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Multistage Long-Term Expansion Planning of Electrical Distribution Systems Considering Multiple Alternatives

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***Abstract*—This paper presents a new mixed-integer linear programming (MILP) model to solve the multistage long-term expansion planning problem of electrical distribution systems (EDSs) considering the following alternatives: increasing the capacity of existing substations, constructing new substations, allocating capacitor banks and/or voltage regulators, constructing and/or reinforcing circuits, and modifying, if necessary, the system's topology. The aim is to minimize the investment and operation costs of the EDS over an established planning horizon. The proposed model uses a linearization technique and an approximation for transforming the original problem into an MILP model. The MILP model guarantees convergence to optimality by using existing classical optimization tools. In order to verify the efficiency of the proposed methodology, a 24-node test system was employed.**

***Index Terms*—Distribution system expansion planning, mixed-integer linear programming, multistage long-term planning.**

I. Nomenclature

The notation used throughout this paper is reproduced below for quick reference.

A. Sets

Ω_a Set of conductor types.

Ω_b Set of nodes.

Ω_c Set of alternatives for substations.

Ω_g Set of alternatives for distributed generations.

Ω_{bp} Set of transfer nodes.

Ω_s Set of substation nodes.

Ω_l Set of branches.

Ω_u Set of stages.

B. Constants

$\overline{\Delta}_i^S$ Upper bound for each block of $\Delta_{i,y,u}^{Ps}$ and $\Delta_{i,y,u}^{Qs}$.

$\overline{\Delta}^G$ Upper bound for each block of $\Delta_{ij,y,u}^{Pg}$ and $\Delta_{ij,y,u}^{Qg}$.

Y Number of blocks in the piecewise linearization.

ϕ_l Load factor.

ϕ_s Loss factor.

τ Interest rate.

α Number of hours in one year (8760 h).

$\overline{b}$ Upper bound for the variable $b_{ij,u}$.

c^{cb} Installation cost of the capacitors (US$).

$c_{i,c}^s$ Substation fixed cost at node i using the alternative c (US$).

$c_{ij,a}^f$ Construction cost of circuit ij using conductor type a (US$).

c^e Energy cost of substations (US$/kWh)

c_g^{edg} Energy cost of distributed generator considering the alternative g (US$/kWh).

c_g^{dg} Installation cost of distributed generator considering the alternative g (US$).

c^{mod} Cost of each standard capacitor unit (US$).

c^{vr} Installation cost of the voltage regulator (US$).

$c_{i,c}^r$ Repowering cost of the substation at node i using the alternative c (US$).

c_i^v Operation cost of the substation at node i (US$/(kW)^2/h).

$\overline{I_a}$ Maximum current flow magnitude of conductor type a (A).

Fc_g^{dg} Power factor for the distributed generator considering the alternative g.

K Number of years in each stage.

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l_{ij} Length of circuit ij (km).

m_y^G Slope of the yth block of the piecewise linearization for the power flow of a circuit.

m_y^S Slope of the yth block of the piecewise linearization for the generated power of a substation.

$\overline{G}$ Maximum number of distributed generators that can be installed in the system.

$\overline{M}$ Maximum number of capacitor banks that can be installed in the system.

$\overline{N}$ Maximum number of standard capacitor units that can be installed at a node of the system.

$\overline{O}$ Maximum number of voltage regulators units that can be added to the system.

$P_{i,u}^D$ Active power demand at node i at stage u (kW).

$Q_{i,u}^D$ Reactive power demand at node i at stage u (kVA).

Q_{bc}^{esp} Reactive power of each standard capacitor unit (kVA).

$R_{ij}^{\%}$ Regulator range of voltage regulators.

R_a Resistance per length of conductor type a (Ω/km).

$Rg'_{i,c}$ Apparent power capacity for the repowering of the substation considering the alternative c at node i (kVA).

$Sg'_{i,c}$ Apparent power capacity for a new substation considering the alternative c at node i (kVA).

S_g^{DG} Apparent power capacity for a distributed generator considering the alternative g at node i (kVA).

$\underline{V}$ Lower voltage magnitude limit (kV).

$\overline{V}$ Upper voltage magnitude limit (kV).

X_a Reactance per length of conductor type a (Ω/km).

Z_a Impedance per length of conductor type a (Ω/km).

C. Variables

$\Delta_{ij,y,u}^{Pg}$ Discretization variable of the yth block for $\left|\widehat{P}_{ij,u}\right|$.

$\Delta_{i,y,u}^{Ps}$ Discretization variable of the yth block for $P_{i,u}^S$.

$\Delta_{ij,y,u}^{Qg}$ Discretization variable of the yth block for $\left|\widehat{Q}_{ij,u}\right|$.

$\Delta_{i,y,u}^{Qs}$ Discretization variable of the yth block for $Q_{i,u}^S$.

$\beta_{i,u}^{cb}$ Binary variable associated with the allocation of a capacitor bank at node i at stage u.

$\beta_{i,g,u}^{dg}$ Binary variable associated with the allocation of a distributed generator at node i considering the alternative g at stage u.

$\alpha_{ij,u}^{vr}$ Binary variable associated with the operation of a voltage regulator in circuit ij at stage u.

$\beta_{ij,u}^{vr}$ Binary variable associated with the allocation of a voltage regulator in circuit ij at stage u.

$\varepsilon_{i,u}$ Binary variable associated with the use of transfer node i at stage u.

$\delta_{ij,a,u}$ Binary variable associated with constructing/reconductoring circuit ij using conductor type a at stage u.

$\sigma_{i,c,u}$ Binary variable associated with constructing a substation at node i using the alternative c at stage u.

$r_{i,c,u}$ Binary variable associated with repowering a substation at node i using the alternative c at stage u.

$\varpi_{i,u}$ Integer number of standard capacitor units installed at node i at stage u.

$\psi_{i,u}$ Integer number of standard capacitor units operating at node i at stage u.

$I_{ij,a,u}^{sqr}$ Square of the current flow magnitude in circuit ij associated with conductor type a at stage u.

$\widehat{I}_{ij,u}^{sqr}$ Square of the current flow magnitude in circuit ij at stage u.

$P_{i,g,u}^{DG}$ Active power by distributed generation at node i at stage u considering the alternative g (kW).

$P_{i,u}^S$ Active power by substation at node i at stage u (kW).

$P_{ij,a,u}$ Active power flow in circuit ij associated with conductor type a at stage u (kW).

$\widehat{P}_{ij,u}$ Active power flow in circuit ij at stage u (kW).

$P_{ij,u}^+$ Auxiliary variable used in the calculation of $\left|\widehat{P}_{ij,u}\right|$ (kW).

$P_{ij,u}^-$ Auxiliary variable used in the calculation of $\left|\widehat{P}_{ij,u}\right|$ (kW).

$Q_{i,g,u}^{DG}$ Reactive power from distributed generation at node i at stage u considering the alternative g (kVAr).

$Q_{i,u}^S$ Reactive power from substation at node i at stage u (kVAr).

$Q_{ij,a,u}$ Reactive power flow in circuit ij associated with conductor type a at stage u (kVAr).

$\widehat{Q}_{ij,u}$ Reactive power flow in circuit ij at stage u (kVAr).

$Q_{ij,u}^+$ Auxiliary variable used in the calculation of $\left|\widehat{Q}_{ij,u}\right|$ (kVAr).

$Q_{ij,u}^-$ Auxiliary variable used in the calculation of $\left|\widehat{Q}_{ij,u}\right|$ (kVAr)

$Sg_{i,u}^{sqr}$ Square of the apparent power supplied by substation at node i at stage u (kVA).

$V_{i,u}^{sqr}$ Square of the voltage magnitude at node i at stage u.

$\widetilde{V}^{sqr}_{i,u}$ Square of the not regulated voltage magnitude at node i at stage u.

$y^{+}_{ij,u}$ Binary variable associated with the forward direction of circuit ij at stage u.

$y^{-}_{ij,u}$ Binary variable associated with the backward direction of circuit ij at stage u.

$w_{i,c,u}$ Binary variable that indicates whether the substation at node i, considering the alternative c, is active at stage u.

$z_{ij,a,u}$ Binary variable associated with operating circuit ij using conductor type a at stage u.

$b_{ij,u}$ Variable used in the calculation of the voltage magnitude drop of circuit ij at stage u.

II. INTRODUCTION

THE main objective of the electrical distribution system (EDS) is to provide reliable service to consumers, while ensuring the quality of the power supply at minimum cost. Increased demand on the system, along with the installation of new loads, requires utilities to expand their EDSs in order to satisfy this new demand. Thus, the purpose of the distribution system expansion planning (DSEP) problem is to develop strategies to fulfill the new demand, while maintaining the safe operation of the EDS. In the DSEP problem, multiple objective functions must be considered: the cost of installing new equipment, the operating costs of the substations, the reliability of the distribution system, and active power losses. Over the years, researchers have contributed significantly to solving the DSEP problem using various mathematical models and solution techniques [1].

If the economic and physical characteristics of the DSEP problem are considered realistically, this problem becomes a large-scale mixed-integer nonlinear programming (MINLP) problem. The DSEP problem has been solved using different techniques, such as heuristic algorithms, classical optimization techniques (including linear, nonlinear, and integer programming), and, in recent years, metaheuristic algorithms [1]. These solution techniques have demonstrated different levels of performance depending on the nature of the mathematical model and the size of the EDS [1].

