Multisite, multifrequency tensor decomposition of magnetotelluric data

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ABSTRACT

INTRODUCTION

1993;
1993;
1997;
1991),
(1995,

1C 2 3,

shear and *twist*, *regional strike* and the *regional impedances*.

2-

(1977)

A

GROOM-BAILEY DECOMPOSITION

(1977)

$$\mathbf{Z}_{measured} = \mathbf{RCZ}_{2D}\mathbf{R}^T \quad (1)$$

where $\mathbf{R}$ … $\mathbf{C}$ … $\times 2$ … $\mathbf{Z}_{2D}$ …

B … (1992) … θ, $\mathbf{Z}_{2D}$) … B …

B … $\mathbf{C}$ …

$$\mathbf{C} = g\mathbf{TSA}, \quad (2)$$

where g … $\mathbf{T}$, $\mathbf{S}$, … $\mathbf{A}$ … *twist, shear,* … *anisotropy*, … $\mathbf{C}$, … $\mathbf{T}$) … ($\mathbf{S}$) … ^{-1}t, … x- … y- … ^{-1}e. … e … 1 … -1 … 45° … $-45°$), … e … e and t …

g) and … $\mathbf{A}$) … *static shifts* … g …

A … g and … s …

$$\mathbf{Z}_{measured} = \mathbf{RTSZ}_{regional}\mathbf{R}^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1-te & e-t \\ e+t & 1+te \end{bmatrix} \times \begin{bmatrix} 0 & A \\ -B & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \quad (3)$$

where $\mathbf{Z}_{regional}$ … ($g\mathbf{AZ}_{2D}$). … e … s … ° and $-45°$ … 1 and -1). … e … s …

θ, A and B (Z_{xy} and Z_{yx}). t and e, χ^2,

$$\alpha_0 = Z_{xx} + Z_{yy}$$
$$\alpha_1 = Z_{xy} + Z_{yx}$$
$$\alpha_2 = Z_{yx} - Z_{xy} \quad (4)$$
$$\alpha_3 = Z_{xx} - Z_{yy}.$$

$$\alpha_0 = t\sigma + e\delta$$
$$\alpha_1 = (\delta - et\sigma)\cos 2\theta - (t\delta + e\sigma)\sin\theta$$
$$\alpha_2 = -\sigma + et\delta \quad (5)$$
$$\alpha_3 = -(t\delta + e\sigma)\cos 2\theta - (\delta - et\sigma)\sin\theta,$$

$$\sigma = A + B$$

$$\delta = A - B$$

Regional strike estimation

$$\mathbf{Z}_{meas}(\theta_{regional}) = \begin{bmatrix} -(e-t)B & (1-te)A \\ -(1+te)B & (e+t)A \end{bmatrix}, \quad (6)$$

t) and (e) B

$$minimize|Z_{xx} - Z_{yy}|^2 \quad (7)$$

$$\theta' = \theta_{regional} + \frac{1}{2}\tan^{-1}\left(\frac{t\delta + e\sigma}{\delta - et\sigma}\right). \quad (8)$$

θ' (1987), δ,

EXTENSION OF GB DECOMPOSITION FOR MULTIPLE SITES AND MULTIPLE FREQUENCIES

$S \times N \times 8$ … $S \times (N \times 4 + 2) + 1$ … $S \times N$ … A … B), … $S \times 2$ … t … e), … θ).

… $S \times (N \times 4 - 2) - 1$ …

… (5), … $S \times N \times 8$ … 4 … $S \times N$ …

$$\gamma^2(a) = \sum_{k=1}^{SN} \left(\sum_{i=0}^{3} \left[\frac{Re(\alpha_i^{obs}) - Re(\alpha_i^{model}(a))}{\sigma_{\alpha_i}} \right]^2 + \sum_{i=0}^{3} \left[\frac{Im(\alpha_i^{obs}) - Im(\alpha_i^{model}(a))}{\sigma_{\alpha_i}} \right]^2 \right) \quad (9)$$

… σ …

… $S \times (N \times 4 + 2) + 1$ …

$$\gamma^2(\mathbf{a}) = \gamma^2(\mathbf{P}) + \sum_i \frac{\partial \gamma^2}{\partial a_i} a_i + \frac{1}{2} \sum_{i,j} \frac{\partial^2 \gamma^2}{\partial a_i \partial a_j} a_i a_j + \cdots$$
$$\approx c + \mathbf{J} \cdot \mathbf{a} + \frac{1}{2} \mathbf{a} \cdot \mathbf{H} \cdot \mathbf{a}, \quad (10)$$

… **a** … **c** … **P**, **J** … **H** …

$$\nabla \gamma^2 \approx \mathbf{H} \cdot \mathbf{a} + \mathbf{J}. \quad (11)$$

$$\mathbf{H} \cdot \mathbf{a}_{min} + \mathbf{J} = 0. \quad (12)$$

$$\nabla \gamma^2(\mathbf{P}) = \mathbf{H} \cdot \mathbf{P} + \mathbf{J}. \quad (13)$$

$$\mathbf{a}_{min} = \mathbf{P} - \mathbf{H}^{-1} \cdot \nabla \gamma^2(\mathbf{P}). \quad (14)$$

