = q) D_{k|k-1}(\mathbf{x}_k, r_k = q \mid Z_{1:k-1}) d\mathbf{x}_k.$$
$$(12)$$

Although there is no explicit mode probability update in the MMPHD filter, the update stage implicitly updates the mode probabilities as well. Therefore, the updated mode probability for a particular model can be calculated by integrating the mode-dependent updated PHD divided by the total expected number of targets. The total number of targets can be found by summing up all the integrals of the updated mode-dependent PHDs. The updated mode probability for a particular target, assuming that the target densities do not overlap, can be found by integrating the mode-dependent PHD over the region in which that target is represented. However, it is not necessary to calculate the numerical value of the updated mode probability in the recursive MMPHD filter since the mode-dependent density as a whole is used in the mixing stage to feed the filter.

Using multiple-model algorithms for benign nonmaneuvering targets might degrade the performance of the tracker and increases the computational load. However, with higher target

maneuverability, a multiple-model approach is needed. The decision about whether a multiple model is preferred or not is done based on the maneuvering index [7], which quantifies the maneuverability of the target in terms of the process noise, sensor measurement noise, and sensor revisit interval. A study that compares the IMM estimator with the Kalman filter based on the maneuvering index is given in [22].

C. Particle Implementation

This section describes the SMC approach to the MMPHD filter. This approach provides a mechanism to represent the posterior MMPHD by a set of random samples or particles, which consists of state and model information with associated weights. The key idea in this implementation is to include the model index parameters in the sample set with the associated weights to represent the mode-dependent posterior density. Although it does not appear explicitly, there are several PHD filters, each matched to different target dynamics, running in parallel in the MMPHD filter. The model index parameter in a sample directs the MMPHD filter to choose the PHD filter that matches the associated target model. Further, the number of samples used in each PHD filter is not necessarily the same for different target models. Since the mode probabilities are updated through the update step of the filter, the PHD filter that matches the target motion will contain the higher number of samples. This makes the filter computationally efficient compared with a filter that uses an equal number of samples to represent the PHD matched to each model.

The particle filter implementation of the proposed algorithm can be considered as a special case of the algorithm given in [3] with some modifications. The target state is augmented with the model index parameter and the prediction and update operators of the algorithm given in [3] are modified in order to include target motion model uncertainty.

The SMC implementation considered here is structurally similar to the sampling importance resampling (SIR) type of particle filter [2]. Let the posterior MMPHD $D_{k-1|k-1}(\mathbf{x}_{k-1}, r_{k-1} \mid Z_{1:k-1})$ be represented by a set of particles $\{w_{k-1}^{(s)}, \mathbf{x}_{k-1}^{(s)}, r_{k-1}^{(s)}\}_{s=1}^{L_{k-1}}$. That is,

$$D_{k-1|k-1}(\mathbf{x}_{k-1}, r_{k-1} \mid Z_{1:k-1})$$

$$= \sum_{s=1}^{L_{k-1}} w_{k-1}^{(s)} \delta(\mathbf{x}_{k-1} - \mathbf{x}_{k-1}^{(s)}, r_{k-1} - r_{k-1}^{(s)}) \quad (13)$$

where $\delta(\cdot)$ is the Dirac Delta function.

In contrast to particle filters, the total weight $\sum_{s=1}^{L_{k-1}} w_{k-1}^{(s)}$ is not equal to one; instead, it gives the expected number of targets n_{k-1}^X at time step $(k-1)$,

which follows from the property that the integral of the PHD over the state space gives the expected number of targets. Further, the posterior model probabilities corresponding to a particular target are approximately equal to the proportion of the total sample weights in the index set $\{r_{k-1}^{(s)}\}_{s=1}^{L_{k-1}}$ for each model corresponding to that target.

1) *Prediction*: The MMPHD filter prediction step involves predicting the models in addition to state prediction. The model prediction for existing targets is performed based on the model transition probabilities $f_{k|k-1}(r_k \mid r_{k-1})$. Model samples $\{r_{k|k-1}^{(s)}\}_{s=1}^{L_{k-1}}$ from the model-predicted MMPHD $\tilde{D}_{k|k-1}(\mathbf{x}_{k-1}, r_k \mid Z_{1:k-1})$ are generated by importance sampling from a proposal density $\pi_k(\cdot \mid r_{k-1})$. We generate independent and identically distributed (IID) model samples $\{r_{k|k-1}^{(s)}\}_{s=L_{k-1}+1}^{L_{k-1}+J_k}$ corresponding to new spontaneously born targets by sampling from another proposal density $\beta_k(\cdot)$. Then,