There are two types of DSEP models: static and multistage [2]. In the static approach, optimal planning is aimed at accommodating the demand projected for the end of the planning period. The multistage approach defines not only the ideal location, type, and capacity of the investments, but also the most appropriate time to make such investments. In this way, the continuous growth of the demand is always absorbed by the EDS in an optimal way. This approach looks at the expansion of the EDS over several stages, which represents the natural course of an expansion problem. Due to the coupling between the various stages, it is much more difficult to formulate and solve the multistage DSEP problem. The solution achieved for the multistage DSEP, however, is usually better than the one found using the static approach.

The multistage DSEP problem is an MINLP problem that involves the optimization of binary investment variables, which represent the construction and/or allocation of new equipment, and continuous variables, which represent the steady-state operation of the EDS [2]. One reason for not using the multistage planning approach is the added complexity of the mathematical model. However, current research shows that it is entirely feasible to work with the multistage model using available optimization techniques. In addition, in order to fulfill the voltage limit constraint, it is increasingly important to incorporate additional expansion alternatives, such as the allocation of capacitors and voltage regulator banks, into long-term planning. In a basic DSEP model, the solution for low voltages is to use conductors with lower impedance, but this leads to an increase in the cost of the expansion plan. By contrast, the formulation proposed in this paper considers the allocation of capacitor banks, voltage regulators, and/or distributed generation (DG) as alternatives that can be used to satisfy the voltage constraints with a lower investment cost.

Within this context, [3] developed a pseudo-dynamic method to solve the multistage DSEP problem in which power losses are represented using linear sections. A multistage model for the DSEP problem, including distributed generation, was presented in [4]. The objective function in [4] was the cost of the installation, operation, and maintenance of the EDS, along with the cost of distributed generation. The paper presented an extension of the linear disjunctive formulation, representing the inclusion, exclusion, and replacement of circuits, and a generalization of constraints related to the creation of new paths, which could be applied in more complex topologies. The authors in [4] also guaranteed that it would be possible to find an optimal solution for the resulting mixed-integer linear programming (MILP) model by using a branch and bound algorithm. In [5], an MILP model for the DSEP problem was proposed in order to minimize the costs of investment, maintenance, and power losses of the EDS. In order to choose the solution with the lowest cost, a pool of solutions was obtained, and the cost of interruptions, due to failures in the circuits, was determined for each solution. Multistage models for the DSEP problem were also presented in [6]–[11].

In [12], the authors presented an MINLP model for the static DSEP problem, which could be solved using classical optimization techniques. The model also considered the possibility of using transfer nodes to define the final system's topology. Due to its significant computational complexity, the model was not able to solve large-scale planning problems in a reasonable amount of time. [13] presented a conic programming model for the DSEP problem, analyzing two formulations: the single-circuit and the parallel-circuit equivalent. In addition, constraints to eliminate loops were proposed with the aim of obtaining a tight formulation and reducing the computational effort needed to solve the DSEP problem. The authors in [14] proposed a quadratically constrained model that considered the construction/ reinforcement of substations, the construction/reconductoring of circuits, the allocation of capacitor banks, and radial topology modification. The formulation proposed in

[14] used voltage magnitudes and power flows to represent the steady-state operation of the EDS, in contrast to [13] in which new variables were introduced.

Furthermore, the DSEP problem has been solved through heuristic algorithms, which have produced good solutions with relatively low computational effort. Popular approaches from this category include the branch exchange technique that was used in [15] and [16], and the constructive heuristic algorithm that was implemented in [17]. Reference [17] used a local improvement phase and a branching technique to improve the quality of the solution. In order to enhance the performance of heuristic methods, in recent years, numerous metaheuristic algorithms have been proposed to solve the DSEP problem. An evolutionary algorithm was presented in [18]. In [6] and [19], a genetic algorithm was applied to the DSEP problem. A method based on ant colony systems was developed in [20], and simulated annealing was conducted in [21] and [22].

In [23], an expert system based on neural networks was used to transform the original DSEP problem into a directed graph planning problem, whereas [24] used a particle swarm algorithm. Although metaheuristics are robust, flexible, and achieve good results, they also present many problems, such as high computational demand, the need for adjusting and fine-tuning the parameters, and the definition of a stop criterion. In addition, they cannot guarantee convergence to a global optimum, nor can they indicate the quality of the final solution, because they do not provide a distance indicator to the optimal solution.

Some papers about the DSEP problem have independently addressed the construction of circuits and substations [5], [6], [8], [10], [13], [15]–[20], [23], and [24]; the allocation of capacitor banks [25]–[28]; the allocation of voltage regulators [29]–[31]; and the joint allocation of capacitor banks and voltage regulators [32]–[35]. The construction of circuits and substations, and the installation of capacitor banks were considered in [14]. The authors demonstrated that better solutions could be found by including the allocation of fixed capacitor banks in the DSEP problem, thereby reducing power losses and investments in circuits.

Currently, alternative options for expanding the system, such as DG, can be implemented as possible solutions in DSEP. Relevant works addressing the expansion planning of distribution networks in the presence of distributed generation are found in [4], [9], and [12]. However, in these works, the location and timing of the DG were considered to be parameters, rather than variables of the optimization process. Relevant approaches are also found in [36]–[39]. In [36] an MINLP model for the DSEP problem was proposed and solved using commercially available software; due to the nonlinear formulation, global optimality was not guaranteed. In [37], the model described in [36] was extended and solved using a genetic algorithm. Finally, another evolution-inspired heuristic, namely an evolutionary particle swarm optimization, was applied in [38] and [39] to solve the MINLP formulation for the DSEP problem. The methods in [36]–[39] neither achieve global optimality nor provide a measure of the distance to the optimum. In addition, in [36]–[39], investment decisions associated with the distribution network are exclusively related to the reinforcement of feeders and substations, and the construction of new distribution assets is disregarded. Moreover, the models described in [36]–[39] only consider DG comprised of conventional units. Reference [11] presented an approach to expand the distribution network and the DG simultaneously. Investment decisions considered both the reinforcement of assets and the installation of new equipment. This modeling aspect required the incorporation of specific constraints in order to impose the radial operation on the resulting network, as per [12]. DG was comprised of not only conventional units, but also wind power generation. Appendix A shows a summary of the technical characteristics of the references cited herein.

According to the authors' knowledge, in the specialized literature, no works have taken into account the construction and/or installation of the multiple expansion alternatives in the solution of the multistage long-term expansion planning problem of EDSs. The consideration of multiple alternatives in the DSEP problem has the advantage of assessing which equipment is the most appropriate to be installed in the EDS. In this paper, an MINLP model for the multistage long-term DSEP problem is presented. This formulation considers the expansion and/or reinforcement of the substations and circuits; the allocation of capacitor banks, voltage regulators, and/or distributed generation; and a change to the system's topology. A linearization technique and an approximation are used to obtain an MILP model that can be solved using classical optimization techniques, guaranteeing the optimal solution. The planning horizon is divided into several stages, and the proposed optimization model indicates the best time at which to make the investments. A 24-node test system was used to demonstrate the efficiency of the proposed method. The main contributions of this work are given here.

1) A new model for the DSEP problem that takes into account the construction of new substations, the increase of the capacity of existing substations, the allocation of capacitor banks and voltage regulators, the construction or reinforcement of circuits, and a change to the system's topology.
2) An MILP formulation for the multistage DSEP problem that can be solved using classical optimization techniques to obtain its optimal solution.

III. Expansion Planning Problem of EDSs

A. Steady-State Operation of a Radial Distribution System

The equations used to represent the operation of a radial EDS are based on [14]. In these equations, the variables have been changed: $\widetilde{V}^{sqr}_{j,u} = \widetilde{V}^{2}_{j,u}$ and $I^{sqr}_{ij,a,u} = I^{2}_{ij,a,u}$, in order to obtain a suitable MILP model. The model is applied under the following assumptions: 1) the EDS is operating in steady-state; 2) the loads are modeled as a constant power type; and 3) the EDS is balanced and represented by a single-phase equivalent:

$$\sum_{ki\in\Omega_l}\sum_{a\in\Omega_a} P_{ki,a,u} - \sum_{ij\in\Omega_l}\sum_{a\in\Omega_a}(P_{ij,a,u} + R_a l_{ij} I^{sqr}_{ij,a,u}) + P^{S}_{i,u} + \sum_{g\in\Omega_g} P^{DG}_{i,g,u} = P^{D}_{i,u} \quad \forall i\in\Omega_b, u\in\Omega_u \tag{1}$$

$$\sum_{ki\in\Omega_l}\sum_{a\in\Omega_a} Q_{ki,a,u} - \sum_{ij\in\Omega_l}\sum_{a\in\Omega_a}(Q_{ij,a,u} + X_a l_{ij} I^{sqr}_{ij,a,u}) + \psi_{i,u} Q^{esp}_{bc} + Q^S_{i,u} + \sum_{g\in\Omega_g} Q^{DG}_{i,g,u} = Q^D_{i,u} \quad \forall i \in \Omega_b, u \in \Omega_u \tag{2}$$

$$\widetilde{V}^{sqr}_{j,u}\widehat{I}^{sqr}_{ij,u} = \widehat{P}^2_{ij,u} + \widehat{Q}^2_{ij,u} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{3}$$

$$V^{sqr}_{i,u} - \widetilde{V}^{sqr}_{j,u} = \sum_{a\in\Omega_a}[2(R_a P_{ij,a,u} + X_a Q_{ij,a,u})l_{ij} + Z^2_a l^2_{ij} I^{sqr}_{ij,a,u}] + b_{ij,u} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{4}$$

$$\widehat{I}^{sqr}_{ij,u} = \sum_{a\in\Omega_a} I^{sqr}_{ij,a,u} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{5}$$

$$\widehat{P}_{ij,u} = \sum_{a\in\Omega_a} P_{ij,a,u} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{6}$$

$$\widehat{Q}_{ij,u} = \sum_{a\in\Omega_a} Q_{ij,a,u} \quad \forall ij \in \Omega_l, u \in \Omega_u. \tag{7}$$

Equations (1) and (2) represent the real and reactive power flow balance in each node. Equation (3) establishes the relationship between the real and reactive power flows of a circuit, the square of the voltage magnitude at the end of the circuit, and the square of the current flow magnitude of the circuit. Equation (4) calculates the voltage drop in the circuits. Since the real and reactive power flows ($P_{ij,a,u}$ and $Q_{ij,a,u}$) and current magnitudes ($I^{sqr}_{ij,a,u}$) are associated with the selection of conductor type a, and (3) is written in terms of the square of the total current flow magnitudes and the total real and reactive power flows of circuit ij, which are calculated using (5)–(7).

