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Multiple-point principle with a scalar singlet extension of the standard model

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We suggest a scalar singlet extension of the standard model, in which the multiple-point principle (MPP) condition of a vanishing Higgs potential at the Planck scale is realized. Although there have been lots of attempts to realize the MPP at the Planck scale, a realization maintaining naturalness is quite difficult. Our model can easily achieve the MPP at the Planck scale without large Higgs mass corrections. It is worth noting that the electroweak symmetry can be radiatively broken in our model. In the naturalness point of view, the singlet scalar mass should be of $\mathcal{O}(1)$ TeV or less. We also consider a right-handed neutrino extension of the model for neutrino mass generation. The model does not affect the MPP scenario, and might keep the naturalness with the new particle mass scale beyond TeV, thanks to accidental cancellation of Higgs mass corrections.

Subject Index B32, B40, B53, C01

1. Introduction

The observed mass of the Higgs boson may imply that Higgs self-coupling vanishes at a high energy scale in the framework of the standard model (SM). About twenty years ago, Ref. [1] suggested the multiple-point principle (MPP) at the Planck scale, and predicted a Higgs boson mass of 135 ± 9 GeV with 173 ± 5 GeV for the top quark mass. The MPP means that there are two degenerate vacua in the SM Higgs potential, $V(v_H) = V(M_{\rm Pl}) = 0$ and $V'(v_H) = V'(M_{\rm Pl}) = 0$, where V is the effective Higgs potential, $v_H = 246$ GeV is the vacuum expectation value (VEV) of the Higgs doublet, and $M_{\rm Pl} = 2.44 \times 10^{18}$ GeV is the reduced Planck scale. One is our vacuum at the electroweak (EW) scale, and the other vacuum lies at the Planck scale, which can be realized by the Planck-scale boundary conditions of vanishing effective Higgs self-coupling, $\lambda_H(M_{\rm Pl}) = 0$, and its beta function, $\beta_{\lambda_H}(M_{\rm Pl}) = 0$. Furthermore, an asymptotic safety scenario of gravity [2] predicted a 125 GeV Higgs boson mass with a few GeV uncertainty. This scenario also pointed out that $\lambda_H(M_{\rm Pl}) \sim 0$ and $\beta_{\lambda_H}(M_{\rm Pl}) \sim 0$ (see also Refs. [3–14] for more recent analyses).

Although Ref. [1] was able to predict the approximate Higgs boson mass, the MPP condition cannot fit the observed 125 GeV Higgs boson mass with the recent data inputs. In fact, within the context of the SM, the MPP condition at the Planck scale leads to a Higgs boson mass of 129.1 ± 1.5 GeV by using $173.10 \pm 0.59_{\rm exp} \pm 0.3_{\rm th}$ GeV for the world-averaged top quark mass [12]. There have been lots of attempts to realize the MPP at the Planck scale so far [15–33]. For example, in Ref. [18] the MPP at the Planck scale is achieved by introducing a scalar dark matter and a large Majorana mass of the right-handed neutrino. In this case, the masses of dark matter and the right-handed neutrino can be predicted. However, there is a tension from the viewpoint of naturalness, since the Higgs mass corrections via the heavy particles well exceeds the EW scale. Actually, it turns out to be quite difficult to realize the MPP at the Planck scale while keeping naturalness.

The difficulty is related with the renormalization group (RG) running of the Higgs self-coupling. In order to satisfy $\lambda_H(M_{\rm Pl}) = 0$ and $\beta_{\lambda_H}(M_{\rm Pl}) = 0$ simultaneously, there should exist one or more new particles which change β_{λ_H} adequately from the SM case. In almost all cases, such new particles need to be much heavier than the EW scale, as long as the Higgs self-coupling is "continuous" during the RG running. However, when a new scalar field couples with the Higgs doublet and develops nonzero VEV, the Higgs self-coupling has a tree-level threshold correction [34–37].[1] The correction causes a gap between the Higgs self-coupling in the extended model and the one in the effective theory, which is identified as the SM one. It has been shown that using the gap, the EW vacuum can be stabilized in a scalar singlet extended model [35] and type-II seesaw model [37]. Most importantly, even if the new scalar particle is as light as a TeV scale, the gap can appear. Then, the model does not affect the naturalness in the sense of Bardeen [38].

Here, we comment on the naturalness. According to Bardeen's argument, in quantum corrections quadratic divergences can be treated as an unphysical quantity, so that only logarithmic divergences should be concerned. In this sense, there is no hierarchy problem within the SM, which possesses an approximate scale invariance and its stability is guaranteed by the smallness of the logarithmic corrections. Since the logarithmic corrections can be taken into account as a beta function of the Higgs mass parameter, the naturalness can be evaluated with the solution of its RG equation. Namely, it is natural if the Higgs mass parameter does not significantly change during the RG running. We will apply this sense of naturalness to our model.

In this paper, we will investigate the MPP condition in a scalar singlet extended model, which can be consistent with the 125 GeV Higgs boson mass. Our model is explained in the next section, in which we show the gap explicitly. Numerical analyses of the MPP scenario are given in Sect. 3. We will find that the EW symmetry can be radiatively broken in our model. We also discuss the naturalness of the Higgs mass. In Sect. 4, we will introduce right-handed neutrinos into the scalar singlet extended model to incorporate active neutrino masses. We will show that in the presence of the right-handed neutrinos, the MPP scenario can be realized. It will be pointed out that even if the singlet scalar and the right-handed neutrinos are much heavier than the EW scale, the model might keep the naturalness thanks to an accidental cancellation of Higgs mass corrections coming from them. Finally, we will summarize our results in Sect. 5.

