Multipoint Moment Matching Model For Multiport Distributed Interconnect Networks

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Abstract

In this paper, we provide a multipoint moment matching model for multiport distributed interconnect networks. We introduce a new concept: integrated-congruence transform which can be applied to the partial differential equations of a distributed line and generate a passive finite order system as its model. Moreover, we also provide an efficient algorithm based on the L^2 Hilbert space theory so that exact moment matching at multiple points can be obtained.

1 Introduction

With the rapid increase of signal frequency and decrease of feature sizes of high speed electronic circuits, interconnect has become a dominating factor in determining circuit performance and reliability in deep submicron designs. Not only interconnect delay may dominate gate delay, but also transmission line effects, such as reflection, distortion, dispersion and crosstalk, have severe impact on circuit performance. In recent years, interconnect modeling, simulation and design has become a hot topic in the research of advanced CAD techniques [1], and model order reduction techniques have been advanced quickly to meet with the requirement of the fast simulation and design of interconnects. Especially, the symmetric $Padé\ Via\ Lanczos$ algorithm and the congruence transform together with the $Arnoldi$ algorithm have been successfully applied to the model order reduction with multipoint moment matching of RC and RLC lumped circuits [2]-[6].

At high frequencies, a long on-chip wire should be modeled as a lossy transmission line [7]. One way to use the Lanczos/Arnoldi algorithm to find the reduced order model of transmission lines is to do discretization for the lines, as these algorithm can only be applied to finite order systems [8] [9]. But discretization is not ideal either in theory or in practice. For example, for a nearly lossless line terminated with resistors matching the characteristic impedance of the line at high frequencies and excited by a pulse source, when discrete model is used, there are ripples in the waveform of the output voltage, which can not be seen in practice. Also, because of the discretization, the moment matching model generated cannot be exact. Moreover, the error of the discretization increases in the modeling of gobal wires where we have very high frequency and quite obvious inductance effects. The development of an algorithm to do model order reduction for distributed networks without any discretization has interested many researchers.

In this paper, we provide a new algorithm to do model order reduction of distributed interconnect networks. Our algorithm consists of two main steps. In the first step, each distributed line is modeled by a finite order system with passivity preservation and multipoint moment matching of its input admittance matrix. In the second step, an Arnoldi-based congruence transform is applied to the network to form its reduced order model.Our algorithm can guarantee the passivity of the reduced order model and provide the multi-point moment matching as required. The main contribution of this paper is that we provide a passive reduced order model algorithm for distributed lines with multipoint moment matching and avoid any discretization of the lines. We developed an integrated-congruence transform, which can directly be applied to the partial differential equations of a line and generate a passive finite order system so that the discretization step can be eliminated. To meet with the moment matching requirement, we extend the Krylov subspace algorithm for finite dimensional space to an approach based on the L^2 Hilbert space theory [11] , which can provide an exact moment matching model. Our algorithm is new in theory, and experiments show it works well in practice.

2 Integrated-congruence Transform and Passivity Preservation

2.1 General form of transmission line equations

We consider a transmission line system consisting of m coupled lines. Suppose that R', L', G' and C' are the resistance, inductance, conductance and capacitance matrix per unit length of the line, respectively, and d is its length. We first normalize the length to 1, and let $R = R'd$, $L = L'd$, $G = G'd$ and $C = C'd$ be the normalized resistance, inductance, conductance and capacitance matrix per unit length, respectively, of the normalized line [1]. Let z be the axis along the line, and $z = 0$ and $z = 1$ correspond to its near and far end, respectively. Let $I(z,s)$ and $V(z,s)$ be the current and voltage vector in the frequency domain along the line. Then, the equations of the line can be written as follows:

$$(sM + N + T\frac{d}{dz})X(z,s) = 0 \tag{1}$$

where

$$X(z,s) = \begin{bmatrix} I(z,s) \\ V(z,s) \end{bmatrix} \tag{2}$$

$$M = \begin{bmatrix} L & \\ & C \end{bmatrix} \tag{3}$$

$$N = \begin{bmatrix} R & \\ & G \end{bmatrix} \tag{4}$$

and

$$T = \begin{bmatrix} & I \\ I & \end{bmatrix} \tag{5}$$

where I is an $m \times m$ identity matrix. When we are interested in the input admittance of the matrix, it is assumed that the system is driven by voltage sources, so the boundary conditions of the line are

$$\begin{bmatrix} V(0,s) \\ V(1,s) \end{bmatrix} = V_s(s) \tag{6}$$

where $V_s(s)$ is a $2m$-dimension source voltage vector.