$$r_{k|k-1}^{(s)} \sim \begin{cases} \pi_k(\cdot \mid r_{k-1}) & s = 1, \ldots, L_{k-1} \\ \beta_k(\cdot) & s = L_{k-1}+1, \ldots, L_{k-1}+J_k \end{cases}$$

$$(14)$$

where $\pi_k(\cdot \mid r_{k-1})$ and $\beta_k(\cdot)$ are probability mass functions. Then, a discrete weighted approximation to the model predicted MMPHD $\tilde{D}_{k|k-1}(\mathbf{x}_{k-1}, r_k \mid Z_{1:k-1})$ is given by

$$\tilde{D}_{k|k-1}(\mathbf{x}_{k-1}, r_k \mid Z_{1:k-1}) = \sum_{s=1}^{L_{k-1}+J_k} \varpi_{k|k-1}^{(s)} \delta(\mathbf{x}_{k-1} - \mathbf{x}_{k-1}^{(s)}, r_k - r_{k|k-1}^{(s)})$$

$$(15)$$

where

$$\varpi_{k|k-1}^{(s)} = \begin{cases} \dfrac{f_{k|k-1}(r_{k|k-1}^{(s)} \mid r_{k-1}^{(s)})}{\pi_k(r_{k|k-1}^{(s)} \mid r_{k-1}^{(s)})} w_{k-1}^{(s)} \\ \qquad\qquad\qquad\qquad s = 1, \ldots, L_{k-1} \\ \dfrac{\theta_k(r_{k|k-1}^{(s)})}{\beta_k(r_{k|k-1}^{(s)})} \dfrac{1}{J_k} \\ \qquad\qquad\qquad s = L_{k-1}+1, \ldots, L_{k-1}+J_k \end{cases}.$$

$$(16)$$

In (16), the probability mass function $\theta_k(\cdot)$ denotes the model distribution of spontaneous target births at time step k. The number of new particles J_k can be a function of time step k to accommodate a varying number of targets.

We now apply importance sampling to generate state samples that approximate the predicted MMPHD $D_{k|k-1}(\mathbf{x}_k, r_k \mid Z_{1:k-1})$. We generate $\{\mathbf{x}_{k|k-1}^{(s)}\}_{s=1}^{L_{k-1}}$ state samples from the proposal density $q_k(\cdot \mid \mathbf{x}_{k-1}, r_{k|k-1}, Z_k)$ and IID state samples $\{\mathbf{x}_{k|k-1}^{(s)}\}_{s=L_{k-1}+1}^{L_{k-1}+J_k}$ corresponding to new spontaneously born targets from another proposal density $p_k(\cdot \mid r_{k|k-1}, Z_{k-1})$. That is,

$$\mathbf{x}_{k|k-1}^{(s)} \sim \begin{cases} q_k(\cdot \mid \mathbf{x}_{k-1}, r_{k|k-1}, Z_k) & s = 1, \ldots, L_{k-1} \\ p_k(\cdot \mid r_{k|k-1}, Z_k) & s = L_{k-1}+1, \ldots, L_{k-1}+J_k \end{cases}. \tag{17}$$

Then, the weighted approximation of the predicted MMPHD is given by

$$D_{k|k-1}(\mathbf{x}_k, r_k \mid Z_{1:k-1}) = \sum_{s=1}^{L_{k-1}+J_k} w_{k|k-1}^{(s)} \delta(\mathbf{x}_k - \mathbf{x}_{k|k-1}^{(s)}, r_k - r_{k|k-1}^{(s)}) \tag{18}$$

where

$$w_{k|k-1}^{(s)} = \begin{cases} \dfrac{e_{k|k-1}(\mathbf{x}_{k|k-1}^{(s)}) f_{k|k-1}(\mathbf{x}_{k|k-1}^{(s)} \mid \mathbf{x}_{k-1|k-1}^{(s)}, r_{k|k-1}^{(s)}) + b_{k|k-1}(\mathbf{x}_{k|k-1}^{(s)} \mid \mathbf{x}_{k-1|k-1}^{(s)}, r_{k|k-1}^{(s)})}{q_k(\mathbf{x}_{k|k-1}^{(s)} \mid \mathbf{x}_{k|k-1}^{(s)}, r_{k|k-1}^{(s)}, Z_k)} \varpi_{k|k-1}^{(s)} & s = 1, \ldots, L_{k-1} \\[4mm] \dfrac{\gamma_k(\mathbf{x}_{k|k-1}^{(s)} \mid r_{k|k-1}^{(s)})}{p_k(\mathbf{x}_{k|k-1}^{(s)} \mid r_{k|k-1}^{(s)}, Z_k)} \varpi_{k|k-1}^{(s)} & s = L_{k-1}+1, \ldots, L_{k-1}+J_k \end{cases}. \tag{19}$$

