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Multiplicity Distribution under the Clustering Assumption in High Energy Hadron Collisions

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Recently it has been shown[1] that the multiplicity distribution in high energy pp collisions is broader than the Poisson distribution, and obeys the Koba-Nielsen-Olesen scaling law[2] above 50 GeV/c up to 300 GeV/c of the initial momentum. We calculate the multiplicity distribution assuming that in high energy hadron collisions, some clusters are formed and then decay into secondary particles. To obtain the distribution function of secondary particles emitted from each cluster, we follow the argument of photon counting distribution from a thermal source.[3],[4]

Let a_k be the annihilation operator for the particle with momentum k. It is well known that the coherent state $|z\rangle$ corresponding to it satisfies the following equation:

$$a_k|z\rangle = z_k|z\rangle\,, \tag{1}$$

where

$$|z\rangle = \sum_{n=0}^{\infty} \exp\left(-\frac{1}{2}\,|z_k|^2\right)\frac{(z_k)^n}{\sqrt{n!}}\,|n\rangle\,. \tag{2}$$

We will identify this state with the final particle state emitted from a cluster. Here the particles are assumed to be neutral and scalar for simplicity. Then, the probability for n particles finding in the final state is given as[5]

$$P_c(n) = |\langle n|z\rangle|^2 = \frac{|z_k|^{2n}}{n!}\exp(-|z_k|^2)\,. \tag{3}$$

Therefore, the squared value of $|z_k|$ specifying the coherent state $|z\rangle$ represents the average multiplicity.

We further assume that the coherent state of secondary particles emitted from a cluster is distributed at random analogously to photon distribution from a thermal source. Then the density operator of the coherent state is given as follows:

$$\rho = \int \varphi(z)\,|z\rangle\langle z|\,d^2z \tag{4}$$

and

$$\varphi(z) = \frac{1}{\pi}\,\frac{1}{\langle n\rangle}\exp\left(-\frac{|z_k|^2}{\langle n\rangle}\right). \tag{5}$$

Here d^2z denotes $d(\mathrm{Re}\,z)\,d(\mathrm{Im}\,z)$, and $\langle n\rangle$ is the average of $|z_k|^2$. Because of this distribution of the coherent state the probability given by equation (3) is changed to the following:

$$P(n) = \int P_c(n)\,\varphi(z)\,d^2z = \frac{\langle n\rangle^n}{(1+\langle n\rangle)^{n+1}}\,. \tag{6}$$

Equation (6) is known as a geometric distribution or Bose distribution,[3],[4] which can be obtained from the generating function $Q(\lambda)$ defined as

$$Q(\lambda) \equiv \sum_{n=0}^{\infty}(1-\lambda)^n P(n) = (1+\lambda\langle n\rangle)^{-1}. \tag{7}$$

In the above derivation only one mode is treated. Extension to general modes is straightforward, but we will not consider the momentum dependence in this short note.

Now let us consider the case in which the number of produced clusters is α. If the multiplicity distribution from each cluster obeys Eq. (6) and the average number is the same, the generating function takes the form

$$Q_\alpha(\lambda) = \left(1+\lambda\,\frac{\langle n\rangle}{\alpha}\right)^{-\alpha}. \tag{8}$$

Here $\langle n\rangle$ denotes the average total multiplicity in the hadron collisions. Then, from Eq. (8) the probability $P_\alpha(n)$ is given as

$$P_\alpha(n)=\frac{1}{n!}(-1)^n\frac{\partial^n}{\partial\lambda^n}Q_\alpha(\lambda)|_{\lambda=1}$$

$$=\frac{\Gamma(\alpha+n)}{\Gamma(n+1)\cdot\Gamma(\alpha)}\left(1+\frac{\langle n\rangle}{\alpha}\right)^{-\alpha}$$

$$\times\left(1+\frac{\alpha}{\langle n\rangle}\right)^{-n}. \quad (9)$$ [4),6)]