2.2 Integrated-congruence transform

Suppose that $U(z)$ is a $2m \times n$ matrix, which can be expressed as

$$U(z) = \begin{bmatrix} u_1(z) & u_2(z) & \ldots & u_n(z) \end{bmatrix} = \begin{bmatrix} U_I(z) \\ U_V(z) \end{bmatrix}$$

$$= \begin{bmatrix} U_{I,1}(z) & U_{I,2}(z) & \ldots & U_{I,n}(z) \\ U_{V,1}(z) & U_{V,2}(z) & \ldots & U_{V,n}(z) \end{bmatrix} \tag{7}$$

where $u_j(z)$ is $U(z)$'s j-th column vector and $U_I(z)$ and $U_V(z)$ are two $m \times n$ submatrices of matrix $U(z)$. $u_j(z)$ consists of two subvectors

$$u_j(z) = \begin{bmatrix} U_{I,j}(z) \\ U_{V,j}(z) \end{bmatrix} \tag{8}$$

[1] In the case of an RC line, matrices L and G are zero matrices.

each of which has m components, i.e.,

$$U_{I,j}(z) = [U_{I,1j}(z), U_{I,2j}(z), \ldots, U_{I,mj}(z)]^T \tag{9}$$

and

$$U_{V,j}(z) = [U_{V,1j}(z), U_{V,2j}(z), \ldots, U_{V,mj}(z)]^T \tag{10}$$

Note that matrix $U(z)$ is a function of variable z but not a function of s.
Let

$$X(z,s) = U(z)\hat{x}(s) \tag{11}$$

where vector $\hat{x}(s) = [\hat{x}_1(s), \hat{x}_2(s), \ldots, \hat{x}_n(s)]^T$ is of dimension n. Substitute Eq(11) to Eq(1), premultiply $u^T(z)$ on both sides of the equations and integrate them w.r.t. variable z from 0 to 1, we obtain

$$(s\hat{M} + \hat{N}_1 + \hat{T})\hat{x}(s) = 0 \tag{12}$$

where

$$\hat{M} = \int_0^1 U^T(z)M(z)U(z)dz$$

$$= \int_0^1 (U_I^T(z)L(z)U_I(z) + U_V^T(z)C(z)U_V(z))dz \tag{13}$$

$$\hat{N}_1 = \int_0^1 U^T(z)N(z)U(z)dz$$

$$= \int_0^1 (U_I^T(z)R(z)U_I(z) + U_V^T(z)G(z)U_V(z))dz \tag{14}$$

and

$$\hat{T} = \int_0^1 U^T(z)T\frac{dU(z)}{dz}dz$$

$$= \int_0^1 (U_I^T(z)\frac{dU_V(z)}{dz} + U_V^T(z)\frac{dU_I(z)}{dz})dz \tag{15}$$

The transform from Eq(1) to Eq(12) is called an integrated-congruence transform (w.r.t. the transformation matrix $U(z)$). Note that the order of the system described by Eq(12) is n, and $\hat{x}(s)$ can be regarded as the state vector of the finite order system.
We divide $\hat{T}$ into two matrices: $\hat{T} = \hat{N}_2 + P$, where

$$P = \int_0^1 \frac{d(U_I^T(z)U_V(z))}{dz}dz = U_I^T(1)U_V(1) - U_I^T(0)U_V(0) \tag{16}$$

and

$$\hat{N}_2 = \int_0^1 (U_V^T(z)\frac{dU_I(z)}{dz} - \frac{dU_I^T(z)}{dz}U_V(z))dz \tag{17}$$