The functions that characterize the Markov target transition density $f_{k|k-1}(\cdot)$, target spawning $b_{k|k-1}(\cdot)$ and entry of new targets $\gamma_k(\cdot)$ in (19) are conditioned on the particular motion model. Thus, although it is not explicitly explained by the equations, there are N_r PHD filters running in parallel in this implementation.

2) *Update*: With the available set of measurements Z_k at time step k, the updated particle weights can be calculated by

$$w_k^{*(s)} =$$

$$\left[(1 - p_D(\mathbf{x}_{k|k-1}^{(s)})) + \sum_{i=1}^{N_k^Z} \frac{p_D(\mathbf{x}_{k|k-1}^{(s)}) f_{k|k}(\mathbf{z}_k^i \mid \mathbf{x}_{k|k-1}^{(s)}, r_{k|k-1}^{(s)})}{\lambda_k c_k(\mathbf{z}_k^i) + \Psi_k(\mathbf{z}_k^i)} \right] w_{k|k-1}^{(s)}$$

where $\tag{20}$

$$\Psi_k(z_k^i) = \sum_{s=1}^{L_{k-1}+J_k} p_D(\mathbf{x}_{k|k-1}^{(s)}) f_{k|k}(\mathbf{z}_k^i \mid \mathbf{x}_{k|k-1}^{(s)}, r_k^{(s)}) w_{k|k-1}^{(s)}. \tag{21}$$

The single-target/single-sensor measurement likelihood function $f_{k|k}(\cdot)$ in (20) and (21) is written as dependent on the model, considering a general case in which the measurement model can also be mode dependent.

3) *Resample*: To perform resampling, since the weights are not normalized to unity in PHD filters, the expected number of targets is calculated by summing up the total weights, i.e.,

$$\hat{n}_k^X = \sum_{s=1}^{L_{k-1}+J_k} w_k^{*(s)}. \tag{22}$$

Then the updated particle set $\{w_k^{*(s)}/n_k^X, \mathbf{x}_{k|k-1}^{(s)}, r_{k|k-1}^{(s)}\}_{s=1}^{L_{k-1}+J_k}$ is resampled to get $\{w_k^{(s)}/n_k^X, \mathbf{x}_k^{(s)}, r_k^{(s)}\}_{s=1}^{L_k}$ such that the total weight after resampling remains

n_k^X. Now, the discrete approximation of the updated posterior MMPHD at time step k is given by

$$D_{k|k}(\mathbf{x}_k, r_k \mid Z_{1:k}) = \sum_{s=1}^{L_k} w_k^{(s)} \delta(\mathbf{x}_k - \mathbf{x}_k^{(s)}, r_k - r_k^{(s)}). \tag{23}$$

The mode-dependent posterior PHDs can be easily identified by grouping the particles based on model index parameters.

IV. SIMULATIONS

A. Scenario

This section presents a two-dimensional tracking example to compare the MMPHD filter's performance with that of the single-model PHD filters, namely, a constant-velocity model PHD filter and a coordinated-turn model PHD filter. The test scenario is constructed as follows.

1) The observations are taken from four fixed bearing-only sensors located at $(0,0)$ m, $(0, 1 \times 10^4)$ m, $(1 \times 10^4, 0)$ m, and $(1 \times 10^4, 1 \times 10^4)$ m. The measurements are available at discrete-time sampling interval $T = 60$ s. The target-generated measurements corresponding to target j on sensor i are given by

$$z_k^{i,j} = \tan^{-1}\left(\frac{y_k^j - y_S^i}{x_k^j - x_S^i} \right) + v_k^{i,j} \tag{24}$$

