

# Multisite, multifrequency tensor decomposition of magnetotelluric data

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**ABSTRACT**

Abstract text is rendered as a series of scattered, illegible characters and symbols, including fragments like 'A u i', 'w,', 'y', 'u u s', 'B i', 'g', 'A s,', 'w s', 'w s', 'u n y-', and 'i'.

**INTRODUCTION**

Introduction text is rendered as a series of scattered, illegible characters and symbols, including fragments like 'D u i', 'g', '1993;', '1993;', '1997;', '1991)', '(1995,', 'B i', and 'g'.

\* *mus i*  
 \* *us y C n d ,*  
 † *g*  
 © 2001 *i*

*mus i*  
 E *h i*  
 1C2 3,  
*i*

shear twist,  
 regional strike regional impedances.

2-  
 45° -45°),  
 static shifts

GROOM-BAILEY DECOMPOSITION

(1977)  
 (1992)  
 $Z_{measured} = RCZ_{2D}R^T$

$C = gTSA,$

$Z_{measured} = RTSZ_{regional}R^T$

$= \begin{bmatrix} \theta & -\theta \\ \theta & \theta \end{bmatrix} \begin{bmatrix} 1-te & e-t \\ e+t & 1+te \end{bmatrix}$

$\times \begin{bmatrix} 0 & A \\ -B & 0 \end{bmatrix} \begin{bmatrix} \theta & \theta \\ -\theta & \theta \end{bmatrix},$

(3)



EXTENSION OF GB DECOMPOSITION FOR MULTIPLE SITES AND MULTIPLE FREQUENCIES

Let  $\mathbf{a}$  be a vector of size  $S \times (N \times 4 + 2) + 1$ . The GB decomposition of the tensor  $\mathbf{y}$  is given by

$$\mathbf{y} = \mathbf{P} \mathbf{a} \mathbf{B} \quad (5)$$

where  $\mathbf{P}$  is a tensor of size  $S \times N \times 8$ ,  $\mathbf{a}$  is a vector of size  $S \times (N \times 4 + 2) + 1$ , and  $\mathbf{B}$  is a tensor of size  $S \times N \times 8$ . The decomposition is extended to multiple sites and multiple frequencies by considering the real and imaginary parts of the observed data  $\alpha_i^{obs}$  and the model  $\alpha_i^{model}(a)$ .

$$\gamma^2(a) = \sum_{k=1}^{SN} \left( \sum_{i=0}^3 \left[ \frac{Re(\alpha_i^{obs}) - Re(\alpha_i^{model}(a))}{\sigma_{\alpha_i}} \right]^2 + \sum_{i=0}^3 \left[ \frac{Im(\alpha_i^{obs}) - Im(\alpha_i^{model}(a))}{\sigma_{\alpha_i}} \right]^2 \right) \quad (9)$$

The variance  $\sigma_{\alpha_i}$  is assumed to be constant for all sites and frequencies. The model  $\alpha_i^{model}(a)$  is defined as

$$\alpha_i^{model}(a) = \frac{\alpha_i^{obs} - \alpha_i^{model}(a)}{\sigma_{\alpha_i}} \quad i = 1, 2, \dots, S \times N \times 8, \quad (15)$$

$$\gamma^2(\mathbf{a}) = \gamma^2(\mathbf{P}) + \sum_i \frac{\partial \gamma^2}{\partial a_i} a_i + \frac{1}{2} \sum_{i,j} \frac{\partial^2 \gamma^2}{\partial a_i \partial a_j} a_i a_j + \dots \approx c + \mathbf{J} \cdot \mathbf{a} + \frac{1}{2} \mathbf{a} \cdot \mathbf{H} \cdot \mathbf{a}, \quad (10)$$

where  $\mathbf{P}$ ,  $\mathbf{J}$ , and  $\mathbf{H}$  are tensors of size  $S \times N \times 8$ . The gradient of  $\gamma^2$  with respect to  $\mathbf{a}$  is given by

$$\nabla \gamma^2 \approx \mathbf{H} \cdot \mathbf{a} + \mathbf{J}. \quad (11)$$

The Hessian  $\mathbf{H}$  is a tensor of size  $S \times N \times 8$ . The condition for a minimum is given by

$$\mathbf{H} \cdot \mathbf{a}_i + \mathbf{J} = 0. \quad (12)$$

The gradient of  $\gamma^2$  with respect to  $\mathbf{P}$  is given by

$$\nabla \gamma^2(\mathbf{P}) = \mathbf{H} \cdot \mathbf{P} + \mathbf{J}. \quad (13)$$

The model  $\mathbf{a}_i$  is given by

$$\mathbf{a}_i = \mathbf{P} - \mathbf{H}^{-1} \cdot \nabla \gamma^2(\mathbf{P}). \quad (14)$$

The model  $\alpha_i^{model}(a)$  is defined as

$$\alpha_i^{model}(a) = \frac{\alpha_i^{obs} - \alpha_i^{model}(a)}{\sigma_{\alpha_i}} \quad i = 1, 2, \dots, S \times N \times 8, \quad (15)$$