Note that

$$\hat{N}_2^T = -\hat{N}_2 \tag{18}$$

Let $\hat{N} = \hat{N}_1 + \hat{N}_2$, then Eq(12) becomes

$$(s\hat{M} + \hat{N})\hat{x}(s) = -P\hat{x}(s) \tag{19}$$

From Eq(16),

$$-P\hat{x}(s) = U_I^T(0)U_V(0)\hat{x}(s) - U_I^T(1)U_V(1)\hat{x}(s)$$

and from Eq(11),

$$U_V(0)\hat{x}(s) = V(0,s) = V_{s1}(s)$$

and

$$U_V(1)\hat{x}(s) = V(1,s) = V_{s2}(s)$$

so that

$$-P\hat{x}(s) = \hat{B}V_s(s) \tag{20}$$

where

$$\hat{B} = \begin{bmatrix} U_I(0) \\ -U_I(1) \end{bmatrix}^T \tag{21}$$

Substitute Eq(20) to Eq(19), we have the state equations for the state vector $\hat{x}$ as follows:

$$(s\hat{M} + \hat{N})\hat{x}(s) = \hat{B}V_s(s) \tag{22}$$

When we are interested in the input admittance of the line, the output vector $y(s)$ consists of $I(0,s)$ and $-I(1,s)$. From Eq(11), we have

$$y(s) = \begin{bmatrix} I(0,s) \\ -I(1,s) \end{bmatrix} = \begin{bmatrix} U_I(0) \\ -U_I(1) \end{bmatrix} \hat{x}(s) \tag{23}$$

Compared with Eq(21), it can be seen that

$$y(s) = \hat{B}^T\hat{x}(s) \tag{24}$$

and the input admittance matrix of the transformed system can be expressed as

$$\hat{Y}(s) = \hat{B}^T(s\hat{M} + \hat{N})^{-1}\hat{B} \tag{25}$$

2.3 Passivity of the reduced order system

Theorem 1
Suppose that the transformation matrix $U(z)$ is of full rank and each element of $U(z)$ in the region $z \in [0,1]$ is in C^1, then the reduced order model generated by using an integrated-congruence transform w.r.t. matrix $U(z)$ on an RLGC transmission line system is passive.
The proof of Theorem 1 can be found in [10].

3 Moment Matching

3.1 Moment matrices

An m-coupled line system is a $2m$ port. To find its input admittance, we can apply a set of $2m$ input voltage vectors with the j-th source voltage vector being the unit vector e_j. Suppose that $X_j(z,s)$ be the solution to Eq(1) when the source vector e_j is applied, and let the admittance matrix be

$$Y(s) = [Y_0(s), Y_1(s), \ldots, Y_{2m}(s)] \tag{26}$$

where $Y_j(s) = [I_j(0,s)^T, -I_j(1,s)^T]^T$.
Let $\tilde{I}(z,s) = [I_1(z,s), I_2(z,s), \ldots, I_{2m}(z,s)]$, $\tilde{V}(z,s) = [V_1(z,s), V_2(z,s), \ldots, V_{2m}(z,s)]$, and $W(z,s) = [\tilde{I}(z,s)^T, \tilde{V}(z,s)^T]^T$. Then, from Eq(1), we have

$$(sM + N + T\frac{d}{dz})W(z,s) = 0 \tag{27}$$

This is the block form of the line equations, and its boundary conditions are

$$\begin{bmatrix} \tilde{V}(0,s) \\ \tilde{V}(1,s) \end{bmatrix} = I \tag{28}$$

where I is the $2m \times 2m$ identity matrix. Let matrix $W(z,s)$ be expanded into Tylor series at some point $s = s_0$, $W(z,s) = \sum_{k=0}^{\infty} W^{(k)}(z,s_0)(s-s_0)^k$, then $W^{(k)}(z,s_0)$ is called the k-th order moment of $W(z,s)$ at $s = s_0$, which satisfies the following equations. For $k = 0$,

$$(TN(s_0) + \frac{d}{dz})W^{(0)}(z,s_0) = 0 \tag{29}$$

where

$$TN(s_0) = \begin{bmatrix} & G + s_0C \\ R + s_0L & \end{bmatrix} \tag{30}$$

The boundary conditions of this equation are

$$\begin{bmatrix} \tilde{V}^{(0)}(0,s_0) \\ \tilde{V}^{(0)}(1,s_0) \end{bmatrix} = I \tag{31}$$