$S \times N \times 8$

$$\gamma_i(a) = \frac{\alpha_i^{obs} - \alpha_i^{model}(a)}{\sigma_{\alpha_i}} \quad i = 1, 2, \ldots, S \times N \times 8, \quad (15)$$

$$\gamma^2(a) = \sum_{i=1}^{SN8} [\gamma_i(a)]^2. \quad (16)$$

… $S \times$

$N \times 8 \times [S \times (N \times 4 + 2) + 1$

$$\frac{\partial \gamma_i(a)}{\partial a_j} = \frac{-1}{\sigma_{\alpha_i}} \frac{\partial \alpha_i^{model}(a)}{\partial a_j} \qquad i = 1, 2, \ldots, S \times N \times 8$$
$$j = 1, 2, \ldots, S \times (N \times 4 + 2) + 1. \tag{17}$$

$S \times (N \times 4 + 2) + 1$

$S \times N \times 8$

$$\frac{\partial \gamma^2(a)}{\partial a_i} = 2 \sum_{j=1}^{S \times N \times 8} \gamma_j(a) \frac{\partial \gamma_j(a)}{\partial a_i}$$
$$i = 1, 2, \ldots, S \times (N \times 4 + 2) + 1. \tag{18}$$

$S \times (N \times 4 + 2) + 1 \times [S \times (N \times 4 + 2) + 1$

$$\frac{\partial^2 \gamma^2}{\partial a_i \partial a_j} = 2 \sum_{k=1}^{SN8} \frac{1}{\sigma_{\alpha_k}^2} \left[\frac{\partial \alpha_k^{model}(a)}{\partial a_i} \frac{\partial \alpha_k^{model}(a)}{\partial a_j} + \left[\alpha_k^{obs} - \alpha_k^{model}(a)\right] \frac{\partial^2 \alpha_k^{model}(a)}{\partial a_i \partial a_j} \right]. \tag{19}$$

$[\alpha^{obs} - \alpha^{model}$

$$\frac{\partial^2 \gamma^2(a)}{\partial a_i \partial a_j} \approx 2 \sum_{k=1}^{SN8} \frac{\partial \gamma_k(a)}{\partial a_i} \frac{\partial \gamma_k(a)}{\partial a_j}$$
$$i, j = 1, 2, \ldots, S(N4 + 2) + 1. \tag{20}$$

α_i,

χ^2,

Error estimation

D

B

Failure of decomposition analysis

$\pm 45^\circ$ (e ± 1)

s)

e),

$$Z_{meas}(\theta_{regional}) = \begin{bmatrix} -(1-t)B & (1-t)A \\ -(1+t)B & (1+t)A \end{bmatrix} \tag{21}$$

$^{-1} e$)

t

$$t(\theta) = t - (\theta - \theta_{regional}) \tag{22}$$

$(\theta - \theta_{regional})$

$$Z_{meas}(\theta_{regional}) = \begin{bmatrix} -(e-t)B & 0 \\ -(1+te)B & 0 \end{bmatrix} \tag{23}$$

$$Z_{meas}(\theta_{regional}) = \begin{bmatrix} 0 & (1-te)A \\ 0 & (e+t)A \end{bmatrix}. \tag{24}$$

y
w g

EXAMPLES OF APPLICATION

Synthetic example 1

u
y (1993) w g
(AC) u
1990),
y.
u ,

$$\mathbf{Z} = \mathbf{C}\mathbf{Z}_{2-D} = \begin{bmatrix} 1.26 & 0.44 \\ 0.53 & 0.86 \end{bmatrix} \times \begin{bmatrix} 0 & (4.72, 4.05) \\ (-8.25, -3.10) & 0 \end{bmatrix} \times 10^{-4}(\Omega) = \begin{bmatrix} (-3.63, -1.36) & (5.95, 5.10) \\ (-7.10, -2.67) & (2.51, 2.15) \end{bmatrix} \times 10^{-4}(\Omega). \quad (25)$$

D
g) 1. *s*)
0. −2. ° ($t = -0.037$),
24. ° ($e = 0.47$).
°.
u
(25) u 31 31
y
u
u u
g
y
g
y .
y
y ,
u
−2.2°, °,
0. °, −2.1°, °, °,
g
u y- y
.

−7° ±4° (,

Synthetic example 2

w
2-
B
y ,
2-
3- y
B
u
(E w
u u .
0.
g
y-
°.
(1993) w
2-
u D,
w g y
g
w g

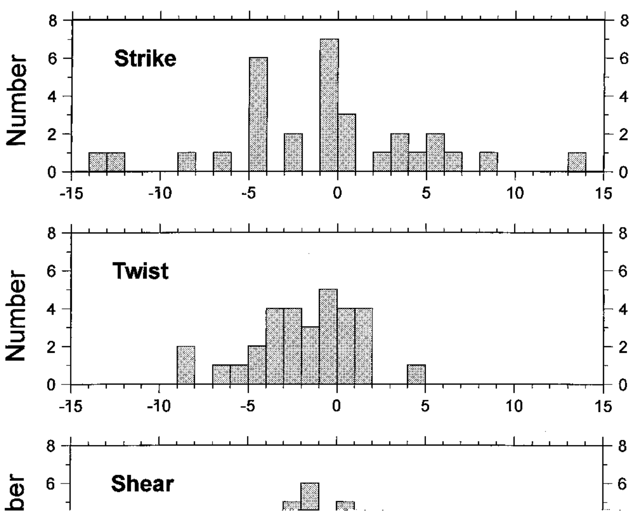

FIG. 1.
u y y

g
g
u u ,
w −12° g ° (.
1993).
2-
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w
g
g
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y
°, °, −11°).
u
w
y
°, −13.0°, °.
69 g
χ^2 u 90. χ^2
u
Z_{yx} (g
)
Z_{yx}
u g Z_{yx}
g Z_{yx}
.