We also obtain $F^{(k)}\equiv\langle n(n-1)\cdots(n-k+1)\rangle$ as

$$F^{(k)}=(-1)^k\frac{\partial^k}{\partial\lambda^k}Q_\alpha(\lambda)|_{\lambda=0}$$

$$=\frac{\Gamma(\alpha+k)}{\alpha^k\cdot\Gamma(\alpha)}\langle n\rangle^k. \quad (10)$$

Defining $g^k\equiv F^{(k)}/\langle n\rangle^k$ and $C^k\equiv\langle n^k\rangle/\langle n\rangle^k$, we get

$$C^k=g^k+\frac{k(k-1)}{2}\frac{1}{\langle n\rangle}+O\left(\frac{1}{\langle n\rangle^2}\right). \quad (11)$$

Under the assumption $n_{\rm ch}=\frac{2}{3}n$ as usual, we put naturally

$$C^k=\langle n^k\rangle/\langle n\rangle^k=\langle n_{\rm ch}^k\rangle/\langle n_{\rm ch}\rangle^k,$$

where $n_{\rm ch}$ is the charged multiplicity. For n, $\langle n\rangle\gg1$ multiplicity distribution becomes as follows:

$$P_\alpha(n)\simeq\left(\frac{\alpha}{\langle n\rangle}\right)^\alpha\frac{1}{\Gamma(\alpha)}n^{\alpha-1}\exp\left(-\alpha\frac{n}{\langle n\rangle}\right)$$

$$=\frac{1}{\langle n\rangle}\frac{\alpha^\alpha}{\Gamma(\alpha)}z^{\alpha-1}\exp(-\alpha z)$$

$$=\frac{1}{\langle n\rangle}\psi_\alpha(z), \quad (12)$$

where we put $z=n/\langle n\rangle=n_{\rm ch}/\langle n_{\rm ch}\rangle$. This is the KNO scaling law, unless α depends on energy. In this limit C^k is equal to g^k as is clear from Eq. (11).

Finally we will compare this model predictions with the experimental data. For the choice of free parameter $\alpha=5$ and some cases of $\langle n_{\rm ch}\rangle$, we obtain numerical values shown in Table I together with the experimental data. From Fig. 1 and Table I, the multiplicity distribution and moments predicted by the model will be consistent with the experimental data and the KNO scaling law will be satisfied. There is,

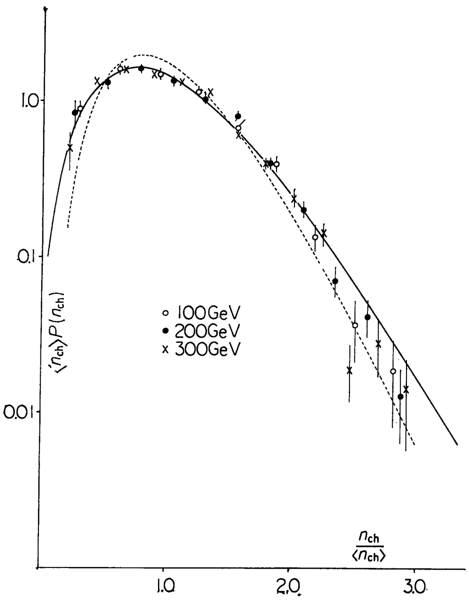

Fig. 1. The solid line is the multiplicity distribution from Eq. (9) with $\langle n_{\rm ch}\rangle=8$ and $\alpha=5$ and the dotted line is the multiplicity distribution in the high energy limit from Eq. (11) with $\alpha=5$.

Table I.

	for $\langle n_{\rm ch}\rangle=8$	for $\langle n_{\rm ch}\rangle=10$	for $n_{\rm ch}$, $\langle n_{\rm ch}\rangle\gg1$	Data[1]
c^2	1.283	1.266	1.2	1.244
c^3	1.987	1.924	1.68	1.813
c^4	3.587	3.397	2.688	2.973
c^5	7.380	6.82	4.838	5.36
c^6	16.95	15.34	9.677	10.43

however, another possibility that the number of clusters α may vary with the energy of incident particle. If the average number of secondaries emitted from a cluster is constant, α will increase with energy, for example, $\alpha \propto \langle n \rangle \propto \log s$. In this case Eq. (9) will approach the Gaussian distribution in the high energy limit.

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