where I is an $2m \times 2m$ identity matrix.
For $k > 0$, we have

$$(TN(s_0) + \frac{d}{dz})W^{(k)}(z,s_0) = -TMW^{(k-1)}(s_0,z) \tag{32}$$

where

$$TM = \begin{bmatrix} & C \\ L & \end{bmatrix} \tag{33}$$

and the boundary conditions of the equation are

$$\begin{bmatrix} \tilde{V}^{(k)}(0,s_0) \\ \tilde{V}^{(k)}(1,s_0) \end{bmatrix} = 0 \tag{34}$$

From these equations, the moemnt matrices $W^{(k)}(z,s_0)$ can be exactly computed [10].

3.2 Moment matching theorem

The moment matching theorem is based on the Hilbert L^2 space theory [11].

Theorem 2

Let $W(k, z, s_0) = \{W^{(0)}(z, s_0), \dots, W^{(k)}(z, s_0)\}$, i.e., $W(k, z, s_0)$ is a set of moment matrices of $W(z, s)$ at $s = s_0$ from order 0 to k. Let $U(z) = [u_1(z), u_2(z), \dots, u_n(z)]$ be the transformation matrix of the integrated-congruence transform. If matrix $U(z)$ is orthonormal and $W(k, z, s_0) \in colspan(U(z))$, i.e., each element in the set $W(k, z, s_0)$ can be expressed as a linear combination of the column vectors of matrix $U(z)$, then

$$\hat{Y}^{(j)}(s_0) = Y^{(j)}(s_0) \qquad 0 \le j \le k \tag{35}$$

The proof of the theorem can be found in [10].

3.3 Multipoint moment matching algorithm

Based on Theorem 2, we give the multipoint moment matching algorithm as follows. For a matching point s_i and its matching order k_i, we define a matching pair $m_i = (s_i, k_i)$, and we define a matching set $MS = \{m_1, m_2, \dots, m_p\}$. Given the matching set, the algorithm can be stated as follows.

Multipoint Moment Matching Algorithm 1

```
{ Input: Line number m, line parameters R, L, G and
    C and matching set MS.
  Output: Transformation matrix U.
  U = φ; n = 0;
  for i = 1 to |MS| do
    {(s_i, k_i) = m_i;
     for j = 0 to k_i do
       {compute [r_1(z), r_2(z), ..., r_2m(z)] = W^(j)(z, s_i);
        for k = 1 to 2m do
          {if(s_i is real)
             n=orthonormal(U, r_k(z), n);
           else
             {r_a = real(r_k(z));    r_b = imag(r_k(z));
               n=orthonormal(U, r_a, n);
               n=orthonormal(U, r_b, n);
             }
          }
       }
    }
}
```

In the above algorithm, the function orthonormal(u, r, n) is based on the Modified Gram-Schmidt(MGS) process and can be stated as follows.

function orthonormal(U(z),r(z),n)

```
 {for i=1 to n do
   {λ = ∫_0^1 r^T(z) u_i(z) dz;
     r(z) = r(z) − λ u_i(z);
   }
   a = ∫_0^1 r^T(z) r(z) dz;
   n = n + 1;
   u_n = r/√a;
   return(n);
 }
```

4 Model Order Reduction of Combined Lumped and Distributed Network

4.1 MNA equations

Suppose that we have a p-port distributed network, which consists of n distributed lines, some linear resistors, capacitors and inductors. This distributed network is driven by p voltage sources and its input admittance matrix is of interest. We first apply the model order reduction algorithm to all the lines. Suppose that for the i-th line, its state equations and output equations of the reduced order model are as follows:

$$F_{ai}(s) x_{ai} = B_{ai} V_{ai} \tag{36}$$

and

$$I_{ai} = B_{ai}^T x_{ai} \tag{37}$$

Where V_{ai} and I_{ai} are the port voltage and current vector of the line, respectively. Let V_n be the node voltage vector of the entire network, and A_{ai} be the node-branch incidence matrix of the port branches of the line. Then, $V_{ai} = A_{ai}^T V_n$, and Eq(36) can be rewritten in the following form:

$$F_{ai}(s) x_{ai} = C_{ai} V_n \tag{38}$$

where

$$C_{ai} = B_{ai} A_{ai}^T \tag{39}$$

and the contribution of I_{ai} to the KCL equations of the nodes can be expressed as

$$A_{ai} I_{ai} = C_{ai}^T x_{ai} \tag{40}$$

Let $x_a = [x_{a1}^T, x_{a2}^T, \dots, x_{an}^T]^T$, then we have

$$F_a(s) x_a = C_a V_n \tag{41}$$

where

$$F_a(s) = block\_diag(F_{a1}(s), F_{a2}(s), \dots, F_{an}(s)) \tag{42}$$

and

$$C_a = [c_{a1}^T, c_{a2}^T, \dots, c_{an}^T]^T \tag{43}$$

and the contribution of $I_{ai},\ \ 1 \le i \le n$ to the KCL equations can be expressed as $C_a^T x_a$.
Suppose that the nodal conductance and capacitance matrix of lumped resistors and capacitors are G and C, respectively. Suppose that there are resistor-inductor branches with the branch resistance and inductance matrix R and L respectively, the branch current vector I_L and the node- branch incidence matrix A_L. Let the port voltage and current vector be V_s and I_s, respectively, and their node-branch incidence matrix be A_s. Note that the current of each voltage source flows out of its positive terminal. Then, the unknown vector of the MNA equations of the network $x = [x_a^T, V_n^T, I_L^T, I_s^T]^T$, and the MNA equations can be written as follows:

$$H(s)x = BV_s \tag{44}$$

where

$$H(s) = \begin{bmatrix} F_a & -Q \\ Q^T & F_b \end{bmatrix} \tag{45}$$

with

$$Q = \begin{bmatrix} C_a & 0 & 0 \end{bmatrix} \tag{46}$$

$$F_b = \begin{bmatrix} sC+G & A_L & -A_s \\ -A_L^T & SL+R & \\ A_s^T & & \end{bmatrix} \tag{47}$$

and

$$B = [0 \quad 0 \quad 0 \quad I]^T \tag{48}$$

The output equations of the network are

$$I_s = B^T x \tag{49}$$

and the input admittance matrix of the network is

$$Y(s) = B^T H(s)^{-1} B \tag{50}$$

Theorem 3
Matrix $H(s)$ in Eq(50) is positive-real.

4.2 Model order reduction

Suppose that $|x| = r$, and we apply a congruence transform with the transformation matrix $V \in R^{r \times q}$ with $q < r - |I_s|$ on Eq(44), with

$$x = V\hat{x} \tag{51}$$

$$\hat{H}(s) = V^T H(s) V \tag{52}$$

and

$$\hat{B} = V^T B \tag{53}$$

then we have a q-th order system as follows:

$$\hat{H}(s)\hat{x} = \hat{B}V_s \tag{54}$$

The output equations become

$$I_s = \hat{B}^T \hat{x} \tag{55}$$

and the input admittance matrix of the reduced order model is

$$\hat{Y}(s) = \hat{B}^T \hat{H}(s)^{-1} \hat{B} \tag{56}$$

Theorem 4
If the transform matrix V is of full rank, than $\hat{Y}(s)$ is positive-real and the reduced order model is passive.

4.3 Moment matching

As in the case of modeling a line, we apply p sets of input voltages such that the j-th input voltage vector is the unit vector e_j, and let x_j be the solution to Eq(44) in this case. Let

$$X = [x_1, x_2, \ldots, x_p] \tag{57}$$

Then, the block form of the MNA equations will be

$$H(s)X = BV_s \tag{58}$$

and the output equations become

$$Y(s) = B^T X \tag{59}$$

We have the moment matching theorem as follows.
Theorem 5
Let $H(s) = sM + N$, $N(s_0) = s_0 M + N$, $X^{(j)}(s_0) = (-N(s_0)^{-1}M)^j N(s_0)^{-1} B$ and $K(n, s_0) = \{X^{(0)}(z, s_0), X^{(1)}(z, s_0), \ldots, X^{(n)}(z, s_0)\}$. If the congruence transform matrix V is orthonormal and $K(n, s_0) \in colspan(V)$, then

$$\hat{Y}^{(j)}(s_0) = Y^{(j)}(s_0), \qquad 0 \le j \le n \tag{60}$$

Based on Theorem 5, given the moment matching set MS, the algorithm for the formation of congruence transform matrix V is as follows.