2. Scalar singlet extension

We consider a simple extension of the SM with a real singlet scalar field. The scalar potential is given by [36]

$$V(H,S) = \frac{\lambda_H}{2}(H^\dagger H)^2 + m_H^2 H^\dagger H + \frac{\lambda_S}{8}S^4 + \frac{\mu_S}{3}S^3 + \frac{m_S^2}{2}S^2 + \frac{\lambda_{HS}}{2}S^2 H^\dagger H + \mu_{HS} S H^\dagger H, \quad (1)$$

where H and S are the Higgs doublet and the scalar singlet fields, respectively. In this paper, we consider the case with $m_S^2 > |m_H^2|$ and $\mu_{HS} > 0$, and omit a linear term of the singlet scalar field, which can vanish by a shift of the field. Note that we do not assume an ad hoc Z_2 symmetry, and

[1] When a new heavy fermion couples with the Higgs doublet, there is a one-loop threshold correction, but it is usually negligibly small.

then, we will find that μ_{HS} plays an important role for the vacuum stability and the EW symmetry breaking. In the unitary gauge, the scalar fields are written by

$$H = \left(0, \ \frac{v_H + h}{\sqrt{2}}\right)^T, \quad S = v_S + s, \tag{2}$$

where v_H and v_S are vacuum expectation values. The Higgs VEV is $v_H = 246\,\mathrm{GeV}$, and v_S has a negative small value in our setup, as will be discussed below.

The minimization conditions of the potential are given by

$$\frac{\partial V}{\partial h}\Big|_{h\to 0, s\to 0} = \frac{v_H}{2}\left(\lambda_H v_H^2 + 2m_H^2 + \lambda_{HS} v_S^2 + 2\mu_{HS} v_S\right) = 0, \tag{3}$$

$$\frac{\partial V}{\partial s}\Big|_{h\to 0, s\to 0} = \frac{1}{2}\left[v_S\left(\lambda_S v_S^2 + 2\mu_S v_S + 2m_S^2 + \lambda_{HS} v_H^2\right) + \mu_{HS} v_H^2\right] = 0. \tag{4}$$

From Eq. (3), the Higgs VEV is obtained by

$$v_H^2 = -\frac{1}{\lambda_H}\left(2m_H^2 + \lambda_{HS} v_S^2 + 2\mu_{HS} v_S\right). \tag{5}$$

To realize the EW symmetry breaking, the Higgs mass term m_H^2 is negative at the EW scale, and $2(-m_H^2) > \lambda_{HS} v_S^2 + 2\mu_{HS} v_S$ should be satisfied. Without any fine-tuning, we can expect $\mu_{HS} \simeq m_S$ by a naive dimensional analysis. Thus, $|v_S|$ should be much smaller than v_H for $m_S^2 \gg |m_H^2|$.

The nonzero Higgs VEV induces a tadpole for the singlet scalar due to the μ_{HS} term. If we neglect the cubic term of S, Eq. (4) is approximated by $m_S^2 v_S + \mu_{HS} v_H^2 \approx 0$ for $\lambda_S \leq \mathcal{O}(1)$, $\lambda_{HS} \leq \mathcal{O}(1)$. This gives the singlet VEV as

$$v_S \approx -\frac{\mu_{HS} v_H^2}{2m_S^2}, \tag{6}$$

and its order of magnitude is $\mathcal{O}(v_H^2/m_S)$ for $\mu_{HS} \simeq m_S$. In the no tadpole limit $\mu_{HS} \to 0$, v_S vanishes. The assumption of $\mu_S = 0$ seems to be unnatural, but it is necessarily required by the MPP condition, as discussed later. Actually, we will find that λ_S and λ_{HS} also vanish by the MPP condition.

The mass matrix for the scalar fields is expressed by the second derivatives of the potential at the VEVs:

$$(h, \ s)\begin{pmatrix} m_{hh}^2 & m_{hs}^2 \\ m_{hs}^2 & m_{ss}^2 \end{pmatrix}\begin{pmatrix} h \\ s \end{pmatrix} = (\phi_1, \ \phi_2)\begin{pmatrix} m_{\phi_1}^2 & 0 \\ 0 & m_{\phi_2}^2 \end{pmatrix}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \tag{7}$$

with

$$\frac{\partial^2 V}{\partial h^2}\Big|_{h\to 0, s\to 0} = m_{hh}^2 = \frac{3}{2}\lambda_H v_H^2 + m_H^2 + \frac{1}{2}\lambda_{HS} v_S^2 + \mu_{HS} v_S, \tag{8}$$

$$\frac{\partial^2 V}{\partial h \partial s}\Big|_{h\to 0, s\to 0} = m_{hs}^2 = \lambda_{HS} v_H v_S + \mu_{HS} v_H, \tag{9}$$

$$\frac{\partial^2 V}{\partial s^2}\Big|_{h\to 0, s\to 0} = m_{ss}^2 = \frac{3}{2}\lambda_S v_S^2 + 2\mu_S v_S + m_S^2 + \frac{1}{2}\lambda_{HS} v_H^2, \tag{10}$$

and

$$m_{\phi_1}^2 = \frac{1}{2}\left(m_{hh}^2 + m_{ss}^2 - \sqrt{(m_{hh}^2 - m_{ss}^2)^2 + 4m_{hs}^4}\right), \tag{11}$$

$$m_{\phi_2}^2 = \frac{1}{2}\left(m_{hh}^2 + m_{ss}^2 + \sqrt{(m_{hh}^2 - m_{ss}^2)^2 + 4m_{hs}^4}\right). \tag{12}$$