The total  $\gamma^2(a)$  is given by

$$\gamma^2(a) = \sum_{i=1}^{SN8} [\gamma_i(a)]^2. \quad (16)$$

The model  $\alpha_i^{model}(a)$  is defined as

$$\frac{\partial \gamma_i(a)}{\partial a_j} = \frac{-1}{\sigma_{\alpha_i}} \frac{\partial \alpha_i^{model}(a)}{\partial a_j} \quad i = 1, 2, \dots, S \times N \times 8$$

$$j = 1, 2, \dots, S \times (N \times 4 + 2) + 1. \quad (17)$$

$$\frac{\partial \gamma^2(a)}{\partial a_i} = 2 \sum_{j=1}^{S \times N \times 8} \gamma_j(a) \frac{\partial \gamma_j(a)}{\partial a_i}$$

$$i = 1, 2, \dots, S \times (N \times 4 + 2) + 1. \quad (18)$$

$$\frac{\partial^2 \gamma^2}{\partial a_i \partial a_j} = 2 \sum_{k=1}^{S \times N \times 8} \frac{1}{\sigma_{\alpha_k}^2} \left[ \frac{\partial \alpha_k^{model}(a)}{\partial a_i} \frac{\partial \alpha_k^{model}(a)}{\partial a_j} + [\alpha_k^{obs} - \alpha_k^{model}(a)] \frac{\partial^2 \alpha_k^{model}(a)}{\partial a_i \partial a_j} \right]. \quad (19)$$

$$\frac{\partial^2 \gamma^2(a)}{\partial a_i \partial a_j} \approx 2 \sum_{k=1}^{S \times N \times 8} \frac{\partial \gamma_k(a)}{\partial a_i} \frac{\partial \gamma_k(a)}{\partial a_j}$$

$$i, j = 1, 2, \dots, S(N \times 4 + 2) + 1. \quad (20)$$

Assuming  $\chi^2$  error estimation

Define  $B$  and  $y$  as

g  
u  
i  
y  
s  
u  
A  
B

Failure of decomposition analysis

w  
y  
g  
±45° (e ±1)  
s  
y  
e),

$$Z_{meas}(\theta_{regional}) = \begin{bmatrix} -(1-t)B & (1-t)A \\ -(1+t)B & (1+t)A \end{bmatrix} \quad (21)$$

$$t(\theta) = t - (\theta - \theta_{regional})$$

$$(22)$$

$$Z_{meas}(\theta_{regional}) = \begin{bmatrix} -(e-t)B & 0 \\ -(1+te)B & 0 \end{bmatrix} \quad (23)$$

$$Z_{meas}(\theta_{regional}) = \begin{bmatrix} 0 & (1-te)A \\ 0 & (e+t)A \end{bmatrix}. \quad (24)$$

g



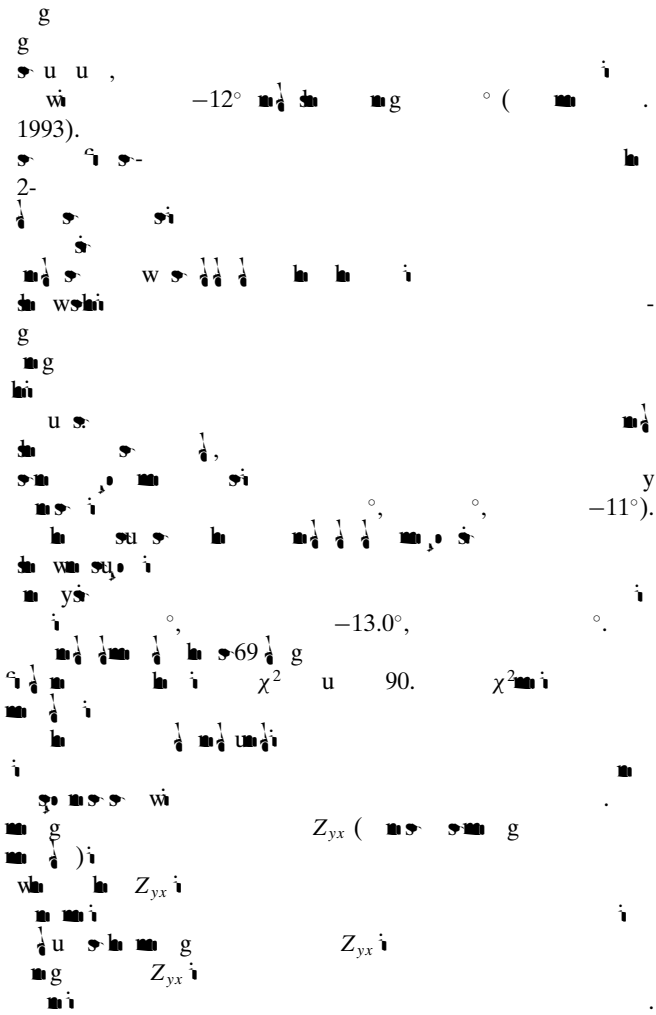


FIG. 2.

