MULTIPLE POSITIVE SOLUTIONS OF KIRCHHOFF–TYPE EQUATIONS WITH CONCAVE TERMS

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Abstract. In this paper, we study the existence of positive solutions for the Kirchhoff equations with concave terms

$$\begin{cases} -\left(a+b\int_\Omega |\nabla u|^2 \, dx\right)\Delta u = f(x,u) - \lambda |u|^{q-2}u, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases} \tag{0.1}$$

where Ω is a bounded domain with a C^2-boundary $\partial\Omega$ in R^N $(N = 1, 2, 3)$, and $a, b > 0$, $1 < q < 2$. By applying variational methods, we show that there exists a constant $\lambda^* > 0$ such that for any $\lambda \in (0, \lambda^*)$, problem (0.1) has at least two positive solutions.

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