

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

Multiple Scattering of Light by Particles

This volume provides a thorough and up-to-date treatment of multiple scattering of light and other electromagnetic radiation in media composed of randomly and sparsely positioned particles. For the first time in monographic literature, the radiative transfer theory (RTT) is systematically and consistently presented as a branch of classical macroscopic electromagnetics. The book traces the fundamental link between the RTT and the effect of coherent backscattering (CB) and explains their place in the context of a comprehensive hierarchy of electromagnetic scattering problems. Dedicated sections present a thorough discussion of the physical meaning and range of applicability of the radiative transfer equation (RTE) and compare the self-consistent microphysical and the traditional phenomenological approaches to radiative transfer. The work describes advanced techniques for solving the RTE and gives examples of physically based applications of the RTT and CB in noninvasive particle characterization and remote sensing. This thorough and self-contained book will be valuable for science professionals, engineers, and graduate students working in a wide range of disciplines including optics, electromagnetics, remote sensing, atmospheric radiation, astrophysics, and biomedicine.

MICHAEL I. MISHCHENKO is a senior scientist at the NASA Goddard Institute for Space Studies in New York City. After gaining a Ph.D. in physics in 1987, he has been principal investigator on several NASA and DoD projects and has served as topical editor and editorial board member of several leading scientific journals. Dr. Mishchenko is a recipient of the Henry G. Houghton Award of the American Meteorological Society and is an elected Fellow of the American Geophysical Union, the Optical Society of America, and the Institute of Physics. His research interests include electromagnetic scattering, radiative transfer, and remote sensing.

LARRY D. TRAVIS is presently Associate Chief of the NASA Goddard Institute for Space Studies. He gained a Ph.D. in astronomy at Pennsylvania State University in 1971. Dr. Travis has acted as principal investigator on several NASA projects and was awarded a NASA Exceptional Scientific Achievement Medal. His research interests include the theoretical interpretation of remote sensing measurements of polarization, planetary atmospheres, atmospheric dynamics, and radiative transfer.

ANDREW A. LACIS is a senior scientist at the NASA Goddard Institute for Space Studies, and teaches radiative transfer at Columbia University. He gained a Ph.D. in physics at the University of Iowa in 1970 and has acted as principal investigator on numerous NASA and DoE projects. His research interests include radiative transfer in planetary atmospheres, the absorption of solar radiation by the Earth's atmosphere, and climate modeling.

Among the numerous scientific publications by these authors is the monograph on *Scattering, Absorption, and Emission of Light by Small Particles* published by Cambridge University Press in 2002.

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

Multiple Scattering of Light by Particles

Radiative Transfer and
Coherent Backscattering

Michael I. Mishchenko

Larry D. Travis

Andrew A. Lacis

NASA Goddard Institute for Space Studies, New York



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press

The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9780521834902

© NASA 2006

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without the written
permission of Cambridge University Press.

First published 2006

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-83490-2 Hardback

Cambridge University Press has no responsibility for the persistence or
accuracy of URLs for external or third-party internet websites referred to in
this publication, and does not guarantee that any content on such websites is,
or will remain, accurate or appropriate. Information regarding prices, travel
timetables, and other factual information given in this work is correct at
the time of first printing but Cambridge University Press does not guarantee
the accuracy of such information thereafter.

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

Contents

Preface xi**Dedication and acknowledgments** xv**Chapter 1 Introduction** 1

- 1.1 Electromagnetic scattering by a fixed finite object 1
- 1.2 Actual observables 5
- 1.3 Foldy–Lax equations 6
- 1.4 Dynamic and static scattering by random groups of particles 7
- 1.5 Ergodicity 9
- 1.6 Single scattering by random particles 10
- 1.7 Multiple scattering by a large random group of particles 12
- 1.8 Coherent backscattering 14
- 1.9 Classification of electromagnetic scattering problems 16
- 1.10 Notes and further reading 18