u n y n y s

Synthetic example 3

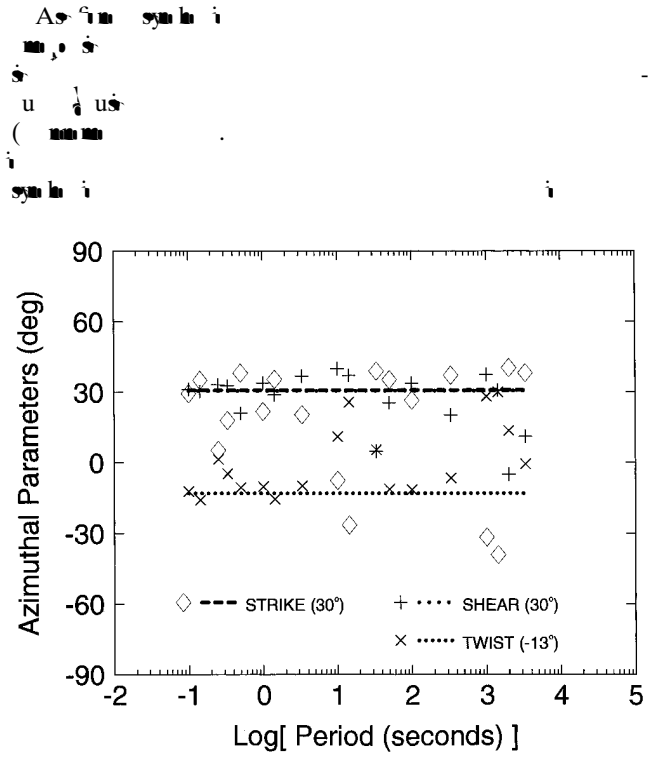


FIG. 3.

(s, s) w i

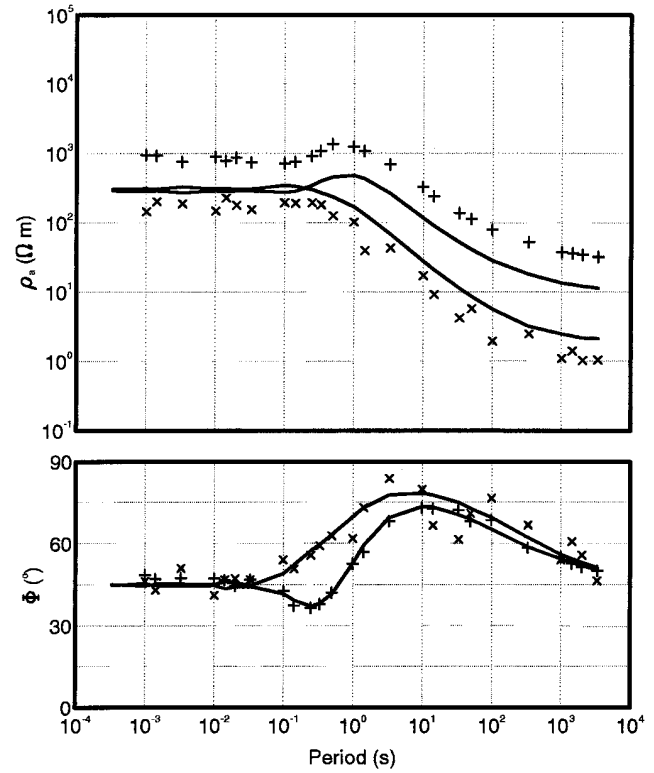
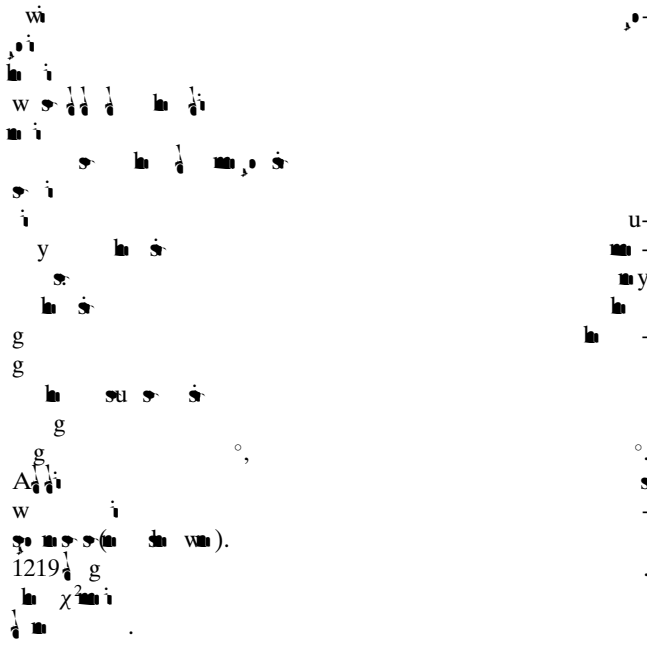


FIG. 4.

u n y n y s



Real data example—Papua New Guinea

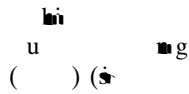


Table 1. Results of decomposition of synthetic 2-D data set. Joint decomposition found a regional strike of 30.3°, close to the real value of 30.0°.

i	u		D	
	(°)	(°)	(°)	(°)
001	20.	-20.	-10.	-20.
002	-10.		-10.	
003	25.	-15.	-25.	-15.
004	40.		-25.	
005	-25.	-40.	-25.	-40.
006	-20.		-20.	
007	-35.	-50.	-34.	-50.
008	25.	-10.	-10.	
009	35.	-5.	-5.	
010	15.			