where $v_k^{i,j}$ is an IID sequence of zero-mean Gaussian variables with standard deviation 0.01 rad. (x_k^j, y_k^j) and (x_S^i, y_S^i) denote the locations of target j and sensor i at time step k, respectively. The average false alarm rate $\lambda_k = 4 \times 10^{-3}$ rad^{-1}, the volume of observation of each sensor $= 2\pi$ rad, the false alarms are distributed uniformly within the sensor field of view, and the
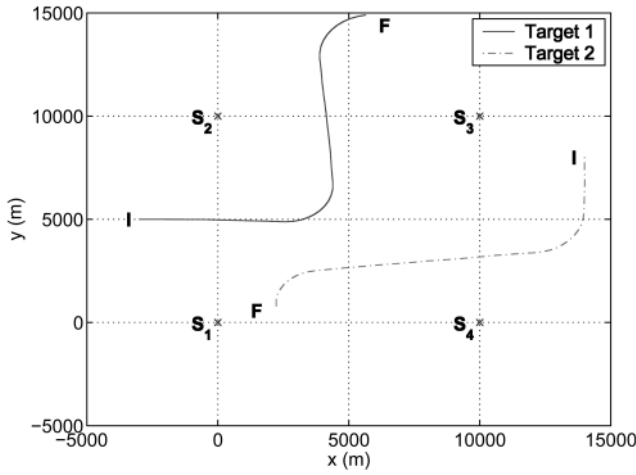
Fig. 1.   Target trajectory (I—initial, F—final, S_i—sensor).

probability of target detection $p_D(\mathbf{x}_k) = 1$, regardless of the target state $\mathbf{x}_k$.

2) Two maneuvering targets, namely, target 1 and target 2, are travelling from initial target positions $(-3 \times 10^3, 5 \times 10^3)$ m and $(1.4 \times 10^4, 8 \times 10^3)$ m, respectively. This simulation does not include any spawning of new targets from existing ones. Target 1 moves eastward for 20 min at a nearly constant velocity with an initial velocity of 5 ms^{-1}, before executing a 9°/min (which amounts to an acceleration of about 0.1 ms^{-2}) coordinated turn [7] in the anticlockwise direction for 10 min. Then it moves northward for another 20 min, followed by a clockwise 9°/min coordinated turn for 10 min. Target 2 moves southward for 10 min at a nearly constant velocity with an initial velocity of 5 ms^{-1}, before executing a coordinated turn of 9°/min in the clockwise direction for 10 min. Then it moves westward at nearly a constant velocity for 30 min followed by an anticlockwise coordinated turn of 9°/min for 10 min. The target trajectories are shown in Fig. 1. This scenario leads to a maneuvering index[1] of more than 1.

3) The single target Markov transition equation that characterizes the constant velocity target dynamics ($r_k = 1$) is given by

$$\mathbf{x}_k^j = A_{1,k}^j \mathbf{x}_{k-1}^j + \mathbf{w}_{1,k}^j \qquad (25)$$

where $\mathbf{x}_k^j = [x_k^j, \dot{x}_k^j, y_k^j, \dot{y}_k^j]$ is the state of the jth target, which consists of target position (x_k^j, y_k^j) and target velocity $(\dot{x}_k^j, \dot{y}_k^j)$ at time step k, and $\mathbf{w}_{1,k}^j$ is an i.i.d sequence of zero-mean Gaussian vectors with covariance Σ_k^j. The matrices $A_{1,k}^j$ and $\Sigma_{1,k}^j$ are given

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[1]The measurement contribution part of the posterior Cramér-Rao lower bound is used to obtain the measurement standard deviation in calculating the maneuvering index.

as follows

$$A_{1,k}^j = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (26)$$

$$\Sigma_{1,k}^j = \begin{bmatrix} \dfrac{T^3}{3} & \dfrac{T^2}{2} & 0 & 0 \\ \dfrac{T^2}{2} & T & 0 & 0 \\ 0 & 0 & \dfrac{T^3}{3} & \dfrac{T^2}{2} \\ 0 & 0 & \dfrac{T^2}{2} & T \end{bmatrix} l \qquad (27)$$

where $l = 1 \times 10^{-4}$ m^{2}s^{-3} is the level of the power spectral density of the corresponding continuous process noise.