Chapter 2 Maxwell equations, electromagnetic waves, and Stokes parameters 20

- 2.1 Maxwell equations and constitutive relations 20
- 2.2 Boundary conditions 23
- 2.3 Time-harmonic fields 26
- 2.4 The Poynting vector 28
- 2.5 Plane-wave solution 31
- 2.6 Coherency matrix and Stokes parameters 37
- 2.7 Ellipsometric interpretation of the Stokes parameters 41

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

vi	Multiple Scattering of Light by Particles	
2.8	Rotation transformation rule for the Stokes parameters	47
2.9	Quasi-monochromatic light	48
2.10	Measurement of the Stokes parameters	54
2.11	Spherical-wave solution	58
2.12	Coherency dyad of the electric field	62
2.13	Historical notes and further reading	64
Chapter 3	Basic theory of electromagnetic scattering	66
3.1	Volume integral equation and Lippmann–Schwinger equation	67
3.2	Scattering in the far-field zone	71
3.3	Scattering dyadic and amplitude scattering matrix	78
3.4	Reciprocity	80
3.5	Scale invariance rule	84
3.6	Electromagnetic power and electromagnetic energy density	87
3.7	Phase matrix	93
3.8	Extinction matrix	99
3.9	Extinction, scattering, and absorption cross sections	102
3.10	Coherency dyad of the total electric field	105
3.11	Other types of illumination	109
3.12	Variable scatterers	110
3.13	Thermal emission	112
3.14	Historical notes and further reading	114
Chapter 4	Scattering by a fixed multi-particle group	115
4.1	Vector form of the Foldy–Lax equations	115
4.2	Far-field version of the vector Foldy–Lax equations	118
Chapter 5	Statistical averaging	123
5.1	Statistical averages	124
5.2	Configurational averaging	126
5.3	Averaging over particle states	126
Chapter 6	Scattering by a single random particle	131
6.1	Scattering in the far-field zone of the trap volume	131
6.2	“Near-field” scattering	136
Chapter 7	Single scattering by a small random particle group	140
7.1	Single-scattering approximation for a fixed group of particles	141
7.2	Far-field single-scattering approximation for a fixed particle group	142
7.3	Far-field uncorrelated single-scattering approximation and modified uncorrelated single-scattering approximation	145

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)**Contents**

vii

7.4	Forward-scattering interference	147
7.5	Energy conservation	151
7.6	Conditions of validity of the far-field modified uncorrelated single-scattering approximation	151
7.7	First-order-scattering approximation	158
7.8	Discussion	163
Chapter 8	Radiative transfer equation	165
8.1	The Twersky approximation	166
8.2	The Twersky expansion of the coherent field	171
8.3	Coherent field	173
8.4	Transfer equation for the coherent field	180
8.5	Dyadic correlation function in the ladder approximation	181
8.6	Integral equation for the ladder specific coherency dyadic	191
8.7	Integro-differential equation for the diffuse specific coherency dyadic	195
8.8	Integral and integro-differential equations for the diffuse specific coherency matrix	197
8.9	Integral and integro-differential equations for the diffuse specific coherency column vector	198
8.10	Integral and integro-differential equations for the specific intensity column vector	199
8.11	Summary of assumptions and approximations	200
8.12	Physical meaning of the diffuse specific intensity column vector and the coherent Stokes column vector	203
8.13	Energy conservation	208
8.14	External observation points	209
	8.14.1 Coherent field	210
	8.14.2 Ladder coherency dyadic	211
	8.14.3 Specific intensity column vector	213
	8.14.4 Discussion	214
	8.14.5 Illustrative example: first-order scattering	216
8.15	Other types of illumination	217
8.16	Phenomenological approach to radiative transfer	218
8.17	Scattering media with thermal emission	224
8.18	Historical notes and further reading	225
Chapter 9	Calculations and measurements of single-particle characteristics	227
9.1	Exact theoretical techniques	227
9.2	Approximations	234
9.3	Measurement techniques	237