B. Mathematical Modeling of Voltage Regulators, Capacitor Banks, and Distributed Generation

For the allocation of voltage regulators, it is assumed that all voltage regulators (VRs) have the same regulator range, as well as the same number of tap steps. Taking into account the multistage long-term nature of the DSEP problem, the tap position of the VRs can be calculated approximately. So, in this work, the tap position of the VRs is assumed to be a continuous variable. Constraints

$$\left(1 - R^{\%}_{ij}\right)^2 \widetilde{V}^{sqr}_{j,u} \leq V^{sqr}_{j,u} \leq \left(1 + R^{\%}_{ij}\right)^2 \widetilde{V}^{sqr}_{j,u} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{8}$$

$$V^{sqr}_{j,u} - \widetilde{V}^{sqr}_{j,u} \leq \left|\overline{V}^2 - \underline{V}^2\right| \alpha^{vr}_{ij,u} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{9}$$

represent these assumptions. In addition, VRs can only operate after their allocation, as shown in

$$\alpha^{vr}_{ij,u} \leq \sum_{k=1}^{u} \beta^{vr}_{ij,k} \quad \forall ij \in \Omega_l, u \in \Omega_u. \tag{10}$$

Meanwhile, the following equation limits the number of units that can be allocated in the EDS over the planning horizon:

$$\sum_{u\in\Omega_u}\sum_{ij\in\Omega_l} \beta^{vr}_{ij,u} \leq \overline{O}. \tag{11}$$

The allocation of capacitor banks (CBs) is formulated according to [32]. The allocation of a CB has two associated costs: 1) a fixed cost related to the decision to allocate a capacitor at a node of the EDS, as defined by variable $\beta^{cb}_{i,u}$ and corresponding to the structure, protection, and other installation costs; and (b) a cost that depends on the number of modules installed, as defined by the variable $\varpi_{i,u}$. Constraint:

$$\varpi_{i,u} \leq \overline{N} \sum_{k=1}^{u} \beta^{cb}_{i,k} \quad \forall i \in \Omega_b, u \in \Omega_u \tag{12}$$

establishes that the installation of standard capacitor units at a given stage is possible only if the decision to allocate capacitors at the node in question has already been made in that or any previous stage. Constraint

$$\sum_{u\in\Omega_u} \varpi_{i,u} \leq \overline{N} \quad \forall i \in \Omega_b \tag{13}$$

limits the number of modules installed in a node according to the maximum number allowed and the prior allocation of equipment, while

$$\psi_{i,u} \leq \sum_{k=1}^{u} \varpi_{i,k} \quad \forall i \in \Omega_b, u \in \Omega_u \tag{14}$$

limits the number of modules operating at every demand level so that they do not exceed the number of modules already installed. No more than one CB can be allocated at each node

$$\sum_{u\in\Omega_u} \beta^{cb}_{i,u} \leq 1 \quad \forall i \in \Omega_b \tag{15}$$

while the maximum number of CBs installed in the EDS is limited by

$$\sum_{u\in\Omega_u}\sum_{i\in\Omega_b} \beta^{cb}_{i,u} \leq \overline{M}. \tag{16}$$

It is assumed that in the planning of the distribution system, investment decisions about new distributed generators can be made. Constraints

$$0 \leq P^{DG}_{i,g,u} \leq S^{DG}_g Fc^{dg}_g \beta^{dg}_{i,g,u} \quad \forall i \in \Omega_b, g \in \Omega_g, u \in \Omega_u \tag{17}$$

$$\left|Q^{gd}_{i,g,u}\right| \leq S^{DG}_g \sin\left(\arccos Fc^{dg}_g\right) \beta^{dg}_{i,g,u} \quad \forall i \in \Omega_b, g \in \Omega_g, u \in \Omega_u \tag{18}$$

respectively limit the active and reactive power that can be generated by DG. Constraint

$$\sum_{g\in\Omega_g} \beta^{dg}_{i,g,u} \leq 1 \quad \forall i \in \Omega_b, u \in \Omega_u \tag{19}$$

guarantees that the model can only choose one alternative of equipment to be allocated, while

$$\sum_{u\in\Omega_u}\sum_{g\in\Omega_g} \beta^{dg}_{i,g,u} \leq 1 \quad \forall i \in \Omega_b \tag{20}$$

limits the number of DGs allocated at each node to one. Constraint

$$\sum_{u\in\Omega_u}\sum_{i\in\Omega_b}\sum_{g\in\Omega_g} \beta^{dg}_{i,g,u} \leq \overline{G} \tag{21}$$

limits the total number of DGs installed in the system over the planning horizon.

C. Mixed-Integer Non-Linear Programming Model for the DSEP Problem

An MINLP formulation to solve the DSEP problem is developed in this section. The equations used in this model are based on the formulations presented in [4], [5], [12], and [14].

1) Objective Function: The objective function (30) considers the investment and operation costs, and is based on [17]. The first part is the cost associated with the investment, and the second part corresponds to the costs associated with the active power losses and operating costs. The function $f(\tau, K) = 1 - (1+\tau)^{-K}/\tau$ in (27) and (29) calculates the present value of an annualized cost that has a duration of K years in terms of interest rate τ.

Investments in circuits (IC):

$$\sum_{ij \in \Omega_l} \sum_{a \in \Omega_a} c^{f}_{ij,a} \delta_{ij,a,u} l_{ij}. \tag{22}$$

Investments in substations (IS):

$$\sum_{i \in \Omega_s} \sum_{c \in \Omega_c} \left(c^{s}_{i,c} \sigma_{i,c,u} + c^{r}_{i,c} r_{i,c,u} \right). \tag{23}$$

Investments in voltage regulators (IVR):

$$\sum_{ij \in \Omega_l} c^{vr} \beta^{vr}_{ij,u}. \tag{24}$$

Investments in capacitor banks (ICB):

$$\sum_{i \in \Omega_s} \left(c^{cb} \beta^{cb}_{i,u} + c^{mod} \varpi_{i,u} \right). \tag{25}$$

Investments in distributed generation (IDG):

$$\sum_{i \in \Omega_s} \sum_{g \in \Omega_g} c^{dg}_{g} \beta^{dg}_{i,g,u}. \tag{26}$$

Production costs of electricity from substations (CES):

$$\sum_{i \in \Omega_s} \alpha \phi_l c^{e} P^{S}_{i,u} f(\tau, K). \tag{27}$$

Production costs of electricity from DG (CEDG):

$$\sum_{i \in \Omega_s} \sum_{g \in \Omega_g} \alpha \phi_l c^{edg}_{g} P^{DG}_{i,g,u} f(\tau, K). \tag{28}$$

Operating costs of substations (OS):

$$\sum_{i \in \Omega_s} \alpha \phi_s c^{v}_{i} Sg^{sqr}_{i,u} f(\tau, K). \tag{29}$$

Thus, the objective function

$$\text{Min} \sum_{u \in \Omega_u} [IC + IS + IVR + ICB + \text{IDG} + \text{CES} + \text{CEDG} + OS](1+\tau)^{-(u-1)K} \tag{30}$$

corresponds with the sum of the terms related to the investments in circuits, substations, voltage regulators, and capacitor banks, as well as the terms associated with the cost of power losses in the circuits and the operating cost of the substations.

2) Constraints: The constraints related to the investment and operation variables along the planning horizon are represented by

$$\sum_{a \in \Omega_a} \delta_{ij,a,u} \leq 1 \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{31}$$

$$\sum_{u \in \Omega_u} \delta_{ij,a,u} \leq 1 \quad \forall ij \in \Omega_l, a \in \Omega_a \tag{32}$$

$$z_{ij,a,u} \leq \sum_{h=1}^{u} \delta_{ij,a,h} \quad \forall ij \in \Omega_l, a \in \Omega_a, u \in \Omega_u \tag{33}$$

$$\sum_{c \in \Omega_c} \sigma_{i,c,u} \leq 1 \quad \forall i \in \Omega_s, u \in \Omega_u \tag{34}$$

$$\sum_{c \in \Omega_c} r_{i,c,u} \leq 1 \quad \forall i \in \Omega_s, u \in \Omega_u \tag{35}$$

$$\sum_{u \in \Omega_u} \sum_{c \in \Omega_c} \sigma_{i,c,u} \leq 1 \quad \forall i \in \Omega_s \tag{36}$$

$$\sum_{u \in \Omega_u} \sum_{c \in \Omega_c} r_{i,c,u} \leq 1 \quad \forall i \in \Omega_s \tag{37}$$

$$w_{i,c,u} \leq \sum_{h=1}^{u} \sigma_{i,c,h} \quad \forall i \in \Omega_s, c \in \Omega_c, u \in \Omega_u \tag{38}$$

$$r_{i,c,u} \leq \sum_{h=1}^{u} \sigma_{i,c,h} \quad \forall i \in \Omega_s, c \in \Omega_c, u \in \Omega_u. \tag{39}$$

These constraints are adapted from [3]–[5]. The operation state of a circuit at each stage is represented by two binary variables in order to improve the performance of the solution

$$\sum_{a \in \Omega_a} z_{ij,a,u} = y^{+}_{ij,u} + y^{-}_{ij,u} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{40}$$

$$y^{+}_{ij,u} + y^{-}_{ij,u} \leq 1 \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{41}$$

as proposed in [40] to solve the reconfiguration problem of EDSs.