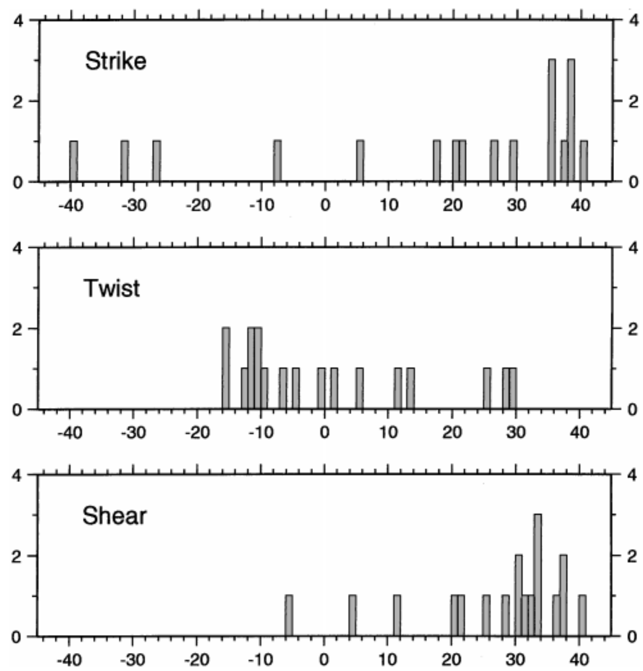

FIG. 2.
u y y

Synthetic example 3

As
,
u us
(.

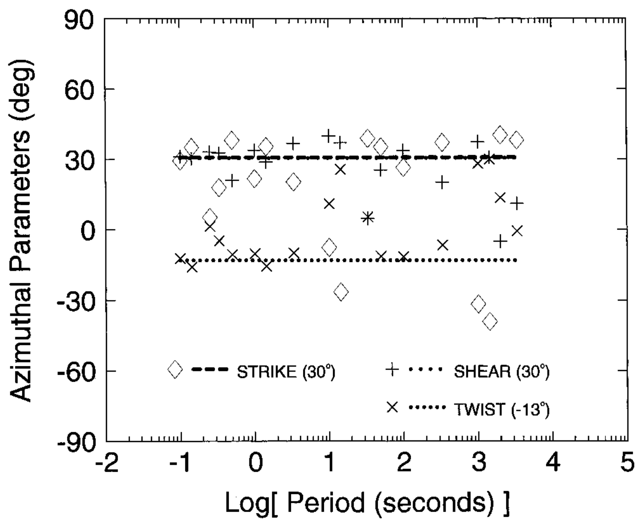

FIG. 3.
( ) w

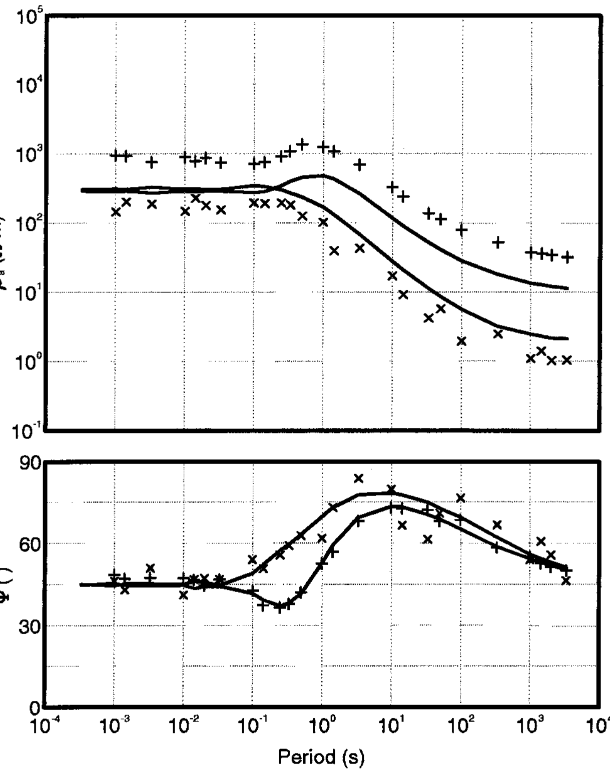

FIG. 4.
u
, 2.

0.

(30°E)

1219

χ^2

Real data example—Papua New Guinea

Table 1. Results of decomposition of synthetic 2-D data set. Joint decomposition found a regional strike of 30.3°, close to the real value of 30.0°.