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

viii	Multiple Scattering of Light by Particles
9.4	Further reading 239
Chapter 10	Radiative transfer in plane-parallel scattering media 240
10.1	The standard problem 240
10.2	The propagator 243
10.3	The general problem 245
10.4	Adding equations 247
10.5	Invariant imbedding equations 255
10.6	Ambarzumian equation 258
10.7	Reciprocity relations for the reflection and transmission matrices 259
10.8	Notes and further reading 260
Chapter 11	Macroscopically isotropic and mirror-symmetric scattering media 261
11.1	Symmetries of the Stokes scattering matrix 262
11.2	Macroscopically isotropic and mirror-symmetric scattering medium 265
11.3	Phase matrix 266
11.4	Forward-scattering direction and extinction matrix 270
11.5	Backward scattering 273
11.6	Scattering cross section and asymmetry parameter 275
11.7	Thermal emission 276
11.8	Spherically symmetric particles 277
11.9	Effects of nonsphericity and orientation 278
11.10	Normalized scattering and phase matrices 279
11.11	Expansion in generalized spherical functions 282
11.12	Circular-polarization representation 286
11.13	Illustrative examples 291
Chapter 12	Radiative transfer in plane-parallel, macroscopically isotropic and mirror-symmetric scattering media 302
12.1	The standard problem 302
12.2	The general problem 304
12.3	Adding equations 306
12.4	Invariant imbedding and Ambarzumian equations 311
12.5	Successive orders of scattering 313
12.6	Symmetry relations 315
12.6.1	Phase matrix 315
12.6.2	Reflection and transmission matrices 316
12.6.3	Matrices describing the internal field 317
12.6.4	Perpendicular directions 317
12.7	Fourier decomposition 318

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

	12.7.1	Fourier decomposition of the VRTE	318
	12.7.2	Fourier components of the phase matrix	319
12.8		Scalar approximation	321
12.9		Notes and further reading	322
Chapter 13 Illustrative applications of radiative transfer theory 324			
13.1		Accuracy of the scalar approximation	324
	13.1.1	Rayleigh-scattering slabs	325
	13.1.2	Polydisperse spherical particles and spheroids	337
13.2		Directional reflectance and spherical and plane albedos	347
13.3		Polarization as an effect and as a particle characterization tool	357
13.4		Depolarization	362
13.5		Further reading	362
Chapter 14 Coherent backscattering 365			
14.1		Specific coherency dyadic	366
14.2		Reflected light	371
14.3		Exact backscattering direction	373
14.4		Other types of illumination	379
14.5		Photometric and polarimetric characteristics of coherent backscattering	380
	14.5.1	Unpolarized incident light	380
	14.5.2	Linearly polarized incident light	381
	14.5.3	Circularly polarized incident light	382
	14.5.4	General properties of the enhancement factors and polarization ratios	383
	14.5.5	Spherically symmetric particles	385
	14.5.6	Benchmark results for Rayleigh scattering	386
14.6		Numerical results for polydisperse spheres and polydisperse, randomly oriented spheroids	386
14.7		Angular profile of coherent backscattering	395
14.8		Further discussion of theoretical and practical aspects of coherent backscattering	402
14.9		Applications and further reading	404
Appendix A Dyads and dyadics 407			
Appendix B Spherical wave expansion of a plane wave in the far-field zone 409			
Appendix C Euler rotation angles 411			
Appendix D Integration quadrature formulas 413			
Appendix E Stationary phase evaluation of a double integral 416			

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)x **Multiple Scattering of Light by Particles**

Appendix F Wigner functions, Jacobi polynomials, and generalized spherical functions 418

- F.1 Wigner d -functions 418
F.2 Jacobi polynomials 422
F.3 Orthogonality and completeness 422
F.4 Recurrence relations 423
F.5 Legendre polynomials and associated Legendre functions 424
F.6 Generalized spherical functions 425
F.7 Wigner D -functions, addition theorem, and unitarity 426
F.8 Further reading 428

Appendix G Système International units 429**Appendix H Abbreviations 431****Appendix I Glossary of symbols 433****References 442****Index 469**

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

Preface

Since the seminal papers by Lommel (1887), Chwolson (1889), and Schuster (1905), the radiative transfer equation (RTE) has been widely used in diverse areas of science and engineering to describe multiple scattering of light and other electromagnetic radiation in media composed of randomly and sparsely distributed particles. Analytical studies of the RTE have formed a separate branch of mathematical physics. However, despite the importance and the widespread use of the radiative transfer theory (RTT), its physical basis had not been established firmly until quite recently.