4) The single-target Markov transition equation that characterizes the coordinated turn target dynamics ($r_k = 2$) is given by

$$\mathbf{x}_k^j = A_{2,k}^j \mathbf{x}_{k-1}^j + \mathbf{w}_{2,k}^j \qquad (28)$$

where $\mathbf{x}_k^j = [x_k^j, \dot{x}_k^j, y_k^j, \dot{y}_k^j, \Omega_k^j]$ is the augmented state vector of the jth target, which consists of target position (x_k^j, y_k^j), target velocity $(\dot{x}_k^j, \dot{y}_k^j)$, and turn rate Ω_k^j at time step k, and $\mathbf{w}_{2,k}^j$ is an i.i.d sequence of zero-mean Gaussian vectors with covariance $\Sigma_{2,k}^j$. The matrices $A_{2,k}^j$ and $\Sigma_{2,k}^j$ are given as follows

$$A_{2,k}^j = \begin{bmatrix} 1 & \dfrac{\sin\Omega_{k-1}^j T}{\Omega_{k-1}^j} & 0 & -\dfrac{1-\cos\Omega_{k-1}^j T}{\Omega_{k-1}^j} & 0 \\ 0 & \cos\Omega_{k-1}^j T & 0 & -\sin\Omega_{k-1}^j T & 0 \\ 0 & \dfrac{1-\cos\Omega_{k-1}^j T}{\Omega_{k-1}^j} & 1 & \dfrac{\sin\Omega_{k-1}^j T}{\Omega_{k-1}^j} & 0 \\ 0 & \sin\Omega_{k-1}^j T & 0 & \cos\Omega_{k-1}^j T & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (29)$$

$$\Sigma_{2,k}^j = \begin{bmatrix} \dfrac{T^3 l_1}{3} & \dfrac{T^2 l_1}{2} & 0 & 0 & 0 \\ \dfrac{T^2 l_1}{2} & T l_1 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{T^3 l_1}{3} & \dfrac{T^2 l_1}{2} & 0 \\ 0 & 0 & \dfrac{T^2 l_1}{2} & T l_1 & 0 \\ 0 & 0 & 0 & 0 & T l_2 \end{bmatrix} \qquad (30)$$

where the levels of the power spectral densities are $l_1 = 1 \times 10^{-4}$ m^{2}s^{-3} and $l_2 = 1 \times 10^{-10}$ rad^{2}s^{-3}. Since the turn rate Ω_k^j is unknown, it is included in the state vector and must be estimated.

5) The Markovian model transition probability matrix in the MMPHD filter is taken as

$$[h_{pq}] = \begin{bmatrix} 1 - \dfrac{T}{\tau_1} & \dfrac{T}{\tau_1} \\ \dfrac{T}{\tau_2} & 1 - \dfrac{T}{\tau_2} \end{bmatrix} \qquad (31)$$

with sojourn times [7] $\tau_1 = 20$ min and $\tau_2 = 10$ min. The initial model probabilities for both models are 0.5.

6) We assume that the probability of target survival = 0.99, the probability of target spawning = 0, and the probability of spontaneous target birth = 0.01. The birth model that describes the entry of new targets is taken as follows.

The x and y position elements of the state vector of newborn particles are normally distributed centered around every two intersecting measurements with a standard deviation of 200 m. This one point initialization is done to all intersecting measurement pairs within the surveillance region, which is a square region defined by $(-5 \times 10^3, -5 \times 10^3)$ m and $(1.5 \times 10^4, 1.5 \times 10^4)$ m as the lower left and upper right corners, respectively. The velocity and turn rate elements of the state vector for the newborn particles are distributed uniformly within the interval $[-5, 5]$ ms^{-1} and $[-0.02, 0.02]$ rads^{-1}, respectively.

7) 1000 particles are used to represent one target and $J_k = (10 \times$ number of intersecting measurement points within the surveillance region) particles are used to represent new spontaneously born targets in all the tracking algorithms. Before simulations begin, all PHD filters are initialized with 1000 particles normally distributed with mean $[-3 \times 10^3$ m, 5 ms^{-1}, 5×10^3 m, 0 ms^{-1}, 0 rads$^{-1}]$ and covariance diag$[4 \times 10^4$ m^2, 1 m^{2}s^{-2}, 4×10^4 m^2, 1 m^{2}s^{-2}, 4×10^{-4} rad^{2}s$^{-2}]$ representing target 1, and the model index parameters of the initial particles are taken as 1, i.e., the constant-velocity model, in the MMPHD filter.

8) The PHD filter does not provide a mechanism to get the target state estimates directly. One way of getting the estimates is by identifying the local maxima of the density. An expectation maximization based peak extraction algorithm to obtain the target state estimates can be found in [34]. For simplicity and to avoid added computational load we use the K-means clustering algorithm [19] to associate particles to targets[2] based on the position elements of the particle state vectors. The targets are associated to tracks using the nearest neighbor approach based on the mean of each target cluster.