001	20.	−20.		−20.
002	−10.		−10.	
003	25.	−15.	−25.	−15.
004	40.			
005	−25.	−40.	−25.	−40.
006	−20.		−20.	
007	−35.	−50.	−34.	−50.
008	25.	−10.		−10.
009	35.	−5.		−5.
010	15.			

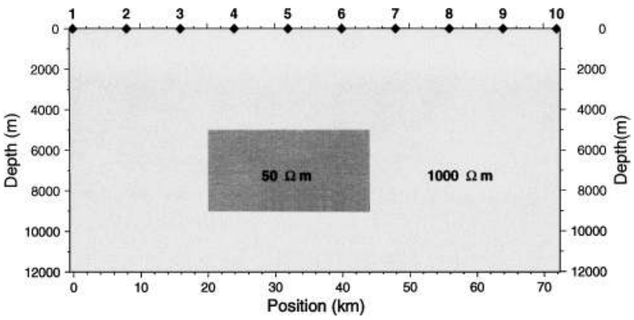

FIG. 5.
50

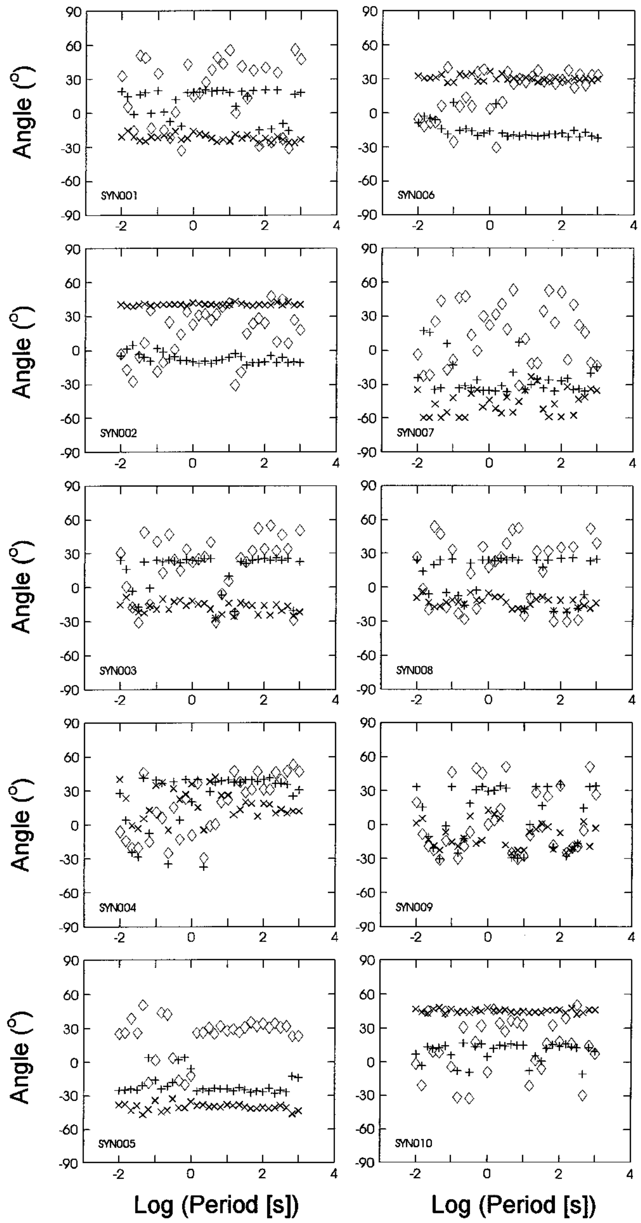

FIG. 6.
(+), ×)

(30°E).

21–

(1997)

>20 s)

121,

5 s;

122

3-

25°–

°.

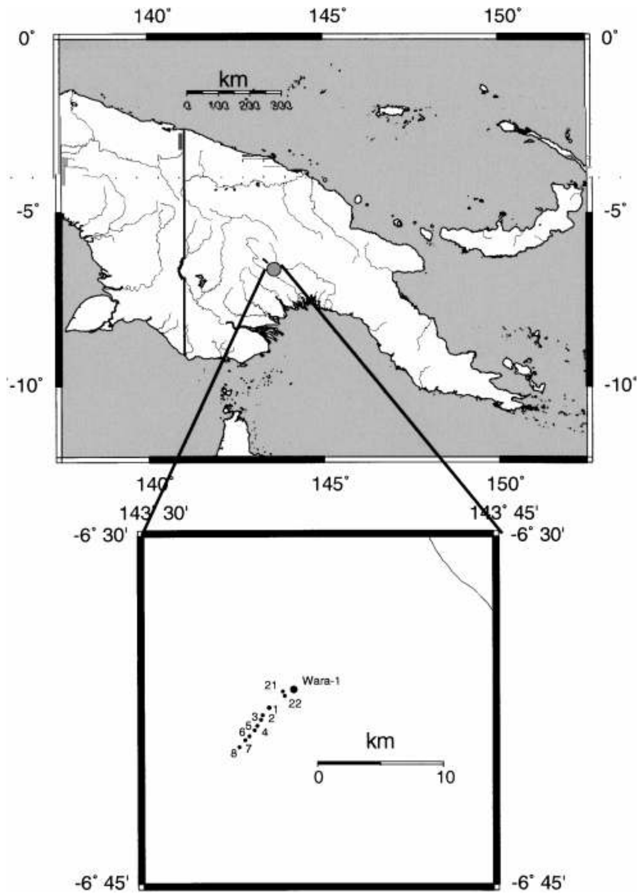

FIG. 7.