Indeed, the traditional “phenomenological” way to introduce the RTE has been to invoke an eclectic combination of principles borrowed from classical radiometry (i.e., intuitively appealing arguments of energy balance and the simple heuristic concepts of light rays and ray pencils), classical electromagnetics (electromagnetic scattering, Stokes parameters, and phase and extinction matrices), and even quantum electrodynamics (“photons”). Furthermore, the phenomenological approach has always relied on an illusive concept of an “elementary (or differential) volume element” of the discrete scattering medium. To sew together these motley concepts, one needs a set of postulates that appear to be plausible at first sight but turn out to be artificial upon close examination.

This inconsistent approach to radiative transfer is quite deceptive since it implies that in order to derive the RTE for media composed of elastically scattering particles one needs postulates other than those already contained in classical electromagnetics. The phenomenological “derivation” becomes especially questionable when one attempts to include the effects of polarization described by the so-called vector RTE and/or to take into account the effects of particle nonsphericity and orientation. Furthermore, it does not allow one to determine the range of applicability of the RTE and

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

trace the fundamental link between the RTT and the effect of coherent backscattering.

During the past few decades, there has been significant progress in studies of the statistical wave content of the RTT. This research has resulted in a much improved understanding of the basic assumptions leading to the RTE and has indeed demonstrated it to be a corollary of the Maxwell equations. Hence, the main goal of this monograph is to consistently present the RTT as a branch of classical electromagnetics as applied to discrete random media and to clarify the relationship between radiative transfer and coherent backscattering.

Another motivation for writing this book was the recognition of the scarcity of comprehensive monographs describing the fundamentals of polarized radiative transfer and its applications in a way intelligible to graduate students and non-expert scientists.¹ This factor has significantly impeded the development and wide dissemination of physically-based remote sensing and particle characterization techniques. Hence, the additional purpose of this volume is to present a broad and coherent outline of the subject and to make the technical material accessible to a larger audience than those specializing in this research area. Consistent with this purpose, our presentation assumes minimal prior knowledge of the subject matter and the relevant theoretical approaches. We expect, therefore, that the book will be useful to science professionals, engineers, and graduate students working in a broad range of disciplines: optics, electromagnetics, atmospheric radiation and remote sensing, radar meteorology, oceanography, climate research, astrophysics, optical engineering and technology, particle characterization, and biomedical optics.

This volume is a natural continuation of our recent monograph on *Scattering, Absorption, and Emission of Light by Small Particles* (Mishchenko *et al.*, 2002; hereinafter referred to as MTL²) in that it consistently uses the same general methodology and notation system while applying them to multiple scattering by random particle ensembles. However, the present book contains all the necessary background material and is self-contained.

As in MTL, we usually denote vectors using the Times bold font and matrices using the Arial bold font. Unit vectors are denoted by a caret, whereas dyads and dyadics are denoted by the symbol \leftrightarrow . The Times italic font is reserved for scalar quantities, important exceptions being the square root of minus one, the differential sign, and the base of natural logarithms, which are denoted by Times roman characters *i*, *d*, and *e*, respectively. Another exception is the relative refractive index, which is denoted by a sloping sans serif *m*. For the reader's convenience, a glossary listing the symbols used, their meaning and dimension, and the section where they first appear is provided at the end of the book (Appendix I). Appendix H contains a list of abbreviations.

¹ The recent book by Hovenier *et al.* (2004) is a notable exception.

² By agreement with Cambridge University Press, MTL is now publicly available in the .pdf format at <http://www.giss.nasa.gov/~crmim/books.html>.