We identify the lighter eigenstate ϕ_1 with the SM-like Higgs, and its mass eigenvalue m_{ϕ_1} corresponds to the observed Higgs boson mass $M_h = 125\,\text{GeV}$. In our numerical calculation, we will take into account a renormalization group effect for the Higgs mass. The scalar-mixing matrix is defined by

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix} \quad \text{with} \quad \tan 2\alpha = \frac{2m_{hs}^2}{m_{ss}^2 - m_{hh}^2}. \tag{13}$$

For $|m_H^2| \ll m_S^2 \simeq \mu_S^2$, the mixing coupling is obtained by $\sin\alpha \approx \mu_{HS} v_H / m_S^2$, and it must be lower than the experimental bound $|\sin\alpha| \leq 0.36$ given by the LHC Run 1 data [39]. This constraint induces $m_S \simeq \mu_{HS} \gtrsim 685\,\text{GeV}$, and also $|v_S| \lesssim 45\,\text{GeV}$ from Eq. (6).

In the low energy effective theory, the tree-level effective Higgs potential is given by [36]

$$V_{\text{eff}}(H) = m_{\text{SM}}^2 H^\dagger H + \frac{1}{2}\lambda_{\text{SM}}(H^\dagger H)^2 + \frac{1}{3}\eta_6 (H^\dagger H)^3 + \frac{1}{8}\eta_8 (H^\dagger H)^4, \tag{14}$$

with

$$m_{\text{SM}}^2 = m_H^2, \quad \lambda_{\text{SM}} = \lambda_H - \frac{\mu_{HS}^2}{m_S^2}, \quad \eta_6 = \frac{3\lambda_{HS}\mu_{HS}^2}{2m_S^4} - \frac{\mu_S \mu_{HS}^3}{m_S^6}, \quad \eta_8 = \frac{\lambda_S \mu_{HS}^4}{2\mu_S^8}. \tag{15}$$

Note that the Higgs self-coupling has a nontrivial gap $\Delta\lambda \equiv \mu_{HS}^2/m_S^2$, which can play a crucial role to make the EW vacuum stable as in a scenario in Refs. [35,37]. In particular, the Higgs self-coupling λ_H can vanish at the UV scale, e.g. the Planck scale, as well as the effective Higgs self-coupling λ_{SM} explaining the observed Higgs boson mass, which has been studied in a type-II seesaw model [34]. This scenario indicates that

$$\lambda_H(M_{\text{Pl}}) = 0 \quad \text{and} \quad \lambda_{\text{SM}}(v_H) = \frac{M_h^2}{v_H^2}. \tag{16}$$

We show the RG running of the Higgs self-coupling in Fig. 1, where we have used the beta functions given in Appendix A. The vertical and horizontal axes show the Higgs self-coupling and renormalization scale μ, respectively. Here, we have considered m_S as the cutoff of the SM, and taken the boundary condition $\lambda_{\text{SM}} = \lambda_H - \Delta\lambda$ at $\mu = m_S = 1\,\text{TeV}$. Figure 1 shows that the Higgs self-coupling remains positive up to the Planck scale, and thus the EW vacuum can be stabilized.

3. Multiple-point principle

The MPP condition requires that all scalar-quartic couplings vanish, and a simultaneous vanishing of their beta functions at the UV scale. In particular, $\beta_{\lambda_H}(M_{\text{Pl}}) = 0$ with $\lambda_H(M_{\text{Pl}}) = 0$ requires the top Yukawa coupling as $y_t(M_{\text{Pl}}) \simeq 0.388$. In this paper, when we solve the RG equations we use boundary conditions Eqs. (A12)–(A16). Then, to realize $y_t(M_{\text{Pl}}) \simeq 0.388$, the top pole mass M_t should be taken as 172.322 GeV, 172.687 GeV, and 173.052 GeV for the fixed strong coupling $\alpha_s(M_Z) = 0.1179$, 0.1185, and 0.1191, respectively. For measurements of the top pole mass, $M_t = 172.99 \pm 0.91\,\text{GeV}$ [40] and $M_t = 172.44 \pm 0.48\,\text{GeV}$ [41] are obtained by the ATLAS and CMS collaborations, respectively. Thus, our result expected by the MPP is consistent with the current experimental data. In the following, we take $\alpha_s(M_Z) = 0.1185$ and $M_t = 172.687\,\text{GeV}$ as reference values.

Imposing the MPP condition in the scalar singlet extended model, λ_S and λ_{HS} remain zero during the RG runnings. Then, the MPP condition also requires a vanishing triple coupling of the singlet

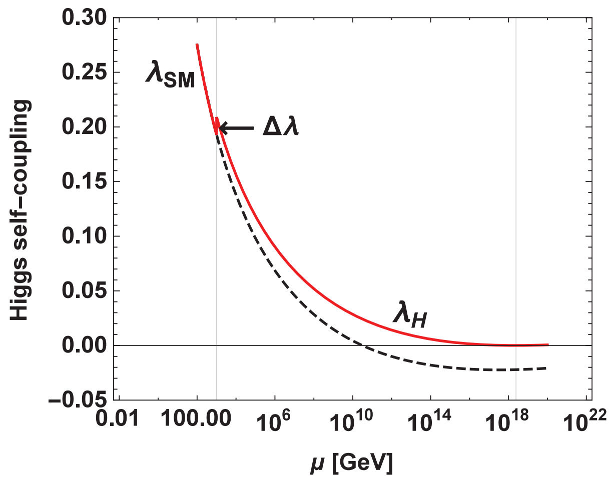

Fig. 1. Renormalization group running of the Higgs self-coupling in our model (red). The black dashed line shows the running of the Higgs self-coupling in the SM. The vertical lines correspond to $m_S = 1$ TeV and $M_{\rm Pl}$, respectively. We have used $M_h = 125.09$ GeV, $M_t = 172.687$ GeV, and $\alpha_s = 0.1185$ as the reference values.