Although we have assumed Gaussian process noise in this particular example, the MMPHD algorithm presented in Section III is able to deal with any process noise as long as the samples can be drawn from the process noise distribution. Since the PHD recursion given in this paper is valid only for single sensor, the measurements are grouped from each sensor and used to update the filter iteratively.

B. Simulation Results

Fig. 2 illustrates the model-switching property of the MMPHD filter by plotting number of particles in each model against time. The particles considered here are of equal weight, so the proportion of the particles in each model of a particular target gives the approximate posterior model probabilities of that target. Fig. 2 also illustrates the number of targets estimated from the PHD filter. At the beginning of the scenario, all PHD filters assume that there is only one track, i.e. target 1, within the surveillance region and the particles corresponding to target 1 in the MMPHD filter are shown in Fig. 2(a). However, it can be seen in Fig. 2(b) that the total number of particles corresponding to target 2 increases with time at the beginning until it reaches nearly 1000, which is the number of particles assigned to represent one target. Once the track is formed, the MMPHD filter clearly displays the model switching property for target 2 and assigns more particles to the model that matches with the target motion.

Figs. 3 and 4 show the root-mean-squared errors (RMSE) of position and velocity estimates, respectively. All figures indicate that the overall performance of the MMPHD filter is significantly better than that of both single-model PHD filters. Further, the single-model PHD filters show poor adaptation to target maneuvers and yield larger estimation errors. Fig. 5 compares the RMSEs of turn rate estimates of the MMPHD filter and the single-model PHD filter that uses the coordinated-turn model. The single-model PHD filter that uses the constant-velocity model is not considered since there is no turn rate estimate in that filter. Table I gives the average of the estimation errors; note that the values for the MMPHD filter show significant improvements over the single-model PHD filters.

The average computational times (based on 100 Monte Carlo runs) on dual 2.4 GHz Intel Xeon processors for a tracking period of 60 time steps are 205 s, 201 s, and 199 s with standard deviation 2.6 s, 1.7 s, and 2.5 s for multiple model, constant-velocity model, and coordinated-turn model PHD filter algorithms, respectively. All filters were implemented in MATLAB. Thus, if the same number of sample points are used, the MMPHD filter is only marginally more expensive compared with single-model PHD filters since the only additional calculation required in the MMPHD filter is to generate model index parameters in the mixing stage.

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[2]We assume that the targets are sufficiently far apart that a particle represents only one target at any particular time.
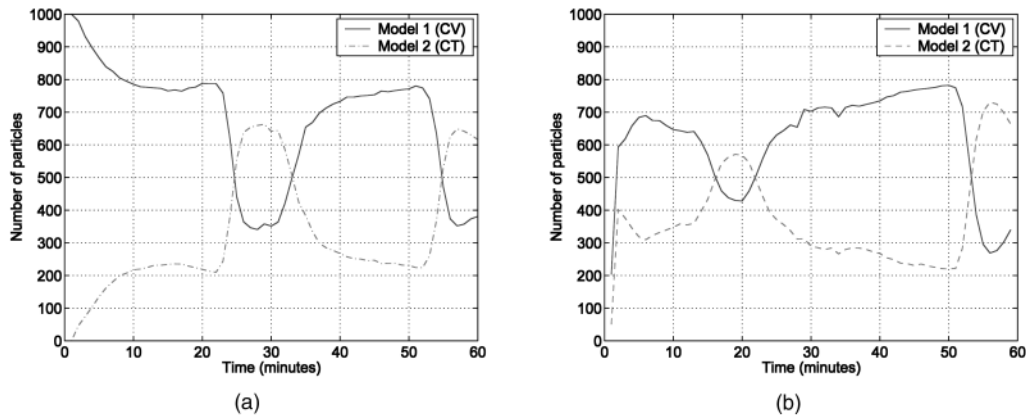
Fig. 2. Model switching in MMPHD filter (100 Monte Carlo runs). (a) Target 1. (b) Target 2.
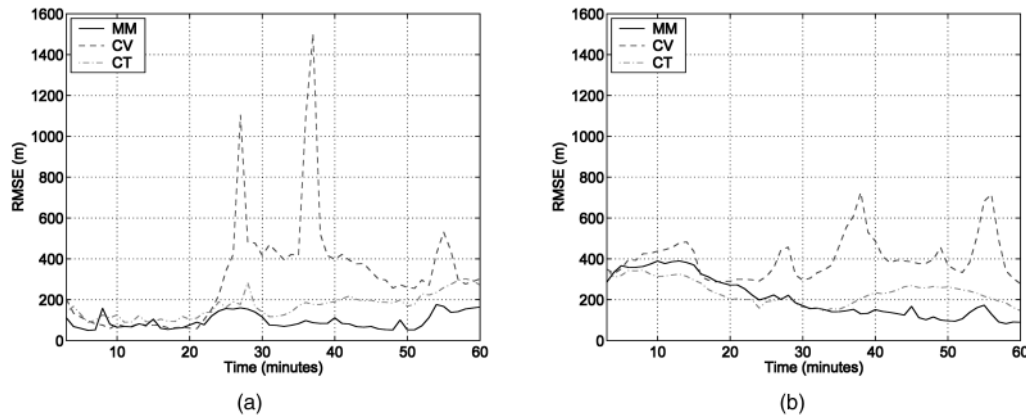