Constraints

$$\sum_{ij \in \Omega_l} \left(y^{+}_{ij,u} + y^{-}_{ij,u} \right) = |\Omega_b| - |\Omega_s| - \sum_{i \in \Omega_{bp}} \varepsilon_{i,u} \quad \forall u \in \Omega_u \tag{42}$$

$$\sum_{ij \in \Omega_l} \left(y^{+}_{ij,u} \right) + \sum_{ki \in \Omega_l} \left(y^{-}_{ki,u} \right) \geq 2\varepsilon_{i,u} \quad \forall i \in \Omega_{bp}, u \in \Omega_u \tag{43}$$

$$y^{+}_{ij,u} + y^{-}_{ij,u} \leq \varepsilon_{i,u} \quad \forall ij \in \Omega_l, i \in \Omega_{bp}, u \in \Omega_u \tag{44}$$

$$y^{+}_{ji,u} + y^{-}_{ji,u} \leq \varepsilon_{i,u} \quad \forall ji \in \Omega_l, i \in \Omega_{bp}, u \in \Omega_u \tag{45}$$

together with (1) and (2)ensure the radial operation of the system and are natural extensions of the work done in [12], adapted to account for the presence of transfer nodes and the particularities of the DSEP problem.

Constraints

$$Sg_{i,u}^{sqr} = {P_{i,u}^{S}}^{2} + {Q_{i,u}^{S}}^{2} \quad \forall i \in \Omega_s, u \in \Omega_u \tag{46}$$

$$Sg_{i,u}^{sqr} \leq \left(\sum_{h=1}^{u} \sum_{c \in \Omega_C} Sg'_{i,c} \sigma_{i,c,h} + \sum_{h=1}^{u} \sum_{c \in \Omega_c} Rg'_{i,c} r_{i,c,h} \right)^2 \quad \forall i \in \Omega_s, u \in \Omega_u \tag{47}$$

represent the limits of transformer capacity of substations. Note that (46) is a quadratic constraint; as the operation cost of the substations is minimized in (30), the square of the apparent power (in the optimal solution) must be equal to the sum of the square of the real and reactive power supplied by the substations. Constraint

$$\underline{V^2} \leq V_{i,u}^{sqr} \leq \overline{V^2} \quad \forall i \in \Omega_b, u \in \Omega_u \tag{48}$$

limits the voltage magnitudes. Constraints

$$0 \leq I_{ij,a,u}^{sqr} \leq \overline{I}^2 z_{ij,a,u} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{49}$$

$$0 \leq I_{ij,a,u}^{sqr} \leq \overline{I}^2 \left(y_{ij,u}^{+} + y_{ij,u}^{-} \right) \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{50}$$

represent the current magnitude limits of circuits in terms of the capacity of each conductor type and the stage of the circuit, respectively. Constraint

$$|b_{ij,u}| \leq \overline{b} \left(1 - y_{ij,u}^{+} - y_{ij,u}^{-} \right) \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{51}$$

limits the voltage magnitude drop in a circuit according to its stage; if circuit ij is active at stage u, then the auxiliary variable $b_{ij,u}$ is zero, otherwise it remains limited by $\overline{b}$. Constraints

$$|P_{ij,a,u}| \leq \overline{VI}_a \left(y_{ij,u}^{+} + y_{ij,u}^{-} \right) \quad \forall ij \in \Omega_l, a \in \Omega_a, u \in \Omega_u \tag{52}$$

$$|Q_{ij,a,u}| \leq \overline{VI}_a \left(y_{ij,u}^{+} + y_{ij,u}^{-} \right) \quad \forall ij \in \Omega_l, a \in \Omega_a, u \in \Omega_u \tag{53}$$

$$|P_{ij,a,u}| \leq \overline{VI}_a z_{ij,a,u} \quad \forall ij \in \Omega_l, a \in \Omega_a, u \in \Omega_u \tag{54}$$

$$|Q_{ij,a,u}| \leq \overline{VI}_a z_{ij,a,u} \quad \forall ij \in \Omega_l, a \in \Omega_a, u \in \Omega_u \tag{55}$$

limit the power flow of a circuit according to its stage and the corresponding type of conductor. These constraints are natural extensions of the work done in [14] to include the multistage planning.

Constraints

$$w_{i,c,u} \in \{0,1\} \quad \forall i \in \Omega_s, c \in \Omega_c, u \in \Omega_u \tag{56}$$

$$r_{i,c,u} \in \{0,1\} \quad \forall i \in \Omega_s, c \in \Omega_c, u \in \Omega_u \tag{57}$$

$$\sigma_{i,c,u} \in \{0,1\} \quad \forall i \in \Omega_s, c \in \Omega_c, u \in \Omega_u \tag{58}$$

$$z_{ij,a,u} \in \{0,1\} \quad \forall ij \in \Omega_l, c \in \Omega_c, u \in \Omega_u \tag{59}$$

$$\beta_{i,g,u}^{dg} \in \{0,1\} \quad \forall i \in \Omega_b, g \in \Omega_g\, u \in \Omega_u \tag{60}$$

$$y_{ij,u}^{+}, y_{ij,u}^{-} \in \{0,1\} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{61}$$

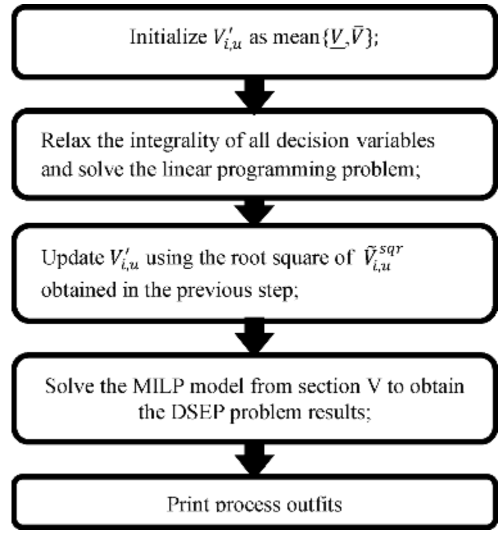

Fig. 1. Procedure to obtain an estimation of the voltage magnitude.

$$\alpha_{ij,u}^{vr} \in \{0,1\} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{62}$$

$$\beta_{ij,u}^{vr} \in \{0,1\} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{63}$$

$$\varepsilon_{i,u} \in \{0,1\} \quad \forall i \in \Omega_b, u \in \Omega_u \tag{64}$$

$$\beta_{i,u}^{cb} \in \{0,1\} \quad \forall i \in \Omega_b, u \in \Omega_u \tag{65}$$

$$\varpi_{i,u} \in \mathbb{Z}_{\geq 0} \quad \forall i \in \Omega_b, u \in \Omega_u \tag{66}$$

$$\psi_{i,u} \in \mathbb{Z}_{\geq 0} \quad \forall i \in \Omega_b, u \in \Omega_u \tag{67}$$

represent the binary and integer nature of the decision variables.

The MINLP model that represents the DSEP problem is described by (1)–(67). Due to the complexity of this model, in the next section it will be linearized in order to obtain a robust and efficient mathematical formulation for the DSEP problem.

IV. Linearization of the DSEP Mathematical Model

Note that (3), (46), and (47) contain nonlinear expressions. Here, those constraints will be linearized in order to obtain an MILP model for the DSEP problem.

A. Linearization of the Left Member of (3)

The linearization of the product $\widetilde{V}_{j,u}^{sqr} \widehat{I}_{ij,u}^{sqr}$ can be performed by using an estimated voltage magnitude $V'_{i,u}$, as shown in

$$\widetilde{V}_{j,u}^{sqr} \widehat{I}_{ij,u}^{sqr} \approx \left(V'_{i,u} \right)^2 \widehat{I}_{ij,u}^{sqr}. \tag{68}$$

This simplification is an approximation with minimal error due to the limited range of the voltage magnitude variation $\left[\underline{\mathrm{V}}, \overline{\mathrm{V}} \right]$; results in Section V show the low approximation error of the power losses.

In order to obtain a suitable value for the parameter $V'_{i,u}$, the variables first take the value equal to the average voltage limits. Then, a relaxed version (linear programming) of the model in which the binary variables are relaxed is solved; it is assumed that the initial value of the parameter $V'_{i,u}$ corresponds to the average value of the voltage limits. The voltage magnitudes found by solving this relaxed model are used to fix the corresponding values of $V'_{i,u}$. Finally, the proposed MILP model is solved. The procedure shown in the first three steps of Fig. 1 is performed.