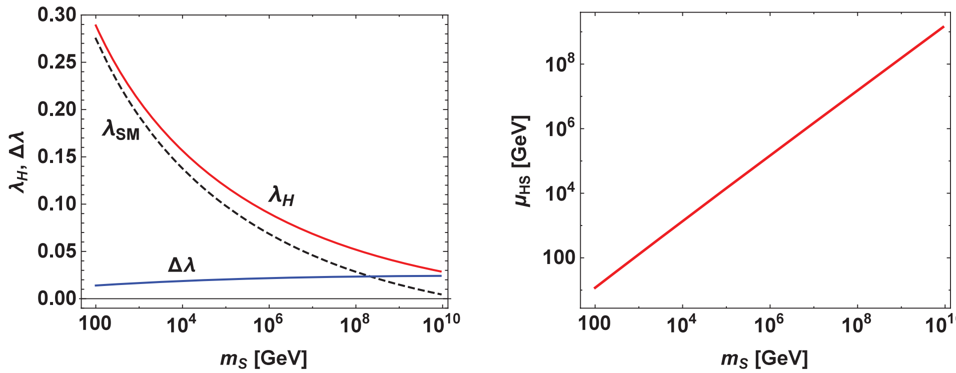

Fig. 2. Left: m_S dependence of $\Delta\lambda$ (blue). The red line and black dashed line show $\lambda_H(m_S)$ and $\lambda_{\rm SM}(m_S)$, respectively. Right: m_S dependence of μ_{HS}.

scalar (μ_S), because the highest term of S must be an even function to realize the degenerate vacua. Once μ_S vanishes, it also remains zero. In the rest of this paper, we can take away λ_S, λ_{HS}, and μ_S from our discussion. Note that for the vacuum around the Planck scale, $v_H \sim M_{\rm Pl}$, the stationary condition (4) suggests $v_S \sim M_{\rm Pl}$ with $\lambda_S(M_{\rm Pl}) \sim \mu_{HS}(M_{\rm Pl})/M_{\rm Pl}$. This value of $\lambda_S(M_{\rm Pl})$ is extremely small, and in practice we can use the MPP condition as $\lambda_S(M_{\rm Pl}) = 0$.

It is worth noting that $\Delta\lambda$ is uniquely determined for a given m_S, once the MPP condition and Eq. (16) are required. Then, μ_{HS} is determined by $\mu_{HS}^2 = \Delta\lambda\, m_S^2$. In addition, v_S is exactly obtained by Eq. (6) because of $\lambda_S = \lambda_{HS} = 0$ and $\mu_S = 0$. As a result, our model is controlled by only one free parameter. In the following, we choose m_S as the free parameter.

The left panel of Fig. 2 shows the m_S dependence of $\Delta\lambda$ as the blue line. The red line and black dashed line show $\lambda_H(m_S)$ and $\lambda_{\rm SM}(m_S)$, respectively. We find that $\Delta\lambda$ is almost constant, and thus $\mu_{HS}(m_S)$ is roughly proportional to m_S as shown in the right panel of Fig. 2. To stabilize the EW vacuum, the Higgs self-coupling should remain positive up to the Planck scale. Thus, m_S has to be smaller than 10^{10} GeV, and we do not consider the heavier case.

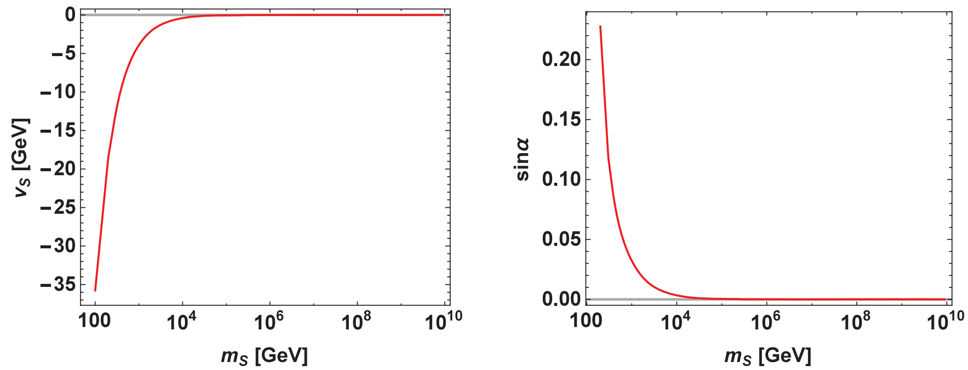

Fig. 3. v_S (left) and $\sin\alpha$ (right) as functions of m_S.

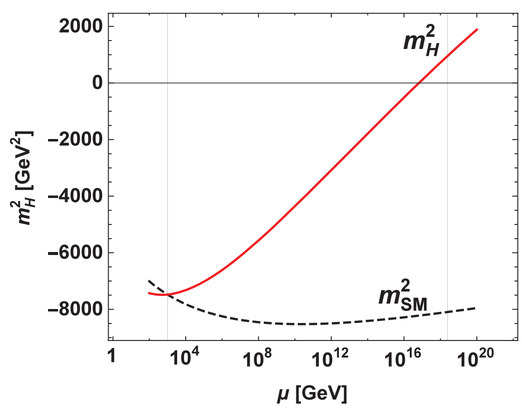

Fig. 4. Renormalization group running of m_H^2 (red). The black dashed line shows the running of the Higgs mass parameter in the SM. The vertical lines correspond to $m_S = 1$ TeV and $M_{\rm Pl}$, respectively.