Fig. 3. Target position estimation RMSE (100 Monte Carlo runs). (a) Target 1. (b) Target 2.
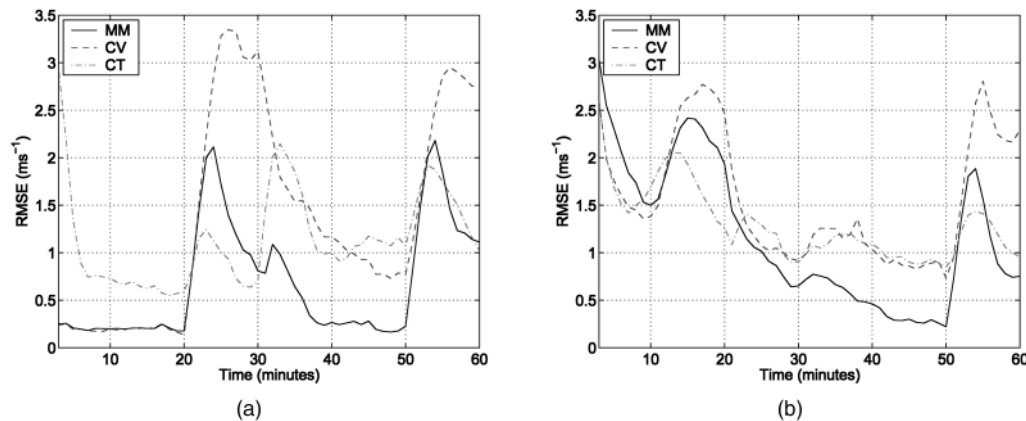

Fig. 4. Target velocity estimation RMSE (100 Monte Carlo runs). (a) Target 1. (b) Target 2.

V. CONCLUSIONS

This paper considered the problem of tracking multiple maneuvering targets with nonlinear, non-Gaussian dynamics in a cluttered environment. The filter proposed to solve this problem is a multiple-model extension to the PHD filter using the SMC approach. The PHD filter is a computationally efficient solution to multitarget tracking problems, and the SMC implementation makes the filter applicable in general scenarios with nonlinear, non-Gaussian

systems. This paper presented a new multiple-model approach to enhance the PHD filter's capability to handle target maneuvers.

The MMPHD filter was compared with single-model PHD filters. It resulted in significantly better tracking performance compared with the single-model PHD filters. The single-model PHD filters that matched only one kind of target dynamics showed poor adaptation to maneuvers, resulting in large estimation errors when the target maneuverability is high. We can conclude that a

PUNITHAKUMAR ET AL.: MULTIPLE-MODEL PROBABILITY HYPOTHESIS DENSITY FILTER   95
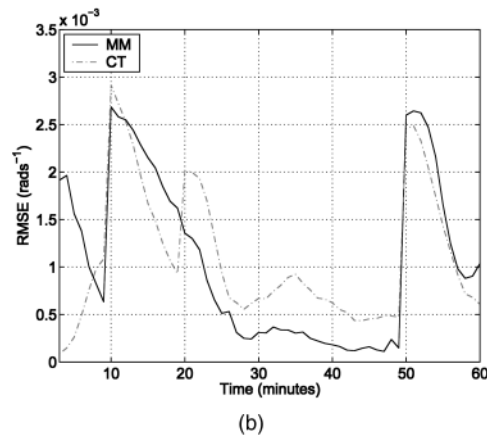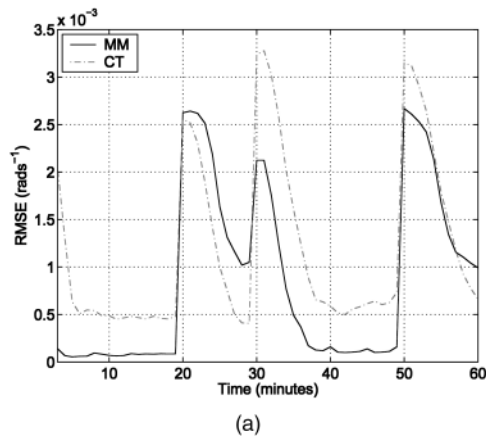
Fig. 5.   Target turn rate estimation RMSE (100 Monte Carlo runs). (a) Target 1. (b) Target 2.