B. Linearization of the Right Member of (3)

The following equations are linear expressions that approximate the right member of (3):

$$\widehat{P}_{ij,u}^2 + \widehat{Q}_{ij,u}^2 \approx \sum_{y=1}^{Y} m_y^G \Delta_{ij,y,u}^{Pg} + \sum_{y=1}^{Y} m_y^G \Delta_{ij,y,u}^{Qg} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{69}$$

$$\widehat{P}_{ij,u} = P_{ij,u}^+ - P_{ij,u}^- \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{70}$$

$$\widehat{Q}_{ij,u} = Q_{ij,u}^+ - Q_{ij,u}^- \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{71}$$

$$P_{ij,u}^+ + P_{ij,u}^- = \sum_{y=1}^{Y} \Delta_{ij,y,u}^{Pg} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{72}$$

$$Q_{ij,u}^+ + Q_{ij,u}^- = \sum_{y=1}^{Y} \Delta_{ij,y,u}^{Qg} \quad \forall ij \in \Omega_l, u \in \Omega_u \tag{73}$$

$$\Delta_{ij,y,u}^{Pg} \le \overline{\Delta}^G \quad \forall ij \in \Omega_l, u \in \Omega_u, y = 1,..,Y \tag{74}$$

$$\Delta_{ij,y,u}^{Qg} \le \overline{\Delta}^G \quad \forall ij \in \Omega_l, u \in \Omega_u, y = 1,..,Y \tag{75}$$

The terms on the right-hand side of (69) are linear approximations of $\widehat{P}_{ij,u}^2$ and $\widehat{Q}_{ij,u}^2$, which are formulated according to the discretization for quadratic expressions used in [32]. Equations (70) and (71) represent $\widehat{P}_{ij,u}$ and $\widehat{Q}_{ij,u}$, respectively, by using non-negative auxiliary variables. Constraints (72) and (73) establish that $\left|\widehat{P}_{ij,u}\right|$ and $\left|\widehat{Q}_{ij,u}\right|$ are the sums of the discretization variables $\Delta_{ij,y,u}^{Pg}$ and $\Delta_{ij,y,u}^{Qg}$ respectively. Constraints (74) and (75) impose limits on the values that can take the discretization variables. The following equations calculate the values of the parameters used in the discretization:

$$m_y^G = (2y-1)\,\overline{\Delta}^G \quad y = 1,..,Y \tag{76}$$

$$\overline{\Delta}^G = \frac{\overline{V}}{Y} \max\left\{I_a, a \in \Omega_a\right\}. \tag{77}$$

These constraints are adapted from [32].

C. Linearization of (46)

Using the same linearization technique presented above, (46) is approximated as shown in

$$Sg_{i,u}^{sqr} = \sum_{y=1}^{Y} m_y^S \Delta_{i,y,u}^{Ps} + \sum_{y=1}^{Y} m_y^S \Delta_{i,y,u}^{Qs} \quad \forall i \in \Omega_s, u \in \Omega_u \tag{78}$$

$$P_{i,u}^S = \sum_{y=1}^{Y} \Delta_{i,y,u}^{Ps} \quad \forall i \in \Omega_s, u \in \Omega_u \tag{79}$$

$$Q_{i,u}^S = \sum_{y=1}^{Y} \Delta_{i,y,u}^{Qs} \quad \forall i \in \Omega_s, u \in \Omega_u \tag{80}$$

$$\Delta_{i,y,u}^{Ps} \le \overline{\Delta}^S \quad \forall i \in \Omega_s, u \in \Omega_u, y = 1,..,Y \tag{81}$$

$$\Delta_{i,y,u}^{Qs} \le \overline{\Delta}^S \quad \forall i \in \Omega_s, u \in \Omega_u, y = 1,..,Y. \tag{82}$$

The following equations calculate the values of the parameters used in the discretization:

$$m_y^S = (2y-1)\,\overline{\Delta}^S \quad y = 1,..,Y \tag{83}$$

$$\overline{\Delta}^S = \frac{\overline{V}}{Y} \max\left\{Sg'_{i,c} + Rg'_{i,c}, c \in \Omega_c, i \in \Omega_s\right\}. \tag{84}$$

D. Linearization of (47)

Constraint (47) can be linearized taking into account that: 1) only one substation type is chosen for the installation of a substation in a single stage, as guaranteed by (36); 2) only one substation type is chosen for the repowering of a substation in a single stage, as guaranteed by (37); 3) a repowering can only occur after the construction of the substation [a planning condition modeled in (39)]; and 4) the decision variables related to the construction or repowering of a substation are binary variables. Consequently, the operational limit of the substations can be replaced by an equivalent linear constraint, as shown in

$$Sg_{i,u}^{sqr} \le \sum_{h=1}^{u} \sum_{c \in \Omega_C} {Sg'_{i,c}}^2 \sigma_{i,c,h} + \sum_{h=1}^{u} \sum_{c \in \Omega_C} {Rg'_{i,c}}^2 r_{i,c,h} + 2\sum_{h=1}^{u} \sum_{c \in \Omega_C} Sg'_{i,c} Rg'_{i,c} r_{i,c,h} \quad \forall i \in \Omega_s, u \in \Omega_u. \tag{85}$$

E. MILP Model for the DSEP Problem

The proposed algorithm is summarized in the flowchart in Fig. 1. Taking into account the linearization presented in Section IV, the MINLP model presented in Section III can be transformed into an MILP model defined as

$$\text{Min } (30)$$

$$\text{Subject to } (1)-(2), (4)-(67), (70)-(85)$$

$$\left(V'_{i,u}\right)^2 \widehat{I}_{ij,u}^{sqr} = \sum_{y=1}^{Y} m_y^G \Delta_{ij,y,u}^{Pg} + \sum_{y=1}^{Y} m_y^G \Delta_{ij,y,u}^{Qg} \quad \forall ij \in \Omega_l, u \in \Omega_u. \tag{86}$$

V. Numerical Results

A 24-node distribution system, based on [41], was used to show the performance of the proposed formulation. The system consisted of 24 nodes (4 substations and 20 load nodes) and 34 branches operating at a nominal voltage of 13.8 kV. The planning horizon was divided into three stages. The proposed model was implemented in the modeling language AMPL [42] and solved with CPLEX [43] using a Dell PowerEdge R910x64 computer with six processors at 1.87 GHz and 128 GB of RAM memory.

The initial topology of the EDS is shown in Fig. 2, in which the rectangles denote the substations, the circles represent the nodes, the branches drawn as continuous lines indicate the initial network, and the branches drawn as dashed lines are candidates for expansion. Table I presents the loads for the three stages of the planning horizon. Table II shows all circuit data and includes the lengths and initial conductor types of each circuit. At the initial stage, substations 21 and 22 were built with

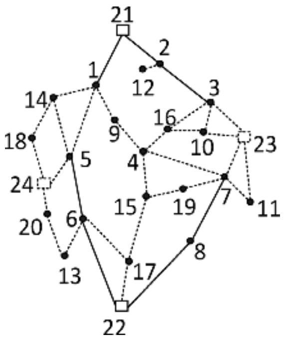

Fig. 2. Initial topology of the 24-node distribution system.

TABLE I
LOAD DATA (KVA)

Node	Stage 1	Stage 2	Stage 3	Node	Stage 1	Stage 2	Stage 3
1	4050	4735	5420	13	0	1150	1350
2	780	995	1210	14	0	3050	3160
3	2580	3380	3980	15	0	1620	1620
4	320	410	490	16	0	0	1220
5	280	370	470	17	0	2160	2400
6	1170	1305	1440	18	0	0	2100
7	4040	4200	4360	19	0	0	1810
8	720	830	940	20	0	0	3790
9	1140	1455	1770	21	0	0	0
10	1560	2040	2400	22	0	0	0
11	0	1910	2800	23	0	0	0
12	0	930	1290	24	0	0	0

TABLE II
CIRCUIT DATA

N°	i	j	l_{ij}	ct^0	N°	i	j	l_{ij}	ct^0	N°	i	j	l_{ij}	ct^0
1	1	5	3.885	-	13	4	15	2.800	-	25	10	16	1.400	-
2	1	9	2.100	-	14	4	16	2.275	-	26	10	23	2.275	-
3	1	14	2.100	-	15	5	6	4.200	1	27	11	23	2.800	-
4	1	21	3.850	1	16	5	24	1.225	-	28	13	20	2.100	-
5	2	3	3.500	1	17	6	13	2.100	-	29	14	18	1.750	-
6	2	12	1.925	-	18	6	17	3.850	-	30	15	17	2.100	-
7	2	21	2.975	1	19	6	22	4.550	1	31	15	19	2.800	-
8	3	10	1.925	-	20	7	8	3.500	1	32	17	22	2.625	-
9	3	16	2.100	-	21	7	11	1.925	-	33	18	24	2.625	-
10	3	23	2.100	-	22	7	19	2.800	-	34	20	24	1.575	-
11	4	7	4.550	-	23	7	23	1.575	-					
12	4	9	2.100	-	24	8	22	3.500	1					

N°≡ circuit number; l_{ij} in km; ct^0: initial conductor type

types 1 and 2, respectively, while the other two substations were alternatives that could be built only with type 3. Table III shows the different investment alternatives for substations and conductors. The costs related to the substations and conductors were adapted from [14], the costs related to the CBs and VRs were adapted from [32], and data related to the DGs were adapted from [11].

Furthermore, c_g^{dg} is equal to US\$1000/ kVA, S_g^{DG} is equal to 3000 kVA, $\overline{G}$ is equal to 5, c^{cb} is equal to US\$1000, the unit cost c^{mod} is equal to US\$900, Q_{bc}^{esp} is equal to 300 kVAr, $\overline{N}$ is equal to 4, and $\overline{M}$ is equal to 6. Finally, c^{vr} is equal to US\$8000, $R_{ij}^{\%}$ is equal to 10%, and $\overline{O}$ is equal to 4. This work adopted a planning horizon of 15 years, subdivided into periods of five years. The interest rate was set at 10%, Upper and lower voltage magnitude limits were 1.05 and 0.95 p.u., respectively. The price of energy generated by the substations was 0.10 US\$/kWh, the price of energy generated by the DGs was 0.04 US\$/kWh, and the load power factor was equal to 0.9 and 0.95 for DGs.