Figure 3 shows m_S dependences of v_S and $\sin\alpha$ in the left and right panels, respectively. Imposing the MPP condition, Eq. (6) becomes exactly equal, where values of v_S are obtained as $v_S = -\sqrt{\Delta\lambda} v_H^2/(2m_S)$. Since $\Delta\lambda$ is almost constant, v_S is almost inversely proportional to m_S. We find that $-35\,\text{GeV} \lesssim v_S < 0\,\text{GeV}$, and in particular $|v_S| < 1\,\text{GeV}$ for $m_S > 4\,\text{TeV}$. The scalar-mixing angle is obtained by

$$\tan 2\alpha = \frac{2\mu_{HS} v_H}{m_S^2 - \lambda_H v_H^2} \quad \longrightarrow \quad \sin\alpha \approx \alpha \approx \frac{\mu_{HS} v_H}{m_s^2} = 2\frac{-v_S}{v_H} \quad \text{for } m_S^2 \gg |m_H^2|. \tag{17}$$

Thus, we can estimate $\sin\alpha < 0.01$ for $m_S > 4\,\text{TeV}$. Note that the whole parameter region is safe from the LHC Run 1 constraint $|\sin\alpha| \leq 0.36$ [39]. This result is different from the estimation discussed below Eq. (13). The reason is that the estimation comes from $\mu_{HS} \simeq m_S$, while the MPP condition requires $\mu_{HS} \simeq 0.1\, m_S$.

It is remarkable that the EW symmetry is radiatively broken in our model. The beta function of m_H^2 is dominated by the μ_{HS}^2 term for $|m_H^2| \ll \mu_{HS}^2$. Its RG solution is approximately given by

$$m_H^2(\mu) \approx m_H^2(M_{\rm Pl}) - \frac{\mu_{HS}^2}{16\pi^2} \ln\left(\frac{M_{\rm Pl}}{\mu}\right)^2 \quad \text{for } m_S \leq \mu \leq M_{\rm Pl}. \tag{18}$$

To realize the EW symmetry breaking, m_H^2 should be negative at the EW scale, while m_H^2 is positive at the Planck scale as $m_H^2(M_{\rm Pl}) \sim \mu_{HS}^2$. This behavior is explicitly shown in Fig. 4. Here, we have

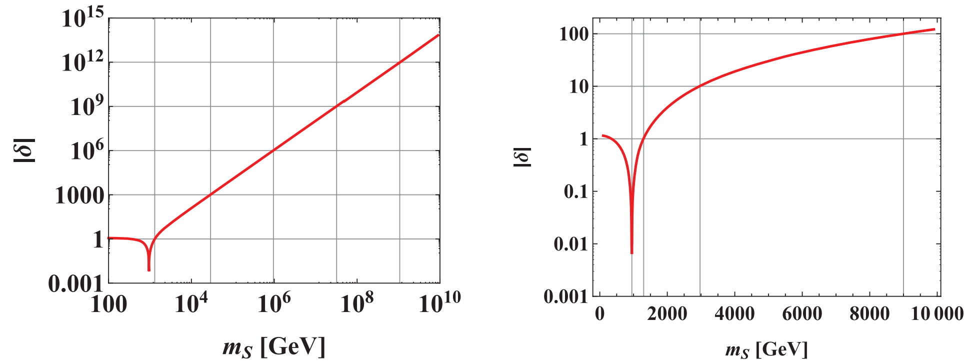

Fig. 5. m_S dependence of δ. The right panel concentrates on $100\,\mathrm{GeV} \leq m_S \leq 10\,\mathrm{TeV}$.

taken the cutoff of the SM at $\mu = m_S(m_S) = 1$ TeV, and then $\Delta\lambda \simeq 0.0166$, $\mu_{HS}(m_S) \simeq 129$ GeV, and $v_S \simeq -3.90$ GeV.

At the end of this section, we mention the naturalness of the Higgs mass. When m_S is much higher than the EW scale, it induces $|m_H^2(v_H)| \ll m_H^2(M_{\mathrm{Pl}})$, that is, the RG running of m_H^2 is highly tuned to realize the observed Higgs mass. Here, we define the fine-tuning level as $\delta \equiv m_H^2(M_{\mathrm{Pl}})/|m_H^2(v_H)| = 2m_H^2(M_{\mathrm{Pl}})/M_h^2$, where $M_h = 125$ GeV. For example, $\delta = 10$ indicates that we need to fine-tune the Higgs mass squared at an accuracy level of 10%. Figure 5 shows the m_S dependence of δ, and we find $\delta = 1$, 10, and 100 correspond to $m_S \simeq 1.3$ TeV, 3.0 TeV, and 9.0 TeV, respectively. Therefore, from the naturalness point of view, there should exist the singlet scalar at $\mathcal{O}(1)$ TeV scale. We have found that $m_H^2(M_{\mathrm{Pl}})$ vanishes for $m_S \simeq 950$ GeV, and becomes negative in the lower m_S region, in which the radiative EW symmetry breaking does not occur. For a tadpole diagram which contributes Higgs mass correction, it is tiny due to the heavy mass of m_S.[2]

4. Additional extension with right-handed neutrinos

In addition to the singlet scalar, we can introduce right-handed neutrinos to explain the active neutrino masses. The interaction parts of the Lagrangian including right-handed neutrinos are given by

$$-\mathcal{L}_N = Y_\nu^\dagger \overline{L}\tilde{H}N + Y_N S\overline{N}N + \frac{1}{2}M_N\overline{N^c}N + \mathrm{h.c.}, \tag{19}$$

where L and N are lepton doublet and right-handed neutrino fields, respectively. Imposing the MPP condition at the Planck scale, $\beta_{\lambda_S}(M_{\mathrm{Pl}}) = 0$ is required, and then Y_N vanishes at all energy scales (see Appendix B). Therefore, the new parameters are only Y_ν and M_N, the same as the usual type-I seesaw model [42–45]. These parameters should satisfy the seesaw relation $m_\nu = Y_\nu^T M_N^{-1} Y_\nu v_H^2/2$, where m_ν is the active neutrino mass matrix calculated by mass eigenvalues and the PMNS matrix [46,47].