FIG. 5. 50 Ω m - 1000 Ω m

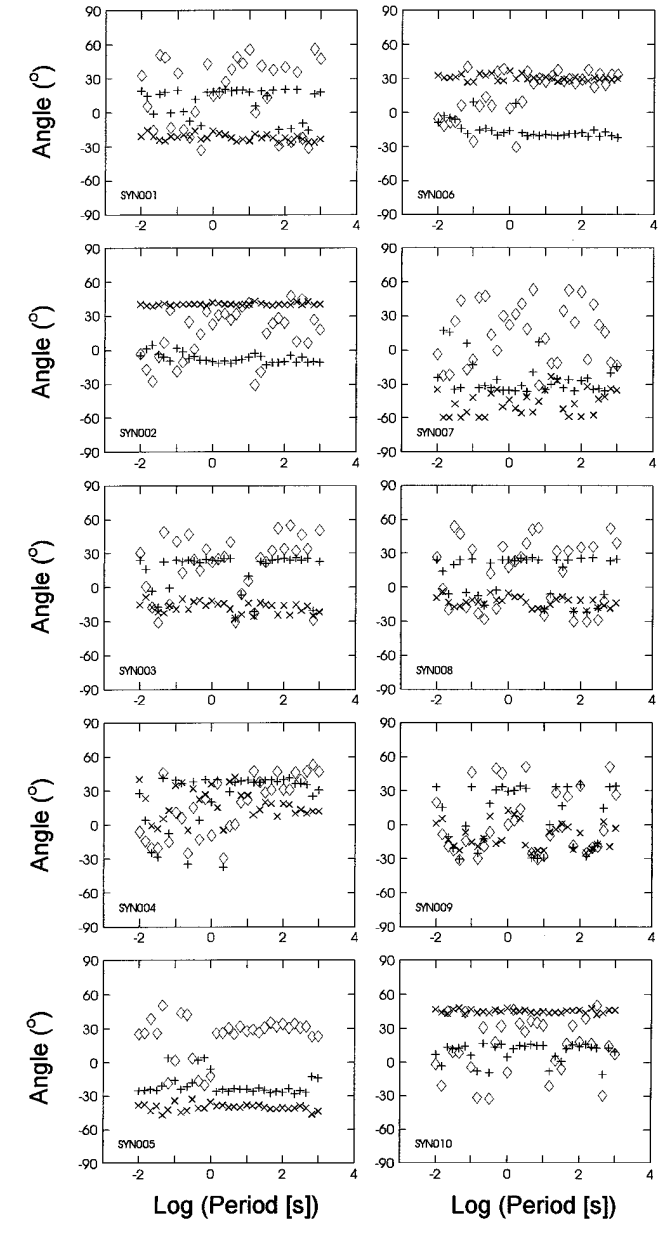
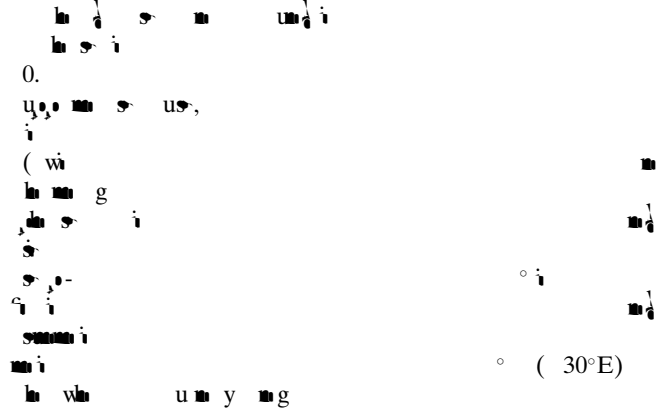


FIG. 6. (+), (x) m s<sup>-1</sup>



us... (30°E).  
 21-  
 (1997) du w...  
 >20...  
 121,  
 5...  
 3-  
 25°-  
 g  
 i  
 w

g  
 121  
 122  
 500-  
 2500  
 (±0.2°),  
 34°  
 3

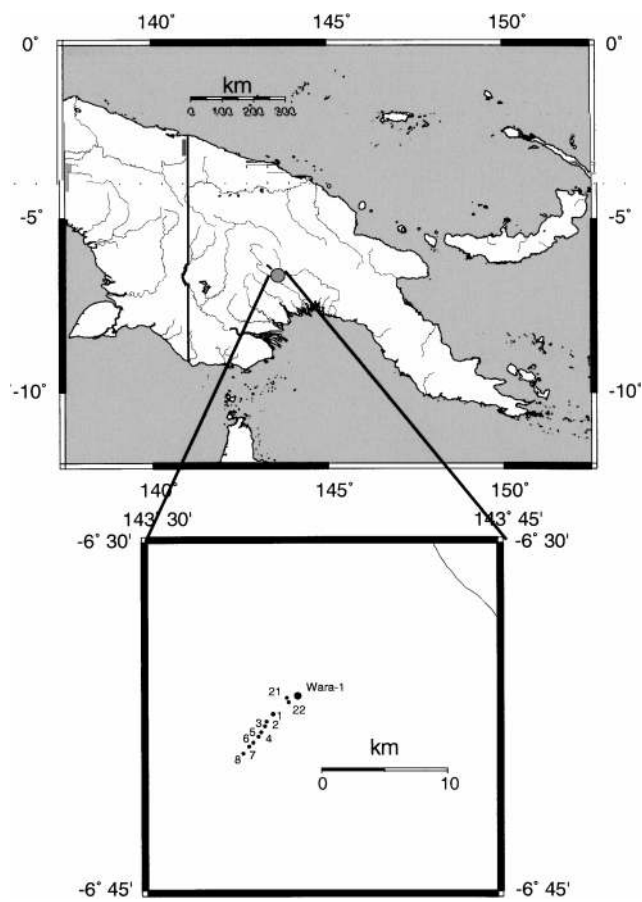


FIG. 7.