121 122

+30°,

3

500–

2500 (>2 s).

°(±0.2°),

1–

34°.

ROTATION OF MT IMPEDANCE TENSOR

Z_{xx} Z_{yy})

(Z_{yy}/Z_{xy} Z_{xx}/Z_{yx})

h g i
y, m g i
y i g
d s u h s i i d i
m d w m s i
D m , s (, , m s

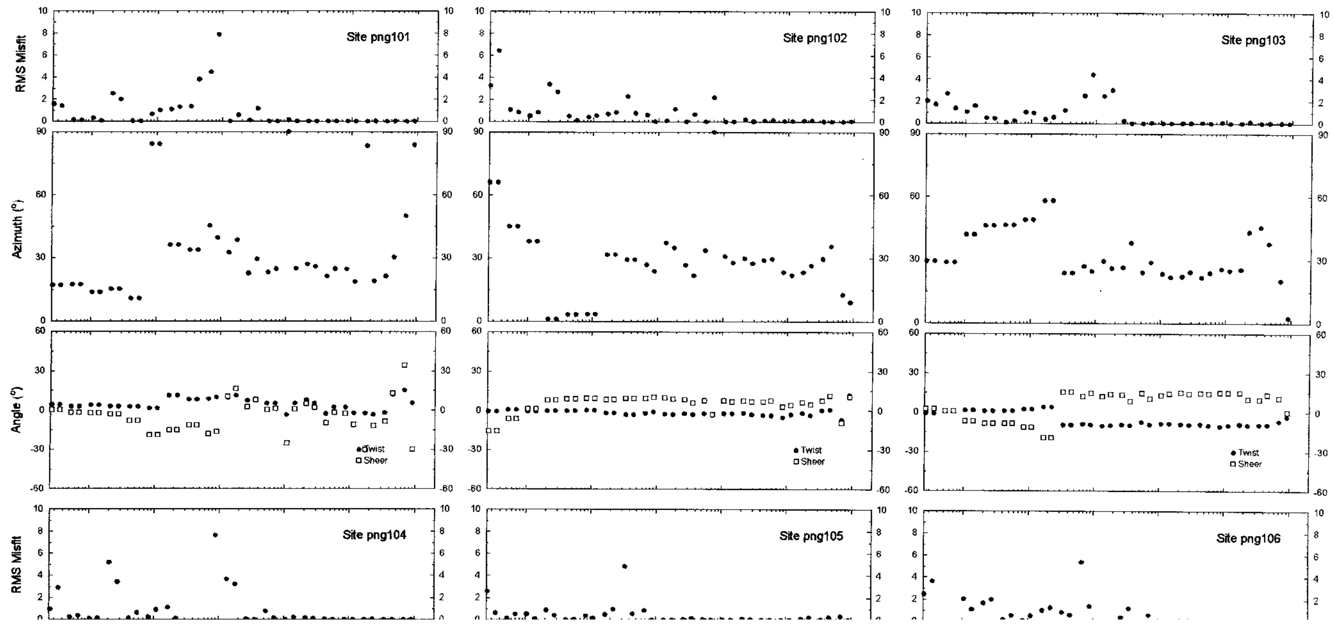

FIG. 8. 01–

g °.
−35°, °,
w

us
u u y (−35.0°, °, °).
y w

EXTENSION TO INCLUDE MAGNETIC EFFECTS

A
g y
u ,
g
y w
g
y (1991)
D
u
y
,
g us

g
(D)
Z g
(1991),
2 × 2 D [Q
. C

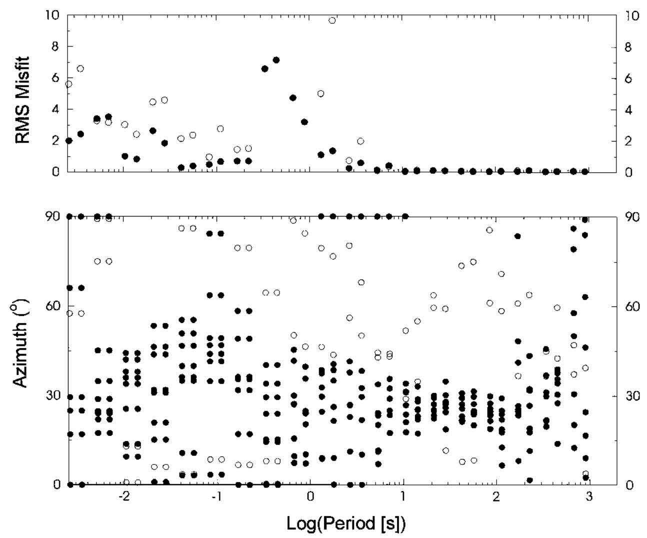

FIG. 10.