When we consider the $Y_\nu \ll \mathcal{O}(1)$ (or equivalently $M_N \ll \mathcal{O}(10^{14})$ GeV) case, right-handed neutrino contributions are negligible in runnings of the scalar-quartic couplings. Thus, the MPP scenario remains the same as the one without right-handed neutrinos.[3] However, only the RG running

[2] For $\mu_S \neq 0$, there is a finite Higgs mass correction by a tadpole diagram of the singlet scalar. However, we need not consider it because of $\mu_S = 0$ coming from the MPP condition.

[3] When the neutrinos are Dirac fermions, there are no Majorana masses and $Y_\nu \ll \mathcal{O}(1)$. Then, the MPP scenario can be realized as in the previous section.

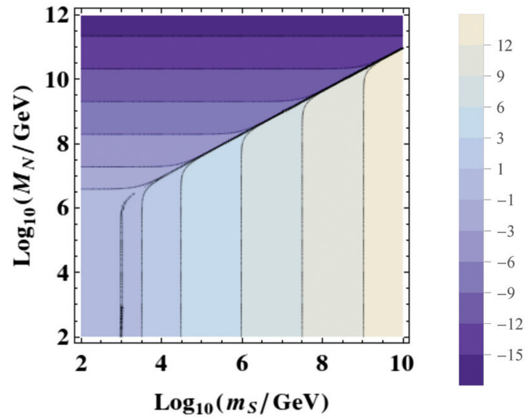

Fig. 6. Contour plot of δ in the (m_S, M_N) plane. The values shown in the bar on the right are $\mathrm{sign}[\delta]\,\mathrm{Log}_{10}|\delta|$, where we have defined $\mathrm{sign}[\delta] \equiv \delta/|\delta|$.

of m_H^2 might change significantly. Including contributions of the right-handed neutrinos, Eq. (18) is rewritten by

$$m_H^2(\mu) \approx m_H^2(M_{\mathrm{Pl}}) - \frac{\mu_{HS}^2}{16\pi^2}\ln\left(\frac{M_{\mathrm{Pl}}}{\mu}\right)^2 + \frac{4N_\nu m_{\mathrm{eff}} M_N^3}{16\pi^2 v_H^2}\ln\left(\frac{M_{\mathrm{Pl}}}{M_N}\right)^2 \quad \text{for } m_S \le \mu \le M_N, \tag{20}$$

or

$$m_H^2(\mu) \approx m_H^2(M_{\mathrm{Pl}}) - \frac{\mu_{HS}^2}{16\pi^2}\ln\left(\frac{M_{\mathrm{Pl}}}{m_S}\right)^2 + \frac{4N_\nu m_{\mathrm{eff}} M_N^3}{16\pi^2 v_H^2}\ln\left(\frac{M_{\mathrm{Pl}}}{\mu}\right)^2 \quad \text{for } M_N \le \mu \le m_S, \tag{21}$$

where, using the seesaw relation, we have defined $\mathrm{Tr}(Y_\nu^\dagger M_N^2 Y_\nu) \equiv 2N_\nu m_{\mathrm{eff}} M_N^3/v_H^2$ (on the right-hand side M_N is a number not a matrix). The effective neutrino mass m_{eff} is typically given by the heaviest active neutrino mass, and N_ν means the relevant number of right-handed neutrinos. Since the singlet scalar and the right-handed neutrinos oppositely contribute to m_H^2, the Higgs mass corrections might be accidentally canceled (at the one-loop level).

We show a contour plot of δ in Fig. 6, where the horizontal and vertical axes show m_S and M_N, respectively. For the calculation of Eqs. (20) and (21), we have taken $N_\nu = 1$ and $m_{\mathrm{eff}} = 0.05\,\mathrm{eV}$ as reference values. The positive δ region, in which the singlet scalar contribution is dominant, can drive the radiative EW symmetry breaking as mentioned above. When the right-handed neutrino mass becomes larger, the value of δ becomes smaller and vanishes at a specific point. From Eqs. (20) and (21), the point is estimated by

$$\log_{10}\left(\frac{M_N}{\mathrm{GeV}}\right) \approx 4 + \frac{2}{3}\log_{10}\left(\frac{m_S}{\mathrm{GeV}}\right). \tag{22}$$

If this relation is realized, δ can be small and hence our scenario can be natural even for the masses of singlet scalar and right-handed neutrinos $\gg$1 TeV.

5. Summary

We have investigated the scalar singlet extension of the SM with the MPP condition, in which the scalar potential has two degenerate vacua at the EW and UV scales. The condition requires all scalar-quartic couplings to vanish, with simultaneously vanishing beta functions at the UV scale, which we have taken as the Planck scale. In particular, $\beta_{\lambda_H}(M_{\mathrm{Pl}}) = 0$ with $\lambda_H(M_{\mathrm{Pl}}) = 0$ can determine the top pole mass as 172.322 GeV, 172.687 GeV, and 173.052 GeV for $\alpha_s(M_Z) = 0.1179$, 0.1185, and 0.1191, respectively. These values are consistent with the current experimental data $M_t = 172.99 \pm 0.91$ GeV by the ATLAS collaboration [40] and $M_t = 172.44 \pm 0.48$ GeV by the CMS collaboration [41]. The MPP conditions strongly restrict our model parameters, and there is only one free parameter left in our analysis, which we have taken as the singlet mass m_S. We have shown the m_S dependence of some model predictions, and found that our model is consistent with the LHC Run 1 results for the SM Higgs boson properties.