ROTATION OF MT IMPEDANCE TENSOR

$Z_{xx}$   $Z_{yy}$   
 $(Z_{yy}/Z_{xy}$   $Z_{xx}/Z_{yx})$   
 $(\pm 0.2^\circ)$   
 $34^\circ$

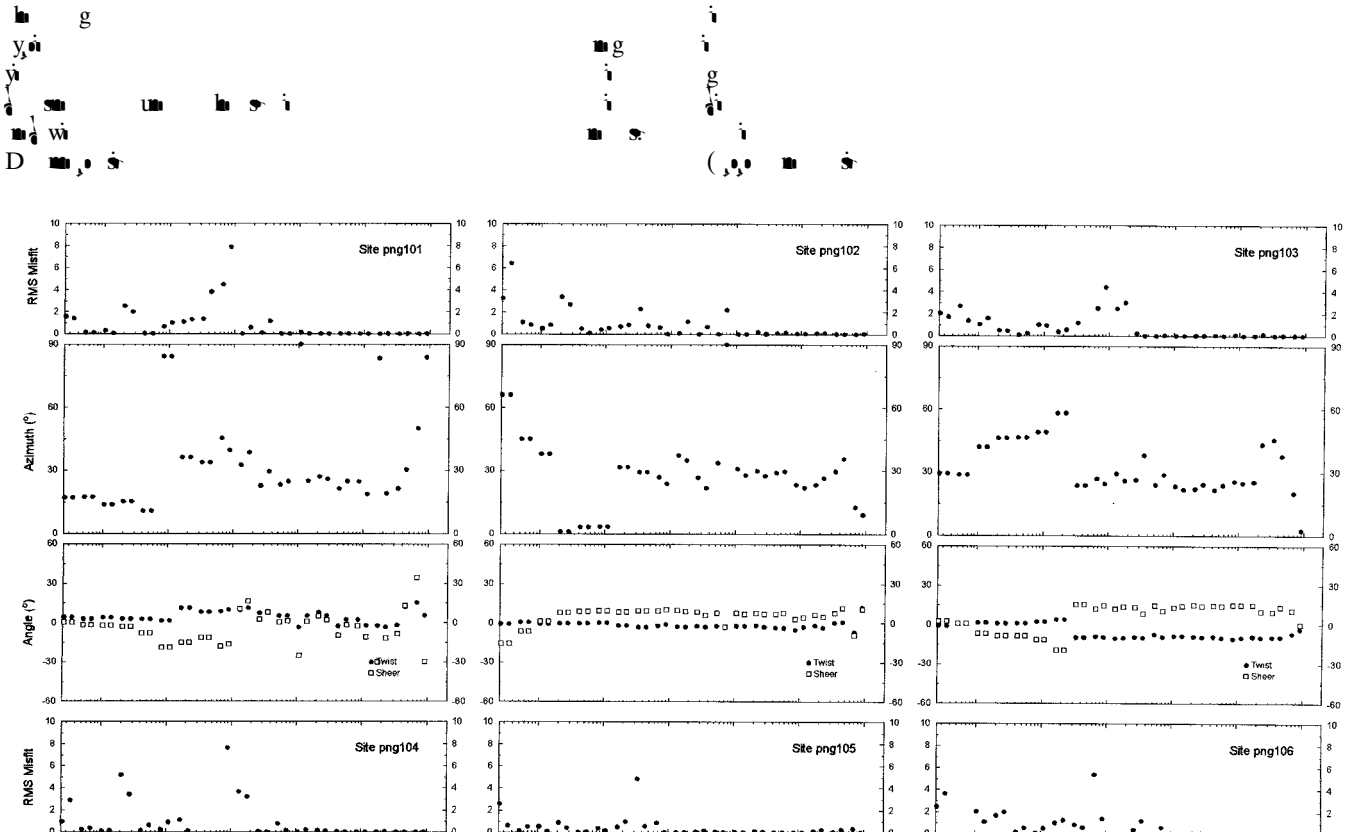
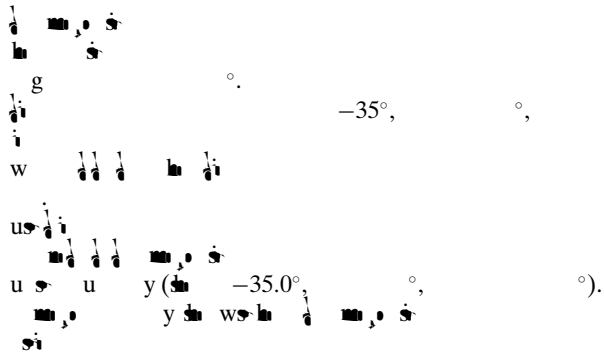


FIG. 8.