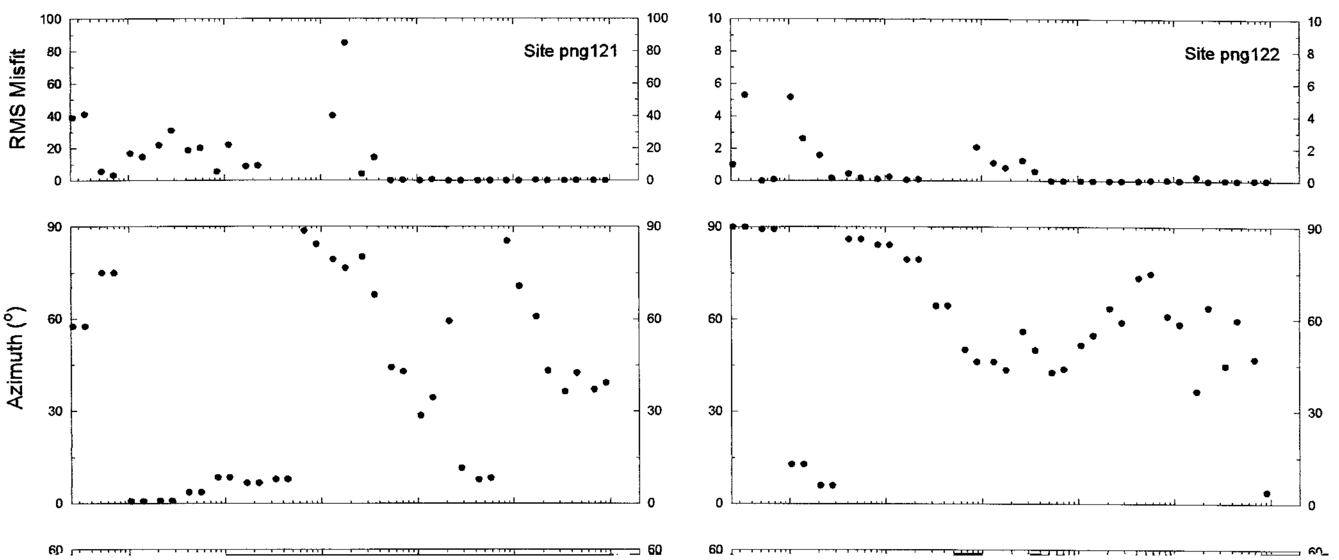

FIG. 9.

21–

$$\mathbf{Z}_{measured} = \mathbf{R}\mathbf{C}\mathbf{Z}_{2D}(\mathbf{I} + \mathbf{D}\mathbf{Z}_{2D})^{-1}\mathbf{R}^{T}. \qquad (26)$$

D,

g

g

u

u

C, **D**,

y w $\mathbf{Z}_{2D}$, θ.

w , ;

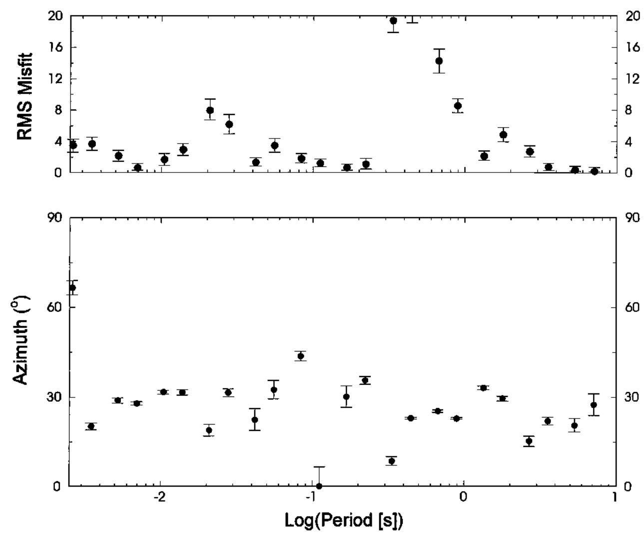

FIG. 11.

w g

w - **D**.

y y

$\mathbf{Z}_{meas}$),

g -

(1993),

-

w

g

A

B

2)

, -

g

y- +60° 1000

+30° y 10 ,

° −12°

w

“ ” y u

g

u (29.3° ± 0.2°, 30.5° ± 0.05°, −12.1° ± 0.05°, °, °, −12°,

>1) u

100 (

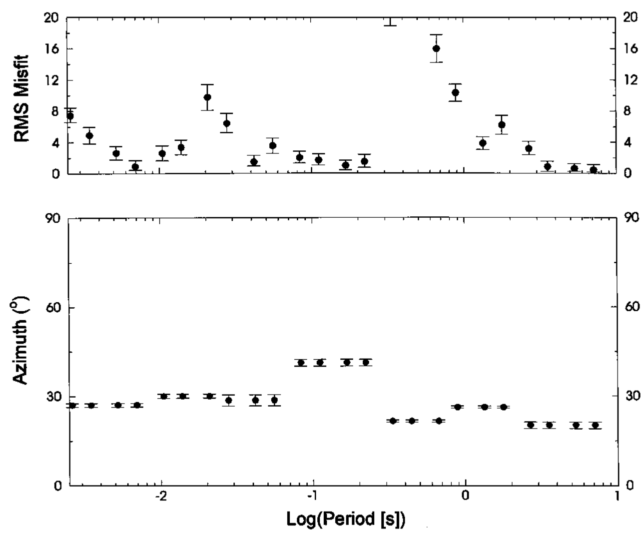

FIG. 12.