TABLE I
Average Target State Estimation RMSE
(Based on 100 Monte Carlo Runs)

	Multiple Model PHD	Constant Velocity Model PHD	Coordinated Turn PHD	Units
Target 1 (position)	95.48	311.56	168.59	m
Target 2 (position)	210.89	401.88	237.09	m
Target 1 (velocity)	0.69	1.38	1.15	ms^{-1}
Target 2 (velocity)	1.17	1.57	1.29	ms^{-1}
Target 1 (turn rate)	0.00086	—	0.00121	$rads^{-1}$
Target 2 (turn rate)	0.00106	—	0.00110	$rads^{-1}$

multiple-model approach is essential to ensure
satisfactory performance in such cases.

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K. Punithakumar (S'06) was born in Sri Lanka in 1976. He received the B.Sc.Eng. degree in electronic and telecommunication engineering from University of Moratuwa, Sri Lanka, in 2001, and the M.A.Sc degree in electrical and computer engineering from McMaster University, Canada, in 2003.

He was an instructor in the Department of Electronic and Telecommunication Engineering at the University of Moratuwa, Sri Lanka, from 2001 to 2002. Since 2002 he has been a graduate student/teaching assistant in the Department of Electrical and Computer Engineering at McMaster University, Canada, in pursuit of a Ph.D. His research interests are in target tracking, sensor management and nonliner filtering.

T. Kirubarajan (S'95—M'98) was born in Sri Lanka in 1969. He received the B.A. and M.A. degrees in electrical and information engineering from Cambridge University, England, in 1991 and 1993, and the M.S. and Ph.D. degrees in electrical engineering from the University of Connecticut, Storrs, in 1995 and 1998, respectively.

Currently, he is an assistant professor in the Electrical and Computer Engineering Department at McMaster University, Hamilton, Ontario, Canada. He is also serving as an adjunct assitant professor and the Associate Director of the Estimation and Signal Processing Research Laboratory at the University of Connecticut. His research interests are in estimation, target tracking, multisource information fusion, sensor resource management, signal detection and fault diagnosis. His research activities at McMaster University and at the University of Connecticut are supported by US Missile Defense Agency, US Office of Naval Research, NASA, Qualtech Systems, Inc., Raytheon Canada Ltd., and Defense Research Development Canada, Ottawa. In September 2001, he served on a DARPA expert panel on unattended surveillance, homeland defense and counterterrorism. He has also served as a consultant in these areas to a number of companies, including Motorola Corporation, Northrop-Grumman Corporation, Pacific-Sierra Research Corporation, Lockhead Martin Corporation, Qualtech Systems, Inc., Orincon Corporation and BAE systems. He has worked on the development of a number of engineering software programs, including BEARDAT for target localization from bearing and frequency measurements in clutter, FUSEDAT for fusion of multisensor data for tracking. He has also worked with Qualtech Systems, Inc., to develop an advanced fault diagnosis engine.

Dr. Kirubarajan has published about 100 articles in the areas of his research interests, in addition to one book on estimation, tracking and navigation and two edited volumes. He is also a recipient of Ontario Premier's Research Excellence Award (2002).

Abhijit Sinha received his B.S. degree in physics from the University of Calcutta, India, in 1994. He received his M.S. degree in electrical communication engineering from Indian Institute of Science, Bangalore, India, in 1998, and received his Ph.D. degree in electrical and computer engineering form the University of Connecticut, Storrs, in 2002.

He worked as a Postdoctoral Fellow at the University of Connecticut from 2002 to 2003. Currently, he is working as a research associate at McMaster University, Canada. His research interests include signal processing, target tracking and communications.