EXTENSION TO INCLUDE MAGNETIC EFFECTS

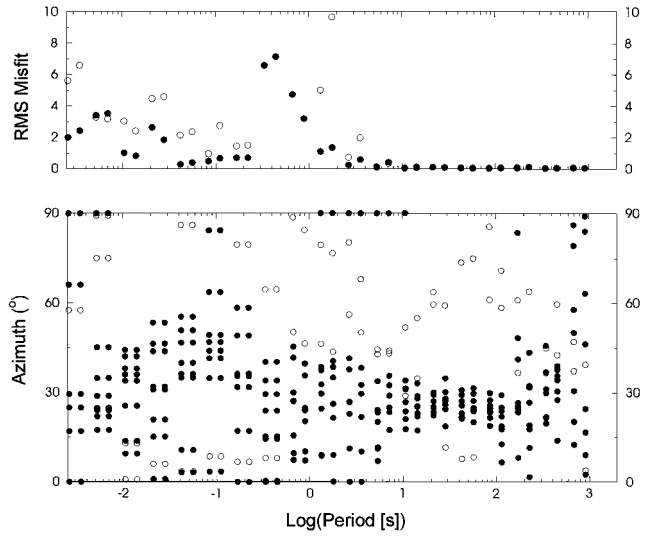
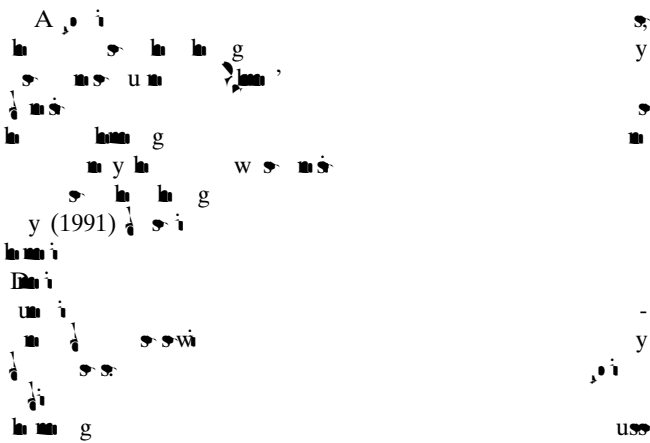


FIG. 10.

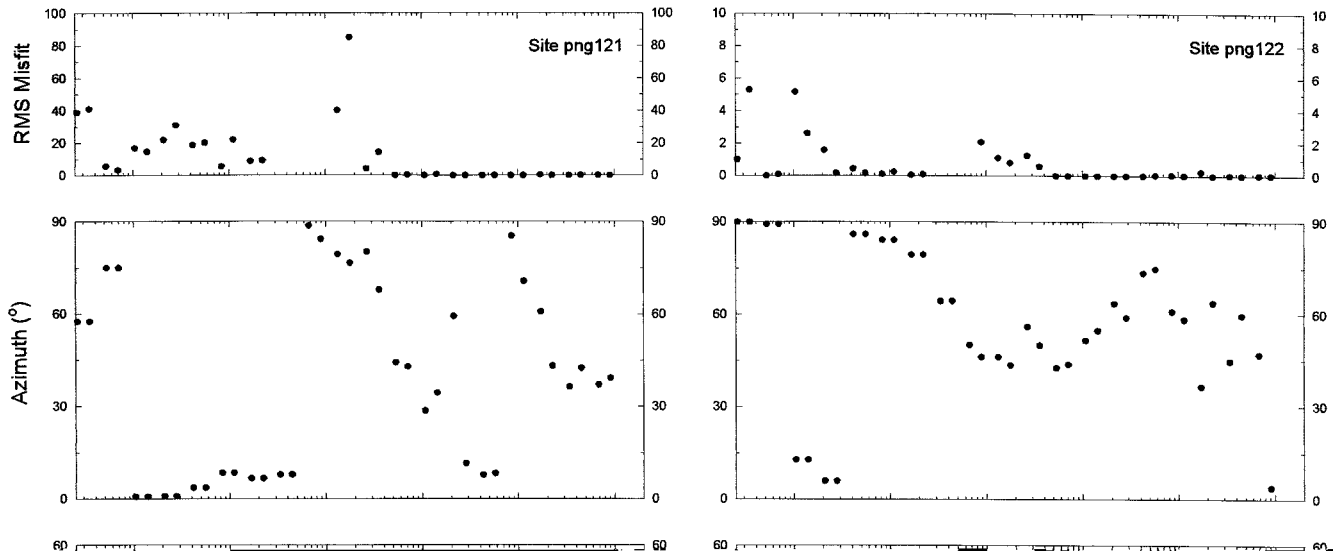


FIG. 9.

$$\mathbf{Z}_{measured} = \mathbf{RCZ}_{2D}(\mathbf{I} + \mathbf{DZ}_{2D})^{-1}\mathbf{R}^T. \quad (26)$$

Figure 11 shows the results of the extended MT tensor decomposition. The top plot displays the RMS Misfit versus Log(Period [s]), and the bottom plot displays the Azimuth (°) versus Log(Period [s]). The RMS Misfit is generally low, indicating a good fit, and the Azimuth is relatively constant around 30 degrees. The data points are shown with error bars.

FIG. 11.