Multipoint Moment Matching Algorithm 2
{ Input: Port number p, matrix $H(s) = sM + N$, matrix B and matching set MS.
  Output: Transformation matrix V.
  $V = \phi$; $n = 0$;
  for $i = 1$ to $|MS|$ do
    $\{(s_i, k_i) = m_i$;
     for $j = 0$ to k_i do
       {compute $[r_1, r_2, \ldots, r_p] = X^{(j)}(s_i)$;
        for $k = 1$ to p do
          {if(s_i is real)
            n=orthonormal1(V, r_k, n);
          else

```
            {r_a = real(r_k); r_b = imag(r_k);
             n=orthonormal1(V, r_a, n);
             n=orthonormal1(V, r_b, n);
            }
          }
        }
      }
}
```

In the above algorithm, the function $orthonormal1$ is as follows:

function $orthonormal1(V, x, n)$
{for $i = 1$ to n do
  $x = x - x^T V_i$;
 $a = \sqrt{x^T x}$;
 $n = n + 1$;
 $V_n = x/a$;
 return(n); }

5 Examples

We have successfully tested a number of examples. We show some of them here.
Example 1.
This is a simple example to show the advantage of our model over the discrete model of a transmission line. The circuit consists of a single line with parameters $R = 0.01\Omega/cm$, $L = 2.5nH/cm$, $C = 1ph/cm$, $d = 1cm$, a load resistor and a source resistor of 50Ω, which match the characteristic impedance of the line at high frequencies. The voltage source is a pulse. Fig.1 shows the output voltage waveform V_2 obtained by the SPICE simulation with the segmentation model of the line, where the solid and dashed lines correspond to the number of segments equal to 50 and 100, respectively. The ripple in the waveform is obvious, which should not exist when exact model is used, and when the number of segments increases, the magnitude of the ripple does not decrease much. Fig.2 shows the time domain simulation by using a three point moment matching model generated by our algorithm, where the ripple is missing, and the waveform is nearly exact. This example shows that discretization is not ideal in practice as we stated in “Introduction".
Example 2.
This is a clock net consisting of 73 lossy transmission lines, 2895 resistors and 2777 capacitors driven by a cascade of two inverters, as shown in Fig.3. The waveforms at $PIN117$ are shown in Fig.4, where the solid and dashed lines correspond to the result of SPICE simulation and the time domain simulation with our model, where for each line moment matching at 0 frequency with order 4 and at a high frequency with order 0 is used. These two waveforms are close.
Example 3.
The circuit is shown in Fig.5, where 4 coupled lines with neighbor coupling are presented. The waveforms of V_5 are shown in Fig.6, where the solid and dashed lines correspond to the result of SPICE simulation and the time domain simulation with our model, which is obtained by moment matching at zero frequency with order 4. These two waveforms are close.
Example 4.
This is an example borrowed from [12]. The circuit is shown in Fig.7, where two coupled line systems, each of which consists of three coupled lines, are presented. The frequency domain and time-domain response of V_{out} are shown in Fig.8 and 9, respectively. The solid line represents the exact solution where the coupled lines are modeled by their exact multiport characteristic model. The dashed line corresponds to our model, where a moment matching set $MS = \{(0Hz, 4), (1.5Ghz, 0), (3Ghz, 0), (4Ghz, 0), (5Ghz, 0)\}$ for each line system is used. The solid and dashed lines are indistinguishable. Compared with the model used in [12], not only that our model is guaranteed passive and theirs not, but also that our model order is much lower (a 40-th moment matching model at zero frequency is used for each coupled line system in [12]). This also shows the advantage of a multipoint moment matching model over a single point one.