To simultaneously realize the MPP condition and the observed Higgs mass, the singlet-Higgs–Higgs coupling μ_{HS} plays an important role. Furthermore, this coupling induces the radiative EW symmetry breaking. When the singlet mass is much larger than the Higgs mass, the μ_{HS}^2 term dominates the beta function of the Higgs mass squared $\beta_{m_H^2}$. Then, the sign of m_H^2 can flip during the RG running, that is, m_H^2 becomes negative toward the EW scale while positive at the Planck scale. We have found that this behavior can occur for $m_S > 950$ GeV. On the other hand, too large m_S causes the fine-tuning problem of the Higgs mass. To avoid the problem, there should exist the singlet scalar at $\mathcal{O}(1)$ TeV scale.

In order to incorporate the neutrino masses and flavor mixings to the singlet scalar extended model, we have introduced right-handed neutrinos and investigated the MPP scenario. Here, new parameters Y_ν and M_N are introduced, which are the neutrino Dirac–Yukawa coupling and the right-handed neutrino Majorana mass matrices, respectively, leading to the type-I seesaw mechanism. For $Y_\nu \ll \mathcal{O}(1)$ (or equivalently $M_N \ll \mathcal{O}(10^{14})$ GeV), the running of all couplings except m_H^2 are almost the same as before. Therefore, the model can realize the MPP scenario as well as explaining the active neutrino masses.

It might be possible to solve the fine-tuning problem of the Higgs mass by an accidental cancellation of Higgs mass corrections coming from the singlet scalar and the right-handed neutrinos. We have found its approximate condition as Eq. (22). If the condition is satisfied, the masses of the singlet scalar and right-handed neutrinos can exceed $\mathcal{O}(1)$ TeV.

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Appendix A. Beta functions in the scalar singlet extended model

The one-loop beta functions for the SM are given by

$$\beta_{g_Y} = \frac{g_Y^3}{16\pi^2}\frac{41}{6}, \qquad \beta_{g_2} = \frac{g_2^3}{16\pi^2}\left(-\frac{19}{6}\right), \qquad \beta_{g_3} = \frac{g_3^3}{16\pi^2}(-7), \tag{A1}$$

$$\beta_{y_t} = \frac{y_t}{16\pi^2}\left(-\frac{9}{4}g_2^2 - 8g_3^2 - \frac{17}{12}g_Y^2 + \frac{9}{2}y_t^2\right), \tag{A2}$$

$$\beta_{\lambda_{\rm SM}} = \frac{1}{16\pi^2}\left[\lambda_{\rm SM}\left(12\lambda_{\rm SM} - 9g_2^2 - 3g_Y^2 + 12y_t^2\right) + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2g_Y^2 + \frac{3}{4}g_Y^4 - 12y_t^4\right], \tag{A3}$$

$$\beta_{m_{\rm SM}^2} = \frac{m_{\rm SM}^2}{16\pi^2}\left(6\lambda_{\rm SM} - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2\right). \tag{A4}$$

Here, we omit the Yukawa couplings except for the top Yukawa coupling, since the other Yukawa couplings are small enough to be neglected.

For a real singlet scalar extension of the SM, the one-loop beta functions of the gauge and the top Yukawa couplings do not change. The beta functions of the other couplings are given by

$$\beta_{\lambda_H} = \frac{1}{16\pi^2}\left[\lambda_H\left(12\lambda_H - 9g_2^2 - 3g_Y^2 + 12y_t^2\right) + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2g_Y^2 + \frac{3}{4}g_Y^4 - 12y_t^4 + \lambda_{HS}^2\right], \tag{A5}$$

$$\beta_{\lambda_S} = \frac{1}{16\pi^2}\left(9\lambda_S^2 + 4\lambda_{HS}^2\right), \tag{A6}$$

$$\beta_{\lambda_{HS}} = \frac{\lambda_{HS}}{16\pi^2}\left(6\lambda_H - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2 + 4\lambda_{HS} + 3\lambda_S\right), \tag{A7}$$

$$\beta_{\mu_S} = \frac{1}{16\pi^2}\left(9\lambda_S\mu_S + 6\lambda_{HS}\mu_{HS}\right), \tag{A8}$$

$$\beta_{\mu_{HS}} = \frac{1}{16\pi^2}\left[\mu_{HS}\left(6\lambda_H - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2 + 4\lambda_{HS}\right) + \lambda_{HS}\mu_S\right], \tag{A9}$$

$$\beta_{m_H^2} = \frac{1}{16\pi^2}\left[m_H^2\left(6\lambda_H - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2\right) + \lambda_{HS}m_S^2 + 2\mu_{HS}^2\right], \tag{A10}$$

$$\beta_{m_S^2} = \frac{1}{16\pi^2}\left(3\lambda_S m_S^2 + 4\mu_S^2 + 4\lambda_{HS}m_H^2 + 4\mu_{HS}^2\right). \tag{A11}$$