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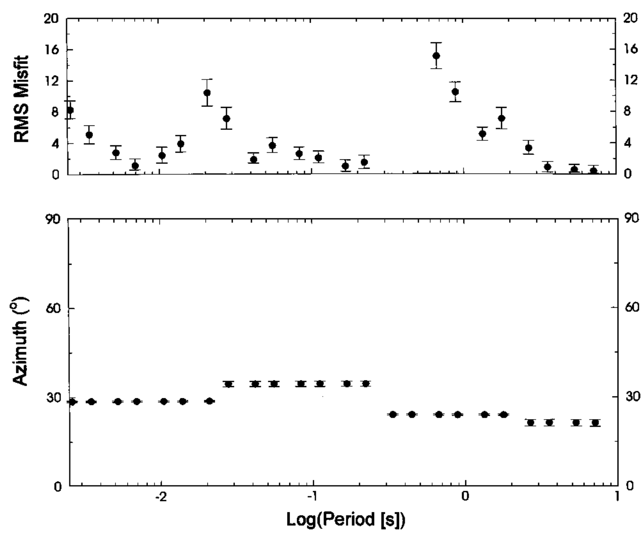

FIG. 13.

.

u y , y

+60°

+30° y

A g +

1000

29.98° ± 0.06°, 29.86° ± 0.04°, −12.03° ± 0.02°

$_1$ = −0.02055 D$_4$ = −0.04374.

+

°, °, −11.8°, −0.021,

−0.043 $_1$, $_2$,

°, °, −7.7°,

0.

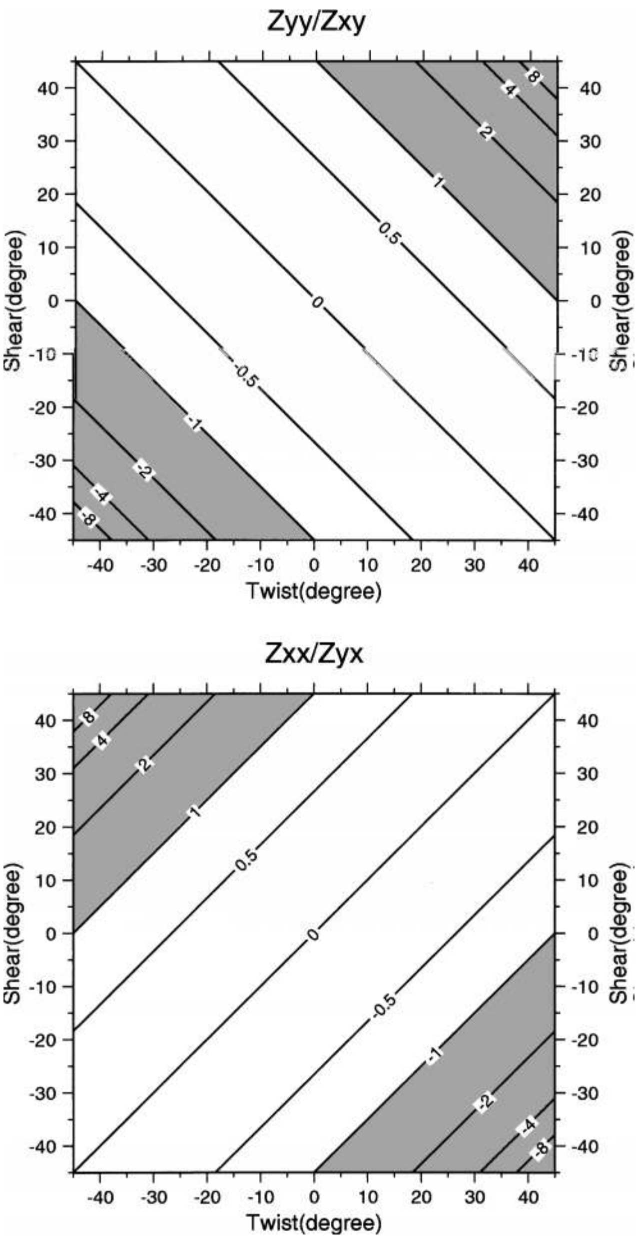

FIG. 14.

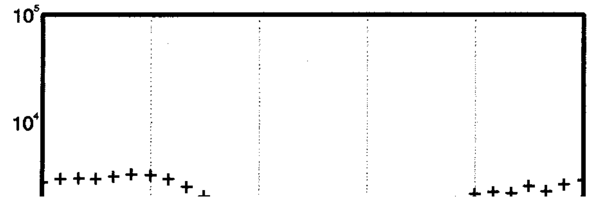

FIG. 15.