TABLE III
DATA OF SUBSTATIONS AND CONDUCTORS

Substation data				
Ω_s	$Sg'_{i,c}$(kVA)	$Rg'_{i,c}$(kVA)	$c_{i,c}^{s}$(US)	$c_{i,c}^{T}$(US)
21	12000	7000	0.00	1000 x10^3
22	15000	0.00	0.00	0.00
23	20000	0.00	3000 x10^3	0.00
24	20000	0.00	3000 x10^3	0.00
Conductor data				
Ω_a	$R_a(\Omega/km)$	$X_a(\Omega/km)$	$\overline{I_a}(A)$	$c_{ij,a}^{f}$(US/km)
1	0.614	0.399	197	25x10^3
2	0.307	0.380	314	35 x10^3

A. Case Studies

Six different cases were carried out for the expansion planning of the 24-node system: 1) a multistage test (MS); 2) a multistage test with CBs (MSCB); 3) a multistage test with VRs (MSVR); 4) a multistage test with DGs (MSDG); 5) a multistage test with both CBs and VRs (MSVRCB); and 6) a multistage test with CBs, VRs, and DGs (MSVRCBDG). The topologies obtained for all cases at each stage are shown in Figs. 3 and 4. In the figures, black lines and blue lines represent circuits built with types 1 and 2, respectively. The symbol represents the allocation of a VR, the symbol represents the allocation of a CB, and the symbol represents the allocation of a DG. The results, summarized inTable IV, show that the installation of control devices, such as VRs and CBs, and DGs, can lead to better solutions. For all cases, the parameter for the piecewise linearization Y was equal to 20. The active power losses were compared to the operation point for the solution of the DSEP problem using a load flow sweep method. The results are summarized in Table V. Note that the approximation errors are negligible, showing the accuracy of the proposed model.

1) Multistage Test: This case was solved in 30.03 min and the solution found had an objective function of US\$ 83,970,980.54. In Stage 1, circuits 4–9 and 4–16 were built with conductor type 1, while circuits 4–15, 10–16, 15–17, and 17–22 were built with conductor type 2; circuits 1–21, and 8–22 were reconductored. In Stage 2, substation 23 was constructed to supply loads that were transferred from substations 21 (node 3) and 22 (nodes 4, 7, 9, 10, and 16). Additionally, circuits 2–12 and 6–13 were built with conductor type 1, and circuits 1–14, 3–23, 7–23, 10–23, and 11–23 were built with conductor type 2. In Stage 3, substation 24 was built to supply new loads. Circuits 7–19 and 13–20 were built with conductor type 1, and circuits 14–18, 18–24, and 20–24 were constructed with conductor type 2. As shown in Fig. 3, considering the conditions proposed in [12] for transfer nodes, in some cases it is possible to avoid constructing circuits to connect nodes without loads, which can lead to a reduction in investment costs. Note that circuits 2–3, 7–8, and 4–15 were

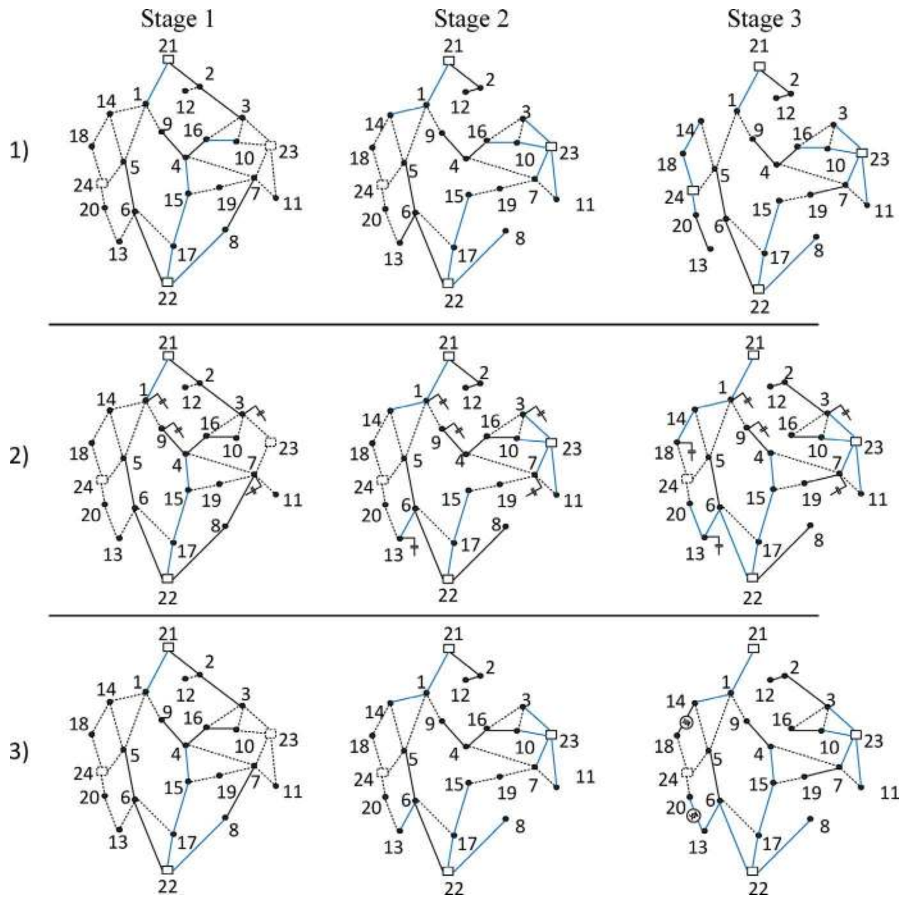

Fig. 3. Topologies For Cases 1–3 of the 24-node Distribution System.

opened in Stage 2; thus, nodes 3, 4, 7, 9, 10, and 16 were transferred to substation 23 through the construction of circuits 3–23, 7–23, and 10–23. In Stage 1, node 16 was used as a transfer node. In Stage 3, circuit 6–13 was opened, and thus node 13 was transferred to substation 24 through the construction of circuits 24–20 and 20–13. Note that, due to reconfigurations between stages, the system topologies for all of the cases change at each stage, which contributes to the reduction in total cost.

2) Multistage Test With CBs: This case was solved in 9.91 min, and the solution found had an objective function of US$ 82,903,536.95 – a lower cost than for the MS case. In Stage 1, circuits 4–9, 4–16, and 10–16 were built with conductor type 1; circuits 4–15, 15–17, and 17–22 were built with conductor type 2; and circuit 1–21 was reconductored. CBs with 1200 kVAr were allocated at nodes 1, 3, and 7, and one CB with 900 kVAr was allocated at node 9. In Stage 2, substation 23 was constructed to supply loads that were transferred from substations 21 (node 3) and 22 (node 4, 7, 9, 10, and 16). Additionally, circuit 2–12 was built with conductor type 1; circuits 1–14, 3–23, 6–13, 7–23, 10–23, and 11–23 were built with conductor type 2, and a CB with 900 kVAr was allocated at node 13. In Stage 3, circuit 7–19 was built using conductor type 1; circuits 13–20 and 14–18 were built using conductor type 2; and circuit 6–22 was reconductored. One CB with 1200 kVAr was installed at node 18, and the capacity of CBs allocated at nodes 9 and 13 was increased to 1200 kVAr. In Stage 2, circuits 2–3, 7–8, and 4–15 were opened, and nodes 3, 4, 7, 9, 10, and 16 were transferred to substation 23 by constructing circuits 3–23, 7–23, and 10–23. In Stage 3, circuit 2–3 was closed, and nodes 2 and 12 were transferred to substation 23 by opening circuit 2–21.

Finally, circuit 4–16 was opened, and nodes 4 and 9 were transferred to substation 22 by closing circuit 4–15. Table VI presents the variation in the number of CB modules over the planning horizon. Comparing the MS and MSCB cases shows that both cases have the same topology for the first stage. However, in the MSCB case, circuits 8–22 and 10–16 operate with conductor type 1. This difference represents a decreased investment for the construction and reconductoring of circuits, compensated by the allocation of the CBs at nodes 1, 3, 7, and 9. In Stage 2, both case studies present the same topologies of operation, except in terms of the conductor type used for circuits 8–22, 10–16, and 6–13. In Stage 3, substation 24 is built in the MS case to supply nodes 13, 14, 18, and 20, whereas in the MSCB case, these nodes are fed as follows: substation 21 feeds nodes 14 and 18, and; substation 22 feeds nodes 13 and 20. The cost difference between the MS and MSCB cases is US$ 1,067,443.59, which represents a 1.30% reduction in total investment costs.