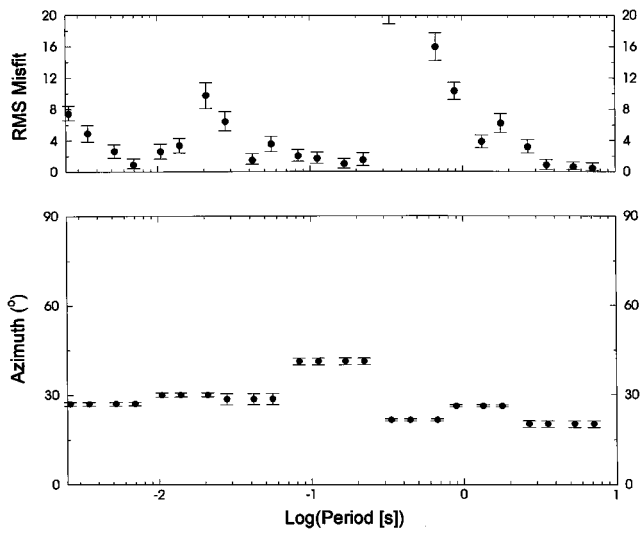


FIG. 12.

Figure 13 shows the results of the extended MT tensor decomposition. The top plot displays the RMS Misfit versus Log(Period [s]), and the bottom plot displays the Azimuth (°) versus Log(Period [s]). The RMS Misfit is generally low, indicating a good fit, and the Azimuth is relatively constant around 30 degrees. The data points are shown with error bars. The figure also includes a diagram of the coordinate system and a list of parameters:  $\theta = 29.3^\circ \pm 0.2^\circ$ ,  $\phi = 30.5^\circ \pm 0.05^\circ$ ,  $\psi = -12.1^\circ \pm 0.05^\circ$ , and  $\chi = -12^\circ$ .

FIG. 13.

$u = y_i, y_i, y_i$   
 $+60^\circ$   
 $+30^\circ$   
 $1000$   
 $29.98^\circ \pm 0.06^\circ, 29.86^\circ \pm 0.04^\circ, -12.03^\circ \pm 0.02^\circ$   
 $D_1 = -0.02055, D_4 = -0.04374.$   
 $-0.043$   
 $0.$

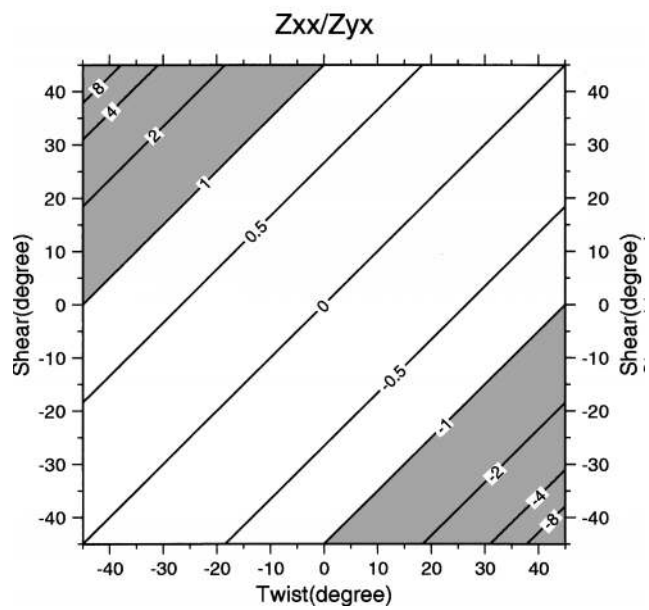
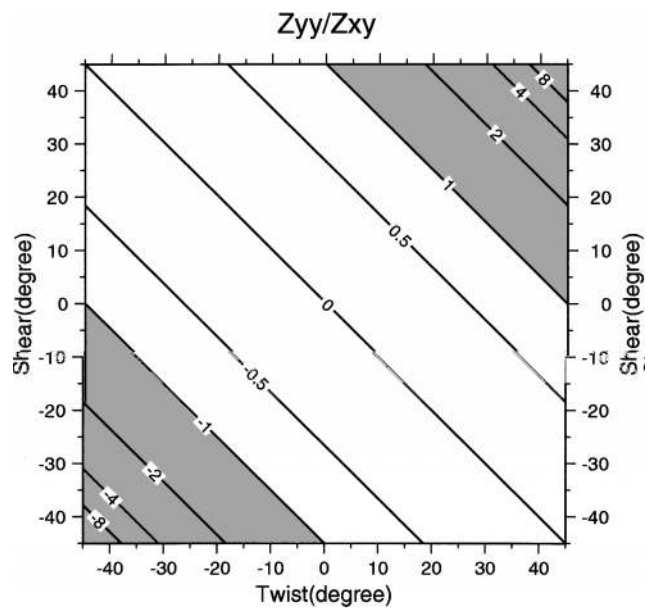
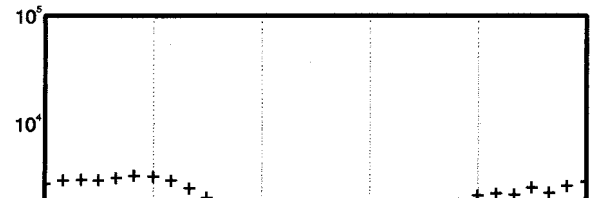


FIG. 14.

FIG. 15.