To solve the RG equations, we take the following boundary conditions [12,48]:

$$g_Y(M_t) = 0.35761 + 0.00011\left(\frac{M_t}{\rm GeV} - 173.10\right), \tag{A12}$$

$$g_2(M_t) = 0.64822 + 0.00004\left(\frac{M_t}{\rm GeV} - 173.10\right), \tag{A13}$$

$$g_3(M_t) = 1.1666 - 0.00046\left(\frac{M_t}{\rm GeV} - 173.10\right) + 0.00314\left(\frac{\alpha_3(M_Z) - 0.1184}{0.0007}\right), \tag{A14}$$

$$y_t(M_t) = 0.93558 + 0.00550\left(\frac{M_t}{\rm GeV} - 173.10\right) - 0.00042\left(\frac{\alpha_3(M_Z) - 0.1184}{0.0007}\right), \tag{A15}$$

$$\alpha_s(M_Z) = 0.1185 \pm 0.0006, \tag{A16}$$

where M_t is the pole mass of the top quark. In our analysis, the top pole mass is determined by the MPP condition: $M_t = 172.322$ GeV, 172.687 GeV, and 173.052 GeV for $\alpha_s(M_Z) = 0.1179$, 0.1185, and 0.1191, respectively.

Appendix B. Beta functions in the scalar singlet extended model with right-handed neutrinos

In addition to the real singlet scalar field, we introduce right-handed neutrinos. The one-loop beta functions of the gauge couplings do not change. The beta functions of the other couplings are given by

$$\beta_{y_t} = \frac{y_t}{16\pi^2}\left(-\frac{9}{4}g_2^2 - 8g_3^2 - \frac{17}{12}g_Y^2 + \frac{9}{2}y_t^2 + {\rm Tr}(Y_\nu^\dagger Y_\nu)\right), \tag{B1}$$

$$\beta_{Y_\nu} = \frac{1}{16\pi^2}\left[Y_\nu\left(-\frac{9}{4}g_2^2 - \frac{3}{4}g_Y^2 + 3y_t^2 + \mathrm{Tr}(Y_\nu^\dagger Y_\nu) + \frac{3}{2}Y_\nu^\dagger Y_\nu\right) + 2Y_N^2 Y_\nu\right], \tag{B2}$$

$$\beta_{Y_N} = \frac{1}{16\pi^2}\left[Y_N\left(4\mathrm{Tr}(Y_N^2) + 12Y_N^2 + (Y_\nu Y_\nu^\dagger)^T\right) + Y_\nu Y_\nu^\dagger Y_N\right], \tag{B3}$$

$$\beta_{\lambda_H} = \frac{1}{16\pi^2}\left[\lambda_H\left(12\lambda_H - 9g_2^2 - 3g_Y^2 + 12y_t^2 + 4\mathrm{Tr}(Y_\nu^\dagger Y_\nu)\right) + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2 g_Y^2 + \frac{3}{4}g_Y^4 - 12y_t^4 + \lambda_{HS}^2 - 4\mathrm{Tr}(Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu)\right], \tag{B4}$$

$$\beta_{\lambda_S} = \frac{1}{16\pi^2}\left[\lambda_S\left(9\lambda_S + 16\mathrm{Tr}(Y_N^2)\right) + 4\lambda_{HS}^2 - 128\mathrm{Tr}(Y_N^4)\right], \tag{B5}$$

$$\beta_{\lambda_{HS}} = \frac{1}{16\pi^2}\left[\lambda_{HS}\left(6\lambda_H - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2 + 4\lambda_{HS} + 3\lambda_S + 2\mathrm{Tr}(Y_\nu^\dagger Y_\nu) + 8\mathrm{Tr}(Y_N^2)\right) - 32\mathrm{Tr}(Y_N^2 Y_\nu^\dagger Y_\nu)\right], \tag{B6}$$

$$\beta_{\mu_S} = \frac{1}{16\pi^2}\left[\mu_S\left(9\lambda_S + 12\mathrm{Tr}(Y_N^2)\right) + 6\lambda_{HS}\mu_{HS} - 96\mathrm{Tr}(M_N Y_N^3)\right], \tag{B7}$$

$$\beta_{\mu_{HS}} = \frac{1}{16\pi^2}\left[\mu_{HS}\left(6\lambda_H - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2 + 4\lambda_{HS} + 2\mathrm{Tr}(Y_\nu^\dagger Y_\nu) + 4\mathrm{Tr}(Y_N^2)\right) + \lambda_{HS}\mu_S - 16\mathrm{Tr}(M_N Y_N Y_\nu^\dagger Y_\nu)\right], \tag{B8}$$

$$\beta_{M_N} = \frac{1}{16\pi^2}\left[M_N(Y_\nu Y_\nu^\dagger)^T + \left(Y_\nu Y_\nu^\dagger\right)M_N + 4\mathrm{Tr}(M_N Y_N)Y_N + 12M_N Y_N^2\right], \tag{B9}$$

$$\beta_{m_H^2} = \frac{1}{16\pi^2}\left[m_H^2\left(6\lambda_H - \frac{9}{2}g_2^2 - \frac{3}{2}g_Y^2 + 6y_t^2 + 2\mathrm{Tr}(Y_\nu^\dagger Y_\nu)\right) + \lambda_{HS}m_S^2 + 2\mu_{HS}^2 - 4\mathrm{Tr}(Y_\nu^\dagger M_N^2 Y_\nu)\right], \tag{B10}$$

$$\beta_{m_S^2} = \frac{1}{16\pi^2}\left[m_S^2\left(3\lambda_S + 8\mathrm{Tr}(Y_N^2)\right) + 4\mu_S^2 + 4\lambda_{HS}m_H^2 + 4\mu_{HS}^2 - 48\mathrm{Tr}(M_N^2 Y_N^2)\right], \tag{B11}$$

where Y_N and M_N are real diagonal matrices. We have used SARAH [49] to obtain these beta functions.

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