3) Multistage Test With VRs: This case was solved in 8.7 min, and the solution found had an objective function of US$ 83,153,544.79 – a lower cost than for the MS case. In Stage 1, circuits 4–9, 4–16, and 10–16 were built with conductor type 1, while circuits 4–15, 15–17, and 17–22 were built with con-

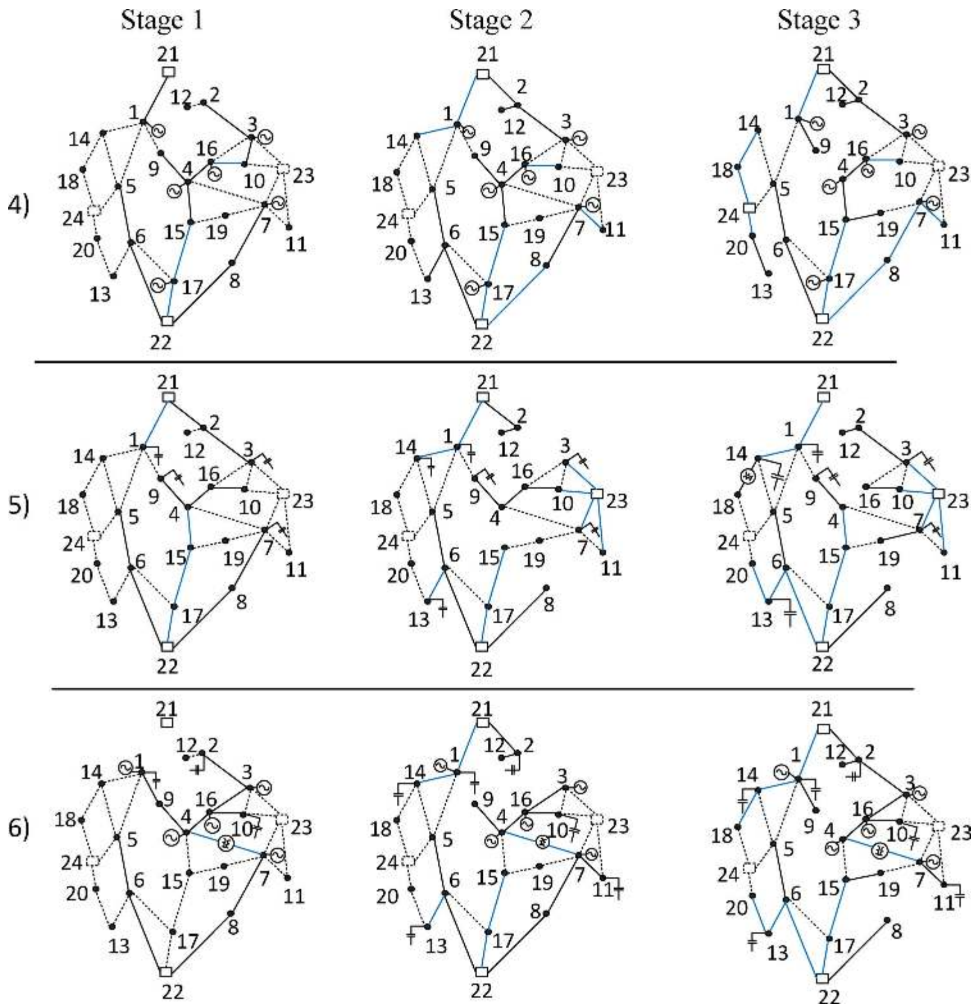

Fig. 4. Topologies for cases 4–6 of the 24-node distribution system.

TABLE IV
RESULTS SUMMARY (COST IN 10^3 US$)

Cost/Cases	1	2	3	4	5	6
IS	3019.4	1862.8	1862.8	1156.6	1862.8	385.5
CES	79846.8	80024.4	80174.7	31856.4	80020.8	31944.4
IC	1104.8	994.2	1109.9	946.6	987.4	886.4
IDG	0.0	0.0	0.0	15000.0	0.0	15000.0
CEDG	0.0	0.0	0.0	18989.3	0.0	18989.3
ICB	0.0	22.3	0.0	0.0	23.4	19.3
IVR	0.0	0.0	6.2	0.0	3.1	8.0
TC	83971.0	82903.5	83153.5	67948.9	82897.4	67233.0

TABLE V
APPROXIMATION ERROR OF THE POWER LOSSES

Cases	Load Flow (kW)	Proposed Model (kW)	Error (%)
1	1206.45	1209.66	-0.27
2	1516.66	1521.99	-0.35
3	1705.15	1701.5	0.21
4	863.46	864.52	-0.12
5	1515.31	1513.51	0.12
6	1086.37	1079.4	0.65

ductor type 2. Circuits 1–21 and 8–22 were reconductored with conductor type 2. In Stage 2, substation 23 was constructed to feed loads that were transferred from substations 21 (node 3)

TABLE VI
VARIATION IN THE NUMBER OF CB MODULES OVER THE PLANNING HORIZON FOR THE MSCB CASE

Stage/ node	1	3	7	9	13	18
1	4	4	4	3	0	0
2	4	4	4	3	3	0
3	4	4	4	4	4	4

and 22 (nodes 4, 7, 9, 10, and 16). In addition, in Stage2, circuit 2–12 was built with conductor type 1, and circuits 1–14, 3–23, 6–13, 7–23, 10–23, and 11–23 were built with conductor type 2. In Stage 3, loads were transferred from substations 21 (nodes 2 and 12) to substation 23, and loads were transferred from substations 23 (nodes 4 and 9) to substation 22. Circuits 7–19 and 14–18 were built with conductor type 1; circuit 13–20 was built with conductor type 2; and circuit 6–22 was reconductored with conductor type 2. VRs were allocated to circuits 14–18 and 13–20, both in position 1 of the tap. In Stage 2, circuits 2–3, 4–15, and 7–8 were opened; nodes 3, 4, 7, 9, 10, and 16 were transferred to substation 23 by constructing circuits 3–23, 10–23, and 7–23. Note that, in Stages 1 and 2, node 16 was used as a transfer node. In Stage 3, circuit 4–16 was opened and nodes 4 and 9 were transferred to substation 22 by connecting circuit 4–15; circuit 2–21 was opened and nodes 2 and 12 were transferred to substation 23 by connecting circuit 2–3. Comparing the MS and MSVR cases shows that the first and second stages have the same topologies. However, in the MSVR case, circuits 6–13 and 10–16 operate with conductor type 2 and 1, respectively; whereas in the MS case these same circuits operate with conductor type 1 and 2, respectively. In Stage 3, the allocation of the VRs makes the allocation of a new substation at node 24 unnecessary, as in the MS case. Therefore, nodes 14 and 18 are fed by substation 21, while nodes 20 and 13 are fed by substation 22. In the MS case, the same nodes are fed by substation 24. Comparing the MSCB and MSVR cases reveals that the same topologies are obtained in all stages, with the exception of the conductor type of circuits 8–22 and 14–18. The cost difference between the MSVR and MSCB cases is US$ 817,435.75, which represents a 0.97% reduction in total investment cost.

4) Multistage Test With DGs: This case was solved in 600 min, and the solution found had an objective function of US$ 67,948,943.57 – a lower cost than for the MS case. In Stage 1, circuits 3–10, 4–9, 4–15, and 4–16 were built with conductor type 1, while circuits 10–16, 15–17, and 17–22 were built with conductor type 2. DGs were allocated at nodes 1, 3, 7, 16, and 17. In Stage 2, circuits 2–12 and 6–13 were built with conductor type 1, while circuits 1–14 and 7–11 were built with conductor type 2; circuits 1–21 and 8–22 were reconductored. In Stage 3, substation 24 was constructed to supply new loads (nodes 18 and 20) and the loads that were transferred from substation 22 (node 13) and substation 21 (node 14). Circuits 1–9, 13–20, and 15–19 were built with conductor type 1; circuits 14–18, 18–24, and 20–24 were built with conductor type 2; and circuit 7–8 was reconductored. For this test, circuit 2–21 was opened in Stage 1, and nodes 2 and 3 were transferred to substation 22 through the construction of circuit 3–10. In Stage 2, circuit 3–10 was opened and nodes 2, 3, and 12 were transferred to substation 21 through the operation of circuit 2–21. In Stage 3, circuit 1–14 was opened and the load from node 14 was transferred to substation 24 by connecting circuits 18–24 and 14–18. Circuit 6–13 was opened, and the load from node 13 was transferred to substation 24 by connecting circuits 20–24 and 13–20. Circuit 4–9 was opened, and the load form node 9 was transferred to substation 21 by connecting of the circuit 1–9. Note that in all stages, node 16 was used as a transfer node. Comparing the MSDG and MS cases shows that, in the first stage of the MSDG case, all of the nodes are fed jointly by substation 22 and the 5 DGs that were allocated at this stage, except for node 1, which is served by substation 21. In the MS case, substations 21 and 22 are used to feed all of the nodes. The main difference observed in Stage 2 of this case is the building of substation 23, which is built to serve loads 3, 4, 7, 9, 10, and 11. In the MSDG case, these loads are fed by substations 21 and 22, and 5 DGs. In Stage 3 of both cases, substation 24 is built to feed nodes 18, 13, 14, and 20. The cost difference between the MS and MSDG cases is US$ 16,022,036.97, which represents a 19.08% reduction in total investment cost. This reduction is explained by the lower cost of the energy generated by the DGs.

5) Multistage Test With CBs and VRs: This case was solved in 10.96 min, and the solution found had an objective function of US$ 82,897,393.01– a lower cost than for the MS case. In Stage 1, circuits 4–9, 4–16, and 10–16 were built with conductor type 1; circuits 4–15, 15–17, and 17–22 were built with conductor type 2; and circuit 1–21 was reconductored. In addition, CBs with 1200 kVAr were allocated at nodes 1, 3, and 7, and one CB with 900 kVAr was allocated at node 9. In Stage 2, substation 23 was constructed to feed loads that were transferred from substations 21 (node 3) and 22 (nodes 4, 7, 9, 10, and 16). Circuit 2–12 was built with conductor type 1, while circuits 1–14, 3–23, 6–13, 7–23, 10–23, and 11–23 were built with conductor type 2. CBs with a capacity of 900 kVAr and 1200 kVAr, respectively, were allocated at nodes 13 and 14. In Stage 3, circuits 7–19 and 14–18 were built with conductor type 1; circuit 13–20 was built with conductor type 2; and circuit 6–22 was reconductored. The capacity of CBs allocated at nodes 9 and 13 was increased to 1200 kVAr. Table VII presents the variation in the number of CB modules over the planning horizon. In addition, one VR was allocated to circuit 14–18 with a tap position of 1. Note that the cost difference between the MS and MSVRCB cases is US$ 1,073,587.57, which represents a 1.28% reduction in total investment cost. Comparing the MS and MSVRCB cases shows that both cases have the same topology for the first stage. However, in the MSVRCB case, circuits 8–22 and 10–16 operate with conductor type 1. This difference represents a decreased investment for the construction and reconductoring of circuit, compensated by the location of the CBs at nodes 1, 3, 7, and 9. In Stage 2, both case studies present the same topologies of operation, except for the conductor type used in circuits 8–22, 10–16, and 6–13. In Stage 3, of the MS case, substation 24 is built to supply nodes 13, 14, 18, and 20. In the MSVRCB case, these nodes are fed as follows: substation 21 feeds nodes 14 and 18, and substation 22 feeds nodes 13 and 20. Comparing the MSCB and MSCBVR cases, the topologies of Stages 1 and 2 are the same. However, in Stage 2 of the MSCBVR case, there is one more CB at node 14. In Stage 3, there are only two differences between the two cases, in the MSVRCB case, one VR is allocated to circuit 14–18, allowing for circuit 14–18 to be constructed with a lower conductor capacity. In the MSCB case, circuit 14–18 is constructed with a conductor type of greater capacity, and a CB is allocated at node 18.