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

Preface

xiii

We did not try to compile a comprehensive and detailed reference list. Instead, preference was given to seminal publications as well as to relevant books and reviews where further references can be found.

We mention several relevant computer programs made publicly available on-line. These programs have been thoroughly tested and are expected to generate reliable results provided that they are implemented as instructed. It is not inconceivable, however, that some of these programs contain errors and/or are not platform-independent. Also, it is possible that users could specify input parameter values that are outside the intended range for which accurate results can be expected. For these reasons the authors of this book and the publisher disclaim all liability for any damage that may result from the use of the programs. Although the authors and the publisher have used their best endeavors to ensure that the URLs for external Internet sites referred to in this book are correct and active at the time of this book going to press, they cannot guarantee that a site will remain live or that its content is or will remain appropriate.

Michael I. Mishchenko
Larry D. Travis
Andrew A. Lacis

New York
September 2005

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

Dedication and acknowledgments

The phenomenological theory of radiative transfer in discrete random media had been widely used for many decades in numerous research and engineering disciplines despite the fact that its physical origin had not been established. This uncomfortable situation has finally changed, and the RTT has become a legitimate branch of classical electromagnetics. It was very exciting for us to be able to write this entire monograph on radiative transfer without ever having to leave the firm grounds of electromagnetic theory. We, therefore, appreciatively dedicate this book to James Clerk Maxwell, whose monumental contribution to physics can be compared only to that of Sir Isaac Newton and whose equations of electromagnetism have been voted by scientists to be the greatest equations ever (Crease, 2004).

Several prominent scientists have made important contributions to the evolving subject of multiple wave scattering by small particles and microphysical justification of the RTT. Our own research has been most influenced by the publications of Yuri Barabanenkov, Anatoli Borovoi, Akira Ishimaru, Leung Tsang, Victor Twersky, and Hendrik van de Hulst to whom we express sincere appreciation.

We are deeply indebted to Joop Hovenier, Michael Kahnert, and Cornelis van der Mee for numerous discussions, continued encouragement, and valuable comments on a preliminary version of this book that resulted in a much improved manuscript. We also gratefully acknowledge illuminating discussions with Yuri Barabanenkov, Anatoli Borovoi, Oleg Bugaenko, Brian Cairns, Barbara Carlson, Zhanna Dlugach, Helmut Domke, James Hansen, Vsevolod Ivanov, Nikolai Khlebtsov, Kuo-Nan Liou, Daniel Mackowski, Bart van Tiggelen, Victor Tishkovets, Gorden Videen, Edgard Yanovitskij, and many other colleagues.

We thank Joop Hovenier for several invitations to visit him at de Vrije Univer-

Cambridge University Press

978-0-521-83490-2 - Multiple Scattering of Light by Particles: Radiative Transfer and Coherent Backscattering

Michael I. Mishchenko, Larry D. Travis and Andrew A. Lacis

Frontmatter

[More information](#)

xvi **Dedication and acknowledgments**

siteit te Amsterdam and de Universiteit van Amsterdam and for generous travel support. MIM thanks Helmut Domke for kind hospitality during a short stay at Zentralinstitut für Astrophysik in Potsdam.

Our research endeavors have been generously funded over the years by grants from the United States Government. We thankfully acknowledge the continuing support from the NASA Glory Mission Project, the NASA Earth Observing System Program, and the Department of Energy Atmospheric Radiation Measurement Program. The preparation of this book was sponsored by a grant from the NASA Radiation Sciences Program managed by Donald Anderson and Hal Maring.

We thank Zoe Wai and Josefina Mora for their help in tracing some of the less accessible publications and acknowledge the fine cooperation that we received from the staff of Cambridge University Press at all stages of the production of this book. We are particularly grateful to Jacqueline Garget for her patience and encouragement and to Jo Tyszka for thorough copy-editing work.

Our special thanks go to Nadia Zakharova who provided invaluable assistance during the preparation of the manuscript and the proofreading stage. She also contributed many numerical results and almost all computer graphics.