6 Conclusions

We have presented a new algorithm for passive model order reduction with multipoint moment matching for distributed interconnect networks. Given the moment matching requirement, for each distributed lines, moment matrix functions are computed, then the Modified Gram-Schmidt(MGS) process is implemented to form an orthonormal transformation matrix in the Hilbert $L^2[0, 1]$ space. By using an integrated-congruence transform on the partial differential equations of the line, a finite order passive system satisfying the moment matching requirement is obtained. Then, the MNA equations of the whole network are formulated, and an orthonormal matrix based on the moment matching requirement is formed. By using a congruence transform with such a matrix, a passive reduced order model of the network with multipoint moment matching is obtained. The whole process can be done exactly in theory, especially as there is no discretization for a distributed line in the algorithm, an exact moment matching model can be obtained. Experiments show that the model generated by the algorithm works well.

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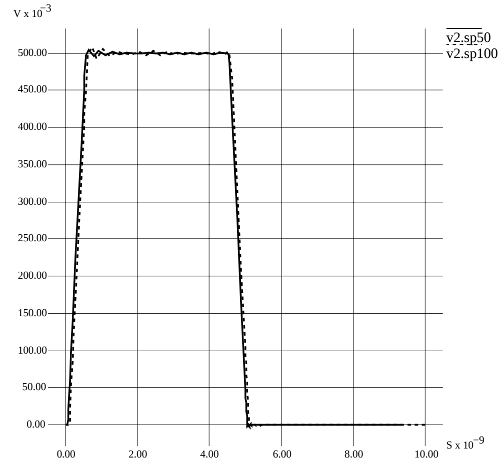
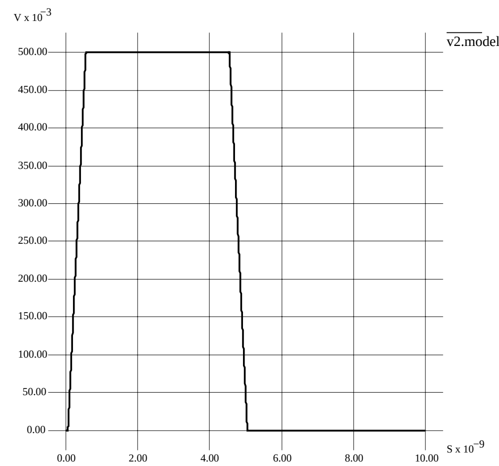


Figure 1.



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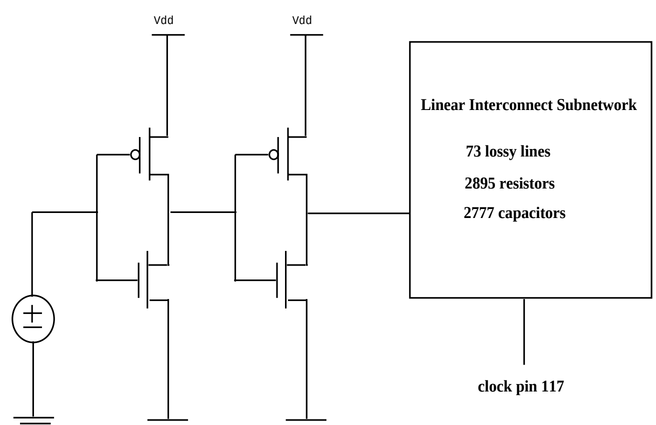

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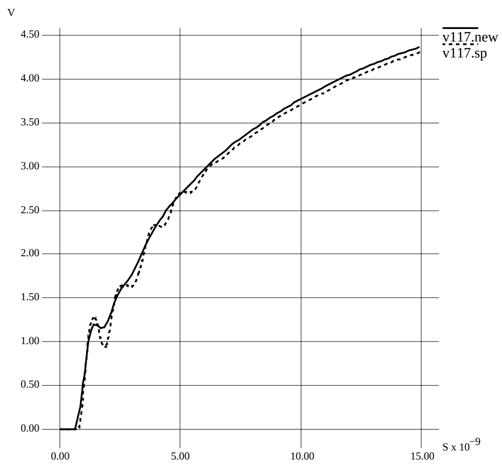