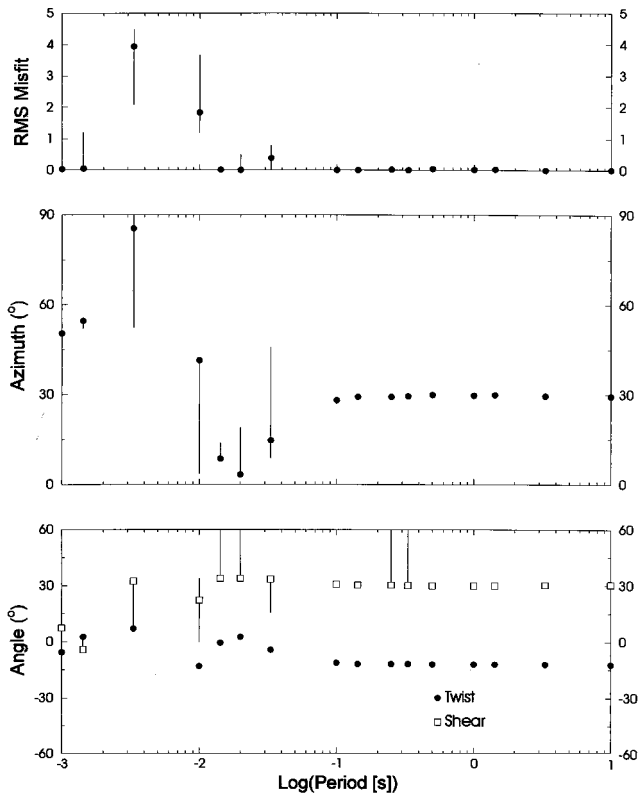


FIG. 16.

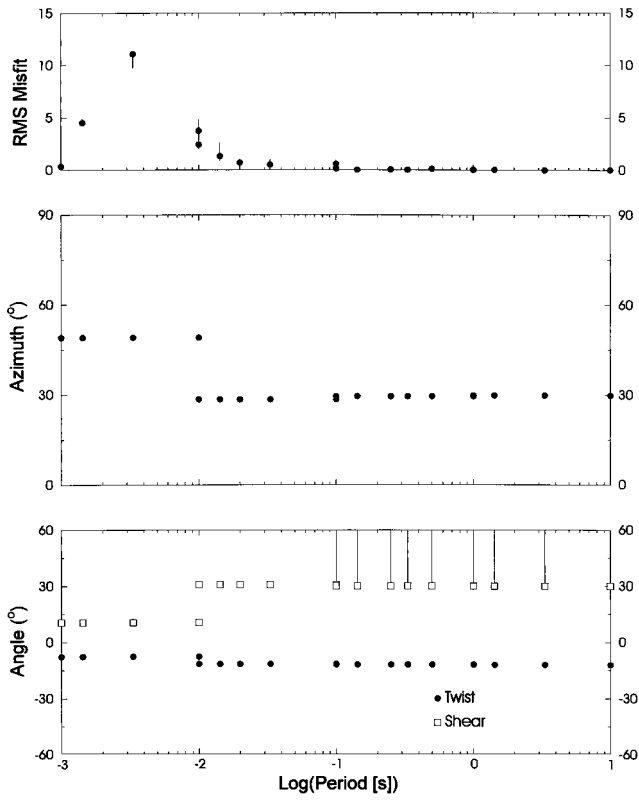


FIG. 17.

(0.

CONCLUSIONS

The results of the extended MT tensor decomposition are presented in this section. The decomposition of the MT tensor into its constituent components is achieved, and the resulting parameters are compared with those obtained from conventional methods. The extended method provides a more accurate and stable decomposition, particularly in the presence of noise and non-linearities. The results demonstrate the effectiveness of the proposed method in extracting the underlying physical parameters from the observed data.

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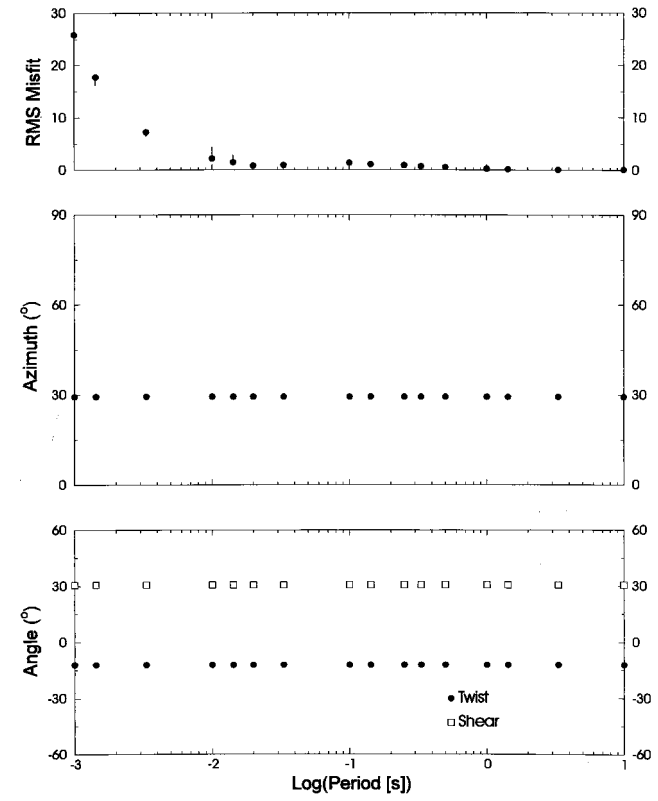


FIG. 18.

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Extended MT Tensor Decomposition

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, hys 50,  
h  
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C m i  
h  
m i Z m g  
s i

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