TABLE VII
VARIATION OF THE NUMBER OF CB MODULES IN THE PLANNING HORIZON FOR THE MSVRCB CASE

Stage/ node	1	3	7	9	13	14
1	4	4	4	3	0	0
2	4	4	4	3	3	4
3	4	4	4	4	4	4

6) Multistage Test With CBs, VRs, and DGs: This case was solved in 600 min, and the solution found had an objective function of US$ 67,233,023.41 – a lower cost than for the MS case. This solution had the lowest cost. The characteristics shown in Fig. 4 emphasize the following facts: in Stage 1, circuits 1–9, 3–16, 4–9, 4–16, and 10–16 were built with conductor type 1, and circuit 4–7 was built with conductor type 2. In addition, CBs with capacities of 900 kVAr, 300 kVAr, and 300 kVAr, respectively, were allocated at nodes 1, 2 and 10. One VR was allocated to circuit 4–7 with a tap position of 1. DGs were allocated at nodes 1, 3, 4, 7, and 16. In Stage 2, substation 21 was used to feed loads that were transferred from substation 22 (nodes 1 and 2), along with new loads (nodes 12 and 14). In addition, circuits 2–12 and 7–11 were built with conductor type 1; circuits 1–14, 6–13, 15–17, and 17–22 were built with conductor type 2, and circuit 1–21 was reconductored. CBs with capacities of 1200 kVAr, 900 kVAr and 1200 kVAr, respectively, were allocated at nodes 11, 13, and 14. The capacity of CBs allocated at nodes 2 and 10 was increased to 900 kVAr, and the CB allocated at node 1 was increased to 1200 kVAr. In Stage 3, circuit 15–19 was built with conductor type 1; circuits 13–20 and 14–18 were built with conductor type 2; and circuit 6–22 was reconductored. Additionally, all of the CBs allocated in the system were increased to 1200 kVA. Table VIII presents the variation in the number of CB modules over the planning horizon. For this test, circuits 1–21 and 2–21 were opened in Stage 1, and nodes 1, 2 and 3 were transferred to substation 22 by constructing circuits 1–9 and 3–16. In Stage 2, circuits 1–9 and 2–3 were opened, and nodes 1, 2, and 12 were transferred to substation 21, through the operation of circuits 1–21 and 2–21. In Stage 3, circuit 4–9 and 7–8 were opened, and nodes 3, 4, 7, 9 10, 11, and 16 were transferred to substation 21 through of the operation of circuits 1–9 and 2–3. Comparing the solutions for all of the different cases, one can see that if more types of equipment are considered in the DSEP problem, better solutions can be obtained and unnecessary investments avoided. Note that the cost difference between the MS and MSVRCBDG cases is US$ 16,737,957.13, which represents a 19.93% reduction in total investment cost. Comparing the MSDGVRCB and MSDG cases, in all three stages the solutions have very different topologies. In the first stage of the MSDGVRCB case all nodes are fed by substation 22 and the 5 DGs installed at this stage; for the MSDG case, substation 21 is used along with substation 22 and 5 DGs to feed node 1, and an additional DGs is allocated at node 17. Naturally, in the MSDGVRCB case, there are allocations of CBs and RTs that are not present in the MSDG case. Furthermore, in the MSDGVRCB case, node 4 is transferred to substation 22 through circuit 4–7; in the MSDG case, the same node is connected to the same substation through circuit 4–15. Similarly, in the MSDGVRCB case, node 3 is transferred to substation 22 through circuit 3–16, while in the MSDG case, the same node is connected through circuit 3–10. In Stage 2 of the MSDGVRCB case, nodes 1 and 2 are transferred to substation 21, while in the MSDG case, nodes 2 and 3 are transferred to the same substation. Additionally, in the MSDGVRCB case, circuit 6–13 is built with conductor type 2, while in the MSDG case, circuit 6–13 is built with conductor type 2. For Stage 3 of the MSDGVRCB case, circuits 14–18 and 13–20 are built to serve the new loads (nodes 18 and 20), and nodes 3, 4, 7, 9, 10, 11, and 16 are transferred to substation 21. On the other hand, for the MSDG case, substation 24 is built to serve both the new loads and node 13. The cost difference between the MSDG and MSVRCBDG cases is US$ 715,920.16, which represents a 1.05% reduction in total investment cost. In this test, the solution obtained is the only one that does not require the construction of new substations, because the loads are handled by the power from the distributed generators. Compared to the other scenarios, this solution requires the fewest number of larger capacity feeders.

TABLE VIII
VARIATION OF THE NUMBER OF CB MODULES IN THE PLANNING HORIZON THE MSDGVRCB CASE

Stage/ node	1	2	10	11	13	14
1	3	1	1	0	0	0
2	4	3	3	4	3	4
3	4	4	4	4	4	4

TABLE IX
SOLUTIONS OBTAINED BY A STATIC METHODOLOGY (COST IN 10^3 US$)

Cases	Static
1	140676.18
2	139004.96
3	140684.93
4	123302.65
5	139058.14
6	122728.06

B. *Comparing Static and Dynamic Planning Approaches*

The model developed in Section V was used to solve the static formulation for the DSEP problem in order to compare static and dynamic planning approaches. In the static formulation, the DSEP problem is solved taking into account the demand in the last stage, but executing the investment decisions in the first year. In order to make a proper comparison with the solutions obtained using the multistage formulation (summarized in Table IV), the performance of the expansion planning found using the static formulation was evaluated in the three stages, with the loads defined by the data in Table I. The results found for these tests are shown in Table IX. It was found that the solutions obtained using the multistage approach (Table IV) had lower costs than those generated using the static approach. This can be explained by the appropriate execution of investments in the multistage formulation.

VI. CONCLUSION

A mixed-integer linear programming model for the multistage long-term expansion planning problem of EDSs was presented. The model considered the construction/ reinforcement of substations, the construction/reconductoring of circuits, the allocation of capacitor banks, the allocation of voltage regulators, the allocation of distributed generators, and a modification of the radial topology including transfer nodes. The results showed that, when considering multiple expansion alternatives in the planning problem, it is possible to avoid unnecessary large investments for meeting new demand conditions.

The use of an MILP model has the following benefits: 1) a robust mathematical model that is equivalent to the MINLP model; 3) efficient computational behavior with MILP solvers; and 2) convergence to optimality guaranteed by using classical optimization techniques.

The results also showed that power losses can be calculated with greater precision than when using the load flow sweep method; furthermore, taking into account multiple expansion alternatives in the DSEP problem makes it possible to evaluate the most appropriate set of equipment to be implemented in the EDS at minimum cost. These results demonstrate the importance of taking into account all expansion alternatives that can be deployed in a network when choosing the expansion plan.

TABLE X
CHARACTERISTICS OF BIBLIOGRAPHIC REFERENCES

Reference	Methodology	Multiobjective Planning	Static Planning	Pseudo-dynamic Planning	Dynamic Planning	IC + IS	IC	ICB	IVR	ICB +I VR	IC + IS + ICB	IDG	IDG + IC + IS	MILP	MIQP	MINLP	Test Systems (bus)
[3]	1			X		X								X			36
[4]	1				X								X	X			18
[5]	1				X								X	X			27
[6]	2				X	X											54
[7]	2	X			X								X			X	33
[8]	1	X			X	X											100
[9]	2	X			X								X				18
[10]	2				X	X											26
[11]	1				X	X							X	X			135
[12]	1		X										X			X	23
[13]	1		X			X									X		136
[14]	1		X								X				X		54
[15]	2				X	X											59
[16]	2		X			X											387
[17]	2		X			X											136
[18]	2		X			X											57
[19]	2		X			X											201
[20]	2		X			X										X	23
[21]	2		X				X										60
[22]	2		X				X										24
[23]	2		X			X											54
[24]	2	X			X	X											100
[25]	2		X					X									135
[26]	2		X					X								X	94
[27]	2		X					X								X	70
[28]	2		X					X								X	70
[29]	2		X						X								229
[30]	2		X						X								229
[31]	2	X	X						X							X	229
[32]	1		X							X							70
[33]	2		X							X							310
[34]	2	X	X							X						X	95
[35]	2		X							X							11
[36]	2		X										X				9
[37]	2		X										X				9
[38]	2		X									X					26
[39]	2		X									X					26
[41]	1				X	X								X			24

Where 1: Classical optimization techniques and 2: Heuristic optimization techniques, including metaheuristics.
MIQP – Mixed Integer Quadratic Problem

Future works should consider reliability constraints in the proposed model.

APPENDIX

Table X shows a summary of the technical characteristics of the bibliographic references.

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