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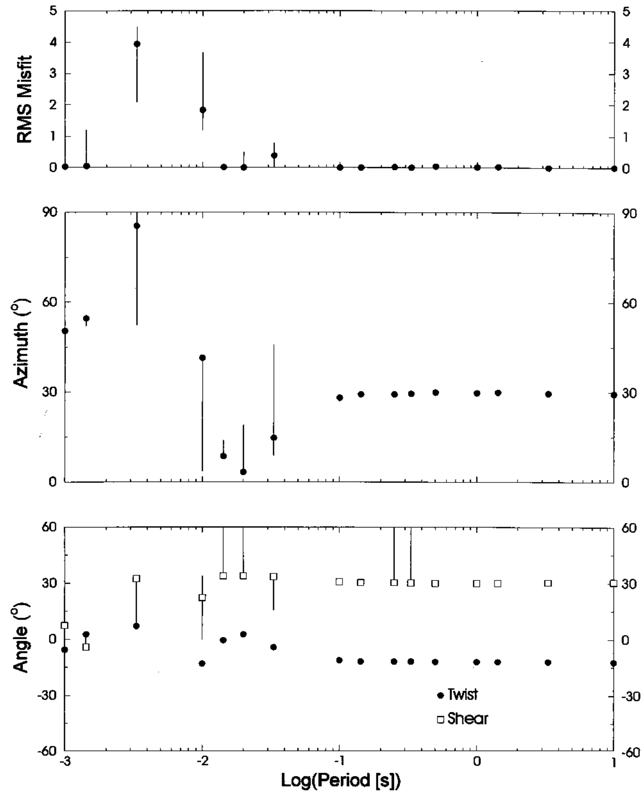

FIG. 16.

CONCLUSIONS

ACKNOWLEDGMENTS

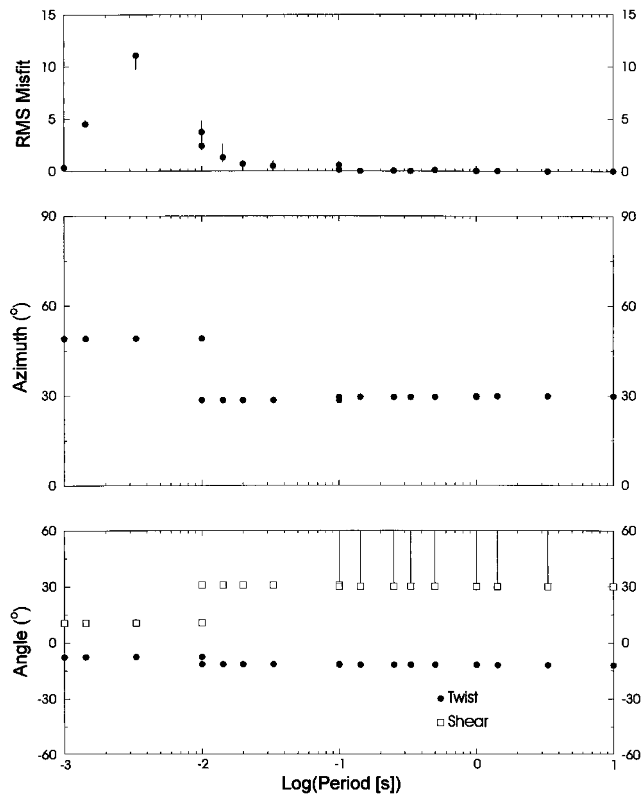

FIG. 17.

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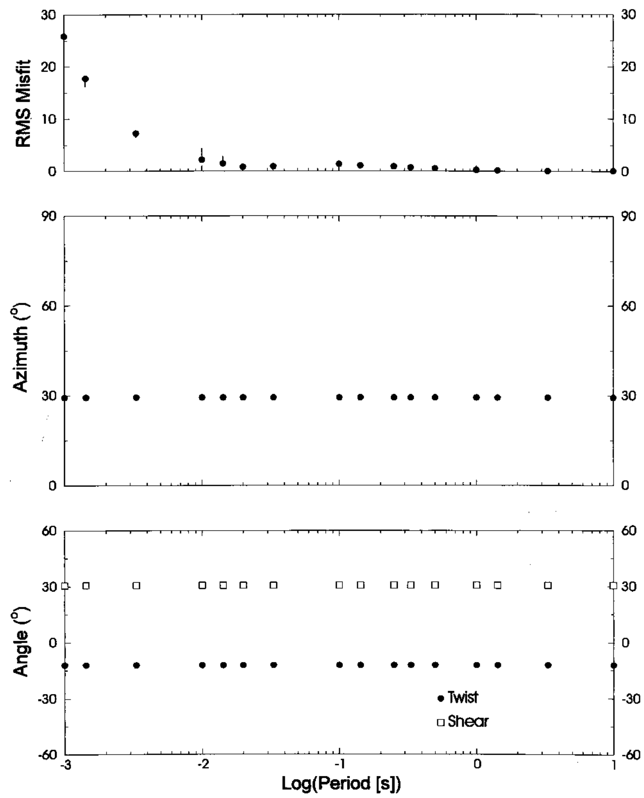

FIG. 18.

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127.
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B ...
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B ... 23,
B ... 87,
C ... 108,
C ... 767– ... 49,
C ... 99,
C ... 49,
E ... 102,
E ... B ... 45,
D ... 13,
341–
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