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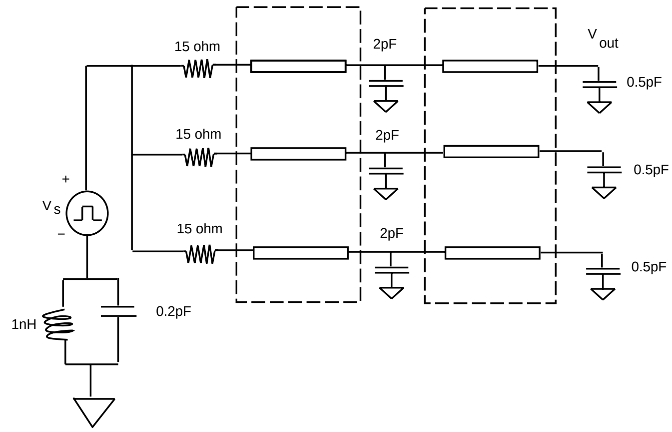

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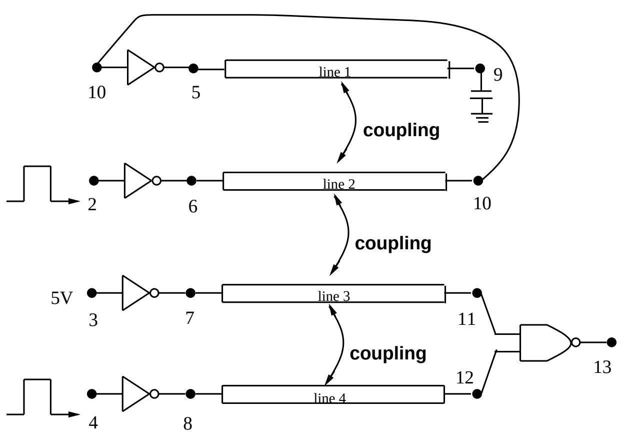

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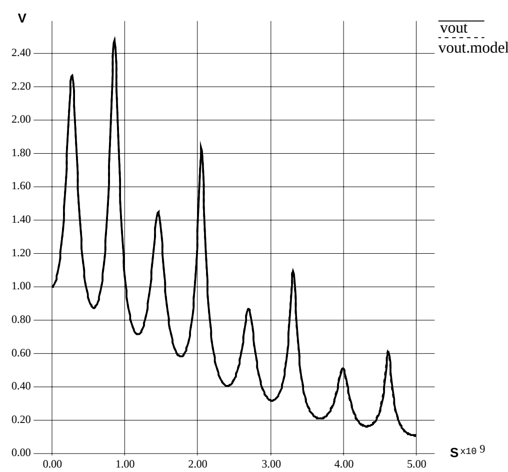

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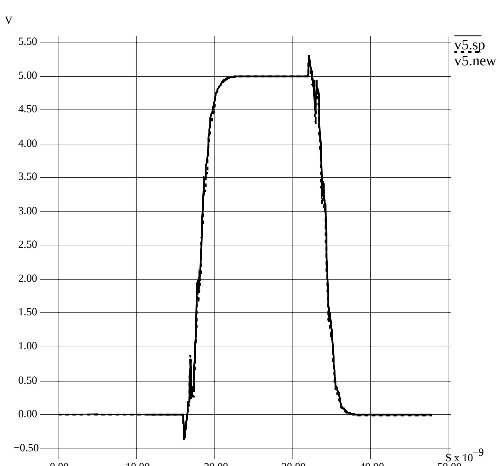

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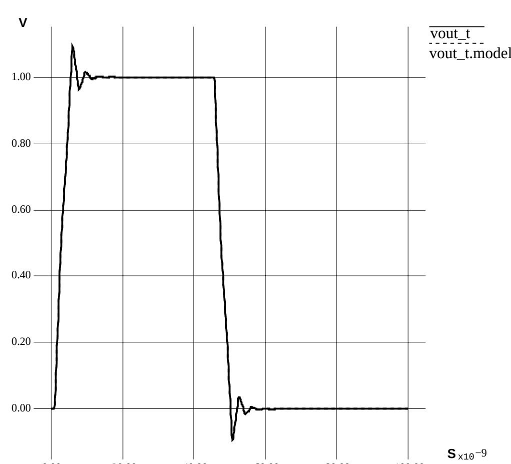

Figure 9.